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Global value chains, trade shocks and jobs: an application to Brexit

Hylke VANDENBUSSCHE, William CONNELL and Wouter SIMONS

International Economics

Faculty of Economics
and Business

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Hylke Vandenbussche, William Connell, Wouter Simons[1]

Abstract

This paper develops a gravity model with sector-level input-output linkages in production. In contrast to a traditional gravity approach, which relies on direct gross exports between bilateral trade partners, our model additionally includes (1) domestic and global value chain linkages between goods and services sectors, (2) bilateral tariffs that affect direct production for a final destination as well as indirect production (shipped via third countries) to a final destination and (3) value added rather than gross production. Including input-output linkages implies that domestic production of intermediates can serve as inputs in foreign products and subsequently be exported "indirectly" to a final destination. Our input-output model can be taken to the sectoral World Input Output Database (WIOD) and can be used to evaluate trade policy shocks. While our framework is entirely general, we use it to predict the impact of the UK's withdrawal from the European Union ("Brexit") in terms of value added production and employment for every individual EU country involved. We find that Brexit hits the UK relatively harder than the EU-27. In contrast to other studies, we find EU-27 losses from Brexit to be substantially higher than hitherto believed.

[1] University of Leuven (KU Leuven) - Department of Economics, Naamsestraat 69, B-3000 Leuven, Belgium (corresponding author: hylke.vandenbussche@kuleuven.be; william.connellgarcia@kuleuven.be; wouter.simons@kuleuven.be).

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1. Introduction

Production processes are increasingly fragmented across national boundaries. The emergence of global production networks imply that one can no longer consider bilateral trade in isolation when evaluating trade policy or idiosyncratic shocks (Johnson, 2014; Acemoglu et al., 2012). A full assessment of trade shocks requires new models that consider global value chain linkages and trade costs worldwide. The starting point of this paper is that a country's production can be exported either directly or indirectly to a final destination. Indirect exports are intermediates that are used in the production process of other countries before they are shipped to the final destination. A traditional gravity model would not take these international production linkages into account and typically only consider the direct shipments between bilateral trade partners[2]. In this paper, we develop and explicitly solve a model that allows us to separately identify all the channels through which tariff changes operate. This results in a gravity model that overcomes the limitations of a traditional gravity approach and that derives closed-form solutions which allow for comparative statics on tariff changes. We then use the model to simulate the impact of the UK's withdrawal from the EU ("Brexit"). Our approach complements existing papers on Brexit that have used either standard gravity or simulated general equilibrium models to estimate the effects of Brexit[3].

The model that we develop is probably closest to the one by Noguera (2012) which features indirect exports in a value added setting. Whereas Noguera (2012) considers aggregate trade flows at the country-level, however, we develop a model with both final and intermediate trade flows at the more disaggregate sector-level. This has a number of important advantages. First, trade tariffs vary substantially across sectors, which means that a failure to account for this heterogeneity across sectors may lead to biased results. Second, as intra-industry trade between countries is substantial, it is important to allow two distinct countries to be active in the same sector producing similar goods. Third, our model allows for differences in the trade elasticity across sectors, meaning that consumers (and firms) can react differently to price changes in different sectors. Fourth, it exploits

[2]Take the example of Brexit, where a traditional gravity approach would only consider the direct impact of a UK tariff on Belgian shipments. However, Brexit also entails a UK tariff on German goods, which indirectly will also affect Belgian production whenever Belgian inputs are embedded in German exports to the UK. This paper considers the impact of tariff changes on both direct and indirect shipments via third countries.

[3]See, for instance, Dhingra et al. (2017) who simulate a computable general equilibrium model with inter-sectoral linkages to which they feed trade flows, trade elasticities and Leontief input-output coefficients. Their focus is solely on UK welfare and income effects which differs from ours. Other papers on Brexit that do not take global intersectoral production linkages into account are Aichele and Felbermayr (2015), Booth et al. (2015) and Lawless and Morgenroth (2016).

the availability of sector-level data such as the World Input-Ouput Database (WIOD), which allows us to include services in the analysis. This is important given that services are increasingly traded as well as embedded in the exports of goods. Disregarding services would therefore miss an important share of global trade. Finally, as the production linkages between two countries typically differ greatly across sectors, our sectoral approach will yield a more precise assessment of the indirect effects of a trade shock. Empirically, we find these indirect effects to be quite substantial and equally important as the direct effects.

The theoretical framework developed in this paper features a Cobb-Douglas-CES nest in production as well as in consumption. On the production side, firms produce output with a Cobb-Douglas technology and fixed expenditure shares on labor and a composite intermediate good, taking goods and factor prices as given. The composite intermediate good is a Cobb-Douglas combination of intermediate goods from all sectors. Each of these sector-specific intermediate goods is a Constant Elasticity of Substitution (CES) aggregate across all the countries the input can be purchased from.

On the consumption side, final consumers derive utility from an aggregate final good, which is a Cobb-Douglas combination of final goods from different sectors. Every sector-specific final good is a CES aggregate across all countries the good can be purchased from. The CES nests on the production and the consumption side rely on the Armington assumption, which means that goods produced by different sources are imperfect substitutes simply because of their origin. The way producers and consumers substitute between goods from different countries within a given sector is directly related to the sector's trade elasticity. The Armington assumption closely mimics the input-output data that shows that in reality similar inputs (from the same sector) are purchased from different countries.

This is different from a Ricardian approach where every input is sourced from only one particular country, as in Dhingra et al. (2015) and Caliendo et al. (2015). How much is sourced from each country depends on relative prices, which is a function of the productive efficiency of the supplier and trade costs. Finally, in our analysis we focus on the value-added share in a country-sector's production and the employment associated with it.

Our theoretical framework predicts that an increase in import tariffs will result in production and job losses losses all along the supply chain. The losses in value added production depend on three parameters, namely the sectoral trade elasticity, the value added shares in production and the Leontief input-output coefficients. The employment losses are obtained from the production losses by means of an employment elasticity. The theory predicts that some of these losses can be mitigated by "trade diversion", resulting from the changes in the multilateral resistance terms. Trade diversion arises, for instance, when a UK tariff on German imports results in Germany redirecting some of its exports to alternative destinations, and the UK importing from other sources that have become relatively cheaper. However, redirecting trade flows typically takes time. In our predictions on the impact of Brexit, we focus on the short-term effects since we cannot empirically observe the change in multilateral resistance terms. While this is a limitation, there are a number of studies such as Magee (2008) that suggest that the trade diversion effects are typically low, compared to the first-order trade effects, which is what we focus on here[4]. Our short-term approach also implies that we do not consider foreign direct investment (FDI) responses to trade policy, which may take longer to materialize. Moreover, we disregard any dynamic effects of Brexit related to investment and innovation, capital mobility and migration.

The database that we use is the World Input-Output Database (WIOD)[5], as in Johnson and Noguera (2012), Foster-McGregor and Stehrer (2013), Timmer et al. (2014), Timmer et al. (2015) and others that have investigated inter-sector and international linkages in global value chains albeit to address different questions. Our approach also differs from David et al. (2013), who assess US employment effects of Chinese import penetration at the regional level but do not consider the input-output linkages between industries. The novelty of our approach is that we consider all the downstream employment effects that stem from a change in domestic value added following a trade shock.

Another line of work in recent years has gone into identifying the welfare gains and losses from trade policy but has been less about inter-sectoral linkages and intermediates (see Costinot and Rodriguez-Clare, 2013 for an overview). An increasing number of papers in trade also turn to input-output data in the context of trade policy but with a different focus, e.g. Blanchard et al. (2016) who show that countries which are more connected in global value chains have lower tariff protection between them, Dhingra et al. (2017) who evaluate Brexit on UK household income levels and Caliendo et al. (2015) who assess the welfare effects of NAFTA. Finally, several studies in trade have now shown that gross trade flows do not necessarily reflect the domestic production underlying

[4]In his basic gravity model, Magee (2008) finds that bilateral trade flows are estimated to increase by 82% after countries engage in a regional agreement and this effect is significant across different econometric specifications. On the contrary, the variable capturing trade diversion reduces imports from outside by 2.9% but is not significant across different econometric specifications, suggesting that trade diversion is small compared to the direct trade effects.

[5]We use the release 2016 of the World-Input-Output Database (WIOD). This sector-level database provides information about the origin and destination of intermediate and final goods and services in 56 sectors using ISIC Rev.4 for 43 countries, and a residual rest of the world between the years 2000 and 2014. Dietzenbacher et al. (2013) describes in great detail the procedure that was followed to construct these World Input-Output Tables.

the trade flow (Koopman et al., 2014; Bernard et al., 2017)[6].

A limitation of the WIOD sector-level data is the lack of information on the underlying firm distribution. Using this database, we cannot know which firms import intermediaries and which firms are the exporting ones. However, whereas firm-level studies with information on their trading activity are often limited in their geographic scope and typically only include firms from one country[7], WIOD has a worldwide coverage that allows for the study of production networks covering all countries. Another advantage of using WIOD is that all upstream and downstream sectors can be identified for any sector in the production network, allowing for the construction of input-output linkages.

WIOD provides us with observations on the main variables required for our analysis of the impact of a trade shock like Brexit, i.e. trade flows, value added shares and production input-output linkages. We complement this data with estimates of sector-level trade elasticities and employment elasticities that we obtain from the literature. We consider both a "soft" Brexit (the "Norwegian scenario"), where the UK continues to be part of the Single Market but faces increased Non-Tariff Barriers (NTBs), as well as a "hard" Brexit scenario where Most-Favored-Nation (MFN) tariffs between the EU-27 and the UK are put in place in addition to the NTBs.

Our model predictions indicate that the UK is hit relatively harder than the rest of the EU-27 in both scenarios. In either case, Brexit reduces economic activity in the UK around three times more than in the EU-27. The UK will experience a drop in value added production as a percentage of GDP of 1.21% under a soft Brexit and up to 4.47% under a hard Brexit scenario. This corresponds to UK job losses of 139,860 jobs in the "soft" Brexit and 526,830 jobs in the "hard" Brexit scenario.

For the EU-27, the absolute job losses are larger, with the numbers of EU-27 jobs lost from Brexit varying between 284,440 jobs and 1,209,470 jobs respectively which corresponds to value added losses as a percentage of GDP of 0.38% for the "soft" and 1.54% for the "hard" Brexit. The losses in value added and jobs differ substantially across EU-27 member states. EU-27 member states that stand to lose most from Brexit are countries with close historical ties to the UK (e.g. Ireland, Malta) and small open economies on the European continent (e.g. Belgium and the Netherlands).

The Brexit impact varies across countries due to differences in sectoral composition. As shown by Acemoglu et al. (2012), it is the network centrality of sectors that determines the impact of an

[6] Bernard et al. (2017) empirically show that many products shipped by manufacturing firms are not produced in-house, but are "carry-along trade", i.e. gross export sales are much larger than the domestic production shipped.

[7] Viegelahn and Vandenbussche (2014) use micro-level data for India and have information on firm-level importing and exporting activities of Indian firms, but do not know the firms they are buying from or are selling to.

aggregate shock through a "cascade effect" in the input-output network. Under Brexit, we consider different tariffs across sectors, and therefore the propagation of tariff shocks differs depending on the sectoral composition of the economy. A sector that only has a few linkages with other sectors may not affect aggregate output much even when it is subject to high tariffs, as opposed to a sector that is very central in the production network. Our results take the network centrality and the number of sectoral production linkages into account when estimating the loss in value added and jobs caused by Brexit.

