MOPOD: A model to investigate fracture porosity development

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Abstract: A model (MOPOD) has been developed to investigate general relationships between simple fracture aperture growth laws and fracture porosity in fractured aquifers. The development of fractures has been studied where growth-rate is proportional to an exponent, *e*, of the flow rate through each fracture. The approach adopted is similar to that of statistical mechanical studies of simple systems, where evolved structures depend on the initial values. The evolved arrays show a range of diverse structures and geometrical phase changes as a function of the aperture growth-rate law and the initial aperture distribution, *i.e*. the standard deviation of the initial aperture distribution. Following an initial growth phase, during which dynamically stable aperture configurations develop, arrays undergo simple amplification. The evolved arrays are highly complex and parameterisation and prediction of their evolution in terms of the initial aperture distributions and growth-rate laws is not trivial.

CONTEXT OF THE STUDY

The spatial variability of flow and transport properties in fractured sedimentary aquifers can be greatly influenced by channelling of flow through preferentially enlarged components of the fracture network over a range of scales. There is a need to generate representative realisations of such fracture networks for use in flow and transport models particularly as it is difficult and costly to characterise fracture arrays in the field. The approach taken to modelling fractured aquifers is most commonly that of stochastic network modelling. Unfortunately, such models may not exhibit the long-range preferential pathways, associated with channelling of flow through fracture networks, which are important for contaminant transport (Berkowitz and Balberg, 1993).

One approach to the generation of more realistic spatial correlations in fracture networks is to explicitly model the development of fractures according to fundamental physical and chemical laws (Bekri et al., 1977; Groves and Howard, 1994; Dreybrodt, 1996; Djik and Berkowitz, 1998; Siemers and Dreybrodt, 1998; Kaufman and Braun, 1999). However, the idealised processes that are modelled, such as dissolution and precipitation, can grossly oversimplify the more complex field situation where hydromechanical plucking, abrasion and microbially mediated processes may be significant.

The work described here is a contribution to the endeavour of finding a middle-way between stochastic and process-based construction of fracture networks for flow and transport modelling, especially where a range of fracture modification processes are active. A particular aim of the study is to develop a generic model to investigate relationships between the growth of fracture apertures and the geometry of evolved fracture apertures using simple growth laws and simple fracture geometries.

THE MOPOD MODEL

The MOPOD model has been formulated as an ‘initial value problem’ expressed by a set of N first-order equations of the form $da_i(t)/dt = \mathrm{G}(a_i, v_i, \mathbf{p})$, where $a_i(t)$ is the aperture of pore i (i = 1,…,N) at time t, subject to the initial aperture values, $a_i(0) = a_{i0}$, G is a user-defined function, **p** is a set of parameters, and v_i is the magnitude of the volumetric flow rate in pore i. For the model runs described in this paper aperture growth-rate was restricted to the form $da_i/dt = v_i^e$, where e is the aperture growth rate exponent.

The model is highly flexible. It is capable of modelling 2-D and 3-D arrays with both regular and random structures, a wide range of initial aperture distributions, and flexible boundary conditions including constant head or flow conditions. All simulations reported in this paper were performed on 20 by 20 arrays of fractures arranged on an orthogonal lattice with a constant unit flux condition across the lateral boundaries and a no-flow condition at the upper and lower boundaries. Also, the flow rates were restricted to being proportional to the cube of the aperture (‘cubic law’) and to the head gradient (Darcy’s law).

We chose to work with the statistics of the logarithms of the apertures, $z_i = \log_{10} a_i$.. All the model runs described here used initial aperture distributions that were spatially un-correlated and lognormal. We denote the initial mean and standard deviation of the z_i values by μ_z and σ_z, respectively: unless otherwise stated, the geometric mean of the initial, a_{i0}, values was set to unity (equivalent to $\mu_z = 0$).

Model runs were performed to investigate the effects of changing the growth-rate exponent, e, and the width of the initial aperture distribution, σ_z. The aperture growth-rate exponent was investigated in the range 0.1 to 0.8. The width of the initial distribution was varied in the range $\sigma_z = 0.1$ to 0.7.

RESULTS

A range of qualitatively distinct evolved geometries can be recognised as a function of the growth-rate laws and the width of the initial aperture distribution. At low growth-rate exponents ($e \leq 0.3$) and moderate values of σ_z ($\sigma_z < 0.4$), there is a homogenisation of apertures oriented parallel to the head gradient with model time. At moderate growth-rate exponents ($0.3 < e < 0.6$) row apertures become increasingly heterogeneous in the evolved arrays, planar heterogeneities develop parallel to the head gradient for low values of σ_z while anastomosing structures develop at higher σ_z values. For growth-rate exponents in the range $0.6 < e < 0.8$ preferentially enlarged array-spanning paths develop. When both e and σ_z are large the array-spanning structures no longer develop, instead isolated enlarged apertures develop. The range of evolved structures is illustrated schematically in Figure 1. Only significantly enlarged components of the orthogonal fracture array are illustrated. Limited additional modelling has shown that similar trends in the geometry of developing arrays can be generated by varying the initial mean (log) aperture, μ_z. For example, smaller values of μ_z lead to the promotion of single array-spanning structures, while higher values of μ_z tend to lead to more homogeneous apertures parallel to the head gradient.

