



# Lay-up optimization for the hull of a racing sailing yacht

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## Abstract

Deformability and buckling load of yacht hulls with fiber reinforced plastic sandwich structure depend on the stack sequence of the skins. In this work an optimization of fiber directions of the laminae for a racing yacht is proposed.

This procedure has been divided into three parts (i.e. material characterization, surface model definition, lay-up optimization). First of all a set of unidirectional specimens has been realized, by using the same fibers and matrix (carbon/epoxy) used for the hull as well as the same procedure and workers, in order to characterize the material according to American Society for Testing and Materials (ASTM) Standard D3039, employing strain gage technique. In the second part, by means of an original software in Turbo-Pascal (which uses the half-width value matrix as an input) linked to Pro/ENGINEER, it has been possible to obtain the body plan and surface and finite element (FE) models of the sailing yacht for the subsequent analyses. In the third step, an optimization procedure that uses the results of FE structural analyses in three different sailing configurations is performed, with the aim of obtaining the fiber directions that are able to minimize the yacht deformability, also taking into account the buckling loads. An approximate analytical model has been used in conjunction with a *sweep* technique in order to evaluate the best of the solutions. © 2001 Elsevier Science Ltd. All rights reserved.

**Keywords:** Lay-up optimization; Sailing yacht; Surface modeling

## 1. Introduction

In recent years composite materials have had a very high diffusion in sailing yacht building. Particularly regarding the competition field where, generally, no budget limit is provided. It is necessary to remind that the old IOR [1] (International Offshore Rule) has been substituted by the IMS (International Measurement System), which has rerouted the design towards light and stiff hulls. In fact the *rating* value [1], i.e. a sort of penalty that every sailing yacht has to pay during a regatta, will be lower for such kind of yacht. In order to have a low rating value, composite materials can be used, since they supply to the structure lightness and relatively high stiffness. Since bending is the prevalent stress, generally the yachts are built by using glass or carbon epoxy sandwich with a PVC core. However, under generic load conditions, except tension, thin sandwich buckles easily [2]; this is the reason why buckling behavior has been considered in this paper.

In order to minimize the yacht deformations, in this work an optimization procedure, integrated by finite element (FE) structural analyses performed using ANSYS code, has been developed; this procedure concerns the lay-up of the sand-

wich (in terms of fiber orientation), also taking into account buckling loads. Three load conditions of the yacht have been examined. The procedure has been set in Ref. [3], where however buckling has not been considered.

## 2. Problem analysis

The yacht has been drawn and designed in a Palermo (Italy) Shipyard by Albergiani and Inzerillo (numerical data have been kindly granted by designers and used by the authors for their calculations). It has the following characteristics:

Length overall ( $L_{oa}$ ) = 7.60 m      Displacement = 1100 kg

Maximum beam ( $B_{max}$ ) = 2.60 m      Ballast = 450 kg

Draft = 1.70 m      Tot. sail area = 35 m<sup>2</sup>

The yacht has been realized to take part in regattas of the IMS class. The structure is a carbon/epoxy sandwich with a PVC core. The total thickness of the sandwich is 20 mm; the core is 15 mm thick, while the six laminae are 0.83 mm each.

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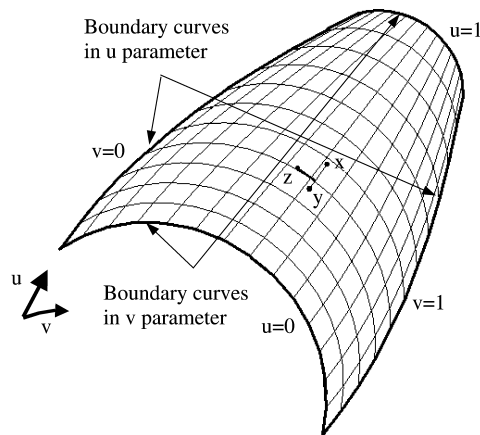


Fig. 1. Blended surface with boundary curves in two directions.

