Theoretical models of optical fibre intensity modulated pressure and displacement sensors

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THEORETICAL MODELS OF
OPTICAL FIBRE INTENSITY MODULATED
PRESSURE AND DISPLACEMENT
SENSORS

BY

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A Master's Thesis
Submitted in partial fulfilment of the requirements
for the award of
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I would like to thank my family for their continuous support and encouragement.
DEDICATED 

TO 

MY PARENTS
SYNOPSIS

In recent years, optical fibres have been widely applied to various applications in communication and sensor systems. Optical fibre sensors have the advantage over electrical transducers in their high sensitivity and immunity from any electromagnetic interference effects. The main work concentrates on the theoretical application of optical fibre sensors in pressure and displacement sensing. The computer modelling of these two systems is achieved via the use of the mathematica software package.

In the pressure sensing system, optical fibre is cut and rejoined in housings which allow for the application of forces near the jointed sections. Models are generated to describe these configurations using geometrical optics. The calculation of the coupling efficiency and sensitivity of the system are then investigated for various fibre geometries.

In the displacement sensor system, models are developed to describe the coupling between launch fibre, reflective surface and receive fibre in which the system geometries and fibre type are variable. Again the coupling efficiency and sensitivity are measured for various geometrical configurations. Initially, the two fibres are positioned parallel to each other with the reflective surface being set normal to the fibre arrangement. Subsequently, the receive fibre is tilted with respect to the transmitting fibre and finally both the fibres are tilted equally with respect to each other.

The main aim of the work is to provide models to facilitate the design of sensor systems with prescribed sensitivities and geometrical limits.
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CHAPTER ONE
1 PREFACE

This thesis is concerned with the development of theoretical models describing two classes of intensity modulated, optical fibre displacement sensor. The first class is based on light coupling between fibres by means of a reflective surface and the second class relates to the transmissive coupling between two fibres through an air gap. In the first class, displacements of the reflective surface are converted to intensity modulations and in the latter class it is the relative movement of the fibres which gives rise to intensity modulations. It is well known that schemes of this sort, can also be used to monitor other measureands such as pressure.

Sensors of the type described above have been investigated both theoretically and practically for some twenty years and a number of commercial systems have been developed. Despite this, existing models describing the reflective type sensor do not include all possible geometries and multimode fibre structures. We have therefore developed a geometrically complete, low order model of reflective displacement sensing, which may be especially useful as the basis of a computer aided design tool for analysing a specific sensor requirement (i.e dynamic range and sensitivity). The transmissive type sensor model aims to describe the coupling between multimode fibres with non-normal endfaces. Our motivation for this study is that such configurations may offer advantages in terms of the sensitivity of coupling efficiency with respect to fibre movement.

Although the models developed apply to intensity modulated, step index, multimode optical fibre displacement sensors, this thesis contains references and discussions of several other sensor types. The primary reason for this is to make note of other solutions to optical displacement sensing, including important designs based on phase modulation. In addition to this we have provided a selective review of some important principles of optical sensor design to provide a wider context for the study.
In chapter two an introduction to the terminology of optical fibre sensors is given. It also introduces the various devices required in general for the design of an optical fibre system. In chapter three we review the main context of our study, namely optical fibre displacement and pressure sensing. In chapter four the transmissive sensor model is described. The design is mainly concerned with the effect of introducing an air gap between two obliquely polished fibres. In chapter five the reflective sensor model is described. The model calculates the coupling efficiency as a function of fibre type and geometry. In chapter six the latter model is investigated with respect to the ranges of sensitivity and performance which can be achieved. We also provide a comparison of the predicted coupling efficiency with measurements. Finally, in chapter seven some conclusions are reached and a specific idea for future study is outlined.
CHAPTER TWO
2.1 THE OPTICAL FIBRE

An optical fibre is a light waveguide which takes the form of a long cylindrical structure consisting of the following three layers shown in Figure(2.1.1):

a: The core.
b: The cladding.
c: The protective jacket.

Figure(2.1.1): Light propagation in an optical fibre.

Both the core and the cladding are usually constructed of either glass or plastic. The external plastic coating (i.e. the protective jacket) provides protection against physical damage. Each of the above mentioned layers have different refractive indices $n_1$ and $n_2$ where

$$n_1 : \text{core refractive index.}$$
$$n_2 : \text{cladding refractive index.}$$

and

$$n_1 > n_2 .$$

The difference in the refractive indices of the two layers, i.e core and cladding, causes light rays to travel inside the core and to be reflected back into the core each time the rays hit the core/cladding
boundary, therefore providing the light rays with long path. The capability of the fibres for collecting light can be deduced from its Numerical Aperture (NA) which is defined as the sine of the half-angle of the acceptance cone shown in Figure(2.1.2).

\[ NA = \sin(\theta) = \sqrt{n_1^2 - n_2^2} \]

![Figure(2.1.2): The representation of the acceptance angle.](image)

**2.2 TYPES OF FIBRES**

In general, there are three different kinds of fibres classified according to the number of modes a fibre can support and they are:

a : Single mode fibre.
b : Multimode step index fibre.
c : Multimode graded index fibre.

The following section describes briefly the various fibres.

**2.2.1 SINGLE MODE FIBRE**

These fibres have very small core size, compared to the cladding, with small difference in their relative indices shown in Figure(2.2.1), the core can have a diameter up to 10 μm [1]. Single mode fibres are especially important in communication purposes,
where is required. In many cases, single mode fibres are fabricated from doped silica.

![Figure(2.2.1): Single-mode step index fibre.]

### 2.2.2 MULTIMODE STEP INDEX FIBRE

These fibres have core diameter in the range of 50μm to 1000μm shown in Figure(2.2.2). They support hundreds to thousands of modes. These fibres suffer from intermodal dispersion [2] which is due to various mode velocities inside the fibre, therefore causing the light pulse to spread out. Multimode step index fibres are quite useful for short distance communication [3].

![Figure(2.2.2): Multimode step index fibre.]

### 2.2.3 MULTIMODE GRADED INDEX FIBRE

These fibres have a variable core refractive index profile shown in Figure(2.2.3). The core refractive index has maximum value at the fibre centre with gradual decrease in its value as the ray reaches the cladding. This causes a decrease in the intermodal dispersion [2].
2.3 OPTICAL FIBRE SYSTEM

There are three different systems associated with optical fibres and they are:

1 : Transmission system.
2 : Distribution system.
3 : Sensor system.

The use of optical fibres for various transmission and distribution systems applications can be advantageous due to its high signal bandwidth and low loss. Some of these applications are concerned with public telephone trunk network, computer network, cable TV network and various military purposes [4]. A fibre optic sensor on the other hand, can offer a great deal of help to sensor systems in general, since optical fibres are highly sensitive and immune from most electromagnetic interference effects, in addition to their ability to retain optical phase information after transmission [5].

2.4 OPTICAL FIBRE SENSOR SYSTEM

An optical fibre sensor system consists of the following components [5] as also shown in Figure(2.4):

a : Optical source
b : Optical interface.
c : Optical fibre.
d : Modulator.
The following are general descriptions of the above mentioned components.

### 2.4.1 OPTICAL SOURCE

The choice of using a certain type of a light source for a certain optical fibre sensor application is very much dependent on the stability of the source and the power loss that occurs through the coupling of the signal to the fibre. LED’s (incoherent light sources) support many modes and are widely used in conjunction with multimode optical fibres [6]. On the other hand, laser sources (coherent light sources) are widely used with single mode fibres because such systems can be useful for applications requiring large bandwidth. The existent disadvantages of LED’s [6] when compared to lasers are:

1. Lower modulation bandwidth.
2. Low optical power coupling into optical fibres.
3. Harmonic distortion.

The advantages of LED’s over lasers are:

1. They are simpler to fabricate.
2 : They are less costly.
3 : They are more reliable than lasers.
2 : They have less temperature dependence.
5 : They require simple derive circuitry.
6 : They produce linear light output against current characteristics.

2.4.2 OPTICAL INTERFACE

The use of certain elements such as lenses, couplers, polarizers, etc., can add a variety of improvements to any sensor system. The use of lenses [7], for example, can improve the coupling of the light power to the fibre through collimation. In systems where more than one fibre is in use, the need for couplers [8,9] is essential. These devices simply provide signal distribution and coupling between the fibres. Couplers can either select certain wavelengths or a certain amount of power dependent on the sensor design.

In certain applications, especially the polarmetric sensors, the implementation of polarizers within the system is useful. The basic principle of operation of these polarizers [10] is to allow light to be polarized in one direction and being transmitted while the orthogonal one is being extinguished. Another important device which always exists in an optical fibre system is a connector. A connector [8,9] is an element required whenever a fibre is to be connected to any device or whenever a break is made within a fibre, this ensures fibre alignment with low insertion loss.

2.4.3 OPTICAL FIBRE

An optical fibre in a sensor system can be used either to transmit light or as an active element achieving both light transmission and optical sensing. Preparation of the end fibres (i.e. polishing) [9] is very essential in order to provide smooth plane surfaces which are normally made perpendicular to the fibre axis unless the application requires otherwise, this can be achieved through the use of a suitably designed
jig. By reasonably polishing the fibre ends, this will provide as little power loss as possible due to interconnection losses.

The preparation of an optical fibre splice [10] which acts as a permanent joint between a fibre and another is necessary. Splices can be extremely useful especially when some kind of damage occurs to long lengths of fibres. There are two techniques used for fibre splicing [10], mechanical and fusion. Mechanical splicing can be achieved by aligning two fibre ends mechanically, for example by surrounding the fibre ends by a tube. Fusion splicing can be achieved by welding two fibre ends via the application of heat. This method is widely applied for silica fibres of very small core diameters.

2.4.4 MODULATOR

Light modulation can be classified into the following categories:

2.4.4.1 INTENSITY MODULATION

Many of the early type of optical fibre sensors employ this technique. There are many ways for achieving light modulation. Many of the intensity modulated fibre optic sensor applications employ fibre bending, light reflection and light absorption techniques [11].

In general, intensity modulated sensors are simple and low in cost. However, errors can arise due to intensity variations of the optical source and losses in optical fibres and connectors [12]. Therefore referencing is required.

2.4.4.2 WAVELENGTH MODULATION

This technique utilises changes in chromatic transmission which result from wavelength dependent variation. Noise due to intensity or phase variations can be reduced. There is not a variety of wavelength phase variations can be reduced. The number of wavelength modulation technique available [12] is small compared to intensity modulation. This is due to the complexity of the demodulation system which could
involve a spectrometer. However, simple demodulation techniques can be achieved by using optical filtering or two wavelength detection and processing [13,14].

2.4.4.3 PHASE MODULATION

This is a highly sensitive method which utilises interference between two light signals: the transmitting signal and the receiving signal. Phase modulated sensors generally require single mode fibres. For example, variations of either the refractive index or the fibre length results in phase changes [5]. In order to measure phase changes, optical interferometers [15,16] are used for the purpose of converting the phase modulated signal to amplitude modulated signal.

Some of the problems with phase modulation are laser instability and phase shifts caused by physical variables which are not to be measured [13]. The cost of some of these sensors is high.

2.4.4.4 POLARIZATION MODULATION

There are two modulation methods associated with these devices:

A: STRAIN-EFFECT BIREFRINGENCE

This method depends on the change in the refractive index of an optical fibre when some kind of a physical strain is applied to it [17]. The change in the refractive index causes a change in the velocity of one of the polarization directions, the one parallel to the direction of the applied strain, therefore causing a resultant difference in the velocities of the above mentioned polarization mode and its orthogonal mode. The velocity change will be extremely low in its value, this is known as the birefringence effect. The phase difference between the two directions per unit length of fibre [17] will be proportional to the applied strain.
B : FARADAY-EFFECT BIREFRINGENCE

This method uses the effect of magnetic field on the polarization state [18]. Whenever a magnetic field is applied to a material through which polarized light is propagating, the plane of polarization is rotated by an angle which is a function of the magnetic field strength. The advantages of using this kind of system is in its great reduced effect of both temperature and induced strain [17]. This system is relatively not expensive.