The fracture aperture distributions appear to become dynamically stable with model time. This is illustrated in Figure 2, which presents nine aperture maps based on $\sigma_z = 0.3$ and e = 0.5. The maps in the left-hand column of the figure are the full fracture aperture maps for arrays at model times t = 0, 10, and 40. The centre and right-hand columns show only the largest 25% and 10% of apertures, respectively. At an early stage in the evolution of the array, the largest apertures are distributed throughout the array with row- and column-parallel orientations. As the array develops the largest fracture apertures form more continuous structures aligned with the head gradient (left to right), which, at higher aperture growth-rate exponents, lead to the development of the preferentially enlarged array-spanning paths. Formation of dynamically stable aperture distributions appears to correspond with re-organisation of the largest fracture apertures at early times in the development of the arrays. Once the stable configuration has developed the aperture field is simply amplified following the aperture growth law. Limited additional modelling, taking evolved apertures arrays as the starting point for new growth models, suggests that changes in growth-rate exponent at later stages only acts to intensify structures already developed.

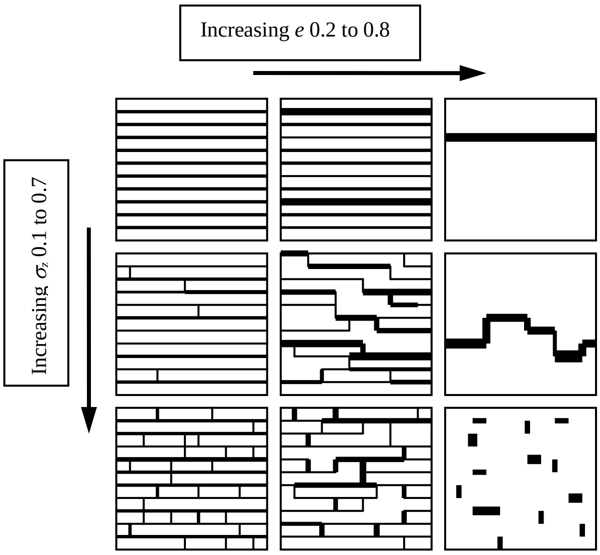

Fig.1 Schematic illustration of the geometry of evolved fracture arrays a function of e and σ_z. Only significantly enlarged components of the fracture array are illustrated.

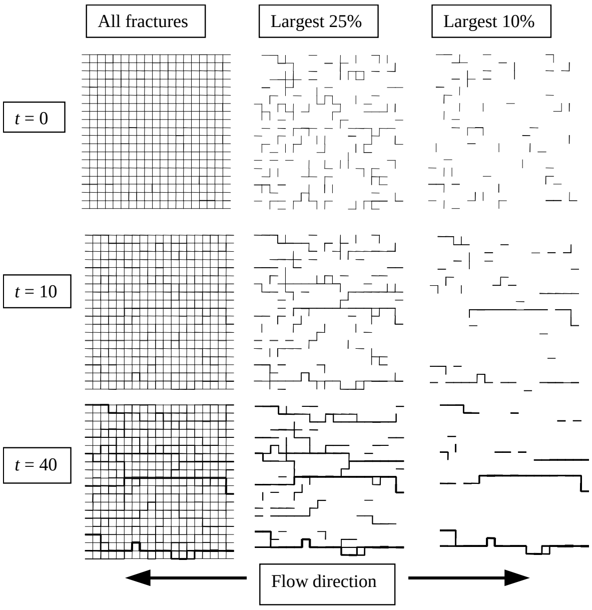

Fig.2 Fracture aperture maps illustrating the development of dynamically stable fracture porosity (see text for details).

DISCUSSION

Many of the evolved MOPOD structures exhibit similar geometries to those developed using dissolutional growth to model karst formation and development in fractured carbonate rocks. There is, however, a fundamental difference between the manner in which porosity develops in the MOPOD model and process-based models. The model is configured so that it is always in a quasi-steady-state of flow and, since aperture growth depend on the magnitude of the flux not its direction, model runs (using identical initial aperture distributions) with the same head gradients or fluxes from left to right and from right to left, develop identical evolved aperture arrays. There is no tracking of ‘packets’ of water through the model. Consequently, MOPOD develops fracture porosity, including the preferentially enlarged array-spanning structures, without propagation of dissolutional fronts under chemical gradients. Instead fracture porosity develops in response to local flow conditions controlled by the aperture field distribution and a (remote) macroscopic head gradient.

A number of approaches have been explored in an attempt to characterise the arrays. Basic visualisation enables a qualitative description of the range of evolved structures, statistics provide quantitative descriptions of the arrays and variograms can be used to characterise the development of spatial correlation in the aperture field. The development of correlations in flow rates between adjacent fractures can also be tracked by comparing the maximum flow into and out of a fracture intersection: increasing correlation in maximum inflow and outflow with time is a characteristic feature of array development. More rigorous parameterisation of the evolving arrays is far more problematic. It has not been possible to identify parameters that can be used either to predict the form of the evolved arrays on the basis of the initial aperture distributions or growth-rate laws.

Analytical solutions to the head and flow equations on some very small arrays consisting of two to five fractures have been developed to predict aperture growth. The analytical model revealed critical exponents. Depending on the geometry, systems with more than about five fractures gave rise to complicated coupled non-linear equations that could not be solved analytically. Consequently, although the simple analytical models may reproduce many of the characteristics of the MOPOD model, they are of limited use.

CONCLUSIONS

A simple model, based on a limited number of assumptions and on simple growth laws applied to idealised fracture arrays with simple initial aperture distributions, can be used to generate long-range correlations (array-spanning paths) in enlarged fracture arrays. In that context, it holds the potential to replicate natural fracture arrays and so be useful in the generation of fracture templates for flow and transport modelling.

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