The remainder of this paper is organized as follows. In Section 2, we develop the theoretical model and obtain an expression for a country-sector's value added production and its determinants on the basis of which we obtain clear predictions on the effects of trade shocks. In Section 3, we explain the methodology and describe the data we use. Section 4 presents the results of the Brexit application. Section 5 compares our results to existing results in the literature and Section 6 concludes.

2. An Input-Output Model of Trade

In the model below we use superscripts to denote the country-sector of origin and subscripts to denote the country-sector of destination. To facilitate understanding, let us consider the following example. The quantity of intermediate steel from Belgium shipped to the German car industry is denoted by $X_{DE,car}^{BE,steel}$. In general, countries are denoted by i, j and k and sectors by r, s and z[8]. Demand for labor by country k's sector z for example is captured by L_{kz}. Throughout this section, upper-case symbols refer to real quantities, whereas lower-case symbols denote their nominal counterparts.

The model is based on the Armington assumption, which means that goods produced by different sources are imperfect substitutes simply because of their origin. As a result, within a sector, goods from different countries can coexist in the same destination market, even though their prices may differ as they are determined by the country-sector's marginal production cost and costs of trade with the destination country[9]. Consumers (and firms) have a love-for-variety and prefer to consume positive amounts of each available variety.

2.1. Consumer Demand

The representative consumer in country k derives utility from consuming quantities of an aggregate final good F_k:

$$U_k = F_k = \prod_{s=1}^{S} \left[F_k^s \right]^{\alpha_k^s} \tag{1}$$

which is a Cobb-Douglas combination of quantities F_k^s consumed of final goods from all sectors $s \in S$, with α_k^s the corresponding share in total expenditures. This sector-specific final good is a CES aggregate across all countries the good can be purchased from,

$$F_k^s = \left[\sum_{i=1}^{N} \left(F_k^{is} \right)^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s - 1}} \tag{2}$$

[8]We need at least three symbols in the model to denote countries and sectors because input-output models typically consider three nodes in a supply chain: (1) the supplier of intermediate inputs, (2) the final producer and (3) the consumer.

[9]As in Noguera (2012), production and trade costs are the only determinants of prices in our model. This does not imply that firms cannot charge markups. In WIOD, however, we have no information on the underlying firm-level distribution within each sector. The absence of markups in the model is assumed at sectoral level.

where $\sigma_s > 1$ is the elasticity of substitution (for final goods) between the countries of origin within sector s[10].

2.2. Producers

In country k's sector z, output Y^{kz} is produced according to a Cobb-Douglas technology combining labor L_{kz} and intermediate inputs X_{kz}[11]:

$$Y^{kz} = (L_{kz})^{1-\beta^{kz}} (X_{kz})^{\beta^{kz}} \tag{3}$$

where β^{kz} represents the share of intermediate expenditures in total sales of country k's sector z. The intermediate goods composite X_{kz} is a Cobb-Douglas combination of intermediate goods from all sectors $s \in S$, X^s_{kz}:

$$X_{kz} = \prod_{s=1}^{S} \left[X^s_{kz} \right]^{\gamma^s_{kz}} \tag{4}$$

where X^s_{kz} denotes the real aggregate demand of intermediates from sector s by country k's sector z, and γ^s_{kz} is the corresponding share in total expenditures on inputs. The sector-specific intermediate good X^s_{kz} is a CES aggregate across all countries the input can be purchased from:

$$X^s_{kz} = \left[\sum_{i=1}^{N} \left(X^{is}_{kz} \right)^{\frac{\rho_s - 1}{\rho_s}} \right]^{\frac{\rho_s}{\rho_s - 1}} \tag{5}$$

where $\rho_s > 1$ is the elasticity of substitution (for intermediate goods) between the countries of origin within sector s[12]. Note that this nested Cobb-Douglas-CES structure is similar to that of the consumer demand aggregates.

2.3. Utility and Profit Maximization

Let w_{kz} denote the price of labor in country k's sector z (L_{kz}) and p^{kz} the price of output from kz (Y^{kz}). Given iceberg-type trade barriers, in order to satisfy country j's demand of one unit of kz, kz needs to produce τ^{kz}_j units, with $\tau^{kz}_j > 1$. The price of one unit of kz's output in destination j

[10] For simplicity, we assume this sector-specific elasticity of substitution to be the same across all countries k.

[11] Following several standard trade models, we allow only for one factor of production. This assumption can be relaxed, for instance by accounting for high-and low skilled labor.

[12] For simplicity, we assume this sector-specific elasticity of substitution to be the same across all countries k.

then equals $p_j^{kz} = \tau_j^{kz} p^{kz}$ accounting for differences in trade costs across destinations j. Note that we typically assume there are no barriers to trade within a country, i.e. $\tau_k^{kz} = 1$.

Firms maximize profits by choosing L_{kz} and X_{kz}^{is} and households maximize utility choosing F_k^{is} subject to their budget which equals $I_k = \sum_{z=1}^{S} w_{kz} L_{kz}$, i.e. their income from supplying labor L_{kz} to each sector z in country k. Firms and households take factor price w_{kz} and goods prices $\tau_j^{kz} p^{kz}$ as given. This results in the optimal nominal counterparts of real demand (which are denoted by a lower-case symbol and that are obtained by multiplying real demand by the corresponding price). Nominal output of kz is represented by $y^{kz} \equiv p^{kz} Y^{kz}$. The CES price index in country k of final goods from sector s equals $P_k^s = \left[\sum_{i=1}^{N} \left(p_k^{is} \right)^{1-\sigma_s} \right]^{\frac{1}{1-\sigma_s}}$. The price of the aggregate intermediate input X_{kz} is given by the Cobb-Douglas price index $PI_{kz} = \prod_{s=1}^{S} (P_k^s)^{\gamma_{kz}^s}$ where P_k^s is the CES price index in country k for intermediate goods from sector s which we assume, for tractability, to be the same as the corresponding price index for final goods (this implies that $\sigma_s = \rho_s$ and that the price of a certain good from sector s is the same whether it is sold as an intermediate or a final good[13]. The (FOB) price[14] of output from kz equals $p^{kz} = (\frac{w_{kz}}{1-\beta^{kz}})^{1-\beta^{kz}} (\frac{PI_{kz}}{\beta^{kz}})^{\beta^{kz}}$. The optimal nominal demands then equal:

$$l_{kz} \equiv w_{kz} L_{kz} = (1 - \beta^{kz}) y^{kz}$$

$$x_{kz} \equiv PI_{kz} X_{kz} = \beta^{kz} y^{kz}$$

$$x_{kz}^s \equiv P_k^s X_{kz}^s = \gamma_{kz}^s \beta^{kz} y^{kz}$$

$$x_{kz}^{is} \equiv p_k^{is} X_{kz}^{is} = \tau_k^{is} p^{is} X_{kz}^{is} = (\frac{\tau_k^{is} p^{is}}{P_k^s})^{1-\sigma_s} \gamma_{kz}^s \beta^{kz} y^{kz} \tag{6}$$

$$f_k^{is} \equiv p_k^{is} F_k^{is} = \tau_k^{is} p^{is} F_k^{is} = (\frac{\tau_k^{is} p^{is}}{P_k^s})^{1-\sigma_s} \alpha_k^s \sum_{z=1}^{S} (1 - \beta^{kz}) y^{kz} \tag{7}$$

[13] The assumption that firms and consumers share the same price elasticities allows us to substantially simplify the analysis, as in Noguera (2012).

[14] The assumption of perfect pass-through inherent to this theoretical framework is a limiting assumption since pass-through depends on firm size with larger firms having lower pass-through rates (Amiti et al. (2014)). However, in the WIOD data we have no information on the underlying firm size distribution within a sector.

2.4. Market Clearing

Let $e_j^{kz} \equiv f_j^{kz} + \sum_{s=1}^{S} x_{js}^{kz}$ denote the nominal gross exports from country-sector kz to (the consumer and producers in) country j. Market clearing requires

$$y^{kz} = \sum_{j=1}^{N} e_j^{kz} \tag{8}$$

Following the same logic as in Anderson and Van Wincoop (2003), we derive gravity equations for final and intermediate goods exports, but now at the sector-level. Denote world nominal output by y^w and country-sector kz's share in world output by $\theta^{kz} \equiv y^{kz}/y^w$. Substituting Equations (6) and (7) into Equation (8) allows to solve for prices p^{is}. Substituting these into the price index P_k^s and plugging the resulting expression for P_k^s into (6) and (7) results in the following gravity equations for intermediate and final bilateral exports and equilibrium price indices:

$$x_{js}^{kz} = \frac{y^{kz}\gamma_{js}^{z}\beta^{js}y^{js}}{y^w}\left(\frac{\tau_j^{kz}}{\Pi^{kz}P_j^z}\right)^{1-\sigma_z} \tag{9}$$

$$f_j^{kz} = \frac{y^{kz}\alpha_j^z\sum_{s=1}^{S}(1-\beta^{js})y^{js}}{y^w}\left(\frac{\tau_j^{kz}}{\Pi^{kz}P_j^z}\right)^{1-\sigma_z} \tag{10}$$

$$P_j^z = \left[\sum_{k=1}^{N}\theta^{kz}\left(\frac{\tau_j^{kz}}{\Pi^{kz}}\right)^{1-\sigma_z}\right]^{\frac{1}{1-\sigma_z}}$$

$$\Pi^{kz} = \left[\sum_{j=1}^{N}\phi_j^{z}\left(\frac{\tau_j^{kz}}{P_j^z}\right)^{1-\sigma_z}\right]^{\frac{1}{1-\sigma_z}}$$

where $\phi_j^z = \sum_{s=1}^{S}\theta^{js}(\gamma_{js}^z\beta^{js} + \alpha_j^z(1-\beta^{js}))$ is a measure of the importance of goods from sector z for producers and consumers in country j. It takes into account (i) the dependence of producers in all sectors s in country j on intermediates from sector z through $\theta^{js}\gamma_{js}^z\beta^{js}$ and (ii) the importance of goods from sector z in the final demand by households in country j (through α_j^z) and the total income these households earn in all sectors s in j (through $\theta^{js}(1-\beta^{js})$).

Equation (9) relates bilateral intermediate trade between firms in country-sector kz and country-sector js to (i) the economic masses of source and destination relative to the world, (ii) the importance of inputs in the destination's production (β^{js}) and the importance of sector z goods within these inputs (γ_{js}^z), (iii) the bilateral trade costs between countries k and j in sector z (τ_j^{kz}), and (iv) outward and inward multilateral resistance terms (Π^{kz} and P_j^z). Similarly, Equation (10) relates bilateral final goods trade between firms in country-sector kz and the consumers in country

j to (i) the economic masses of source (y^{kz}) and destination ($\sum_{s=1}^{S}(1-\beta^{js})y^{js}$)[15] relative to the economic mass of the world (y^w), (ii) the importance of sector z final goods in the destination's consumption (α_j^z), (iii) the bilateral trade costs between countries k and j in sector z (τ_j^{kz}), and (iv) outward and inward multilateral resistance terms (Π^{kz} and P_j^z).