2.4.4.5 TIME RESOLVED

This method is the least familiar one amongst the modulation techniques. It uses time delay analysis to measure variations in the transit-time through a closed loop. The advantages of using such a system is the elimination of both intensity and phase noise [12].

The above various modulation techniques have been described briefly. Many examples on fibre optic sensors which utilises various modulation techniques are presented in Chapter(3).

2.4.5 PHASE DETECTION

In many sensor systems, detection of the output signal in the form of amplitude modulation is desirable. Sensor systems producing output wavelength and polarization modulation signals are usually detected in amplitude modulation form. However, this is not the case with phase modulated sensors. Therefore, optical interferometers are implemented in such sensors for the purpose of converting phase modulated signals to amplitude modulated signals [15].

In general, single mode fibres are implemented in combination with a laser beam source. In these systems, the coupling of a laser source to a single mode fibre [13] is difficult to achieve due to the high efficiency requirement. Interferometers are sensitive to many factors, for example, temperature, vibrations and pressure changes.
Four main types of interferometer exist. The following sections explain briefly the various designs.

2.4.5.1 MACH-ZEHNDER INTERFEROMETRIC SENSORS

These sensors use single mode coherent sources whose output is coupled into an optical fibre [16]. A 3-dB coupler/splitter is used for splitting the light beam into two parts, one propagates inside the sensing fibre, the other propagates inside the reference fibre. The implementation of 3-dB coupler/splitter substitutes the half-silvered mirror in the conventional optical interferometers [12]. The output of these two fibres are again coupled by another 3-dB coupler/splitter, the two output signals are then detected through the use of two photodetectors, this is shown in Figure(2.4.5.1).

![Figure(2.4.5.1): The Mach-Zehnder interferometer sensor.](image)

The sensing fibre can be coated with a special type of coating [12]. Any applied pressure, for example, to the coated sensor fibre, will cause phase shifts and therefore the change in the beam intensity will be detected by a photodetector. Another way for detecting light interference is via fringe counting [19]. When two light beams, with a slight difference in their optical pathlengths interfere, a series of dark and bright fringes results. An example of this kind is illustrated in Chapter(3).
2.4.5.2 MICHELSON INTERFEROMETER SENSORS

This type of interferometer [13] uses a similar principle to the Mach-Zehnder interferometer, the interferometer is shown in Figure (2.4.5.2). A light beam emitted from a laser source propagates through both the sensing and reference fibres through the use of a 3-dB coupler/splitter which splits the beam into two parts. The output of these two fibres are each passed to a mirror. The light will then be propagating back through the two fibres and the signal will then be detected by a photodetector following the 3-dB coupler/splitter. This type of sensor is widely used in pressure and temperature sensing [20].

![Figure (2.4.5.2): The Michelson interferometer sensor.](image)

2.4.5.3 FABRY PEROT INTERFEROMETER SENSORS

This type of interferometer is slightly different from the two above mentioned ones because both the sensing and reference signals are made to propagate into one fibre instead of two [12], shown in Figure (2.4.5.3).
A light beam propagates through the lead fibre and then through the sensing fibre. When both light signals are reflected from the two mirrors, interference will occur and is then detected by a photodetector.

2.4.5.4 SAGNAC INTERFEROMETER SENSORS

This type of sensor also uses one fibre for carrying the sensing and reference signals [21]. This sensor uses a long optical fibre coil, shown in Figure(2.4.5.4).
A 3-dB coupler/splitter is used to split the input light into two parts and allows them to propagate in the fibre coil in opposite directions (i.e. clockwise and counter-clockwise directions).

2.5 EXTRINSIC AND INTRINSIC SENSORS

Optical fibre sensors are usually classified according their mode of operation into two classes. The extrinsic sensors use optical fibres for only light transmission purposes [11]. These sensors for many applications incorporate two optical fibres, one acts as the signal transmitter, the other as the signal detector. The light transmitted by the transmitting fibre is modulated externally by some induced or environmental changes. For example, light interruption by induced pressure, back-reflection from a moving reflector, gas absorption and passage through a temperature-sensitive material can be achieved [11,22]. Most of the early fibre optic sensors employ this kind of sensors.

Intrinsic sensors use optical fibres for both transmission and detection of the output signal [11]. They are widely used for pressure sensing purposes. By applying a kind of pressure to the fibre itself, the light intensity inside the fibre will be reduced because any bending in the fibre will force the light to escape from the core towards the cladding, therefore causing some light loss.

One of the advantages of these sensors [23] over the extrinsic type is that they are not affected by problems such as dirt, dust, and alignment at the transducer. This is because intrinsic sensors modulate light inside the fibre, unlike the extrinsic sensors.
CHAPTER THREE
3 OPTICAL FIBRE DISPLACEMENT AND PRESSURE SENSORS

In this chapter, various applications in the field of optical fibre displacement and pressure sensing based on various modulation techniques will be examined.

3.1 DISPLACEMENT SENSORS

There has been intense research in the field of fibre optic sensors over the last decade. A wide range of displacement sensors has now been described for various application purposes [24-48]. In this section, a variety of displacement sensors employing different modulation techniques are discussed. The main research work which is presented in Chapters(4-6) is based on intensity modulation. Therefore, in this chapter, particular attention is given to intensity modulated sensors.

3.1.1 INTENSITY MODULATED SENSORS

Intensity modulation techniques are generally simple, reliable and low in cost. Referencing is required for achieving high accuracy and stability is referencing [14]. Many techniques have been developed for referencing purposes which in return add to the cost and complexity of the sensor. There is a large number of displacement sensors employing the modulation technique. One of the basic sensor types involves relative displacement of an optical fibre. Displacements could be either transverse, longitudinal, angular or differential [13,25] as shown in Figure(3.1.1.1).
A shutter type of sensor generally utilises either an optical microswitch, an optical wedge, a moving lens or a grating \([13,22,26]\) which can move up and down between two fixed optical fibres (launch and receive fibres).

Photonic or reflective types of sensor \([24]\) are one of the earliest fibre optic sensors used. These sensors consist of two fibre optic arms. One arm transmits light to a reflective surface, the other receives the reflected light and directs it towards a detector. In general, these systems employ either multi fibres \([26-28]\), twin fibres \([5,22]\) or a single fibre with a coupler \([3,5,25]\). Many fibre optic sensors employing this technique have been developed. For example, a displacement sensor employing two fibre bundles as light transmitter and receiver has been investigated \([28]\). A light incident on the sensing element is reflected towards the receive fibre bundle. The reflected light is then divided into two signals which are then transmitted to two photodetectors. A sensitivity of \(\approx 0.27\text{V/\mu m}\) over linear dynamic range of \(60\text{\mu m}\) was achieved. The advantage of this sensor is its insensitivity to the power fluctuations of the optical source and its non contact operation.

The applications of reflective type intensity modulated sensors are quite varied, one of the fields of interest is a fibre optic
vibrometer as used, for example, in monitoring of mechanical equipment for faulty detection purposes. The presence of high voltages will limit the design of conventional vibrometers. Therefore, the use of optical fibre sensors can be advantageous although these can also be expensive. A simple vibrometer design [29,30], used for 100 Hz vibration measurements of an electrical generator, utilises a light modulation reflective technique, as shown in Figure(3.1.1.2).

The vibrometer uses an elastic cantilever for inducing modulation. The sensor device can then be fixed to a conductive bar inside the ventilation channel between the core laminations of the stator. The vibrations of the lever are electrodynamically induced. A sensitivity of 10mV/μm and a linear range up to 0.25mm are achieved.

A widely applied sensor employing reflective techniques is used for level measurements [13], as shown in Figure(3.1.1.3). Light propagates from a transmitter fibre (LF) towards the surface of the liquid. The light is then reflected and received by a second fibre (RF).
Reflective fibre optic sensors can be used for many other applications, for example, measurement of diameter, alignment, thickness, concentricity, etc. [24]. The advantages of using such techniques for sensing is that no contact is required for measurements. One of the disadvantages, however, is the limited dynamic range. This problem can be reduced by the addition of a micro-lens in conjunction with a fibre. An example [31] is shown in Figure(3.1.1.4).

Each fibre (100/140 step index silica fibre) is positioned in such a way that the reflected light is received at its maximum value by these fibres, this position is controlled by the focal length of the lens being used. A linear and referenced response can then be obtained, this normalization process is given by the following expression
\[ NR(z) = \frac{P_1 + P_2}{P_1 + P_2} \]

where

- \( P_1 \): The power collected by the first fibre (fibre 1)
- \( P_2 \): The power collected by the second fibre (fibre 2).

The working dynamic range achieved by this sensor is 46mm compared to non-lensed systems where a few millimetres are achieved. This design is simple and gives a linear response.

A fibre optic displacement sensor which employs a graded index lens has also been demonstrated [32]. This sensor consists of two optical fibres which are coated by a reflective material. The reason for using a graded index lens is that it can expand and collimate the light beam launched by the transmitting fibre. The collimated beam is then received by the receiving fibre. Intensity modulation can be achieved by either transverse or longitudinal displacements of the lens with respect to the two fibres. It was deduced that the former method is twice as sensitive as with the latter one. A displacement of 2.5nm was measured and a dynamic range of 25\( \mu \)m was achieved.

A fourth type of intensity modulated sensor is a fibre loss sensor (evanescent sensor). This type of sensor relies on losses from the core or cladding of an optical fibre [24]. One of the most widely used configurations is a fibre microbending sensor [33,34], as shown in Figure(3.1.1.5).

![Figure(3.1.1.5): Optical fibre microbend displacement sensor.](image)
The above sensor utilises a fibre which is placed between the jaws of a deformer. This is a very sensitive technique, and large losses can be induced with relatively small deformation. A sensitivity of 0.45\(\mu\)V/\(\AA\) can be achieved [34].

In general, there are two kinds of microbending sensors: bright field and dark field sensors [24]. Bright field sensors measure output light intensity from the sensing fibre as is also shown in Figure(3.1.1.5). Dark field sensors measure the light loss through the fibre cladding. This can be achieved by using another fibre (detector fibre) coupled to the sensing fibre via a coupler.

Another method used for designing a microbend sensor is via the use of a spiral of flexible plastic which is wound around the outside of an optical fibre [23]. When the spiral is squashed (displaced), the spiral applies pressure to the fibre and therefore bending of the fibre is achieved. This sensor type [23] is quite useful for both displacement and pressure sensing. It is only required to displace the sensor by 0.035mm to switch off more than 95% of the light travelling inside the fibre. This sensor was tested over a temperature range of -40°C to 80°C and very minute changes in the attenuation level was observed.

An optical fibre rotary sensor which works by exciting low order modes in a multimode composite fibre structure has been investigated [35]. This sensor utilises a multimode fibre which is sandwiched between two fibres (transmitter and receiver fibres) with smaller NA and core radius. Mode excitation is achieved by twisting the multimode fibre, therefore power remaining in the fibre is then received by the receiving fibre. This is an intrinsic sensor and therefore the signal is free from oil, dirt, and moisture effects. The range achieved by this sensor is about 25° (135°-160°) with a sensitivity of 0.029°.

An important requirement of any sensor including microbending sensors is stability. Most fibres are coated with
polymetric materials, however, such types of coating may be damaged under large pressure or increased temperature [24]. As a substitute then, metallic coatings can be used.