### 2.1. Material properties

The sandwich that will be used for the yacht building has been tested [3] according to ASTM D3039 [4] using strain gage technique [5,6]. For the tests, 0 and 90° unidirectional specimens have been realized, employing the same fibers and matrix used for the hull as well as the same procedure and workers. A summary of the test results is shown:

$$v_f = 38\% \quad E_{11} = 85\,500 \text{ MPa} \quad E_{22} = 4200 \text{ MPa}$$

$$G_{12} = 5170 \text{ MPa}$$

$$X_t = 800 \text{ MPa} \quad Y_t = 20 \text{ MPa} \quad S = 60 \text{ MPa}$$

Where  $v_f$  is the volume fiber percentage,  $E_{11}$  and  $E_{22}$  the Young's moduli parallel and normal to the fibers, respectively,  $X_t$  and  $Y_t$  the respective tensile strengths,  $G_{12}$  the shear modulus and  $S$  the shear strength.

### 2.2. Surface and finite element models

The yacht as a whole is defined both by the hull and the superstructures. The mathematical definition of the hull has been a great problem (the so-called *hull equation*) for centuries. However, new CAD software capabilities have partially solved this problem. In fact, for example, it is possible to define a model by means of different parametric surfaces [7] (i.e. blended in one or two directions, swept). In this work a blended surface in two directions has been used. The procedure can be divided into three consecutive, linked steps: (a) sections construction; (b) surface hull construction; and (c) superstructures construction.

The first step has been solved starting from the *half width matrix* [8] and then modeling a specified number of sections by using Bezier's curves [7,9]. This procedure gives the bidimensional sketch of the hull.

An output file (in ASCII format), written by the software, is the input for Pro/ENGINEER. It treats the curves as one feature and, by blending these, gives the parametric surface.

Many pieces of information such as slope and curvature have been extracted from the surface analysis. These data have permitted little changes in the bidimensional sketch in order to increase the fairing. Once the hull has been defined it has been possible to insert the superstructures composed by deck, home-deck and cockpit. The used technique employs boundary curves definition. Fig. 1 shows the generation of the home-deck. It is important to note that with respect to the model used in Ref. [3], in this case, it has been possible to define a *true* 3D model of the yacht.

The FE mesh generation has been carried out by using the Pro/FEM modeler. The strategy used has been to set a specified number of divisions on the boundary curves like keel, wash-board and transom in order to have quasi-rectangular elements with a side practically coincident with the water lines. Only few elements near the bow do not respect this condition, but this approximation is not important due to the stiffeners of this part in the real structure. The above fact is very important, especially if (as foreseen) the yacht is built by using composite materials; the element side (parallel to the water lines) has been assumed to be the origin of a local reference, used to define the composite mechanical properties simplifying both design and realization procedures.

For yacht and sails quadrilateral isoparametric layered shell elements with eight nodes (four nodes for sails) and six degrees of freedom for each node have been used. Many different layers are allowed for this element. Different thickness, fiber orientation and orthotropic material properties can be defined for each layer. The element formulation is based on Midlin's theory. The ANSYS code reduces the "locking" effect [10]. Note that the sails have been modeled without taking into account their effective shape, but simply like plates to which the loads are applied, stressing the structure by means of the shrouds and the mast. The latter has been modeled with beam elements. The shrouds, the fore-and-aft stays, modeled using truss elements, have been prestrained at 0.004–0.005, while the bulb has been assumed to be a concentrated mass attached to the keel. Particular care has been spent in finding a suitable mesh for the yacht, which could assure reliable results and reasonable times of calculation. The final mesh shown in Fig. 2, has been reached after performing the appropriate convergence analyses, and it is composed of about 5700 elements. With respect to Ref. [3], a channel along the keel has been considered for a more realistic determination of the buckling loads. The stiffness of the sails has been increased in order to reduce the eventuality that buckling appears in them. Since they are linked to the structure of the yacht by bars, they should not influence, in a significant way, the results in the hull.

### 2.3. Buckling analyses

Generally two analyses types are available to predict the buckling load with the FE method: nonlinear and linear buckling analyses. Nonlinear analysis is the most accurate

approach: it employs a nonlinear static analysis increasing loads or displacements; with this technique nonlinear effects (plastic behavior, large deflection response, etc.) can be included.