2.5. Input-Output Production Linkages

Dividing both sides of Equation (9) by y^{js} we obtain the technical coefficient a_{js}^{kz} or "dollar's worth of inputs from kz per dollar's worth of output of js":

$$\frac{x_{js}^{kz}}{y^{js}} \equiv a_{js}^{kz} = \frac{y^{kz}\gamma_{js}^{z}\beta^{js}}{y^w}\left(\frac{\tau_j^{kz}}{\Pi^{kz}P_j^z}\right)^{1-\sigma_z} \tag{11}$$

Plugging the technical coefficients into the market clearing in condition in (8), we have

$$\begin{aligned} y^{kz} &= \sum_{j=1}^{N}\left(\sum_{s=1}^{S} x_{js}^{kz} + f_j^{kz}\right) \\ &= \sum_{j=1}^{N}\sum_{s=1}^{S} a_{js}^{kz} y^{js} + \sum_{j=1}^{N} f_j^{kz} \end{aligned}$$

which can be summarized for all countries and sectors as

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{Y} + \sum_{j=1}^{N} \boldsymbol{f_j} \tag{12}$$

where

$$\boldsymbol{Y} = \begin{bmatrix} y^{1,1} \\ y^{1,2} \\ \vdots \\ y^{N,S} \end{bmatrix}; \quad \boldsymbol{A} = \begin{bmatrix} a_{1,1}^{1,1} & a_{1,2}^{1,1} & a_{1,3}^{1,1} & \dots & a_{N,S}^{1,1} \\ a_{1,1}^{1,2} & a_{1,2}^{1,2} & a_{1,3}^{1,2} & \dots & a_{N,S}^{1,2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{1,1}^{N,S} & a_{1,2}^{N,S} & a_{1,3}^{N,S} & \dots & a_{N,S}^{N,S} \end{bmatrix}; \quad \boldsymbol{f_j} = \begin{bmatrix} f_j^{1,1} \\ f_j^{1,2} \\ \vdots \\ f_j^{N,S} \end{bmatrix}$$

where $\boldsymbol{f_j}$ is the $(S*N)$ x 1 vector of country j's final demands and $\boldsymbol{A}$ the (S^*N) x (S^*N) global bilateral input-output matrix at the sectoral level. The system in Equation (12) can be written as

$$(\mathbb{I} - \boldsymbol{A})\boldsymbol{Y} = \sum_{j=1}^{N} \boldsymbol{f_j} \tag{13}$$

[15]This expression reflects the fact that consumers in country j get their income from supplying labor to all sectors s.

with $\mathbb{I}$ the (S^*N) x (S^*N) identity matrix. If $(\mathbb{I} - \boldsymbol{A})$ can be inverted, we can find the solution for nominal output as

$$\boldsymbol{Y} = (\mathbb{I} - \boldsymbol{A})^{-1} \sum_{j=1}^{N} \boldsymbol{f_j} = \boldsymbol{L} \sum_{j=1}^{N} \boldsymbol{f_j} \tag{14}$$

where $\boldsymbol{L}$ is known as the Leontief inverse matrix. Each element L_{is}^{kz} of $\boldsymbol{L}$ is the Leontief coefficient that measures the total of dollars worth of country-sector kz goods required to meet 1 dollar worth of is' final demand. This value combines kz goods used as inputs in is directly as well as kz goods used as inputs in other industries which then also produce inputs for is. Using this, we can obtain country k's nominal output in sector z as

$$\begin{aligned} y^{kz} &= \sum_{i=1}^{N} \sum_{s=1}^{S} L_{is}^{kz} \sum_{j=1}^{N} f_j^{is} \\ &= \sum_{i=1}^{N} \sum_{s=1}^{S} L_{is}^{kz} \sum_{j=1}^{N} \left(\frac{y^{is} \alpha_j^s \sum_{r=1}^{S} (1 - \beta^{jr}) y^{jr}}{y^w} \left(\frac{\tau_j^{is}}{\Pi^{is} P_j^s} \right)^{1-\sigma_s} \right) \end{aligned} \tag{15}$$

where we substituted the gravity relation from Equation (10) for the final value f_j^{is} flowing from country-sector is to the consumer in country j. Finally, we can transform this into value added production. For this purpose, we assume that the value added share of a country-sector's production is the part that is generated by its labor. Looking back at the production function in (3), the value created by country-sector kz after accounting for the intermediates used is captured by the share of labor $1 - \beta^{kz}$. Hence, following Noguera (2012) we find the value added embodied in kz's nominal production y^{kz} as $(1 - \beta^{kz})y^{kz}$ where $1 - \beta^{kz} \equiv v^{kz}$ is the value added to output ratio. The total value added production by kz can thus be written as

$$va^{kz} = v^{kz} \sum_{i=1}^{N} \sum_{s=1}^{S} L_{is}^{kz} \sum_{j=1}^{N} f_j^{is} \tag{16}$$

This value added production (and the jobs depending on it) might be severely impacted in the case of a trade shock, which is the subject of the next section.

2.6. Evaluating Trade Shocks

In this section, we examine the impact of a trade shock such as Brexit on a country-sector's value added production. Equation (16) shows that an import tariff imposed on a specific good will not only affect the producer of the good, but also the suppliers of goods and services whose output is used as an input in the production of the good. This implies that when the UK imposes a tariff on

German cars, the Belgian steel sector which supplies inputs to the German car industry will also be affected, even in the absence of a UK import tariff on Belgian steel. This channel is missing in a traditional gravity approach but can be captured by our model. The impact of a trade shock can be examined by considering what would happen when the variable trade costs (τ) change[16]. Our interest lies in the change $\mathrm{d}va^{kz}$ in country-sector kz's value added production, which we find to equal[17]

$$\begin{aligned} \mathrm{d}va^{kz} &= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}\left\{f_j^{is}+\sum_{r=1}^{S}x_{jr}^{is}\right\} \\ &= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}e_j^{is} \end{aligned} \tag{17}$$

from which we can derive the following general result. Rising trade costs reduce bilateral trade flows e_j^{is} between any country-sector is and j. As kz has an interest L_{is}^{kz} in each of these bilateral flows, va^{kz} will decrease as well. The drop depends on the magnitude of the change in relative trade costs $\hat{\tau}_j^{is}$ between is and j and the corresponding trade elasticity σ_s.

In Equation (17), we defined $\hat{\tau}_j^{is} \equiv \frac{\mathrm{d}\tau_j^{is}}{\tau_j^{is}} - \frac{\mathrm{d}\Pi^{is}}{\Pi^{is}} - \frac{\mathrm{d}P_j^s}{P_j^s}$ as the proportionate change in tariffs τ_j^{is} relative to the proportionate changes in the multilateral resistance (MR) terms. When examining trade policy, it is important to take into account that the multilateral resistance (MR) terms will change along with the tariffs. Therefore, Equation (17) not only examines the impact of $\frac{\mathrm{d}\tau_j^{is}}{\tau_j^{is}}$ but also that of $\frac{\mathrm{d}\Pi^{is}}{\Pi^{is}}$ and $\frac{\mathrm{d}P_j^s}{P_j^s}$. As it is relative tariffs that matter rather than absolute tariffs to determine a country's global competitiveness, individual tariff changes should be compared with changes in the average tariff, which is captured by the multilateral resistance terms. Suppose, for instance, that the UK tariff on Belgian goods goes up with 3%. If the UK further raises its tariffs on all its other trading partners with 2%, the "real" or "relative" increases in the BE-UK tariff is only 1% (3% - 2%). Therefore, what matters for a country-sector's production change $\mathrm{d}va^{kz}$ is the tariff change it faces relative to the tariff change its competitors face.

Under Brexit, the only countries that are likely to face increased tariffs from the UK are the EU-27, whereas the tariffs the UK imposes on its other trading partners such as the US will not change.

[16] In this application, any effect of the exchange rate on EU-UK trade is disregarded. We acknowledge that exchange rates have an important impact on the relative price of UK exports worldwide (and thus in the EU), possibly offsetting any change in tariffs. However, as major exporting firms tend to be major importing firms as well (see, for instance, Amiti et al., 2014), the depreciated pound will increase their production cost which will translate in higher export prices.

[17] See the Appendix for a detailed derivation.

This means that US goods will become relatively less expensive for the UK, even though the UK tariffs on US imports do not change. The reason is that Brexit actually decreases (i.e. $\hat{\tau}_{UK}^{US,s} < 0$) the "relative" US-UK trade costs compared to EU-UK trade costs. As a result, some trade will be diverted from the EU27-UK to the US-UK. The MR changes $\frac{\mathrm{d}\Pi^{is}}{\Pi^{is}}$ and $\frac{\mathrm{d}P_j^s}{P_j^s}$ are essential for trade diversion to happen. We can see this by disentangling the change $\hat{\tau}_j^{is}$ into its different components, namely the tariff change and the MR changes:

$$\mathrm{d}va^{kz} = \underbrace{-v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\frac{\mathrm{d}\tau_j^{is}}{\tau_j^{is}}e_j^{is}}_{\text{trade destruction effect}} + \underbrace{v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\left[\frac{\mathrm{d}\Pi^{is}}{\Pi^{is}}+\frac{\mathrm{d}P_j^s}{P_j^s}\right]e_j^{is}}_{\text{trade diversion effect}} \quad (18)$$

Equation (18) shows that the change in kz's value added production after a change in trade costs τ is a combination of a "trade destruction effect" (-) as a result of higher tariffs and a "trade diversion effect" (+) caused by the change in the multilateral resistance terms.

On the one hand, the "trade destruction effect" measures the drop in va^{kz} that is caused by the reduced trade between any country-sector is and country j. This drop depends on how the output of country-sector kz is used by country i's sector s, as it is the latter sector's exports that will face increased protectionist measures from country j.

The "trade diversion effect", on the other hand, is driven by two channels. First, country-sector is will divert some of its exports away from j to alternative destinations that do not impose tariffs on its goods, since these destinations have now become relatively more attractive (i.e. less expensive) for is to export to. This is caused by the increase in is' outward MR term Π^{is}. Second, the fact that j increases the tariffs on its imports will raise the average price in market j which makes the market less competitive, captured by the increase in j's inward MR term P_j^s. As a result, any country i will find it easier to export to country j. Both the first and second channel of trade diversion increase the exports of is and hence its production, which results in an increase in its demand for inputs from country-sector kz, which in turn increases the latter's value added production va^{kz}. Therefore, the "trade diversion effect" will mitigate some of the negative "trade destruction effect" on va^{kz}. The results can be summarized in the following proposition:

Proposition 1: The change in kz's value added production after a trade shock depends on two effects. First, the negative "trade destruction effect" indicates that the loss in va^{kz} depends on kz's connection with each exporting country-sector is. The drop in va^{kz} will be greater, (i) the higher is the trade elasticity in sector s (higher $(\sigma_s - 1)$); (ii) the greater is the increase in protection imposed by j on sector s goods originating in country i (higher $\frac{\mathrm{d}\tau_j^{is}}{\tau_j^{is}}$); (iii) the greater is the production interlinkage of kz with is (higher L_{is}^{kz}) and (iv) the stronger is the direct bilateral trade relation in both final and intermediate goods between i and j in sector s. Second, these negative effects will be mitigated through the "trade diversion" channel, as some of kz's production will be used in exports that are diverted to different destinations after the trade shock.

Equation (18) sums up the effects of a trade shock on va^{kz}. It characterizes all the different channels through which a trade shock can affect a sector's output. It is clear that the impact of a trade shock such as Brexit on a given sector can be very different depending on a number of determinants that vary by sector. Strong production interlinkages (high L) with a large exporting sector (high e) do not necessarily lead to large production losses (through the "trade destruction effect") in case this sector produces differentiated goods (making it insensitive to price changes, i.e. σ is low) or experiences only minor tariff increases (small $\frac{\mathrm{d}\tau}{\tau}$). Accounting for this sectoral heterogeneity in a model with a closed-form solution on the effects of a tariff shock, contributes to the literature in an important way.