Another type of fibre loss sensor consists of two fibres cemented to a 90° glass prism [23]. This kind of sensor is widely applied for level control, as shown in Figure(3.1.1.6). When the prism is placed in water, light is no longer reflected but is transmitted into the water. One of the problems with such sensors is that the outside surface of the prism should remain clean.

Figure(3.1.1.6): A level control using two optical fibres and a prism.

3.1.2 WAVELENGTH MODULATED SENSORS

Displacement sensors which utilises wavelength modulation techniques can provide referencing [36-42]. Wavelength modulation via the use of prisms [37], diffraction gratings [37-38], zoneplates [39] or chromatic lenses [40] have been reported. The following example which utilises a broadband optical signal, multimode fibre, a chromatic lens and a detector [40] is shown in Figure(3.1.2.1).
An optical signal carried by a transmitting fibre is passed initially through an air gap where displacements of the chromatic modulator occur and a receiving fibre will then detect the signal. These displacements will change the spectral content of the broadband signal as well as its intensity. Therefore the receiver fibre will detect two signals, the broadband non-modulated signal which will pass through the air gap, missing the modulator, and the second signal which is modulated by passing through the chromatic modulator. By suitable processing of the signals, referencing can be achieved. A dynamic range of 0.5mm was achieved.

Rotation sensing has also been achieved via the use of diffraction gratings. Figure(3.1.2.2) shows a sensor system [37] which uses two optical fibres, one as a transmitter and one as a receiver. A white light beam propagates through the transmitter fibre to the sensing element, which is a reflective diffraction grating. At the sensing element, wavelength modulation does occur and the reflected signal will then propagate through the receiving fibre. The signal is then passed to a twin-element photodiode. One of the diodes is filtered providing a referenced signal. The two signals are then amplified and passed to a divider.
A sensitivity of 1mV per one degree rotation is achieved over a range of 45° angular rotation.

The above example has also been applied for linear displacements of diffraction gratings [38]. The reflective diffraction grating was attached to a movable element and therefore allows the determination of displacements. A dynamic range of 10mm was demonstrated.

3.1.3 PHASE MODULATED SENSORS

Many interferometer sensors which are applied for displacement sensing have produced measurements on the nanometer scale [43-46]. A Fabry-Perot type of interferometer which detects cantilever displacements has been demonstrated [43]. Cantilever displacements of <1Å were achieved.
A micro-displacement fibre optic sensor which uses a two-frequency interferometer has been reported [47]. The sensor utilises a polarization maintaining fibre in the sensing arm of a Michelson interferometer, as shown in Figure(3.1.3.1).

The sensor uses He-Ne Zeeman laser which produces two linearly polarized light (orthogonal components) with a frequency difference of 250kHz. The light beam is split into its two components by a polarization beam splitter (PBS). Each component is then passed through a quarter wave plate which transforms the linearly polarized light to circularly polarized light. Each component was incident on a mirror and was then reflected back towards the PBS. One mirror is made fixed, while the other produces displacements. The two components (vertically and horizontally polarized) are transformed to horizontally and vertically polarized states, respectively, at their arrival to the PBS. The two beams were directed towards a polarizer at 45° and the output signal is then detected by a photodetector.
Meanwhile, another arm which carries light power from the laser source through a polarizer at 45° and towards a detector, was added. Finally, the phase shifts due to the mirror displacements were determined by measuring the phase difference between the two photodetectors. Therefore, this results in a linear response of the mirror displacements with respect to the phase difference. Resolution of 0.5μm was achieved.

### 3.1.4 POLARIZATION MODULATED SENSORS

Many of the rotation sensors employ polarization modulation techniques [22,13,48]. A simple example is illustrated in Figure(3.1.4.1).

![Figure(3.1.4.1): A fibre optic rotation sensor based on polarization modulation technique.](image)

By rotating the analyser, light is modulated. This kind of sensor can be modified to include a referencing scheme [13]. This is shown in Figure(3.1.4.2)
Light beam is coupled to a multimode fibre and then split into two parts (50:50). Each beam is then passed through a polarizer whose optical axes are positioned 90° with respect to each other. Another polarizer whose optical axis is positioned 45° to the other two polarizers modulates the light by rotating it.

3.2 PRESSURE SENSORS

In this section a range of pressure sensors is described which employ different modulation techniques.

3.2.1 INTENSITY MODULATED SENSORS

Many of the intensity modulated sensors used for displacement sensing are also applied for pressure sensing. These include transmissive [49,51], reflective [52-54] and microbending techniques [55,56].

Transmissive fibre optic pressure sensors provide modulation by either moving an object between two fixed optical fibres and therefore blocking a certain amount of light, or by moving one fibre with respect to another. A transmissive sensor which utilises two fixed fibres and a grating as the modulator has been reported [51]. This
sensor uses two gratings, one is fixed, the other moves with respect to the fixed one due to the application of pressure on a diaphragm. This will result in light modulation. Very small applied pressure (≈μPa) is required to produce a highly sensitive device.

A metal-embedded fibre optic pressure sensor which utilises an intensity modulation technique has been reported [50], as shown in Figure(3.2.1.1).

Figure(3.2.1.1) : A metal-embedded fibre optic pressure sensor.

Two optical fibres are aligned and separated by a small gap. The fibres are then embedded between two metallic discs which act as a diaphragm. The diaphragm is clamped around the circumference. When a pressure is applied to one side of the diaphragm (i.e. causing a differential pressure between the two sides of the diaphragm), a misalignment of the ends of the two fibres results. Therefore intensity modulated light is achieved. A sensitivity of 0.01V/psi over pressure range of 100psi with maximum deflection of the diaphragm of 0.004inch were reported.

Many reflective techniques employed by fibre optic pressure sensors have been reported [52-54]. A diaphragm type of a pressure sensor which consists of three optical fibres was demonstrated [53]. A light is transmitted to a stainless steel diaphragm of diameter equal to 14.5mm. An applied pressure to the diaphragm causes light modulation and the reflected light is then collected by a receive fibre. A reference fibre transmits light back and forth along the same path as the transmitting and receiving fibres. The reference and receive
signals are then passed through a divider. This sensor produced a sensitivity $= 0.5\text{mA}/\text{MPa}$.

Pressure sensors which utilise a small sized diaphragm have also been demonstrated [52,54]. A reflective type of pressure sensor which utilises one transmitting fibre and a number of receive fibres is shown in Figure(3.2.1.2). A maximum applied pressure of $20\mu\text{Pa}$ on a diaphragm of diameter $812\mu\text{m}$ was demonstrated [54]:

![Fibre optic pressure sensor using reflective technique.](image)

The microbending concept discussed in Section(3.1.1) is widely used by pressure sensors. A high temperature microbend fibre optic pressure sensor which measures diaphragm deflections and uses a reference fibre was demonstrated [55]. Pressure values up to $22.8\text{MPa}$ at $430\degree\text{C}$ were achieved with a sensitivity of $= 0.05\text{V}/\text{MPa}$.

### 3.2.2 PHASE MODULATED SENSORS

Many pressure sensors are based on the Mach-Zehnder configuration, as shown in Figure(2.4.5.1a) of Chapter(2). An applied pressure on the sensing arm results in phase changes of the transmitted signal. This technique is quite sensitive to changes in the optical path. The phase change may be expressed [19] as follows:
\[ \Delta \phi = \beta \Delta L + L \Delta \beta \]

where

- \( L \) : Fibre length.
- \( \beta \) : Propagation constant.

A sensitivity of 40.9\( \mu \)rad/mPa was demonstrated.

One of the applications of a phase modulated pressure sensor is in acoustic sensing [15,16,11]. One of the sensor configurations uses a mandrel and a coated optical fibre [11], as shown in Figure(3.2.2.1).

![Figure(3.2.2.1) : An fibre optic acoustic sensor using a mandrel.](image)

The fibre is wound on a plastic mandrel under tension. An applied pressure on the fibre will reduce the fibre strain. A sensitivity of 0.55\( \mu \)rad/\( \mu \)Pa has been achieved over 0-1000psi pressure range.

### 3.2.3 POLARIZATION MODULATED SENSORS

Many pressure sensors employing polarization techniques use the photoelastic effect [57-65]. A plastic material can be constructed such that birefringence is induced when pressure is applied to the material. An example [49] is shown in Figure(3.2.3.1).
Light is transmitted from a broadband optical source through a launch fibre. The light is then passed through a polarizer which illuminates a suitably birefringent material. The light is then passed through an analyser and then collimated by a lens to a receiver fibre. Pressure values up to 2MPa can be measured.

The above method has also been applied to single mode fibre which acts as the birefringent material [49]. Applications of such sensors include vehicle detection on roads [60] (applied pressures up to 2.5MPa are measured using low birefringent fibres) and high hydrostatic pressure detection [61-63] (applied pressures up to 200MPa are measured using highly birefringent fibres).

A method used for pressure sensing utilises a special type of optical fibre which under the effect of pressure causes an increase in the refractive index of the cladding with respect to the core. An applied pressure, therefore, causes light loss from core to cladding. The output light intensity will then determine the applied pressure value. Pressure values up to = 24μPa have been reported [49].
CHAPTER
FOUR
4 TRANSMISSIVE MODEL

In this chapter, a fibre optic displacement sensor based on intensity modulation is theoretically designed. The main work deals with transmissive type of sensors where two optical fibres (launch and receive fibres) are used. Many types of intensity modulated sensors have been described in Chapter (3). The designed sensor is similar to the one mentioned in Section (3.2.1) of the previous chapter, where an embedded fibre optic pressure sensor was described. In our design, light is modulated by tilting the receive fibre rather than displacing it. The effects of introducing various polishing angles to the fibre on the coupling efficiency and the sensitivity are also investigated.

4.1 THE BASIC DESIGN CONCEPT

The theoretical model of a simple type of a displacement sensor is designed via the use of Mathematica. The basic model is based on the use of one optical fibre, whose small area of its middle part is inserted into a polymer mold. A cut is made through both the upper part of the mold and the optical fibre, leaving the lower part of the mold as it is, i.e no cutting is applied to it as is shown in Figure (4.1.1a-4.1.1c). The result is two optical fibres, one acting as the signal transmitter which is called the launch fibre (LF), the second one acting as the signal receiver and is called the receiver fibre (RF).

![Diagram](a) (b)

Figure (4.1.1a-b): a- an optical fibre, b- a molded optical fibre.
The cutting of the optical fibre is followed by polishing. Different cutting and polishing angles \([66,67]\), ranging from \(0\) to \(70\) degrees, are applied to a series of fibres, which allows us to investigate and compare the different sensitivities and efficiencies of those systems.

There are various ways for operating this sensor. One way could be achieved by tilting RF with respect to LF. Another way is by the application of pressure on the molded fibre, as shown in Figure(4.1.2a-b). The optical fibre is turned upside down, allowing the pressure to be applied from above to the non-sliced part of the mold. The application of pressure will generate an air gap between the two parts of the fibre as is shown in Figure(4.1.2a-4.1.2b). Both ways will produce similar results.

Figure(4.1.2a-b): The optical fibre before the pressure application and after the pressure application.
The final design of the optical fibre pressure sensor requires an optical source and a detection system, shown in Figure(4.1.3).

![Diagram](image)

Figure(4.1.3): The optical fibre pressure sensor.

The theoretical model deals with amount of light received by RF (i.e the coupling efficiency) and the sensitivity of the design model. The model is concerned with looking at various polishing angles.

### 4.2 THE RAY THEORY

The basic concept used in determining the sensor performance, through the proper calculation of the required coupling efficiency for various parameter values, is the ray trace model [40]. It is assumed that two step index multimode optical fibres are used. A large number of light rays propagates inside the launch fibre towards the receive fibre through an air gap. The power carried by each ray is calculated individually and then integrated over the whole light cone. Therefore, \( \cos^2 \theta \) power distribution of the light cone produced at the endface of the launch fibre (\( \theta \) is the angle between a light ray and the axis of the optical fibre) is assumed [68] and easily implemented.