Linear buckling analysis predicts the buckling strength of an ideal linear elastic structure. Large displacements could be also considered, but with the following restrictions [11]: (a) the structure behavior during the prebuckling phase is characterized by negligible displacements and the buckling condition is reached in an unexpected way, showing a deformed state of a completely different nature from that present in the prebuckling phase; and (b) they change linearly with the load; this happens, for example, in beams and slabs subject to compression, in pipes and spherical shells subjected to external pressure. In this paper only linear buckling analyses are performed, considering that the last cases are very close to the one examined and that in a sandwich panel the plastic strain, that could develop up to the failure, is generally negligible [2]; such a choice is justified by the fact that this method requires shorter computing time (this is not negligible considering the high number of analyses to perform), and by the requirement to compare the behavior of the different configurations rather than to calculate absolute values.

Buckling loads so calculated will be upper bounds of the exact ones. The more the previous hypotheses are true, as in the present case, the more will the differences decrease. However, the presence of stiffeners (strings, spar-frame, keelson, etc.), increasing the buckling loads, has been neglected in the calculations.

In linear buckling the limit condition of elastic stability exists when [11]:

$$[K] = 0 \quad (1)$$

where  $[K]$  is the stiffness matrix. In the case of large displacements and elastic material behavior, nonlinearity exists between strain and displacement (geometric nonlinearity), for which the  $[K]$  matrix could be written [10] as the sum of a  $[K_0]$  constant matrix and of a  $[K_\sigma]$  load dependent matrix:

$$[K] = ([K_0] + [K_\sigma]) = 0 \quad (2)$$

If  $[K_\sigma]$  depends linearly on the load it can be written as:

$$([K_0] + \lambda[K_{\sigma 1}]) = 0 \quad (3)$$

where  $[K_{\sigma 1}]$  is evaluated at some arbitrarily chosen level of loading (the exercise loads in this paper) and  $\lambda$  is a load factor, which, multiplied by the loads that generated  $[K_{\sigma 1}]$ , gives the critical load intensity. It is then possible to write:

$$([K_0] + \lambda[K_{\sigma 1}])\{u\} = 0 \quad (4)$$

This relationship defines a typical eigenvalue problem and, therefore, it allows to calculate the eigenvalues  $\lambda$  and the corresponding eigenvectors  $\{u\}$ . In this case the load factor is equal to the minimum value of  $\lambda$ .

In this paper, buckling behavior of the whole sandwich is analyzed, assuming that core material has adequate compressive and shear moduli to prevent the laminate skins from locally buckling [2].

#### 2.4. Optimization procedure

In an optimization procedure design variables are independent quantities that can be modified in order to achieve the optimum design; upper and lower limits are specified to serve as constraints. State variables are dependent variables used to constrain the design; they are response quantities, function of the design variables and may have a maximum and/or minimum limit. In this way design and state variables define a region of feasible designs. Objective function is the dependent variable to minimize in the feasible design region. A procedure based on the construction of approximated

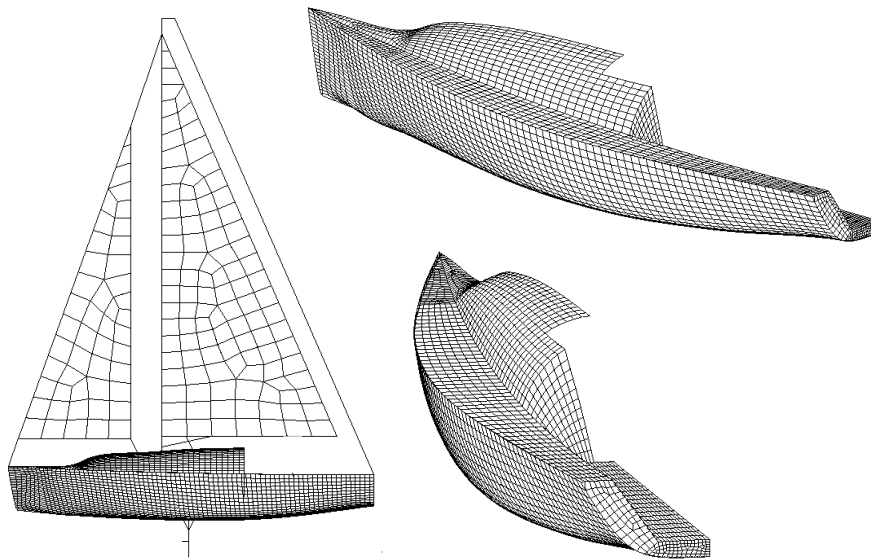


Fig. 2. Finite element model of the yacht obtained with Pro/FEM.