In the next section, we will apply our model to a specific trade shock. We will compute the production and employment effects of Brexit, in which the EU and the UK impose tariffs on each other's goods.

3. Methodology

This section takes the model to the data and simulates the effects of different scenarios of Brexit using input-output data from WIOD. For expositional simplicity, we explain the methodology by focusing on the effects of unilateral UK protection on EU goods, but the analysis is symmetric for EU protection against UK imports. We will investigate the impact on kz's production when the UK imposes tariffs on EU goods using Equation (18)[18][19].

Equation (18) consists of a trade destruction and diversion effect, where the latter derives from the changes in multilateral resistance (MR) terms. These MR terms are not observable, and not controlling for them in gravity estimation is what Baldwin and Taglioni (2006) call "the gold medal of classic gravity model mistakes". Empirically, there are several ways to deal with the issue of MR, see for instance Anderson and Van Wincoop (2003), Baier and Bergstrand (2009) and Novy (2013). Hummels (1999) and Feenstra (2015) suggest to control for MR using directional (exporter and importer) fixed effects in a gravity model based on past data series. However, in our analysis of Brexit, where we simulate the model to engage in future predictions, the inclusion of fixed effects is not an option. The empirical findings in the literature on the magnitude of the trade diversion effect of import tariffs are ambiguous but seem to suggest that trade diversion effects tend to

[18] Note that empirically we account for retaliation i.e. we consider both tariffs imposed by the UK as well as tariffs imposed by the EU-27, when computing losses in value added produced and jobs for each country involved. Put differently, we assume that in the case of a "hard" Brexit, the EU-27 also imposes MFN tariffs on UK goods of the same magnitude as the UK does. To simplify the exposition here, we focus on the case where the UK imposes tariffs because the analysis is completely symmetric for any other EU-27 country involved in Brexit. Further, we make the likely assumption that non-EU relationships remain unchanged after Brexit.

[19] Our model only captures the static effects of a trade shock and it does not include dynamic effects such as access to foreign markets, firm investment and innovation, capital mobility and migration. In terms of the time horizon, we assume all effects to occur immediately after Brexit happens. However, it should be noted that it can take some time for our simulated outcomes to arise. Especially non-tariff barriers (NTBs) can have a lagged effect. Jung (2012) estimated that an adjustment period of 10 to 12 years could be in order. In which case the full effect of our simulated outcomes would be expected around 2030, which is also the time horizon adopted by most other Brexit papers.

be small[20]. Indeed, in order to divert trade, new business contacts have to be established, new contracts negotiated and so on, which takes some time to materialize. In our Brexit application, we therefore concentrate on the short-run effects and restrict Equation (18) to the first term that measures the "trade destruction effect". This is the first-order trade effect, which captures the main effects resulting from the Brexit's tariff changes. The drop in value added production as a result of increased UK trade protection on EU goods (higher $\tau_{UK}^{EU,s}$) under Brexit will thus be approximated by

$$\mathrm{d}va^{kz} \approx -v^{kz} \sum_{i \in EU}^{N} \sum_{s=1}^{S} (\sigma_s - 1) \frac{\mathrm{d}\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}} L_{is}^{kz} e_{UK}^{is}$$

Within this trade destruction effect we can now distinguish two different channels of value added loss by decomposing the trade destruction effect of UK protection into "direct" and "indirect" losses. These refer, respectively, to the losses in value added of country-sector kz stemming from direct bilateral trade (in goods and services) with the UK and the value added losses arising through its production linkages with other affected sectors in other EU-27 countries. For any country-sector kz, the loss in va^{kz} can be decomposed into a "direct" and "indirect" loss as follows

$$\mathrm{d}va^{kz} \approx \underbrace{-v^{kz} \sum_{s=1}^{S} (\sigma_s - 1) \frac{\mathrm{d}\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}} L_{ks}^{kz} e_{UK}^{ks}}_{\text{direct loss}} \underbrace{- v^{kz} \sum_{i \in EU \setminus \{k\}}^{N} \sum_{s=1}^{S} (\sigma_s - 1) \frac{\mathrm{d}\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}} L_{is}^{kz} e_{UK}^{is}}_{\text{indirect loss}} \tag{19}$$

Equation (19) thus captures the effect on va^{kz} of increased UK trade protection on EU-27 goods and services. Similarly, the effects of increased EU-27 protection on UK goods and services can be obtained from equation (19) by simply reversing the country of origin and destination[21]. In Section 4 we present both effects separately and combined to give an idea of the total effect of Brexit on EU-27 and UK value added production and employment.

[20] There appears to be no consensus in the literature on the magnitude of trade diversion effects. Dai et al. (2014) use manufacturing trade data for 64 countries and find that Free Trade Agreements (FTAs) divert trade, particularly on the import-side. However, Magee (2008) using different gravity specifications estimates of the trade diversion effects of regional agreements to be small and their significance to depend on the specification used. Similarly, Soloaga and Wintersb (2001) found evidence of export diversion in a minority of FTAs i.e. only 2 out of 9 FTAs analyzed had substantial trade diversion. Dhingra et al. (2017) estimates the potential gains by non-EU countries arising from the reduced trade between the EU-27 and the UK. The non-EU gains turn out to be very small, approximately between 0.01 and 0.02% of GDP. Therefore, the mitigation effects of trade diversion are likely to be small.

[21] Note that our theoretical framework predicts a loss in UK production even if we only consider trade protection imposed by the UK itself. The main mechanism is that it increases the price of (EU-27) inputs for UK firms and it decreases the demand for UK inputs that are embedded in EU-27 goods and services destined to the UK consumer.

3.1. Value Added Production Losses

In order to obtain an estimate of the value added losses, Equation (19) indicates that five key variables are needed. The five determinants in this equation are retrieved from various sources: (i) the the value added share v^{kz}, the Leontief coefficient L_{is}^{kz} and the direct trade flows e_{UK}^{is} are variables from WIOD; (ii) the trade elasticities at sector-level σ_s are borrowed from the literature and (iii) the change in trade barriers τ are obtained from potential Brexit scenarios that circulate in the literature. In order to obtain the job losses corresponding with the loss in value added in production, we turn to Eurostat data on EU-27 and UK sectoral level employment. Using sectoral employment elasticities from the literature, we obtain the corresponding job losses.

3.1.1. Input-Output Data

The World Input-Output Database (WIOD) contains detailed information on the global value chains of 44 world countries, including an approximation for the rest of the world, and 56 sectors with the latest year being 2014[22].

For each country-sector, WIOD provides its total production, the inputs it needs from other country-sectors and how much of its output is used by other country-sectors in their production process. The first variable that we obtain from WIOD is the value added share of country-sector kz's production, v^{kz}. This captures the value added, obtained as gross output minus gross intermediate inputs, per unit of gross output. We also obtain the Leontief coefficients, L_{is}^{kz} from WIOD, which are obtained using Equation (14). In addition, again from WIOD we obtain the direct trade flows e_j^{is} from country is to country j, which are obtained by summing exports from is that are destined to country j to satisfy its final and intermediate demand.

3.1.2. Trade Elasticities

Another determinant which the losses from Brexit depend on is the sector-level trade elasticity. A trade elasticity measures the proportionate decrease in demand after a 1% increase in trade costs. It captures the idea that higher UK tariffs and Non-Tariff Barriers (NTBs) will increase the price of EU-27 products in the UK (and vice versa), which will lower UK consumers' demand of EU-27 goods as they substitute away to products of cheaper origin. This is captured by the elasticity of

[22] Alternatively, other databases used in the literature are the "Global Trade Analysis Project Database", "OECD Input-Output Tables" or the "WTO-OECD TiVA Database.

substitution σ_s in sector s, from which the trade elasticity is derived as $\sigma_s - 1$. As a result, the extent to which production decreases after Brexit depends on the trade elasticity.

The literature has shown that trade elasticities typically vary both across countries and sectors. For example, Imbs and Méjean (2017) use product-level gross export flows between 1995-2004 to estimate trade elasticities, based on a multi-sector model developed by Arkolakis et al. (2012) and Costinot and Rodriguez-Clare (2013). They confirm that there is considerable heterogeneity in trade elasticities across countries and sectors. Using aggregate data, they find that the average trade elasticity within the EU countries is -2.98 with a minimum of -2.11 for Germany and a maximum of -4.83 for Greece[23]. Using more disaggregated data, they find that, within countries, trade elasticities also vary across products and consequently across sectors. Using their estimates, we find that Germany has an average elasticity across 11 manufacturing sectors of -5.1, with a median of -4.7 and maximum and minimum of -11.1 and -3.2, respectively[24]. In order to allow for the heterogeneity across sectors that is present in the theoretical framework, we use the average trade elasticies across countries at a sectoral level given that Imbs and Méjean (2017) do not report estimates of trade elasticities for every EU country-sector. In this way, we obtain elasticities for 16 different manufacturing sectors. For the remaining sectors we assign a trade elasticity of -4 which is a lower-end estimate of the trade elasticities reported in earlier literature. However, given that we analyze trade in value added rather than gross flows and that our data are at sector-level and not at product-level, we prefer to use the lower-end estimate of the trade elasticity. Therefore, the simulation results that we obtain can be regarded as lower bound estimates[25]. We assume complete pass-through of tariffs into domestic prices (congruent with the model). While our results depend on the choice of the trade elasticity, what has to be kept in mind is that our results vary linearly with the trade elasticity i.e. doubling the trade elasticity in every sector, doubles the value added losses from Brexit. Results depend monotonically on the trade elasticity parameters.

[23]For more information, see Table 4 in Imbs and Méjean (2017).

[24]In our analysis, we use a sectoral aggregation at 2 digit in Nace Rev. 2. For this reason, we use the Reference and Management of Nomenclatures tables (RAMON) provided by Eurostat to find the correspondence of the estimates provided by Imbs and Méjean (2017) who use ISIC3 as their product classification.

[25]Other trade elasticities estimates in the literature confirm this heterogeneity. Baier and Bergstrand (2001) use trade data to estimate a demand elasticity of -6.43, while Broda et al. (2006) use ten-digit HS data to obtain price elasticities of around -12. A recent paper by Coşar et al. (2016) uses a trade elasticity of -5.66. Ossa (2015) estimates sector level trade elasticities which range between -1.54 and -25.05.

3.1.3. Potential Brexit Scenarios

Equation (19) and the resulting losses in value added hinge on the increase in trade barriers i.e. $\frac{d\tau_{UK}^{EU,s}}{\tau_{UK}^{EU,s}}$. We consider two Brexit scenarios, an optimistic ("soft Brexit") and a pessimistic ("hard Brexit") scenario. We refer to Dhingra et al. (2017) for more details on these scenarios. In short, in the "soft Brexit" scenario, the UK continues to belong to the EU Single Market and tariffs remain zero, while non-tariff barriers to trade (NTBs) increase by 2.77%[26]. The scenarios are summarized in Table 1.

Table 1: Imposed tariffs and NTBs in both scenarios of Brexit.

	Soft Brexit	Hard Brexit
Tariff	0%	MFN tariff
Non-tariff barrier	2.77%	8.31%

Note: The scenarios are based on Dhingra et al. (2017).