It is assumed that a point light source is theoretically positioned at \((x_0, y_0)\). Finite number of light rays propagate from LF towards RF (both LF and RF are assumed to be step index multimode fibres) through an air gap. The position of the LF is assumed to be fixed on the x-y axis shown below, and RF is assumed to be moving with respect to LF making an angle \( \theta_g \) with it. The origin of the axis represents the point
where both parts of the fibre are bound together via the polymer mold. The various parameters related to the theory are shown on Figure(4.2.1).

The model assumes propagation of a large number of rays, forming a light cone, which is restricted in its size by the thickness of the fibre core and the refractive indices of both the fibre cladding and core. Each of these rays is traced individually from the assumed point light source \((x_0, y_0)\) inside LF, towards RF through an air gap determined by the amount of applied pressure.

The first requirement for tracing any ray from LF to RF is to find the co-ordinates of the point light source (i.e. the values of both \(x_0\) and \(y_0\)). These values can be found by the formation of two equations for the tangents of the two rays, the two which form the boundary of the light cone, as shown in Figure(4.2.2).
Therefore the two equivalent tangents are represented in terms of the polishing angle and the fibre parameters as follows:

\[ \tan(90-\theta_{c1}) = \frac{d_1-t_1-y_0}{(d_1-t_1)\tan(\theta_p)-x_0} \] ..........(1)

and

\[ \tan(90-\theta_{c1}) = \frac{y_0-t_1}{t_1\tan(\theta_p)-x_0} \] ..........(2)

Eliminating \( y_0 \) from (1) and (2) gives

\[ \cot(\theta_{c1}) \left( t_1\tan(\theta_p)-x_0 \right) + t_1 = d_1-t_1-\cot(\theta_{c1}) \left( (d_1-t_1)\tan(\theta_p)x_0 \right). \]

The above equation is written in terms of \( x_0 \), hence after finding the value of \( x_0 \), the value of \( y_0 \) can be found by substitution into either equation (1) or (2), therefore

\[ x_0 = \frac{2t_1-d_1+d_1\cot(\theta_{c1})\tan(\theta_p)}{2\cot(\theta_{c1})} \]

and

\[ y_0 = \cot(\theta_{c1}) \left( t_1\tan(\theta_p)-x_0 \right) + t_1. \]
The theoretical model represents light rays in terms of vectors, therefore we can now define three unit vectors, relative to the (0,0) position, representing the point light source \((r_0)\), the light ray propagating inside LF \((r_1)\) and the polished surface of LF \((r_{LF})\). These three vectors are given as follows:

\[
\begin{align*}
   r_0 &= \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}, &
   r_1 &= \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix},
\end{align*}
\]

and

\[
r_{LF} = \begin{bmatrix} \sin(\theta_p) \\ \cos(\theta_p) \end{bmatrix}.
\]

The following represents the addition of the above first two vectors in terms of the last vector. Two new unknown constants \(g_1, m_1\) are introduced to satisfy the vector addition, shown in Figure(4.2.3), for an arbitrary LF ray.

\[
\begin{align*}_0 + g_1 r_1 &= m_1 r_{LF}, &
g_1, m_1 : \text{unknown constants.}
\end{align*}
\]

\[
\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + g_1 \begin{bmatrix} \cos(\theta_1) \\ \sin(\theta_1) \end{bmatrix} = m_1 \begin{bmatrix} \sin(\theta_p) \\ \cos(\theta_p) \end{bmatrix},
\]

or alternatively

41
The unknown constants can then be found as follows:

\[
\begin{bmatrix}
\sin(\theta_p) & -\cos(\theta_1) \\
\cos(\theta_p) & -\sin(\theta_1)
\end{bmatrix}
\begin{bmatrix}
m_1 \\
g_1
\end{bmatrix}
= \begin{bmatrix}
x_0 \\
y_0
\end{bmatrix},
\]

where \( |A_1| \) is the determinant of the 2x2 matrix.

\[|A_1| = -\sin(\theta_p) \sin(\theta_1) + \cos(\theta_1) \cos(\theta_p) = \cos(\theta_1 + \theta_p).\]

When the ray vector is propagating along the LF endface, the value of \( |A_1| \) will then be equal to zero, therefore we reject the ray.

We now apply Snell’s law at the LF interface, i.e

\[n_1 \mathbf{n}_{LF} \wedge \mathbf{r}_1 = n_2 \mathbf{n}_{LF} \wedge \mathbf{r}_2. \quad \ldots \ldots (5)\]

where \( \mathbf{r}_2 \) and \( \mathbf{n}_{LF} \), shown in Figure(4.2.4) have the following meaning:

- \( \mathbf{r}_2 \) : the ray vector travelling in air.
- \( \mathbf{n}_{LF} \) : the vector normal to LF endface.

![Figure(4.2.4): The application of the Snell’s law at LF interface.](image-url)
The second vector $\mathbf{n}_{LF}$ is determined by the polishing angle of LF, therefore

$$\mathbf{n}_{LF} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \mathbf{r}_{LF}$$

$$= \begin{bmatrix} \cos(\theta_p) \\ -\sin(\theta_p) \end{bmatrix} .$$

Evaluating the left hand side of the Snell’s law (i.e. equation 5), we get

$$\mathbf{n}_1 \cdot \mathbf{n}_{LF} \times \mathbf{r}_1 = \mathbf{n}_1 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos(\theta_p) & -\sin(\theta_p) & 0 \\ \cos(\theta_1) & \sin(\theta_1) & 0 \end{bmatrix}$$

$$= \mathbf{n}_1 (\cos(\theta_p) \sin(\theta_1) + \cos(\theta_1) \sin(\theta_p)) \mathbf{k}$$

$$= \mathbf{n}_1 \sin(\theta_1 + \theta_p) \mathbf{k} , \quad \ldots \ldots \ldots (6)$$

and the right hand side of the Snell’s law can be written

$$\mathbf{n}_2 \cdot \mathbf{n}_{LF} \times \mathbf{r}_2 = \mathbf{n}_2 \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos(\theta_p) & -\sin(\theta_p) & 0 \\ \cos(\theta_2) & \sin(\theta_2) & 0 \end{bmatrix}$$

where

$$\mathbf{r}_2 = \begin{bmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{bmatrix}$$

so that

$$\mathbf{n}_2 \cdot \mathbf{n}_{LF} \times \mathbf{r}_2 = \mathbf{n}_2 \sin(\theta_p + \theta_2) \mathbf{k} . \quad \ldots \ldots \ldots (7)$$
To find the refractive angle of the ray $\theta_2$, equations (6) and (7) are equated to yield:

$$n_1 \sin(\theta_1 + \theta_p) = n_2 \sin(\theta_2 + \theta_2).$$

Hence

$$\theta_2 = \arcsin\left(\frac{n_1}{n_2} \sin(\theta_1 + \theta_p)\right) - \theta_p.$$

where

$$-1 < \frac{n_1}{n_2} \sin(\theta_1 + \theta_p) < 1.$$

We can now find the ray vector $r_2$ in terms of the angle $\theta_2$, i.e

$$r_2 = \begin{bmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{bmatrix}.$$

Having found $r_2$, we now need to find vector $r_3$ which represents the ray received by RF. Again by applying the vector addition shown in Figure(4.2.5), we form the following expression:

$$m_1 r_{LF} + g_2 r_2 = m_2 r_{RF}, \quad \text{..........(8)}$$

where

$$g_2, m_2: \text{unknown constants.}$$
r_{RF} is expressed in terms of $\alpha$, the angle which lies between RF and the positive x-axes.

\[
\mathbf{r}_{RF} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}.
\]

Angle $\alpha$ is determined by the air gap of angle $\theta_g$, created by the application of pressure to the sensor system. For the time being, let us assume we know the value of $\theta_g$, therefore, we can express $\alpha$ in terms of the polish angle of RF ($\theta_p$) and ($\theta_g$), hence

\[
\alpha = 90 - \theta_p - \theta_g.
\]

By the application of equation (8), we can find the constants $g_2$ and $m_2$

\[
m_1 \begin{bmatrix} \sin(\theta_p) \\ \cos(\theta_p) \end{bmatrix} + g_2 \begin{bmatrix} \cos(\theta_2) \\ \sin(\theta_2) \end{bmatrix} = m_2 \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}
\]

\[
\begin{bmatrix} \cos(\alpha) & -\cos(\theta_2) \\ \sin(\alpha) & -\sin(\theta_2) \end{bmatrix} \begin{bmatrix} m_2 \\ g_2 \end{bmatrix} = m_1 \begin{bmatrix} \sin(\theta_p) \\ \cos(\theta_p) \end{bmatrix}
\]
\[
\begin{bmatrix}
    m_2 \\
    g_2
\end{bmatrix} = \frac{1}{|A_2|} \begin{bmatrix}
    -\sin(\theta_2) & \cos(\theta_2) \\
    -\sin(\alpha) & \cos(\alpha)
\end{bmatrix} \begin{bmatrix}
    m_1 \sin(\theta_2) \\
    m_1 \cos(\theta_2)
\end{bmatrix}
\]

where

\[|A_2| = \sin(\alpha - \theta_2)\]

and is the determinant of the 2*2 matrix.

As in the previous case, if \(|A_2| = 0\) then the ray will be propagating along the RF endface, therefore we reject the ray. Again we apply the Snell’s law at the RF interface, i.e

\[n_2 \, n_{RF} \wedge r_2 = n_3 \, n_{RF} \wedge r_3\]

\[\ldots(9)\]

where \(r_3\) is the ray vector travelling in the air gap, and \(n_{RF}\) is the normal to RF endface, i.e

\[n_{RF} = \begin{bmatrix}
    0 & 1 \\
    -1 & 0
\end{bmatrix} \cdot r_{RF}\]

\[= \begin{bmatrix}
    \sin(\alpha) \\
    -\cos(\alpha)
\end{bmatrix}\]

Applying the left hand side of equation (9), we get

\[n_2 \, n_{RF} \wedge r_2 = n_2 \begin{vmatrix}
    i & j & k \\
    \sin(\alpha) & -\cos(\alpha) & 0 \\
    \cos(\theta_2) & \sin(\theta_2) & 0
\end{vmatrix}\]

\[= n_2 \cos(\theta_2 - \alpha) \, k\]

\[\ldots(10)\]
Now we apply the right hand side of equation (9), hence

\[
\mathbf{n}_3 \cdot \mathbf{n}_{RF} \wedge \mathbf{r}_3 = n_3 \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\sin(\alpha) & -\cos(\alpha) & 0 \\
\cos(\theta_3) & \sin(\theta_3) & 0
\end{vmatrix}
\]

\[= n_3 \cos(\theta_3 - \alpha) \mathbf{k} \quad \text{.........(11)}\]

Next, we need to find the refractive angle \(\theta_3\) by the light ray refracted at the RF surface and travelling inside RF, Figure(4.2.6).

\[
\text{Figure(4.2.6): The ray propagation inside RF.}
\]

Therefore, equating equations (10) and (11) we get

\[n_2 \cos(\theta_2 - \alpha) = n_3 \cos(\theta_3 - \alpha) .\]

Hence

\[\theta_3 = \arccos\left(\frac{n_2}{n_3} \cos(\theta_2 - \alpha) \right) + \alpha ,\]

where

\[-1 < \frac{n_2}{n_3} \cos(\theta_2 - \alpha) < 1 .\]
Now, we can define $r_3$ in terms of $\theta_3$ as follows:

$$r_3 = \begin{bmatrix} \cos(\theta_3) \\ \sin(\theta_3) \end{bmatrix}.$$ 

Having found the direction of the ray travelling inside RF, assuming infinite length of the endface, we need to add the condition that limits the collection of light rays at the endface boundary. Figure(4.2.7) shows the various parameters being used for determining the acceptable range for ray collection.