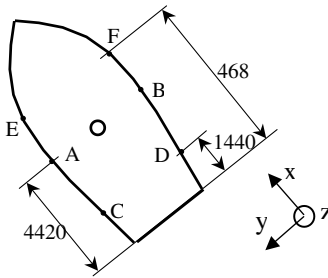


Fig. 3. Points location used for the state variables evaluation (units in mm).

analytical models and on the application of a sweep technique has been employed in order to find the optimal fiber orientation for each layer of the laminate.

The analytical model approximates the dependence of both the objective function and state variables by the design variables. The approximated objective function is minimized instead of the real one. The optimization problem is constrained by limits on design variables. It is converted to an unconstrained problem in order to use more efficient algorithms for the minimum search [12]; the conversion is made by adding penalty terms to the objective function approximation in order to respect the imposed constraints. The procedure is carried out by applying an iterative technique and halts when the convergence is achieved or if the number of specified loops has been performed. The approximated model is obtained by calculating the objective function and the state variables for several sets of design variable values and performing least square fits between the data points [13]. In order to speed up convergence, feasible values of design variables have been initially assigned. Each optimization loop generates a new data point close to the actual minimum so that the approximations are updated. At the end of the procedure the set of data corresponding to the minimum value of the objective function is set as the optimum design. The convergence does not necessarily indicate that a global minimum has been obtained; therefore, it is often expedient to repeat the analyses, starting from different initial values. In this work the approximated analytical models have taken in the form of a fully quadratic representation with cross terms:

$$\mathbf{F} = a + \sum_{n=1}^N b_n \mathbf{x}_n + \sum_{n=1}^N c_n (\mathbf{x}_n)^2 + \sum_{n=1}^{N-1} \sum_{m=n+1}^N d_{nm} (\mathbf{x}_n)(\mathbf{x}_m) \quad (5)$$

In Eq. (5) the quantities  $a$ ,  $b_n$ ,  $c_n$ ,  $d_{nm}$  are calculated with the least squares method,  $\mathbf{x}$  is the set of design variables and  $N$  is the total set number.

With sweep technique the actual optimum of each design variable is joined subsequently to values of the other ones, uniformly swept in the space of design, and the corresponding values of the objective function are calculated.

The optimization analyses are developed in three phases. In the first phase, using a sufficient number of iterations, an

approximated analytical model is found, with the aim to locate the design variables corresponding to a local optimum. In the second, in order to analyze other regions of the feasible domain not explored before, a sweep generation is used. Finally in the third, a new approximated analytical model is developed, starting from the best sets of the design variables calculated in the preceding two phases.

### 2.5. Load conditions

Three load conditions have been studied and for all of these conditions the following have been considered:

1. The weight of both the yacht ( $\sim 1100$  kg) and the crew ( $7 \times 80$  kg = 560 kg) by assigning appropriate densities to the elements and gravity acceleration.
2. The shrouds prestrain at 0.005 (corresponding to  $\sim 13$  000 N) and the fore-and-aft stays prestrain at 0.004 (corresponding to  $\sim 16$  000 N).

The first load condition (marked with #1) considers the yacht in a flat sea with  $30^\circ$  of heel and sailing close-hauled with a true wind of 25 knots. The second configuration (#2) considers the yacht in the same conditions, but supported bow and stern by two consecutive waves; for this purpose some nodes of the extreme bow and stern belonging to the plane of longitudinal symmetry are constrained. In the last one (#3) the yacht is considered in a symmetrical configuration (without heel) again supported by two consecutive waves, but subjected to the weight only.