In a "hard" Brexit scenario, the UK leaves the Single Market and trade between the EU-27 countries and the UK is governed by the World Trade Organization (WTO) rules. This implies an increase in trade tariffs from the current level of 0% to the sectoral "applied tariffs" imposed under the Most Favored Nations (MFN) clause, which differ by sector. These MFN tariffs are the tariffs that are currently imposed on goods traded between the United States and the EU, for instance. In Figure 1, we present the unweighted current MFN tariffs according to WTO rules in the sectors contained in the WIOD database. These are the MFN tariffs from the EU perspective, i.e. those that the EU imposes on imports from abroad. In the "hard Brexit" scenario, we assume EU-UK and UK-EU trade to be subject to an increase in the trade tariffs on goods from 0% to the unweighted average MFN tariff in each sector that ranges from 0% in "Mining and quarrying", "Forestry" and "Electricity and Gas" to 9.1% in the case of Fishing products. Figure 1 gives an overview of the MFN tariffs that currently apply to trade between members of the WTO. Moreover, we assume that under a "hard Brexit" NTBs rise further to a tariff equivalent of 8.31%[27].

[26]This is similar to the case of Norway whose NTBs with the EU are 2.11% higher than for the EU members. The 2.77% is taken from Dhingra et al. (2017). They compute a weighted average tariff equivalent for the current NTBs on US-EU trade, which amounts to 20.4%. Given that only 54% of this tariff equivalent is reducible, they only take into account an NTB tariff equivalent of ca. 11%. In the optimistic Brexit scenario, Dhingra et al. (2017) assume that the EU-UK trade will be subject to a NTB that is only one quarter of the one on EU-US trade, resulting in a tariff equivalent of 2.77%.

[27]This corresponds to three quarters of the NTB on EU-US trade, see Dhingra et al. (2017).

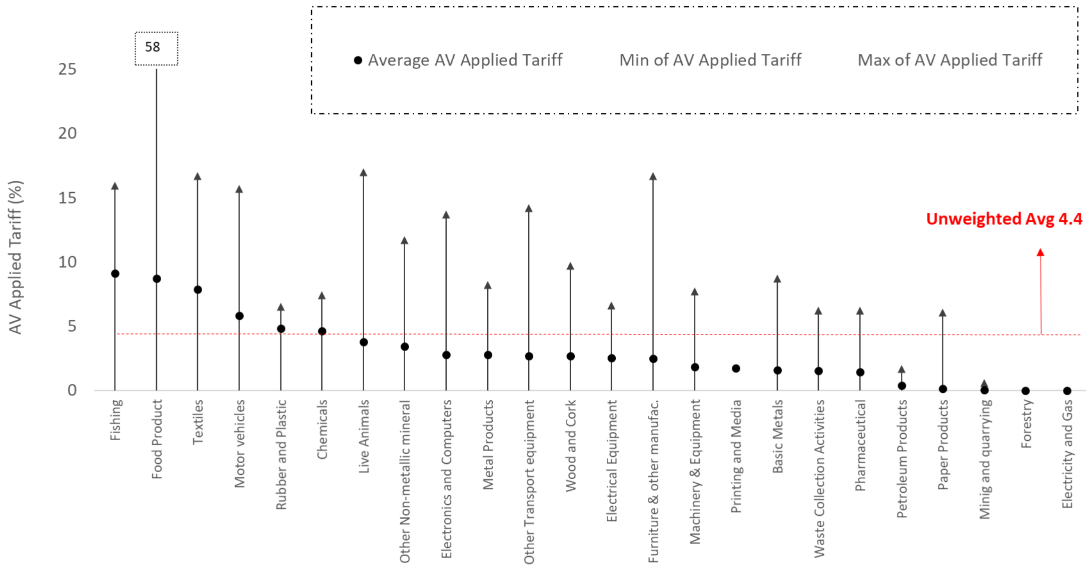

Figure 1: MFN tariffs imposed by the European Union

Note: The upper (lower) bound corresponds to the highest (lowest) tariff imposed within the HS6 classified in a Nace.rev 2 sector. The red dotted line marks the unweighted average tariff of all the HS6 where the European Union reports a tariff to the Most-Favored-Nations (MFNs). Information on the current tariffs applied are collected using the WTO Integrated Data Base (IDB). This database contains information on the applied tariffs at the standard codes of the Harmonized System (HS) for all the WTO Members. In this exercise, we use the Reference and Management of Nomenclatures (RAMON) correspondence tables to classify the equivalent Combined Nomenclature (CN) to the respective CPA 2008 code. In 35 of 5051 HS6 codes considered, the HS6 corresponded to multiply CPA 2008 codes.

3.2. Employment Losses

Combining the data gathered in the previous sections with Equation (19), we obtain the total value added production loss dva^{kz} in country k's sector z. In order to transform these production losses in job losses, we need an employment elasticity. This elasticity measures the proportionate drop in employment after a 1% decrease in value added production. In our theoretical framework, our production function is characterized by constant return to scale. In theory, Hamermesh (1986) argued that a production function characterized by constant returns to scale is identified by an elasticity of 1. However, this differs from empirical evidence. Konings and Murphy (2006) use European firm level data and report employment elasticities with respect to value added for manufacturing and non-manufacturing sectors below 1. They find the range of average employment elasticities between 0.57 and 0.72 in manufacturing sectors and 0.33 in non-manufacturing sectors. Given that we also focus on European data, we use the lower bound of these sectoral estimates to obtain the effect of Brexit on employment. This implies that for every 1% drop in domestically produced value added, we assume employment to go down by 0.57 % in manufacturing and 0.33% in non-manufacturing sectors. Similar to the trade elasticities, the Brexit results on employment depend linearly on the choice of the employment elasticity. Thus, once we have obtained the relative drop in employment from the decrease in production, we can compute the absolute number of jobs lost by multiplying by the country-sector's total employment base[28].

[28]Throughout the analysis, we assume that any job lost in the UK is not going to move to the EU-27 and vice versa.

4. Results

Our approach is a simulation exercise rather than a regression type of analysis since we cannot use past data to estimate the hypothetical impact of Brexit given the unprecedented nature of this event[29]. For this reason, we use our theoretical model to make projections about the potential effects from different scenarios of Brexit.

From the theoretical model we know that the UK's consumption of EU-27 products will decrease after the introduction of import tariffs by the UK. Given that EU-27 producers also use UK inputs in their own production, some of the UK's own value added will go lost when it imposes import tariffs on EU-27 goods and services. For example, take a German car manufacturer that uses a UK insurance as part of its inputs. The introduction of an import tariff on German cars by the UK will decrease the demand for German cars in the UK, subsequently also decreasing the demand for UK insurance. The same mechanism applies to the introduction of import tariffs by the EU-27 on UK goods and services.

Tables 2 and 3 summarize the economic impact of tariffs imposed by the UK on EU-27 products in terms of value added and employment. Both tables contain information on the potential losses for the EU-27 individual countries and the UK, which are obtained by summing across all sectors within the country. We distinguish between a "soft" and a "hard" Brexit, respectively. Columns (1) and (2) in both tables show the *absolute* value, in terms of millions of dollars and thousand of people, that would be hypothetically lost in a "soft" Brexit scenario. Whereas Column (1) shows the direct losses for each EU-27 country from lower direct bilateral trade with the UK, Column (2) identifies the losses from reduced indirect trade with the UK via other EU-27 countries. The sum of both channels is found in Column (3). To normalize the magnitude of the loss by country size, Column (4) expresses it as a percentage of the total value added (and employment) of the country[30]. The remainder of the columns document the losses in a "hard" Brexit scenario. It has to be kept in mind that our analysis has been entirely carried out at the sectoral level, whereas in the tables we have aggregated the sector-level effects at the country-level.

For each of the 28 European countries, Table 4 lists the most affected sector in terms of value added and employment. This sector can differ depending on whether we express losses in terms of value added or employment. The reason is that the value added contribution per worker can differ

[29] The majority of the bilateral tariffs between European countries have remained unchanged between the period 2000-2014.

[30] Total Value Added (TVA) for each country is obtained using the WIOD database.

dramatically across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

Similarly to the previous tables, Tables 5, 6 and 7 summarize the economic impact of tariffs imposed by the EU-27 countries on the UK. As mentioned above, sectors in the EU-27 countries will be affected by tariffs imposed by the EU-27 on the UK as a part of the production of the EU-27 sectors is embedded in UK exports back to EU-27 countries.

Tables 8 and 9 display the total losses from Brexit, obtained by summing the effects from both UK protection against EU-27 and EU-27 protection against the UK. The results indicate that the UK is hit *relatively* harder than the rest of the EU-27 in both the "soft" and "hard" Brexit scenario. In either case, Brexit reduces economic activity in the UK three times more than in the EU-27. The UK will experience a drop in value added production as a percentage of GDP of 1.21% under a "soft" Brexit and up to 4.47% under a "hard" Brexit scenario. This corresponds to UK job losses of 139,860 jobs in the "soft" Brexit and 526,830 jobs in the "hard" Brexit scenario. For the EU-27, the *absolute* job losses are larger, with the numbers of EU-27 jobs lost from Brexit varying between 284,440 jobs and 1,209,470 jobs *respectively* which corresponds to value added losses as a percentage of GDP of 0.38% for the "soft" and 1.54% for the "hard" Brexit. The losses in value added and jobs differ substantially across EU-27 member states. EU-27 member states that lose most are countries with close historical ties to the UK (e.g. Ireland, Malta) and small open economies (e.g. Belgium and the Netherlands).

For each EU country, Table 10 shows the sector that will be most affected under a "hard" Brexit scenario, which is based on both the direct and indirect effects. These most affected sectors may differ from the ones mentioned in other Brexit studies because in our analysis we have accounted for input-output linkages between goods and services sectors, allowing us to get a more complete picture of the effects of a trade shock. For example, in terms of value added the most affected sector in Germany is "Motor Vehicles", while in terms of employment losses it is "Machinery & Equipment". For the UK (GBR), "Wholesale Trade" is the sector most affected by Brexit in terms of value added, while in terms of job losses it is the service sector "Administrative and support activities".

5. Discussion

This section compares our results with those found by other papers that investigate the potential impact of Brexit. Emerson et al. (2017) summarize the results of six papers, three academic papers and three from official sources[31]. These studies each consider an optimistic and a pessimistic Brexit scenario that correspond closely to our "soft" and "hard" Brexit scenarios. An important difference is that most of these papers predict the Brexit effects to be fully materialized after a period of about 10 years. Therefore, most of them set the time horizon at the year 2030. In contrast, our approach does not make any assumption on the transition period that is needed to adjust to the new economic climate. The results that we present are the outcome of a static analysis. Therefore, we do not project the impact of Brexit and how that would occur over time, instead we obtain the immediate overall effects that can be expected to materialize.

When we compare our results to the average effect obtained in earlier studies that also consider the impact of Brexit on the EU-27 [32], our simulated value added losses for the EU-27 are approximately three times higher. We find the *absolute* loss in value added production for the EU-27 to be 1.7 times larger than the UK losses. In a way this should not come as a surprise given that the EU-27 is a much larger economy than the UK. Given the size of the EU-27 economy and the UK's large trade deficit with the EU-27, it seems likely that the EU-27 would suffer larger *absolute* losses than the UK, but this is not what other studies have found. In earlier studies, the absolute losses for the UK were always higher than for the EU-27.