![Diagram showing the relationship between parameters](image)

Figure(4.2.7): The limitation on the light rays by the dimensions of RF.

$$\cos(\theta_p) = \frac{t_2}{p_1} \quad \text{also} \quad \cos(\theta_p) = \frac{d_2 - t_2}{p_2},$$

the condition is

$$p_2 > m_2 > p_1,$$

therefore

$$\frac{d_2 - t_2}{\cos(\theta_p)} \geq m_2 \geq \frac{t_2}{\cos(\theta_p)},$$

otherwise all other rays are rejected.
The last remaining condition required for satisfying a successful ray propagation through RF, is limited by the refractive index of RF. The condition is necessary for allowing the light rays to travel inside the fibre without any losses. Therefore we need to find the angle $\phi$ made by the ray vector $r_3$ with the vector normal to the fibre surface $r_p$, as shown in Figure (4.2.8).

![Figure (4.2.8): The limitations on the light ray by the RF refractive index](image)

Therefore

$$\cos(\phi) = r_p \cdot r_3,$$

where

$$r_p = \begin{bmatrix} \sin(\theta_g) \\ \cos(\theta_g) \end{bmatrix},$$

then

$$\cos(\phi) = \begin{bmatrix} \sin(\theta_g) \\ \cos(\theta_g) \end{bmatrix} \begin{bmatrix} \cos(\theta_3) \\ \sin(\theta_3) \end{bmatrix} = \sin(\theta_g + \theta_3),$$

therefore

$$\phi = \arccos(\sin(\theta_g + \theta_3)).$$
The final condition, therefore, is

\[ \text{crit}_2 \leq \phi \leq 180 - \text{crit}_2 \] otherwise the ray is rejected.

4.3 POWER AND COUPLING EFFICIENCY CALCULATIONS

In the previous section, the trace of the light cone of rays from LF to RF was achieved, this allows us to calculate the proportion of the input power received by RF, i.e the coupling efficiency. Figure(4.3.1) shows an \( i \)th light ray, which is traced from LF, represented by the ray vector \( R_1(i) \), through the air gap, represented by the ray vector \( R_2(i) \), towards RF, represented by the ray vector \( R_3(i) \).

\[ \text{Figure(4.3.1): The propagation of the } i^{\text{th}} \text{ ray through the whole fibre.} \]

\[ N \text{ is a constant equal to the total number of rays which form the light cone. Using a light source which is assumed to have a } \cos^2 \text{ distribution, the power of an } i^{\text{th}} \text{ ray, travelling inside LF in a direction making an angle } \theta_1 \text{ with the x-axis, is then given by the following equation:} \]

\[ \text{power}[i] = \cos^2(\theta_1). \]
Hence, the total input power is then equal to

\[
\text{Power}_{\text{in}} = \int_{\theta_{\text{cl}} - 90}^{90 - \theta_{\text{cl}}} \cos^2(\theta) \, d\theta.
\]

The \(i\)th ray travelling inside RF will continue to maintain the same power, having calculated the direction of the \(i\)th ray travelling inside RF, it is then possible to use Trapezoidal rule to calculate the area under the curve, representing the total output power over the range of the output angles as shown in Figure(4.3.2).

\[
\text{power}_{\text{out}} = \sum_{i=1}^{N-1} \frac{1}{2} (\theta_3[i+1] - \theta_3[i]) \ast (p[i+1] - p[i]).
\]

Figure(4.3.2): The use of the Trapezoidal rule for power calculation.
Therefore, at a certain value of the gap angle made between the two fibres LF and RF, the coupling efficiency $\eta$ is given as follows

$$\eta = \frac{\sum_{i=1}^{N-1} \frac{1}{2} (\theta_3[i+1]-\theta_3[i])*(p[i+1]-p[i])}{90-\theta_{cl}} \int_{\theta_{cl}-90}^{90-\theta_{cl}} \cos^2(\theta) \, d\theta}.$$  

The above procedure is repeated for various $\theta_g$ values. This method will allow us to represent the coupling efficiency with respect to the air gap angle $\theta_g$ for various polishing angles.

4.4 RESULTS

The following are the parameters of the polymer optical fibre used in the design of the optical fibre displacement sensor system:

- The core refractive index : 1.492
- The cladding refractive index : 1.417
- The numerical aperture : 0.45

It is important to note that the use of glass optical fibres in these systems requires highly accurate preparations due to their small core diameters, in addition to low power level outputs, resulting from such fibres. Therefore our interest is only limited to polymer fibres.

Figure(4.4.1) shows the variation of the coupling efficiency with respect to the air gap angle $\theta_g$, for 0, 10, 20, 30, 40 and 50 degrees of polish angles. It can be noticed that up to 20 degrees, the maximum possible coupling efficiency obtained, i.e when there is no air gap, was in fact 100%. The coupling efficiency value reduces as the polish angle increases, in addition to a slight decrease in the dynamic range.
The loss in the light power, occurring for large polish angles at zero degree air gap angle, is due to internal reflections occurring inside LF. The exact polish angle value, at which the coupling efficiency starts to drop from the 100% limit, can be deduced through the following procedure as is also shown in Figure(4.4.2):

\[ n_1 \sin(\theta_1 + \theta_p) = \sin(\theta_2) \]

internal reflection does occur when \( \theta_2 = 90 \) degrees, therefore
\[ n_1 \sin(\theta_1 + \theta_p) = 1, \]
hence
\[ \theta_1 + \theta_p = \arcsin\left(\frac{1}{n_1}\right), \]
and
\[ \theta_1 = \arcsin\left(\frac{NA}{n_1}\right) \]
\[ = 17.554 \text{ degrees}. \]

The value of the maximum polish angle required for the production of 100% efficiency, is found by substituting the \( n_1 \) value in the above equation, therefore
\[ \theta_p = 24.532 \text{ degrees}. \]

The plots that are shown above, reveal high sensitivities at small air gap angles, these sensitivities are largely reduced with larger air gap angles, the application of these sensors, therefore, can be quite useful for a small amount of applied pressure.

4.5 APPLICATION OF THE TRANSMISSIVE DISPLACEMENT SENSOR

The design of the above optical fibre sensor is useful in alarm systems [69]. These can be built through the use of a large area of a polymer material, where multiple applications of cutting along the material length would be applied, shown in Figure(4.5.1). This also can be done via the use of a large number of cut fibres arranged in a parallel configuration. The application of a large number of fibre cuttings to the system will enhance the sensitivity of the sensor.
The appropriate value required for applying maximum pressure, is dependent on the type of the polymer mold used. For the application of large values of stress, the required Young's Modulus should not be of small value, otherwise, it may not be possible to detect any signals for certain range of pressure values, and permanent damage to the optical fibres may also result. However, this is not so severe for small applications of stress.
CHAPTER
FIVE
5 REFLECTIVE MODEL

In this chapter, a fibre optic displacement sensor based on intensity modulation is theoretically designed. The reflective sensor utilises two optical fibres (launch and receive fibres) and a good reflective surface. In Chapter(3), a variety of reflective techniques were discussed. Our design considers various fibre geometries and combinations. Many fibre optic reflective sensors which utilise two parallel fibres and a reflector have been demonstrated [24-32,70-73]. Other types of sensor which utilise two tilted fibres have also been demonstrated [29,30,52,52]. Many of these sensors have also considered the implementation of lenses [29-31]. In our design, we have considered all these types of configurations in addition to another fibre configuration which involves tilting one fibre with respect to another. The latter configuration is shown to provide a solution to not only linear displacement sensing but also to rotary displacement sensing. Briefly, our model will be able to provide a solution (if a real solution does exist) of the proper reflective displacement sensor configuration for the given or the required fibre dimensions and numerical apertures, fibre/fibre distance, minimum/maximum fibre/mirror distance, dynamic range, sensitivity and coupling efficiency.

5.1 THE BASIC DESIGN CONCEPT

A theoretical design of an optical fibre displacement sensor, based on intensity modulation, was investigated through the use of various types of geometrical configurations of optical fibres. The sensor consists of an optical source, two optical fibres, a sensing element and a detector system, shown in Figure(5.1.1). In this system, one of the fibres acts as the light transmitter (also is called the launch fibre LF), the other fibre acts as the light receiver (RF). This type of sensor uses a mirror as an appropriate sensing element.
Figure (5.1.1): The design of the optical fibre displacement sensor.

The idea of the displacement sensor is based on light propagation from a light source through LF towards the sensing element, a reflection will occur at the sensing element, and therefore the light will be partially collected by RF and detected by the detection system. As any displacement occurring in the sensing element, in this case the mirror, the intensity of the light cone reflected from the mirror and collected by RF will vary in its amplitude. Therefore a relationship between the mirror displacements and the received light cone by RF can then be formed, and hence determining the sensitivity of the sensor.

The theoretical model assumes an infinite number of modes propagating in a launch multimode fibre which forms a light cone at its endface with uniform light power distribution. In general, the power distribution of the produced light cone across the endface of the fibre follows $\cos^2\theta$ [66]. However, it is being assumed, that because optical fibres have relatively small numerical apertures, the power distribution is assumed to be constant. Other techniques have been demonstrated, these include assumptions such as ray optics [25] where each point on the endface of the launch fibre produces uniform light cone.
The requirement of a high sensitivity sensor performance is essential, in order to achieve this, the use of both LF and RF in combination with a mirror producing displacements perpendicular to the fibre axis were first examined, as shown in Figure(5.1.2a). In the second case, LF was tilted with respect to RF with the mirror displacements being perpendicular to the RF axis, shown in Figure(5.1.2b). In the third case, both LF and RF were tilted equally with respect to the mirror displacements, shown in Figure(5.1.2c). As both case(2) and case(3) satisfy case(1), we only formed two different models for the three various configurations.

![Diagram](attachment:diagram.png)

Figure(5.1.2a-b): Two displacement sensors: a- two parallel fibres b- one tilted fibre.

A rotary sensor was also investigated with the use of the same principles derived in the two above mentioned models with both the fibres LF and RF positioned parallel to each other in combination with a rotating mirror, shown in Figure(5.1.2d).
It will be shown in Section (5.3) that the theoretical model derived for case (2) can also be easily applied to the rotary sensor. As will be deduced in later sections, in case (1), at least half of the input of the reflected light cone will be avoided by RF and lost through air. However, in case (2), the efficiency will improve, and in case (3), the maximum possible efficiency will be achieved.

The major factors which determine the amount of the receiving light cone by RF are:

1: The numerical apertures of both fibres.
2: The distance between the two fibres.
3: The distance between the fibre and the mirror.
4: The diameter of both fibres.

Overall, it was deduced that better results were partially dependent on the distance between the two fibres, the smaller the distance, the better the sensor performance. However, there are some limitations on acquiring the minimum fibre to fibre distance due to the thicknesses of both the cladding and the protective jacket of both LF and RF. In some cases, the separation between the fibres may not be reduced to the minimum required value, the addition of a lens at the endface of LF, for example a convex micro-lens, may allow the
possibility to work on larger fibre to fibre separations and mirror displacements. This system, however, can be equated to a purely fibre/mirror system with suitable choice of LF, i.e. the values of both the numerical aperture and the diameter.

5.2 TILTED LF WITH RESPECT TO RF

In this section, we will examine the coupling efficiency via ray tracing from LF to RF. The two fibres, LF and RF have core radii and numerical apertures of \( r_1, NA_1 \) and \( r_2, NA_2 \) respectively. The fibre separation is \( s \) and the distance of the fibres from the mirror is \( d \), shown in Figure(5.2.1).