For the purposes of the optimization the following variables have been considered:

The design variables: given by the fiber orientations,  $\beta_1$  and  $\beta_2$ , in two layers of the laminate; since a third layer with fibers orientation of  $0^\circ$  is employed (water lines directions), the global sandwich composition is therefore  $[0/\beta_1/\beta_2/\text{core}/\beta_2/\beta_1/0]$ ; the quantities  $\beta_1$  and  $\beta_2$  are variable in the range  $-90-90^\circ$ .

The objective function: represented by the relative displacement in absolute value in the  $z$ -direction,  $u_z$ , (see Fig. 3) in configuration #1 between the edge of the stern and the root of the mast. This function assures the selection of a fiber orientation that guarantees the greatest bending stiffness in longitudinal direction, compatible with the other conditions.

Three state variables given by the relative displacements in absolute value in the  $y$ -direction,  $u_y$ , of the two points of the wash-board belonging to the section of the mast (points A and B in Fig. 3), calculated in the three configurations. These variables assure that the deformations of the main section do not exceed certain values, which by experience have been set equal to:

$$0 \leq u_y(1) \leq 7 \text{ mm} \quad 0 \leq u_y(2) \leq 7 \text{ mm}$$

$$0 \leq u_y(3) \leq 7 \text{ mm}$$

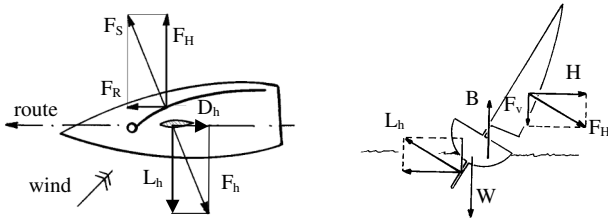


Fig. 4. Loads in a yacht sailing close-hauled.

Two other state variables given by the relative displacements similar to the objective function, but evaluated in configurations #2 and #3, with limits:

$$0 \leq u_z(2) \leq 10 \text{ mm} \quad 0 \leq u_z(3) \leq 10 \text{ mm}$$

Two other state variables that supply limits on the torsional deformation for the configurations #1 and #2. They are evaluated as relative rotations,  $\alpha$ , between the sections containing the points C and D and the points E and F (see Fig. 3). The following limitations are assigned to them:

$$0 \leq \alpha(1) \leq 0.0006 \text{ rad} \quad 0 \leq \alpha(2) \leq 0.0006 \text{ rad}$$

Three other state variables given by the linear buckling load factors. The limits are fixed at 4, according to Ref. [2]:

$$4 \leq \lambda(1) \quad 4 \leq \lambda(2) \quad 4 \leq \lambda(3)$$

### 2.5.1. Loads sailing close-hauled

The #1 load condition is represented in Fig. 4. In equilibrium, the aerodynamic force on the sails,  $F_s$ , is balanced by an equal and opposite load,  $F_h$ , that is produced by the immersed appendices of the hull. The component  $F_R$  (balanced by the drag force  $D_h$ ) permits the progression of the hull. The force  $F_H$  does not contribute to the speed, but only to the sideslip, and forms an upsetting couple with  $L_h$ . This couple must be balanced by the moment produced by the weight  $W$  (including the crew disposed windward) and the buoyancy force  $B$ .

Loads  $F_R$  and  $F_H$  could be estimated by the expressions [14]:

$$F_H = C_H 0.6 A v^2 \quad F_R = C_R 0.6 A v^2$$

where  $A$  is the sail area (equal to  $35 \text{ m}^2$ ),  $v$  the wind speed (25 knots  $\sim 12.9 \text{ m/s}$ ) and  $C_H$  and  $C_R$  are the lift and drag coefficients. These coefficients can be taken to be equal to [14]:

$$C_H = 0.8 \quad C_R = 0.3$$

Finally one obtains:

$$F_H = L_h = 2782 \text{ N} \quad F_R = D_h = 1043 \text{ N}$$

$F_H$  has been distributed as pressure on the sails. The remaining loads have been applied as concentrated forces to the centers of gravity of the keel ( $L_h$ ,  $D_h$ ) and of the sails ( $F_R$ ).