A potential explanation for the larger *absolute* losses for the EU-27 in our study is the inclusion of indirect effects. Throughout the paper, we have extensively argued the importance of considering global value chains and value added trade flows rather than bilateral direct gross flows. The indirect losses from Brexit (e.g. decreased Belgian steel production due to reduced German car exports to the UK) are estimated to be very important, amounting to ca. one third of the direct effects[33]. This is likely to cause the divergence between our results and the other results that circulate in the literature. Our model simulation is bound to yield greater estimated losses given that it captures all

[31] See Ottaviano et al. (2014), Aichele and Felbermayr (2015), the OECD study by Kierzenkowski et al. (2016), Rojas-Romagosa (2016), Booth et al. (2015) and HMTreasury (2016).

[32] See, for instance, Rojas-Romagosa (2016), Aichele and Felbermayr (2015), Booth et al. (2015) and Ottaviano et al. (2014).

[33] Tables 2 and 3 display the predicted losses in terms of value added and employment associated with a tariff imposed by the UK on EU-27 goods and services. In these tables, we distinguish between direct and indirect effects as observed by columns (1), (2), (5) and (6). The relative importance of direct and indirect effects differs greatly across countries.

indirect channels through which a country-sector can be affected in addition to the direct channels captured in the literature. Moreover, our study contains information for all the services sectors, which are typically embedded in many manufactured goods. For sectors within services, the indirect channels are very important which corresponds to an important source of the high losses found in our analysis.

The sector-level dimension of our analysis solves the potential bias obtained in country-level analysis that omits the sectoral structure of an economy. While we present aggregate results at the country-level, it should be noted that these were obtained by summing the sector-level effects, which gives us the total country-level losses from Brexit. This differs from other studies that do not consider the sector-level dimension (see, for instance, the model in Noguera, 2012).

Our results, however, are very comparable in terms of the *relative* losses. In line with other studies, we find the UK to be hit with value added losses that are three times as high as the EU-27 losses, when normalizing by country size.

Our estimates on value added and job losses do not include potential FDI effects of Brexit. Various papers however, seem to suggest that trade effects account for the main part of the Brexit impact[34]. This is reassuring as it suggests that the main effects of Brexit come through the trade channel which is what we focus on in this paper. Not including the FDI effects, suggests that our estimates are lower bound estimates of the true impact that Brexit may have, which should be kept in mind[35].

[34]See the OECD study by Kierzenkowski et al. (2016), HMTreasury (2016), Dhingra et al. (2017), PwC (2016), Oxford-Economics (2016).

[35]For many multinationals, the UK has been an attractive FDI destination as a way to get access to the EU Single Market given its business-friendly climate (See Dhingra et al. (2016) for a detailed discussion on the impact on FDI). With the UK leaving the EU, part of this foreign investment will likely be (temporarily) suspended, or diverted to the other EU-27 member states. For instance, the UK will be a less attractive export platform once it leaves the Single Market, as different regulation and standards will complicate the coordination between the UK headquarter and the EU-27 branches.

6. Conclusion

In this paper we develop and explicitly solve a trade model with worldwide sector-level input-output linkages in production. The model allows us to separately identify all the channels through which tariff changes operate. This results in a new gravity model that overcomes the limitations of a traditional gravity approach and that derives closed form solutions which allow for comparative statics on tariff changes.

In contrast to a traditional gravity approach that solely rests on direct gross exports between bilateral trade partners, our model includes (1) domestic and global value chain linkages between goods and services sectors, (2) bilateral tariffs that affect direct production for a final destination as well as indirect production (shipped via third countries) to a final destination and (3) value added rather than gross production. Including input-output linkages implies that domestic production of intermediates can serve as inputs in foreign products and subsequently be exported "indirectly" to a final destination. Our input-output model can be taken to the sectoral World Input Output Database (WIOD) and can be used to evaluate trade policy. In this paper, we use the theoretical framework to simulate the impact of different scenarios of Brexit in terms of value added and employment for each of the 28 individual countries involved.

From the sectoral World Input Output (WIOD) database, we obtain Leontief input-output coefficients, value added shares in production and trade flows, which we complement with trade and employment elasticities obtained from the literature. These are all the variables needed to simulate the model's predictions on trade destruction effects resulting from the different scenarios of Brexit.

We consider both a "soft" Brexit, where the UK continues to be part of the single market but faces increased non-tariff barriers (NTBs) (the "Norwegian scenario"), as well as a "hard" Brexit scenario where Most-Favored-Nation (MFN) tariffs between the EU-27 and the UK are put in place. Our model simulations indicate that the UK is hit *relatively* harder than the rest of the EU-27 in both scenarios. In either case, Brexit reduces economic activity in the UK three times more than in the EU-27.

The UK will experience a drop in value added production as a percentage of GDP of 1.21% under a "soft" Brexit and up to 4.47% under a "hard" Brexit scenario. This corresponds to UK job losses of around 140,000 jobs in the "soft" Brexit and ca. 530,000 jobs in the "hard" Brexit scenario.

For the EU-27, the *absolute* job losses are larger, with the numbers of EU-27 jobs lost from Brexit varying between ca. 280,000 and 1,200,000 jobs, which corresponds to value added losses as a percentage of GDP of 0.38% for the "soft" and 1.54% for the "hard" Brexit. The losses in value added and jobs differ substantially across EU-27 member states. EU-27 member states that stand to lose most from Brexit are countries with close historical ties to the UK (e.g. Ireland, Malta) and small open economies on the European continent (e.g. Belgium and the Netherlands).

We find that Brexit hits the UK *relatively* harder than the EU-27. Nonetheless, in contrast to other studies, we find the *absolute* losses for the EU-27 to be substantially higher both in terms of value added and jobs lost.

The Brexit impact varies across countries due to differences in sectoral composition. As shown by Acemoglu et al. (2012), it is the network centrality of sectors that determines the impact of an aggregate shock through a "cascade effect" in the input-output network. Under Brexit, we consider different tariffs across sectors, and therefore the propagation of tariff shocks differs depending on the sectoral composition of the economy. A sector that only has a few linkages with other sectors may not affect aggregate output much even when it is subject to high tariffs, as opposed to a sector that is very central in the production network. Our results take the network centrality and the number of sectoral production linkages into account when estimating the loss in value added and jobs caused by Brexit.

Our findings indicate that there are no winners from Brexit, but only losers. Both parties involved would suffer substantial losses if denied free trade access to each other's market. However, while the current belief surrounding Brexit is that especially the UK has a great deal to lose, our sector-level input-output approach clearly shows that the EU-27 also stands to lose substantially and considerably more than previously thought. The reason is that EU-27 production networks are closely integrated, which implies that tariff changes with the UK do not just affect direct trade bilateral flows but also indirect trade flows via third countries. These indirect effects are estimated to be very important (typically amounting to ca. one third of the direct effects), which substantially reinforces the trade destruction effects of Brexit.

7. References

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8. Tables

Table 2: Loss in Value Added (VA) as a result of UK protection against the EU-27

Country	Soft Brexit				Hard Brexit			
	Via domestic sectors (million $) (1)	**Via other EU-27 countries** (million $) (2)	**Total loss** (million $) (3)	(% of total VA) (4)	**Via domestic sectors** (million $) (5)	**Via other EU-27 countries** (million $) (6)	**Total loss** (million $) (7)	(% of total VA) (8)
AUT	-554	-370	-924	-0.23%	-2159	-1571	-3730	-0.92%
BEL	-2156	-536	-2692	-0.54%	-8662	-2284	-10946	-2.18%
BGR	-76	-41	-117	-0.22%	-299	-175	-473	-0.90%
CYP	-31	-32	-63	-0.29%	-103	-104	-207	-0.95%
CZE	-615	-277	-893	-0.45%	-2542	-1203	-3745	-1.89%
DEU	-12021	-2305	-14326	-0.40%	-49640	-9851	-59492	-1.64%
DNK	-1076	-190	-1266	-0.40%	-4138	-780	-4918	-1.55%
ESP	-2126	-456	-2582	-0.20%	-9236	-1982	-11219	-0.86%
EST	-40	-22	-63	-0.25%	-148	-88	-236	-0.95%
FIN	-423	-158	-581	-0.23%	-1491	-649	-2140	-0.86%
FRA	-6599	-1166	-7765	-0.29%	-25704	-5093	-30797	-1.16%
GRC	-175	-39	-214	-0.10%	-598	-162	-760	-0.35%
HRV	-59	-29	-88	-0.17%	-208	-122	-330	-0.65%
HUN	-362	-154	-516	-0.41%	-1429	-676	-2105	-1.66%
IRL	-2766	-149	-2915	-1.23%	-12300	-605	-12905	-5.46%
ITA	-4500	-887	-5388	-0.27%	-19436	-3862	-23298	-1.17%
LTU	-111	-36	-147	-0.32%	-459	-151	-611	-1.32%
LUX	-133	-107	-240	-0.39%	-422	-420	-842	-1.38%
LVA	-63	-21	-83	-0.29%	-229	-85	-314	-1.09%
MLT	-142	-6	-149	-1.52%	-434	-25	-460	-4.69%
NLD	-3873	-1238	-5111	-0.62%	-14578	-5047	-19625	-2.37%
POL	-1470	-501	-1972	-0.39%	-5883	-2178	-8060	-1.57%
PRT	-437	-100	-536	-0.25%	-1919	-440	-2359	-1.09%
ROU	-254	-138	-392	-0.21%	-1079	-593	-1672	-0.89%
SVK	-378	-116	-494	-0.51%	-1341	-498	-1839	-1.89%
SVN	-58	-48	-107	-0.24%	-221	-206	-427	-0.94%
SWE	-1228	-378	-1605	-0.30%	-4487	-1569	-6056	-1.14%
EU-27			**-51227**	**-0.35%**			**-209567**	**-1.44%**
GBR		**-1484**	**-1484**	**-0.05%**		**-6337**	**-6337**	**-0.23%**

Note: See the Appendix for a list of the country name abbreviations.