Figure(5.2.1): The light cone launched from a tilted LF, reflected at the mirror surface and directed towards RF.

Figure(5.2.1) shows the light cone which is traced from LF towards the sensing element, i.e. the mirror, and is reflected back and received by RF. In order to calculate the coupling efficiency of the
system, we need to calculate the area made by the intersection of the reflected light cone with the endface of RF at the plane coinciding with the RF endface. The half angle $\theta$ of the light cone is given by the following equation:

$$\theta = \sin^{-1}(NA_1)$$

and

$$\alpha : \text{LF tilt angle}.$$  

When $\alpha$ is equal to zero, the area of the reflected light cone formed at the plane of intersection will be a circular surface equivalent to the light cone base. When $\alpha$ is not equal to zero, the area of the light cone will be an ellipse. The elliptical surface will then be inclined at an angle $\alpha$ to the circular cone base, the radius of the circular cone base ($R$) is:

$$R = r_1 + \frac{2d+2r_1\sin(\alpha)}{\cos(\alpha+\theta)} \sin(\theta) . \quad \text{......... (1)}$$

From the above value of the base radius, we can deduce the size of the elliptical surface and its $a$ and $b$ values, which represent the major and minor axes respectively, by measuring the various lengths after the projection of the circular base to the plane of intersection. The equation of the ellipse is defined as follows:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$  

Figure(5.2.2a) shows the light cone formed by LF.

let $R = m_1 + m_2$ ,  $\tan(\alpha) = \frac{c}{m_1}$ and  $\tan(\theta) = \frac{m_2}{c}$ .

We, therefore, need to calculate the value of $a$ in the ellipse and this is given as

62
\[ a = \frac{R}{(1+\tan(\alpha)\tan(\theta))\cos(\alpha)} \]

To calculate the value of \( b \), Figure(S.2.2c) shows a section of the lower cone. At the mid point where the two major and minor axes of the ellipse meet, the height of the intersection point to the circular base equals to \( \sin(\alpha) \) which can be deduced from Figure(S.2.2b)

\[ \sin(\alpha) = \frac{2R}{2b} \]

Figure(S.2.2b-c) : The various parameters related to the light cone in Figure(S.2.1).

and therefore

\[ b = \frac{R}{1+\tan(\alpha)\tan(\theta)} \quad \text{or} \quad \cos(\alpha) = \frac{b}{a}, \]

so the equation of the ellipse is defined by the above \( a \) and \( b \) values.

We need now to calculate the area of the intersection of the elliptical surface of the light cone and the circular endface of RF. Figure(S.2.3) shows the intersection at the x-y plane, the two areas \( S_1 \) and \( S_2 \) represent the total intersection area. In this system the equations of the ellipse and RF endface circle are given by
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad (x-x_0)^2 + y^2 = 1 \]

respectively.

Figure (5.2.3): The intersection of the Light cone and the fibre endface.

The centre of the circle is

\[ x_0 = a + r_2 + s - (2d+2r_1\sin\alpha) \tan(\alpha+\theta) . \]
The above two equations of the ellipse and circle are equated at $x=x_1$ where the two intersect, this leads to a quadratic equation, and therefore the value of $x_1$ is deduced. Hence the areas $S_1$ and $S_2$ can be found by integration as follows

\[ S_1 = \frac{2b}{a} \int_{x_1}^{a} \sqrt{a^2 - x^2} \, dx \]

and

\[ S_2 = 2 \int_{x_0-r_2}^{x_1} \sqrt{1 - (x-x_0)^2} \, dx \]

It is essential to note that there are five different possible overlapping geometries for the intersection between the RF endface and the elliptical surface. These possibilities are illustrated in Figure(5.2.4) and are listed in Table(1).
Whenever there is no solution for the quadratic equation of the intersection of the above two mentioned surfaces, then the ellipse and endface regions do not overlap, this geometry is labelled (i). Similarly, when the ellipse fully contains the circular endface, then there is no real solution, however some percentage of coupling does occur, this is labelled as geometry (ii). The third geometry is when the circular endface fully contains the ellipse (i.e no real solution but complete coupling) and the fourth geometry is when there is one real solution (i.e partial coupling). The final geometry corresponds to the case when there are two x-coordinates at which the ellipse and circle overlap, here labelled \( x_2 \) and \( x_3 \). The coupling efficiencies \( \eta \) for geometries (i-v) are summarised below:

The areas \( S_3, S_4 \) and \( S_5 \) shown in Table(1) are given by the following integrals:
\[ S_3 = 2 \int_{r_0-r_2}^{x_2} \sqrt{r_2^2 - (x-r_0)^2} \, dx \]

\[ S_4 = \frac{2b}{a} \int_{x_2}^{x_3} \sqrt{a^2 - x^2} \, dx \]

\[ S_5 = 2 \int_{x_3}^{r_0+r_2} \sqrt{r_2^2 - (x-r_0)^2} \, dx \]
Geometry Coupling Efficiency Range of Validity

(i) zero $s > x_0$
or$x_0 > s + 2r_2 + 2a$

(ii) $\frac{r_2^2}{ab}$ $s + 2r_2 < x_0 < s + 2a$

(iii) 1 $s + 2r_2 > x_0 > s + 2a$

(iv) $\frac{S_1 + S_2}{\pi ab}$ $s + 2r_2 > x_0 > s$
or$s + 2r_2 + 2a > x_0 > s + 2a$

(v) $\frac{S_3 + S_4 + S_5}{\pi ab}$ $s + 2r_2 < x_0 < s + 2a$

Table(1): The solutions for the various intersection geometries for tilted LF with respect to RF.

5.3 EQUIALLY TILTED LF AND RF

In this section, we will be looking at the two fibres, LF and RF, tilted equally with respect to each other. The displacements of the sensing element are identical to the previous section. Again, the model in this section uses the same fibre parameters as in Section(5.2), shown...
in Figure(5.3.1). In this new configuration, we represent the distance between RF and the mirror by the parameter $d$ as before, and $d_2$ represents the shortest distance between the mirror and the point where the LF light cone intersects the plane of the RF endface.

Figure(5.3.1) : The light cone trace and its intersection with RF.
In this new configuration, we will shortly see that the new surface made by the light cone at the plane of intersection, will simply be a circular surface equivalent to the circular cone base. For acquiring the value of \( d_2 \), we must find the intersection of the two lines \( z_1 \) and \( z_2 \) shown in Figure(5.3.1). The first line \( (z_1) \) marks the intersection of the extreme edge of the LF light cone with the x-z plane and the second line \( (z_2) \) is formed by the intersection of the x-z plane with the endface plane of RF.

The equations of these two lines are:

\[
z_1 = x \cot(\alpha + \theta) - (d_1 + 2r_1 \sin \alpha) \tag{2}
\]

and

\[
z_2 = -x \tan \alpha + d + (s + 2r_2 \cos \alpha) \tan \alpha \tag{3}
\]

It is important to note that the position of the mirror surface is assumed to be at the plane \( z=0 \), as in the previous section. However, when \( x=0 \), this coincides with the nearest edge of the tilted LF. Thus the origin of the x-z plane actually moves with tilt angle. Equating the expressions (2) and (3) gives the horizontal coordinate \( x=x_0 \) as

\[
x_0 = \frac{d + (s + 2r_2 \cos \alpha) \tan \alpha + d_1 + 2r_1 \sin \alpha}{\cot(\alpha + \theta) + \tan \alpha} \tag{4}
\]

and it follows that the vertical coordinate of this point of intersection is

\[
d_2 = x_0 \cot(\alpha + \theta) - (d_1 + 2r_1 \sin \alpha) \tag{5}
\]

By going back to Section(5.2), the distance \( R \) shown in Figure(5.3.1) can be obtained by replacing \( 2d \) with \( d_1 + d_2 \) in equation(1), this will give us the following expression for \( R \):

\[
R = r_1 + \left[ \frac{d_1 + d_2 + 2r_1 \sin \alpha}{\cos(\alpha + \theta)} \right] \sin \theta .
\]
By substituting both equations (4) and (5) into the above expression for \( R \), the value of \( R \) is then formed and hence the determination of the expression for the coupling efficiency can then be deduced. As with the previous section, we again calculate the intersection of the two surfaces made by the light cone and the RF endface at the plane of intersection previously shown. This time the two surfaces are both circular, this is due to the equivalent tilt angles made by the two fibres, LF and RF. Similarly, there exists, again, a number of possible overlapping geometries, and in this case, there are four different types of geometries to be considered. Table (2) shows the four possible solutions formed for the various geometries.

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Coupling Efficiency ( \eta )</th>
<th>Range of Validity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>zero</td>
<td>( s &gt; x_0 )</td>
</tr>
<tr>
<td>(ii)</td>
<td>( \frac{r_2^2}{R^2} )</td>
<td>( s+2r_2\cos\alpha &lt; x_0 &lt; s+2R\cos\alpha )</td>
</tr>
<tr>
<td>(iii)</td>
<td>1</td>
<td>( s+2r_2\cos\alpha &gt; x_0 &gt; s+2R\cos\alpha )</td>
</tr>
<tr>
<td>(iv)</td>
<td>( \frac{S_1+S_2}{\pi R^2} )</td>
<td>( s+2r_2\cos\alpha &gt; x_0 &gt; s )</td>
</tr>
</tbody>
</table>

or

\( s+(2r_2+2R)\cos\alpha > x_0 > s+2R\cos\alpha \)

Table (2): The solutions for the various intersection geometries for two equally tilted fibres, LF and RF.
Figure (5.3.2) shows that the circles overlap at the $x$-coordinate given by

$$x_1 = \frac{R^2 + m^2 - r_2^2}{2m}$$

(i) (ii)

(iii) (iv)

Figure (5.3.2): The various configurations for light cone and fibre endface intersection

so that the areas indicated above can be evaluated as

$$S_1 = 2 \int_{x_1}^{R} \sqrt{R^2 - x^2} \, dx$$

and

$$S_2 = 2 \int_{r_0-r_2}^{x_1} \sqrt{r_2^2 - (x-r_0)^2} \, dx$$
5.4 THE ROTARY SENSOR

The design of a sensor, based on intensity modulation principles, was investigated using similar configurations to the one applied in the previous two sections. The theoretical model that was discussed in Section(5.2) can be used to compare to our present system. The design model of the rotary sensor makes use of the same two parallel fibres in combination with a tilted mirror as can be illustrated in Figure(5.4.1).

![Figure(5.4.1) : The light cone trace for the rotary sensor.](image)

The two systems can be proved to be equivalent at $\theta_1 = \theta_2$, this occurs when the light cones produced by both launch fibres are the same for both systems, i.e identical images are formed. The precise dimensions of the two equivalent systems depend on the tilt axes of the LF and mirror but it is easy to demonstrate the equivalence by a geometrical construction. Figure(5.4.2) shows the equivalence of the two systems.
It follows that the exact mirror tilt angle $\alpha_m$ required to equate both systems is simply

$$\alpha_m = \frac{\alpha}{2}.$$  

In certain circumstances this equivalent system may be preferred as the sensor head is rather easier to package and it may resolve particular access difficulties to the moving surface.

5.5 THE IMPLEMENTATION OF MICRO-LENSES

Optical fibre sensors are applied for various applications, some of these applications require a small range, i.e. the sensing element will produce small displacements. However, some of these applications require a larger range that is beyond the system’s capability and one of the ways for solving such a problem is through the use of lenses [31]. Figure(5.5.1) shows a simple example of an optical fibre displacement sensor that makes use of a simple convex lens and is compared to one that does not. The lensed system produces the required range by controlling the distance between the
lens and LF, and therefore by adjusting the lens/fibre distance, the range can be made larger compared to the system that does not use a lens.

Figure (5.5.1): The implementation of micro-lens in an optical fibre sensor system, the dashed line shows the ray trace if the lens is not being used.