The draft has been iteratively calculated by imposing the equilibrium between weight (including the crew) and buoyancy force both in terms of forces and moments. Once the draft is known it has been possible to apply the buoyancy force as a pressure on the hull. Now that the loads system (including the prestrain of fore-and-aft stays and shrouds) is known and the analyses can be carried out.

### 3. Analysis of results

In the following the more interesting results are reported. The remaining ones could be qualitatively deduced from Ref. [3]. Each system of applied loads determines different kinds of deformation, which will be added depending on the sailing condition.

The structure and crew weight, when the hull is in a flat sea, cause the longitudinal inflexion of the yacht with concavity towards the bottom and its stretching in crosswise direction. On the contrary, the shrouds (prestrained and stressed by the sails) and the weight with emerged hull, cause opposite deformations. Results of the analysis seem to show that the analyzed configurations significantly represent different behaviors. The optimization procedure has required relatively few iterations in order to locate, with sufficient reliability, an optimum configuration. In Fig. 5 diagrams of objective function and design variables vs. number of sets are reported. It is observed that the algorithm locates a region of local minimum that begins after eight sets and finishes at the end of the first optimization phase. Afterward the sweep technique for a total of 20 iterations is applied. At the beginning and at the end of the second sweep phase, lower values of  $u_z$  are found for which  $\beta_1 \cong \beta_2 \cong 90^\circ$ . In the third phase, a new approximated analytical model is developed, starting from the 16 best sets of the design variables found in the previous phases. As it is possible to note, the objective function gives the same least value with the same values of  $\beta_1$  and  $\beta_2$ .

Therefore these values define the optimum design of the yacht and the resultant sequence of the sandwich is: [0/90/90/core/90/90/0]. Such a result could be legitimate considering that the relative displacement  $u_z$  depends on the deformability of the yacht in longitudinal and in transverse direction. In a generalized manner, the stiffness in longitudinal direction would have to grow when the fibers in the  $0^\circ$  direction are increased, while increasing fibers at  $90^\circ$  will improve transversal stiffness. However the sluice shape of the yacht and its height already confer it high bending stiffness in longitudinal direction, such that the introduction of other layers at  $0^\circ$  is not able to increase: in fact configurations with

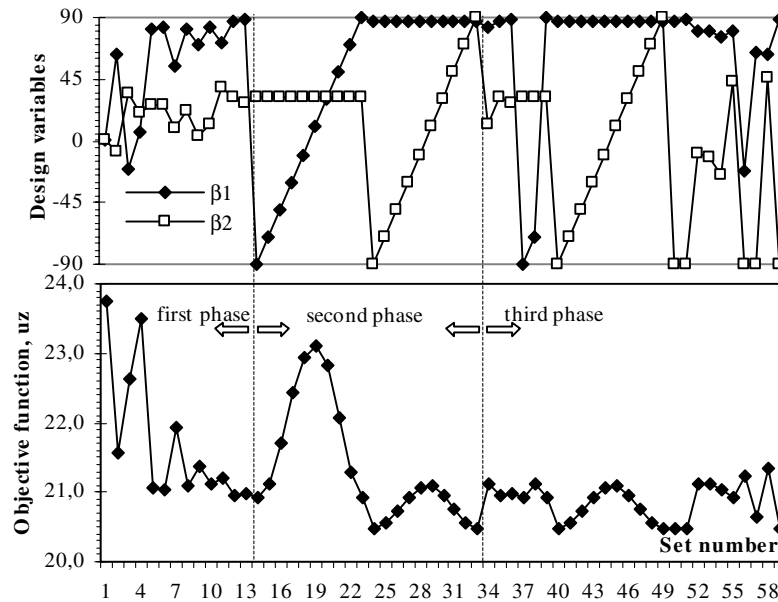


Fig. 5. Objective function (mm) and design variables (degrees) vs. set number.

layers near  $0^\circ$  produce decidedly worse values of the objective function.