Table 3: Loss in Employment (EMP) as a result of UK protection against the EU-27

Country	Soft Brexit				Hard Brexit			
	Via domestic sectors	**Via other EU-27 countries**	**Total loss**		**Via domestic sectors**	**Via other EU-27 countries**	**Total loss**	
	(1000 pers)	(1000 pers)	(1000 pers)	(% of total EMP)	(1000 pers)	(1000 pers)	(1000 pers)	(% of total EMP)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
AUT	-2.36	-1.48	-3.84	-0.09%	-9.52	-6.38	-15.91	-0.37%
BEL	-7.61	-1.77	-9.39	-0.21%	-32.02	-7.65	-39.66	-0.87%
BGR	-2.43	-1.33	-3.75	-0.11%	-10.53	-6.25	-16.78	-0.49%
CYP	-0.17	-0.16	-0.33	-0.09%	-0.63	-0.52	-1.15	-0.32%
CZE	-7.26	-3.21	-10.47	-0.20%	-30.60	-14.00	-44.61	-0.87%
DEU	-54.07	-10.49	-64.56	-0.15%	-228.23	-45.40	-273.63	-0.64%
DNK	-3.34	-0.51	-3.85	-0.14%	-13.69	-2.23	-15.91	-0.57%
ESP	-12.50	-2.46	-14.96	-0.08%	-56.00	-10.82	-66.83	-0.37%
EST	-0.42	-0.21	-0.63	-0.10%	-1.67	-0.84	-2.51	-0.41%
FIN	-1.58	-0.62	-2.20	-0.09%	-5.72	-2.58	-8.31	-0.33%
FRA	-27.38	-4.73	-32.11	-0.12%	-110.88	-21.05	-131.93	-0.48%
GRC	-1.06	-0.27	-1.33	-0.03%	-4.00	-1.19	-5.19	-0.13%
HRV	-0.77	-0.41	-1.19	-0.08%	-2.88	-1.77	-4.65	-0.30%
HUN	-4.83	-1.98	-6.82	-0.16%	-20.09	-8.80	-28.90	-0.69%
IRL	-10.24	-0.40	-10.64	-0.55%	-45.46	-1.71	-47.17	-2.43%
ITA	-24.82	-4.74	-29.55	-0.12%	-111.55	-20.85	-132.40	-0.54%
LTU	-1.17	-0.39	-1.55	-0.12%	-5.32	-1.73	-7.05	-0.54%
LUX	-0.22	-0.20	-0.42	-0.12%	-0.70	-0.79	-1.50	-0.42%
LVA	-0.09	-0.03	-0.12	-0.03%	-0.29	-0.11	-0.40	-0.10%
MLT	-0.51	-0.03	-0.53	-0.37%	-1.57	-0.10	-1.68	-1.16%
NLD	-13.19	-3.92	-17.12	-0.20%	-51.15	-16.30	-67.45	-0.77%
POL	-20.33	-6.43	-26.76	-0.17%	-87.56	-28.64	-116.20	-0.74%
PRT	-5.06	-0.96	-6.02	-0.13%	-24.09	-4.43	-28.52	-0.63%
ROU	-6.10	-2.81	-8.91	-0.10%	-28.59	-12.83	-41.43	-0.48%
SVK	-2.65	-1.14	-3.80	-0.17%	-10.03	-4.95	-14.98	-0.67%
SVN	-0.51	-0.44	-0.96	-0.10%	-2.02	-1.90	-3.92	-0.42%
SWE	-3.57	-1.14	-4.71	-0.11%	-13.61	-4.80	-18.42	-0.41%
EU-27			**-266.50**	**-0.14%**			**-1137.06**	**-0.58%**
GBR		**-6.02**	**-6.02**	**-0.02%**		**-26.09**	**-26.09**	**-0.08%**

Note: See the Appendix for a list of the country name abbreviations.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 4: Most affected sector across countries: UK protection against the EU-27 (Hard Brexit Scenario)

Country	Sector Nace Rev.2			
	Value Added (VA)		**Employment (EMP)**	
	(1)	(2)	(3)	(4)
AUT	Machinery & Equipment	C28	Metal products	C25
BEL	Food Product	C10-C12	Food Product	C10-C12
BGR	Textiles	C13-C15	Live Animals	A01
CYP	Financial Services	K64	Administrative and support act.	N
CZE	Electronics and Computers	C26	Metal products	C25
DEU	Motor vehicles	C29	Machinery & Equipment	C28
DNK	Food Product	C10-C12	Food Product	C10-C12
ESP	Food Product	C10-C12	Live Animals	A01
EST	Wood and Cork	C16	Textiles	C13-C15
FIN	Paper Products	C17	Machinery & Equipment	C28
FRA	Administrative and support act.	N	Administrative and support act.	N
GBR	Wholesale trade	G46	Administrative and support act.	N
GRC	Water transport	H50	Live Animals	A01
HRV	Other services	R_S	Metal products	C25
HUN	Electronics and Computers	C26	Electronics and Computers	C26
IRL	Food Product	C10-C12	Live Animals	A01
ITA	Textiles	C13-C15	Textiles	C13-C15
LTU	Petroleum Products	C19	Textiles	C13-C15
LUX	Financial Services	K64	Administrative and support act.	N
LVA	Wood and Cork	C16	Administrative and support act.	N
MLT	Other services	R_S	Other services	R_S
NLD	Wholesale trade	G46	Administrative and support act.	N
POL	Wholesale trade	G46	Live Animals	A01
PRT	Textiles	C13-C15	Textiles	C13-C15
ROU	Textiles	C13-C15	Textiles	C13-C15
SVK	Real Estate	L68	Metal products	C25
SVN	Metal products	C25	Metal products	C25
SWE	Machinery & Equipment	C28	Machinery & Equipment	C28

Note: See the Appendix for a list of the country name abbreviations and sector codes.

Note: The most affected sector can differ depending on whether we look in terms of value added or employment. The reason is that the value added contribution per worker can differ dramatically across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 5: Loss in Value Added (VA) as a result of EU-27 protection against the UK

Country	**via GBR**			
	Soft Brexit		**Hard Brexit**	
	(million $) (1)	(% of total VA) (2)	(million $) (3)	(% of total VA) (4)
AUT	-71	-0.017%	-286	-0.071%
BEL	-207	-0.041%	-836	-0.167%
BGR	-10	-0.018%	-38	-0.072%
CYP	-4	-0.018%	-15	-0.068%
CZE	-60	-0.030%	-240	-0.121%
DEU	-1038	-0.029%	-4207	-0.116%
DNK	-96	-0.030%	-365	-0.115%
ESP	-168	-0.013%	-683	-0.052%
EST	-5	-0.022%	-21	-0.085%
FIN	-52	-0.021%	-208	-0.084%
FRA	-611	-0.023%	-2392	-0.090%
GRC	-19	-0.009%	-72	-0.033%
HRV	-6	-0.012%	-25	-0.048%
HUN	-38	-0.030%	-151	-0.119%
IRL	-162	-0.069%	-670	-0.283%
ITA	-326	-0.016%	-1301	-0.065%
LTU	-10	-0.022%	-42	-0.092%
LUX	-20	-0.033%	-77	-0.126%
LVA	-7	-0.026%	-29	-0.099%
MLT	-4	-0.045%	-16	-0.166%
NLD	-492	-0.059%	-1897	-0.229%
POL	-138	-0.027%	-557	-0.109%
PRT	-34	-0.016%	-135	-0.063%
ROU	-26	-0.014%	-104	-0.055%
SVK	-26	-0.027%	-100	-0.103%
SVN	-8	-0.019%	-34	-0.075%
SWE	-136	-0.026%	-540	-0.101%
EU-27	**-3777**	**-0.026%**	**-15042**	**-0.103%**
GBR	**-32528**	**-1.159%**	**-119161**	**-4.246%**

Note: See the Appendix for a list of the country name abbreviations.

Table 6: Loss in Employment (EMP) as a result of EU-27 protection against the UK

Country	**via GBR**			
	Soft Brexit		**Hard Brexit**	
	(1000 pers) (1)	(% of total EMP) (2)	(1000 pers) (3)	(% of total EMP) (4)
AUT	-0.27	-0.006%	-1.11	-0.03%
BEL	-0.67	-0.015%	-2.72	-0.06%
BGR	-0.27	-0.008%	-1.11	-0.03%
CYP	-0.02	-0.005%	-0.07	-0.02%
CZE	-0.67	-0.013%	-2.70	-0.05%
DEU	-4.50	-0.011%	-18.29	-0.04%
DNK	-0.26	-0.009%	-0.99	-0.04%
ESP	-0.88	-0.005%	-3.58	-0.02%
EST	-0.05	-0.009%	-0.20	-0.03%
FIN	-0.19	-0.008%	-0.77	-0.03%
FRA	-2.40	-0.009%	-9.39	-0.03%
GRC	-0.10	-0.002%	-0.38	-0.01%
HRV	-0.08	-0.005%	-0.32	-0.02%
HUN	-0.46	-0.011%	-1.85	-0.04%
IRL	-0.68	-0.035%	-3.17	-0.16%
ITA	-1.67	-0.007%	-6.74	-0.03%
LTU	-0.09	-0.007%	-0.38	-0.03%
LUX	-0.04	-0.010%	-0.14	-0.04%
LVA	-0.01	-0.003%	-0.04	-0.01%
MLT	-0.02	-0.012%	-0.07	-0.05%
NLD	-1.48	-0.017%	-5.75	-0.07%
POL	-1.66	-0.011%	-6.75	-0.04%
PRT	-0.30	-0.007%	-1.20	-0.03%
ROU	-0.48	-0.006%	-2.00	-0.02%
SVK	-0.21	-0.009%	-0.81	-0.04%
SVN	-0.07	-0.008%	-0.30	-0.03%
SWE	-0.39	-0.009%	-1.55	-0.03%
EU-27	-17.94	-0.009%	-72.41	-0.04%
GBR	-133.85	-0.435%	-500.74	-1.63%

Note: See the Appendix for a list of the country name abbreviations.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 7: Most affected sector across countries: EU-27 protection against the UK (Hard Brexit Scenario)

Country	**Sector Nace Rev.2**			
	Value Added (VA)		**Employment (EMP)**	
	(1)	(2)	(3)	(4)
AUT	Wholesale trade	G46	Metal products	C25
BEL	Chemicals	C20	Administrative and support act.	N
BGR	Wholesale trade	G46	Live Animals	A01
CYP	Financial Services	K64	Administrative and support act.	N
CZE	Motor vehicles	C29	Metal products	C25
DEU	Chemicals	C20	Administrative and support act.	N
DNK	Mining and quarrying	B	Administrative and support act.	N
ESP	Chemicals	C20	Administrative and support act.	N
EST	Wood and Cork	C16	Metal products	C25
FIN	Paper Products	C17	Paper Products	C17
FRA	Administrative and support act.	N	Administrative and support act.	N
GBR	Administrative and support act.	N	Administrative and support act.	N
GRC	Water transport	H50	Wholesale trade	G46
HRV	Wholesale trade	G46	Metal products	C25
HUN	Motor vehicles	C29	Motor vehicles	C29
IRL	Live Animals	A01	Live Animals	A01
ITA	Administrative and support act.	N	Administrative and support act.	N
LTU	Chemicals	C20	Live Animals	A01
LUX	Financial Services	K64	Administrative and support act.	N
LVA	Wood and Cork	C16	Administrative and support act.	N
MLT	Financial Services	K64	Financial Services	K64
NLD	Wholesale trade	G46	Administrative and support act.	N
POL	Wholesale trade	G46	Retail trade	G47
PRT	Wholesale trade	G46	Administrative and support act.	N
ROU	Land & Pipeline transport	H49	Live Animals	A01
SVK	Real Estate	L68	Metal products	C25
SVN	Metal products	C25	Metal products	C25
SWE	Chemicals	C20	Administrative and support act.	N

Note: See the Appendix for a list of the country name abbreviations and sector codes.