These lenses are quite useful for many applications and the requirement for small sized lenses is essential. These lenses are called micro-lenses and they are fabricated in such a way to fit the endfaces of optical fibres through screwing. Micro-lenses are equally useful for focusing light beams onto optical fibres and therefore avoiding losses.
6 PARAMETRIC STUDY OF REFLECTIVE MODEL

In this chapter, a representation of various results for optical fibre displacement sensors, mentioned in the previous chapter is given. 2D and 3D plots of different LF/RF combinations and arrangements are investigated. We are mainly concerned with the operation of those sensors which are determined by investigating the following variables:

(1) : Coupling Efficiency.
(2) : Range.
(3) : Sensitivity.

The use of two kinds of multimode optical fibres, glass and polymer, are investigated. The sensing element is assumed to be a very good reflective surface such as a mirror surface.

6.1 PARALLEL LF/RF SENSOR

2D-plots of coupling efficiencies with respect to LF/mirror distances, equally the dynamic ranges, are investigated for the parallel LF/RF sensor. Results for glass/glass, plastic/plastic and glass/plastic fibre combinations are given. The following are various parameters used for glass and plastic fibres:

Glass fibre : core radius : 25μm.
cladding diameter : 125μm.
NA = 0.2.

Plastic fibre : core radius : 0.5mm.
cladding thickness : 20μm.
NA = 0.45.

Figure(6.1.1) shows three coupling efficiency plots of a glass/glass fibre combination for LF/RF distances of 0.3, 0.5, and 0.7mm.
From the above plots, the coupling efficiency for the two identical glass fibre combination is shown to be very low (below 1%). The range is also very small in its value (~ 0.1mm) when the sensor is required to operate on the first part of the curve (i.e up to the maximum coupling efficiency value). However, the range becomes larger in its value (>1mm) at the second part of the curve.

Figure(6.1.2) shows the case with plastic/plastic fibre combination for LF/RF distances of 0.1, 0.3 and 0.5mm. The coupling efficiency reaches 11% for small LF/RF distances, however, a decrease in its value does occur when LF/RF distance is increased.
Coupling Efficiency (%)  LF/RF Distance (mm)

(1): 0.1  
(2): 0.3  
(3): 0.5

Distance (mm)

Figure(6.1.2): Two parallel plastic to plastic fibres for various LF/RF distances.

Figure(6.1.3) shows the case with glass/plastic fibre combination for LF/RF distances of 0.4, 0.6, 0.8 and 1mm. The performance of this sensor, thus far is the better one. An increase in the coupling efficiency of the sensor is noted in addition to its sensitivity.

Coupling Efficiency (%)  LF/RF Distance (mm)

(1): 0.4  
(2): 0.6  
(3): 0.8  
(4): 1

Distance (mm)

Figure(6.1.3): Two parallel glass to plastic fibres for various LF/RF distances.

From the above various fibre arrangements, a high sensitivity sensor performance can be obtained by operating on the first part of
the coupling efficiency curves. It can also be noted that glass/plastic fibre combination gives a better result with sensitivity = 10% mm⁻¹.

Figure(6.1.4) shows a 3D-plot of the coupling efficiency of the above glass/plastic sensor as a function of the LF/RF distance and the dynamic range. This plot shows clearly that the better the performance of the sensor, the less is the LF/RF fibre distance.

![3D-plot for two parallel glass to plastic fibres](image)

Figure(6.1.4): A 3D-plot for two parallel glass to plastic fibres.

6.2 ONE TILTED FIBRE WITH RESPECT TO ANOTHER

In this section, three types of fibre/fibre combinations, mentioned in the previous section, will also be investigated. The use of two identical fibres in this kind of sensor arrangement will be a disadvantage. Unnecessary loss in the light intensity is achieved due to the numerical aperture of the fibres. This loss can be avoided by using two different fibres, for example, the use of glass/glass fibre combination, both having different fibre parameters. The following are two glass fibres which are used to produce Figure(6.2.1):
Glass₁(LF): core radius : 25μm.
    cladding Diameter : 125μm.
    NA = 0.2.

Glass₂(RF): core radius : 98μm.
    cladding Diameter : 145μm
    NA = 0.28.

Note that the glass fibre with the smaller core radius and NA, was used as the launch fibre. Figure(6.2.1) shows three plots for tilt angles of 0, 2 and 4.5 Degrees.

Coupling Efficiency(%)

Tilt Angle ( Degree) :

(1) : 0
(2) : 2
(3) : 4.5

Distance (mm)

Figure(6.2.1): A tilted glass fibre to another glass fibre at LF/RF distance = 0.3mm.

Again, the coupling efficiency of this sensor is very low.

In the plastic/plastic fibre arrangement, the following are the various fibre parameters used to produce Figure(6.2.2):

Plastic₁(LF): core radius : 0.5mm.
    cladding thickness : 20μm.
    NA = 0.45.
Plastic\textsubscript{2}(RF) : core radius : 0.6mm.
cladding thickness : 50\mu m
NA = 0.6.

Coupling Efficiency(\%) Tilt Angle (Degree):

(1) : 0
(2) : 5
(3) : 10

Figure(6.2.2): A tilted plastic fibre with respect to another plastic fibre at LF/RF
distance = 0.3mm.

The above plots are for tilt angles of 0, 5 and 10 Degrees. Again, the performance of the sensor is poor, but slightly better than the glass/glass fibre combination.
In the glass/plastic fibre combination, the fibre parameters used to produce Figure(6.2.3) are the ones mentioned in the previous section. The tilt angles are 5, 10 and 15 Degrees. It can be noticed that this system not only provided the better results but also has the advantage over the parallel LF/RF fibre combination, mentioned in the previous section. The performance of this sensor (at the maximum tilt angle of 15 Degrees) can also be seen from Figure(6.2.4).
The above 3D-plot shows the effect of both tilting the LF and the LF/RF distance on the coupling efficiency of the sensor system. The coupling efficiency can reach 100% for very small values of LF/RF distances. However, due to tilting the launch fibre, a shift in the curve towards the left (lower distances) did occur. This can be reduced by increasing the LF/RF distance which, on the other hand, reduces the coupling efficiency.

Figure (6.2.5) shows another 3D-plot of the coupling efficiency with respect to the distance and LF tilt angle, at LF/RF distance of 0.3 mm. As it is expected, this kind of system produces much better sensitivity results (~70% mm⁻¹) when larger tilt angles (up to 15 Degrees) are applied.
Figure(6.2.5): 3D-plot of tilted glass to plastic fibres at LF/RF distance=0.3mm.

6.3 TWO TILTED FIBRES SENSOR

Three kinds of LF/RF combinations are investigated as before in Section(6.1). Figure(6.3.1) shows a glass/glass fibre combination. One of the two plots is for a fibre tilt angle of 45 Degrees, the other is not tilted at all. Although the coupling efficiency of the tilted fibre sensor is low, it provides a better system than the other one.
Figure 6.3.1: Two equally tilted glass to glass fibres for various tilt angles.

Figure 6.3.2 shows plastic/plastic fibre combination for tilt angles of 0, 10, 20, 30 and 40 Degrees, at LF/RF distance of 0.1mm. It can be noticed that by increasing the tilt angle, the sensitivity increases and a shift in the curve towards the negative direction of the range does occur, as was previously deduced.

Figure 6.3.2: Two equally tilted plastic to plastic fibres for various tilt angles, LF/RF distance=0.1mm.
Figure(6.3.3.a) shows a glass/plastic fibre combination for tilt angles of 0, 5, 10 and 15 Degrees at LF/RF distance of 0.3mm. This time, the results are much better than before.

Although this system produces results which are quite similar to the one shown in the previous section (for the same glass/plastic fibre combination), it has the advantage over the other one for not being limited by the numerical aperture of RF concerning light losses. Therefore, it is possible to tilt the fibres for angles up to 85 Degrees as shown in Figure(6.3.3.b). The two plots shows fibre tilt angles of 40 and 50 Degrees. Maximum coupling efficiency is achieved via this system with high sensitivity results (=70%mm⁻¹).
Figure (6.3.3b): Two equally tilted glass to plastic fibres for various tilt angles.

Figure (6.3.4) shows a 3D-plot of the coupling efficiency with respect to the range and LF/RF distance.

Figure (6.3.4): 3D-plot for Two equally tilted glass to plastic fibres, tilt angle=30 Degrees.
Again, it can be noted that higher coupling efficiencies and sensitivities can be achieved via the use of lower LF/RF distances.

Figure (6.3.5) shows the effect of dynamic range and tilting LF and RF on the coupling efficiency at LF/RF distance of 0.3mm.

Again, higher coupling efficiencies (up to 100%) and sensitivities (~180% mm⁻¹) can be achieved with larger applied LF and RF tilt angles.

6.4 ROTATION DISPLACEMENT SENSOR

As was mentioned before in Chapter(5), the displacement sensor can be used as a rotary sensor by rotating the mirror instead of displacing it. It was concluded from the previous section that the use of a glass/plastic fibre combination can give a better sensor performance.
Therefore, it is possible to apply this particular type of combination for rotation sensing purposes.

A glass/plastic fibre combination, identical to the one used in Section(6.4), was used in the rotation sensor. Figure(6.4.1) shows a number of plots of the coupling efficiencies with respect to the rotation angles (produced by rotating the mirror), for LF/RF distances of 0.2, 0.3, 0.4 and 0.5, and LF/mirror distance of 1mm.

![Figure 6.4.1](image)

Figure(6.4.1): Glass to plastic fibres, LF/mirror distance=1mm.

It can be deduced from the above various results that the larger the angle of rotation, the larger is the coupling efficiency. The effect of increasing the LF/RF distance, will only result in shifting the curves towards the positive direction of the tilt angle.

Figure(6.4.2) shows the same thing as before but with LF/mirror distance set to 0.5mm, and LF/RF distances of 0.2, 0.25 and 0.3. It can be deduced that by increasing the LF/mirror distance, the curves will shift towards the left (lower angles of rotation).
Coupling Efficiency (%)  LF/RF Distance (mm) :

(1) : 0.2
(2) : 0.25
(3) : 0.3

Tilt Angle (Degrees)

Figure(6.4.2): Glass to plastic fibres, LF/mirror distance=0.5mm.

Figure(6.4.3) shows a 3D-plot of the coupling efficiency with respect to the rotation angle and fibre/mirror distance, at LF/RF distance of 0.3mm.

Coupling Efficiency (%)  Rotation Angle (Degrees)

Figure(6.4.3): 3D-plot for glass to plastic fibres, LF/RF distance=0.3mm.
Figure (6.4.4) shows a 3D-plot of the coupling efficiency with respect to the rotation angle and LF/RF distance, at LF/mirror distance of 1.2mm. It can be deduced that by increasing the LF/RF distance, the coupling efficiency is reduced.

![3D-plot of coupling efficiency](image)

**Coupling Efficiency (%)**

Rotation Angle (Degrees)

Figure (6.4.4): A 3D-plot for glass/plastic fibres, LF/mirror distance=1.2mm.

### 6.5 IMPLEMENTATION OF MICRO-LENSES INTO SENSORS

The implementation of micro-lenses into optical fibre systems will result not only in an increase in the range but also in an increase in the coupling efficiency of the system, when the proper fibre configuration is used. In the previous sections, it was concluded that by using any two identical fibres in any of the displacement sensor configurations, poor results were achieved. However, these results can be improved via the use of a proper micro-lens at the endface of a fibre. The inclusion of these lenses will result in producing a larger effective core radius of the lensed fibre. Figure (6.5.1a-b) shows a tilted lensed plastic launch fibre with respect to another lensed plastic receive fibre, tilted at angles of 0, 5, 10, 15 and 20 Degrees.
By comparing the above two plots with the one in Section(6.2), it can be concluded that this type of sensor operates on a larger range compared to the non-lensed system (=0.75cm/0.75mm) and with increased coupling efficiency (80%/17.5%). The sensitivity of this type
of system is about 160% cm⁻¹ for a 15 Degree LF tilt angle, this can still be increased for larger LF tilt angles.