The results seem to show, instead, that the longitudinal stiffness improves much if the strain of the generic transversal section is reduced, by introducing layers close to  $90^\circ$ . As far as buckling load factors are concerned, the calculations have given values in the range 5–6; these values are practically scarcely influenced by the change of fibers angles, and the normalized displacements are higher in the sails. Moreover it has been verified that the buckling load factors increase with increasing sails stiffness. The above fact permits to say that the results are valid only from a mathematical point of view and that the corresponding load factors on the hull must be considered higher than those calculated. Buckling load factors of the isolated hull (including superstructures), subject to the loads of configuration #3, with  $\beta_1$  and  $\beta_2$  varying between  $0$  and  $90^\circ$ , are also calculated. The corresponding extreme buckling loads are close to 7 ( $\beta_1 = \beta_2 = 0^\circ$ ) and 17 times ( $\beta_1 = \beta_2 = 90^\circ$ ) the loads on the hull in the real configurations and confirm that the imposed limits on the load factors in the optimization procedure are respected; moreover the configurations with  $\beta_1$  and  $\beta_2$  tending to  $90^\circ$  are still the most stable. In Fig. 6 the buckled shape of the hull with  $\beta_1 = \beta_2 = 90^\circ$  is shown. With respect to this, for

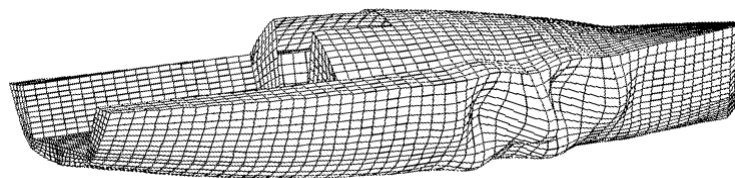


Fig. 6. Buckled shape of the isolated yacht with  $\beta_1 = \beta_2 = 90^\circ$ .

$\beta_1 = \beta_2 = 0^\circ$  the higher normalized displacements are located closer to the bow.

Actually the used optimization procedure fits the purpose very well and allows to find a combination of stiffness in the different directions, with the aim to limit the deformability of the yacht. Another advantage is that the yacht has been analyzed as a whole and not as isolated parts, and that the mutual influences between all the regions have been considered.

The minimum found seems to be a global minimum and so other optimization cycles starting from different initial points have not been performed. Moreover it has been verified that far from the discontinuities the stress state is in safety. The check of resistance of regions with hard discontinuities has not been performed since it has not been considered to be of interest for this paper.

#### 4. Conclusions

The use of Pro/ENGINEER has been a powerful instrument to model the surface of the yacht. It has permitted an easy yacht generation starting from the body plan.

Results of the analysis shows that the three different load configurations analyzed significantly represent different

behaviors. Deformability and buckling loads are influenced by the change of fiber angles, showing that higher values can be achieved only if a proper orientation of the fibers is found.

The optimization method based on the creation of an approximate analytical model, and characterized by the application of a sweep technique, is suitable to solve the problem of finding the optimal fibers disposition, as well as the shape optimization [15]. The minimum value of the objective function has been located in the second step during the sweep phase and it has been found again in the third phase, demonstrating that it is sometimes useful to employ the sweep technique in order to explore all the fields of the feasible domain. Probably similar results could be achieved by repeating the procedure, starting from different imposed initial values.

The resultant sequence from the used optimization procedure was found to be [0/90/90/core/90/90/0]. Laminae with  $\beta_1$  and/or  $\beta_2$  close to  $0^\circ$  could be envisaged, but the latter determines in the hull a behavior more flexible than the corresponding one with fibers close to  $90^\circ$ . This is probably because the closed shape of the hull and its height already confer it a high bending stiffness in the longitudinal direction. The introduction of other laminae at  $0^\circ$  is not able to increase the stiffness; on the contrary, the introduction of fibers at  $90^\circ$  is able to do, by increasing the stiffness in crosswise direction and reducing the relative displacements in the  $z$ -direction of the control points.

It is also found that the limits of 4 on the buckling load factors are not crossed for whatever fiber disposition and that the configurations with  $\beta_1$  and  $\beta_2$  both tending toward  $90^\circ$  are also the most stable.

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