Note: The most affected sector can differ depending on whether we look in terms of value added or employment. The reason is that the value added contribution per worker can differ dramatically across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 8: Total loss in Value Added from Brexit

Country	Soft Brexit		Hard Brexit	
	(million $)	(% of total VA)	(million $)	(% of total VA)
	(1)	(2)	(3)	(4)
AUT	-995	-0.25%	-4016	-0.99%
BEL	-2899	-0.58%	-11782	-2.35%
BGR	-127	-0.24%	-512	-0.97%
CYP	-67	-0.31%	-222	-1.02%
CZE	-952	-0.48%	-3985	-2.01%
DEU	-15364	-0.42%	-63699	-1.76%
DNK	-1362	-0.43%	-5283	-1.67%
ESP	-2749	-0.21%	-11902	-0.91%
EST	-68	-0.28%	-257	-1.04%
FIN	-633	-0.25%	-2348	-0.95%
FRA	-8376	-0.32%	-33190	-1.25%
GRC	-233	-0.11%	-831	-0.38%
HRV	-94	-0.18%	-355	-0.69%
HUN	-554	-0.44%	-2256	-1.78%
IRL	-3077	-1.30%	-13575	-5.74%
ITA	-5713	-0.29%	-24599	-1.23%
LTU	-157	-0.34%	-653	-1.42%
LUX	-260	-0.43%	-919	-1.51%
LVA	-91	-0.31%	-343	-1.19%
MLT	-153	-1.56%	-476	-4.86%
NLD	-5604	-0.68%	-21523	-2.59%
POL	-2110	-0.41%	-8618	-1.68%
PRT	-570	-0.26%	-2494	-1.16%
ROU	-418	-0.22%	-1775	-0.95%
SVK	-520	-0.53%	-1939	-1.99%
SVN	-115	-0.25%	-461	-1.02%
SWE	-1742	-0.33%	-6596	-1.24%
EU-27	**-55004**	**-0.38%**	**-224609**	**-1.54%**
GBR	**-34012**	**-1.21%**	**-125497**	**-4.47%**

Note: See the Appendix for a list of the country name abbreviations.

Table 9: Total loss in Employment from Brexit

Country	Soft Brexit		Hard Brexit	
	(1000 pers) (1)	(% of total EMP) (2)	(1000 pers) (3)	(% of total EMP) (4)
AUT	-4.12	-0.10%	-17.02	-0.40%
BEL	-10.06	-0.22%	-42.39	-0.93%
BGR	-4.02	-0.12%	-17.89	-0.52%
CYP	-0.35	-0.10%	-1.22	-0.34%
CZE	-11.14	-0.22%	-47.31	-0.93%
DEU	-69.06	-0.16%	-291.93	-0.68%
DNK	-4.11	-0.15%	-16.90	-0.61%
ESP	-15.84	-0.09%	-70.41	-0.39%
EST	-0.69	-0.11%	-2.71	-0.45%
FIN	-2.39	-0.10%	-9.08	-0.36%
FRA	-34.50	-0.13%	-141.32	-0.52%
GRC	-1.42	-0.04%	-5.57	-0.14%
HRV	-1.27	-0.08%	-4.97	-0.32%
HUN	-7.28	-0.17%	-30.75	-0.73%
IRL	-11.32	-0.58%	-50.33	-2.59%
ITA	-31.23	-0.13%	-139.14	-0.57%
LTU	-1.64	-0.12%	-7.43	-0.56%
LUX	-0.45	-0.13%	-1.63	-0.46%
LVA	-0.13	-0.03%	-0.44	-0.11%
MLT	-0.55	-0.38%	-1.75	-1.21%
NLD	-18.60	-0.21%	-73.20	-0.84%
POL	-28.42	-0.18%	-122.95	-0.78%
PRT	-6.32	-0.14%	-29.72	-0.66%
ROU	-9.39	-0.11%	-43.43	-0.50%
SVK	-4.00	-0.18%	-15.79	-0.71%
SVN	-1.03	-0.11%	-4.22	-0.45%
SWE	-5.10	-0.11%	-19.97	-0.45%
EU-27	**-284.44**	**-0.15%**	**-1209.47**	**-0.62%**
GBR	**-139.86**	**-0.45%**	**-526.83**	**-1.71%**

Note: See the Appendix for a list of the country name abbreviations.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

Table 10: Most affected sector across countries: Brexit ("Hard" Brexit Scenario)

Country	Sector Nace Rev.2			
	Value Added (VA)		**Employment (EMP)**	
	(1)	(2)	(3)	(4)
AUT	Machinery & Equipment	C28	Metal products	C25
BEL	Food Product	C10-C12	Food Product	C10-C12
BGR	Textiles	C13-C15	Live Animals	A01
CYP	Financial Services	K64	Administrative and support act.	N
CZE	Electronics and Computers	C26	Metal products	C25
DEU	Motor vehicles	C29	Machinery & Equipment	C28
DNK	Mining and quarrying	B	Food Product	C10-C12
ESP	Food Product	C10-C12	Live Animals	A01
EST	Wood and Cork	C16	Wood and Cork	C16
FIN	Paper Products	C17	Administrative and support act.	N
FRA	Administrative and support act.	N	Administrative and support act.	N
GBR	Administrative and support act.	N	Administrative and support act.	N
GRC	Water transport	H50	Live Animals	A01
HRV	Other services	R_S	Metal products	C25
HUN	Electronics and Computers	C26	Electronics and Computers	C26
IRL	Food Product	C10-C12	Live Animals	A01
ITA	Textiles	C13-C15	Textiles	C13-C15
LTU	Petroleum Products	C19	Textiles	C13-C15
LUX	Financial Services	K64	Administrative and support act.	N
LVA	Wood and Cork	C16	Administrative and support act.	N
MLT	Other services	R_S	Other services	R_S
NLD	Wholesale trade	G46	Administrative and support act.	N
POL	Wholesale trade	G46	Live Animals	A01
PRT	Textiles	C13-C15	Textiles	C13-C15
ROU	Textiles	C13-C15	Textiles	C13-C15
SVK	Real Estate	L68	Metal products	C25
SVN	Metal products	C25	Metal products	C25
SWE	Petroleum Products	C19	Machinery & Equipment	C28

Note: See the Appendix for a list of the country name abbreviations and sector codes.

Note: The most affected sector can differ depending on whether we look in terms of value added or employment. The reason is that the value added contribution per worker can differ dramatically across sectors, which means that the same drop in value added might lead to different employment effects in different sectors.

Note: Employment data in Eurostat is missing for some sectors in the following countries: Estonia, Latvia, Lithuania, Luxembourg, Malta and Sweden. Therefore, the presented employment results for these countries will likely underestimate the true impact.

9. Appendix

9.1. Abbreviations

Table 11: Countries and ISO-3 Codes

Country Name	**Code (ISO-3)**	**Country Name**	**Code (ISO-3)**
Austria	AUT	Hungary	HUN
Belgium	BEL	Ireland	IRL
Bulgaria	BGR	Italy	ITA
Cyprus	CYP	Lithuania	LTU
Czech Republic	CZE	Luxembourg	LUX
Germany	DEU	Latvia	LVA
Denmark	DNK	Malta	MLT
Spain	ESP	Netherlands	NLD
Estonia	EST	Poland	POL
Finland	FIN	Portugal	PRT
France	FRA	Romania	ROU
United Kingdom	GBR	Slovakia	SVK
Greece	GRC	Slovenia	SVN
Croatia	HRV	Sweden	SWE

Table 12: Nace Rev. 2 Codes & Labels

Goods		Services	
Nace Rev.2	**Sector Legend (Short)**	**Nace Rev.2**	**Sector Legend (Short)**
A01	Live Animals	F	Construction
A02	Forestry	G45	Wholesale and retail trade
A03	Fishing	G46	Wholesale trade
B	Mining and quarrying	G47	Retail trade
C10-C12	Food Product	H49	Land & Pipeline transport
C13-C15	Textiles	H50	Water transport
C16	Wood and Cork	H51	Air transport
C17	Paper Products	H52	Warehousing
C18	Printing and Media	H53	Postal
C19	Petroleum Products	I	Accommodation & Food serv.
C20	Chemicals	J58	Publishing Act.
C21	Pharmaceutical	J59_J60	Media Production
C22	Rubber and Plastic	J61	Telecom
C23	Other Non-metallic mineral	J62_J63	Computer Programming, consultancy
C24	Basic Metals	K64	Financial Services
C25	Metal products	K65	Insurance
C26	Electronics and Computers	K66	Auxiliary Financial Serv.
C27	Electrical Equipment	L68	Real Estate
C28	Machinery & Equipment	M69_M70	Legal and Accounting
C29	Motor vehicles	M71	Architectural and engineering act.
C30	Transport equipment	M72	Scientific Research
C31_C32	Furniture & other manufac.	M73	Advertising and market research
C33	Installation of machinery	M74_M75	Other professional activities
D35	Electricity & Gas	N	Administrative and support act.
E36	Water Collection Activities	O84	Public admin and defence
E37-E39	Waste Collection Activities	P85	Education
		Q	Health
		R_S	Other services

9.2. Derivations

Equation (17) can be found as follows. From Equation (16), we find $\mathrm{d}va^{kz}$ as

$$\mathrm{d}va^{kz} = \underbrace{v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S} L_{is}^{kz}\sum_{j=1}^{N}\mathrm{d}f_j^{is}}_{\text{final trade effect}} + \underbrace{v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}\mathrm{d}L_{is}^{kz}\sum_{j=1}^{N} f_j^{is}}_{\text{intermediate trade effect}} \tag{20}$$

Next, we apply the following rule to Equation (20): Differentiating $L^{-1}L = \mathbb{I}$ yields $L^{-1}\mathrm{d}L + \mathrm{d}L^{-1}L = 0$ from which it follows that $\mathrm{d}L = -L\mathrm{d}L^{-1}L$. Given that $L = [\mathbb{I} - A]^{-1}$, we have that $\mathrm{d}L^{-1} = -\mathrm{d}A$ and hence $\mathrm{d}L = L\mathrm{d}AL$, from which it is straightforward to obtain the individual elements $\mathrm{d}L_{is}^{kz}$. Hence, we obtain

$$\begin{aligned}
\mathrm{d}va^{kz} &= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is} f_j^{is} + \sum_{h=1}^{N}\sum_{r=1}^{S}\sum_{h'=1}^{N}\sum_{r'=1}^{S}(1-\sigma_r)L_{hr}^{kz}a_{h'r'}^{hr}\hat{\tau}_{h'}^{hr}\sum_{i=1}^{N}\sum_{s=1}^{S}L_{is}^{h'r'}\sum_{j=1}^{N} f_j^{is} \\
&= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is} f_j^{is} + \sum_{h=1}^{N}\sum_{r=1}^{S}\sum_{h'=1}^{N}\sum_{r'=1}^{S}(1-\sigma_r)L_{hr}^{kz}a_{h'r'}^{hr}y^{h'r'}\hat{\tau}_{h'}^{hr} \\
&= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is} f_j^{is} + \sum_{h=1}^{N}\sum_{r=1}^{S}\sum_{h'=1}^{N}\sum_{r'=1}^{S}(1-\sigma_r)L_{hr}^{kz}x_{h'r'}^{hr}\hat{\tau}_{h'}^{hr} \\
&= v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(1-\sigma_s)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is} f_j^{is} + \sum_{i=1}^{N}\sum_{s=1}^{S}\sum_{j=1}^{N}\sum_{r=1}^{S}(1-\sigma_s)L_{is}^{kz}x_{jr}^{is}\hat{\tau}_j^{is} \\
&= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}\left\{f_j^{is} + \sum_{r=1}^{S}x_{jr}^{is}\right\} \\
&= -v^{kz}\sum_{i=1}^{N}\sum_{s=1}^{S}(\sigma_s-1)L_{is}^{kz}\sum_{j=1}^{N}\hat{\tau}_j^{is}e_j^{is}
\end{aligned}$$

where we defined $\hat{\tau}_j^{is} \equiv \frac{\mathrm{d}\tau_j^{is}}{\tau_j^{is}} - \frac{\mathrm{d}\Pi^{is}}{\Pi^{is}} - \frac{\mathrm{d}P_j^s}{P_j^s}$ as the proportionate change in τ_j^{is} net of the proportionate changes in the multilateral resistance (MR) terms.