The case with a one tilted lensed plastic launch fibre with respect to a lensed plastic receive fibre, Figure (6.5.2) shows a 3D-plot of the coupling efficiency with respect to the fibre/mirror distance (up to 10 cm), and the effective Numerical Aperture, LF tilt angle is 5 Degrees. It can be deduced from the 3D-plot that high sensitivity sensor performance can be achieved at numerical aperture values less than 0.05. However, using larger numerical aperture values, this sensor will still operate reasonably well compared to the non-lensed system. The range of operation is larger than 0.5 cm.

![3D-plot for a tilted lensed plastic LF with respect to plastic RF, LF/RF distance=0.7 mm, LF tilt angle=5 Degrees](image)

The case with two equally tilted lensed plastic fibres, Figure (6.5.3) shows a 3D-plot of the coupling efficiency with respect to the fibres tilt angles (up to 40 Degrees) at LF/mirror distance of 1 cm, and the effective Numerical Aperture.
Again, it can be deduced from the above 3D-plot that high sensitivity sensor performance can be achieved at numerical aperture values less than 0.08. The dynamic range of operation is larger than 0.5cm.

6.6 VALIDATION OF THE MODEL

In this section, theoretical and experimental [74] results are compared using the following fibre parameters:

Glass fibre:  
- core diameter = 50 μm  
- cladding diameter = 125 μm  
- numerical aperture = 0.2

Plastic fibre:  
- core diameter = 500 μm  
- cladding thickness = 20 μm  
- numerical aperture = 0.45
Figure (6.6.1) and Figure (6.6.2) show the experimental and theoretical results respectively for the following fibre/fibre arrangements:

a: Parallel plastic to plastic fibres.
b: Parallel glass to plastic fibres.
c: Tilted glass to plastic fibres.

It can be noticed that both the theoretical results and the experimental ones produce increased coupling efficiencies by changing the fibre arrangement from (a) to (c). The theoretical result of the fibre arrangement in (c) produces \( \approx 3.6 \) times better coupling efficiency than the fibre arrangement in (a), compared to \( \approx 3.7 \) produced by the experimental results. However, the theoretical coupling efficiency values are almost twice the experimental ones. Errors associated with the experimental results could arise due to the following factors:

1: Losses in the fibres.
2: Losses at the reflective surface (mirror) due to scattering.
3: Losses at the endface of the fibres due to the polishing techniques used.
4: Misalignment between the two fibres.

By comparing plots (1) and (2) for both the experimental and the theoretical results, a noticeable shift in curve (2) of the theoretical results is observed. Whereas no shift is achieved with the experimental plot. Plot (1) shows the experimental results for a launch fibre with a relatively large numerical aperture and small fibre/fibre distance. On the other hand, plot (2) shows the experimental results for a launch fibre with smaller numerical aperture and larger fibre/fibre distance. Therefore, a shift in curve (2) with respect to curve (1) is expected. This is not the case with the experimental results, and therefore it is likely that this kind of error did occur as a result of the polishing techniques used and misalignment between the two fibres. If, for example, the launch fibre was not perfectly polished normal to the fibre axis, as a result, it is possible that no shift in the curve would occur.
On the other hand, if a misalignment between the fibres did exist, for example, the launch fibre was positioned closer to the mirror surface, then this could also produce no shift in the curve.

However, it is expected that the errors listed above will not solely produce such a large diversity in the theoretical and the experimental results. Problems with the theoretical results could arise due to the assumptions made and mentioned in Chapter(5). It was assumed that due to the relatively small numerical apertures of optical fibres, the power associated with each ray propagating in a fibre is equal. In general, the power distribution of the light cone leaving the endface of an optical fibre follows $\cos^2\theta$. Therefore, a percentage change in the output power from its maximum to its minimum value should result. The percentage change occurring for a glass launch fibre is $\approx 4\%$, compared to $\approx 20\%$ for a plastic launch fibre. On the other hand, the percentage changes between the uniform and the $\cos^2\theta$ power distribution of the output power integrated over the light cone calculated for both power distributions are 19% and 40% for glass and plastic fibres respectively. The problem is more severe for a plastic fibre than a glass fibre. Therefore, because the theoretical model considers a constant power distribution, percentage changes of $\approx 19\%$ and 40% have been ignored. When this concept is taken into consideration, reduced output coupling efficiencies and sensitivities of Figure(6.6.2) are expected.
Figure (6.6.1): The experimental results for various fibre/fibre configurations.

Figure (6.6.2): The theoretical results for the fibre/fibre configurations shown in Figure (6.6.1).
CHAPTER SEVEN
7 CONCLUSIONS

In this thesis, two kinds of displacement sensors were investigated: transmissive and reflective sensors. Theoretical models of each of the above mentioned sensors were introduced in addition to various results, via the use of Mathematica.

7.1 TRANSMISSIVE DISPLACEMENT SENSOR

In the transmissive type of displacement sensing, the effect of introducing a gap between two obliquely polished fibres on the coupling efficiency was investigated. Light modulation occurs by angular displacements of RF with respect to LF. The effect of introducing such angular displacements on the range, coupling efficiency and sensitivity was investigated. The sensor utilised only one polymer optical fibre representing both LF and RF. It is quite difficult, practically, to use two different kinds of fibres. Difficulties can arise due to firstly, possible misalignment between the two different fibres, and secondly, application of molding to the two fibres. Therefore, the use of a reasonably large core radius fibre is preferable (=0.5mm).

An investigation of the effect of introducing various polishing angles to a polymer fibre was presented in Chapter(4). Coupling efficiencies of 100% (at zero air gap angle) were deduced for polish angles ranging from 0° to 24°. The coupling efficiency was then seen to decrease with increased polish angles. No response was predicted at polish angles beyond = 60°. A range of 60° is possible for polish angle of 24 degrees, compared to dynamic ranges of 48° and 33° for polish angles 0° and 50° respectively. The sensitivity of the sensor was seen to be ≈2% per degree for 0 polish angle. A slight decrease in sensitivity was seen for increased polishing angles, up to a reduction in sensitivity of 1% per degree for 50° polish angle.

One of the applications of the transmissive sensor is for the purpose of alarm sensing. An array of fibres can be embedded in some sort of a composite material [75]. Those structurally embedded optical
fibres can also be used for a built-in damage detection system [76]. An example is impact/location detection and micro-crack/location detection [75,76]. Another possible application of these sensors is for a vehicle detection system on roads where applied pressure values of order MPa are required to be measured. In that case, optical fibres could be embedded by metallic materials rather than polymer ones.

7.2 REFLECTIVE DISPLACEMENT SENSOR

For the reflective displacement sensor, an investigation of various geometries fibre configurations was presented. The main work is concerned with the effect of introducing various fibre/fibre geometry on the range, coupling efficiency and sensitivity. Our designed model provides the proper choice of a displacement sensor configuration for given fibre parameters: fibre/mirror distance, fibre/fibre distance, coupling efficiency, sensitivity and range.

Two parallel and tilted fibre/fibre configurations were investigated. By comparing those two configurations it was deduced that the two tilted fibre configuration produced superior results over the parallel fibre in the sense that high coupling efficiencies (up to 100%) were achieved. The implementation of micro-lenses was seen to enable the operation of such sensors on a larger range (i.e: extending the range from mm’s to cm’s).

In the tilt fibre sensor the sensitivity was seen to increase with tilt angles. Sensitivities up to 0.18%\(\mu\)m\(^{-1}\) were deduced. Tilt angles up to 70° can be achieved using glass/plastic fibre combinations. It was also shown that by increasing the distance between the fibres, lower coupling efficiencies and sensitivities were predicted. The minimum possible fibre/fibre distance is determined by the thickness of the cladding of both the LF and RF fibres. The minimum distance achieved was \(\approx 120\mu\)m for glass/plastic fibre combinations. It can also be deduced that choosing a smaller core LF radius and numerical aperture than the RF will result in larger coupling efficiency. Therefore in this
respect, the use of glass/plastic fibre combinations was preferable. This will result in sensitivities up to 0.18%μm⁻¹.

The reflective displacement sensors can be very useful for many application purposes. A possible field of application is the detection of surfaces. The use of a displacement sensor will result in detecting any imperfections on the surface, for example, a crack in a surface will scatter light rather than reflecting it. If, instead, the surface is unlevelled, then this can also be detected via intensity changes in the light. Detection of level changes below 1μm are feasible in principle. Referencing of such sensors can be done through the addition of either a second RF, or a chromatic modulator.

The theoretical results were compared with some real data for a glass/plastic and plastic/plastic fibre combinations. For two parallel plastic fibres, the coupling efficiencies for both the experimental and theoretical results were shown to be 8% and 12% respectively with ranges of 1.5mm and 0.8mm. For the glass/plastic fibre combination, the experimental and theoretical coupling efficiencies were 9% and 18% respectively with ranges of 2mm and 1.5mm. For the two tilt fibres, the experimental and theoretical coupling efficiencies were 27% and 43% respectively with ranges of 2mm and 2.5mm. Increased sensitivities and coupling efficiencies were noticed for both experimental and theoretical results going from the parallel to the tilted fibre configuration. Although theoretical and practical results seem to behave in a similar manner, there is a discrepancy between the two results in that the coupling efficiency is always over estimated. Part of this discrepancy is due to errors in the experimental results, as discussed in Section(6.6). However, the major part of the discrepancy is most probably due to an assumption made at the outset, namely that the fibre output light intensity is constant with angle. The size of the discrepancy was discussed in Chapter(5).

The displacement sensor which uses one tilted fibre with respect to another, can also be used for rotation sensing. The rotation and
reflective displacement sensor were shown to be equivalent. This occurs when the rotation angle is half the tilt angle of RF in the reflective displacement sensor. One of the problems of using this kind of analysis, is that the rotation angle is limited to a certain value which is determined by the numerical aperture of the two fibres (up to 7° for glass/plastic fibre combination). This limitation can be relaxed via the use of micro-lenses which alter the effective numerical aperture and the core radius of the fibres, resulting in the detection of a larger rotation angles. Rotations up to 30° could be achieved in principle using a lensed glass/plastic fibre combination.

7.3 A SUGGESTION FOR FUTURE WORK

A future work concerning the reflective type of sensor could be the implementation of a graded rod lens as a substitute to the reflective mirror. In Section(3.1.1) an example of such a sensor was described. A problem with the sensor however is that it does not provide referencing. A method for eliminating such a problem may be via cementing a hot mirror to the endface of the graded rod lens. This provides chromatic modulation detection by measuring the intensities of two signals of different wavelengths in the manner shown in Figure(7.3.1a-b).

![Figure(7.3.1a): Wavelength modulated fibre optic sensor employing a graded rod lens and a hot mirror.](image-url)
Figure(7.3.1b) shows the effect of introducing displacements on the intensity changes of the output signal at two different wavelengths. The sensor is designed such that by introducing displacements, either an increase in the visible light intensity and a decrease in the infrared intensity are achieved, or vice versa. Any modulation of the light in the source, LF, RF and detector systems will be applied to the whole spectrum. Therefore, any modulations could be eliminated by taking the ratio of the signals in each of the two wavebands.

![Intensity vs Wavelength](image)

**Figure(7.3.1b)**: The sensor response for three different displacements.

The above should provide a stable measurement of nanometer displacement over a range of ≈25μm and is therefore perhaps relevant to acoustic or vibration measurement.
REFERENCES


