Does One Soros Make a Difference?

A Theory of Currency Crises with Large and Small Traders

By

Giancarlo Corsetti, Amil Dasgupta,
Stephen Morris and Hyun Song Shin

DISCUSSION PAPER 372

March 2001

FINANCIAL MARKETS GROUP
AN ESRC RESEARCH CENTRE

LONDON SCHOOL OF ECONOMICS

E·S·R·C
ECONOMIC & SOCIAL RESEARCH COUNCIL

ISSN 0956-8549-372

Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders*

Giancarlo Corsetti
Bologna and Yale University

Amil Dasgupta
Yale University

Stephen Morris
Yale University

Hyun Song Shin
Oxford University

First draft: March 1999
This revision: August 2000

Abstract

Do large investors increase the vulnerability of a country to speculative attacks in the foreign exchange markets? To address this issue, we build a model of currency crises where a single large investor and a continuum of small investors independently decide whether to attack a currency based on their private information about fundamentals. Even abstracting from signalling, the presence of the large investor does make all other traders more aggressive in their selling. Relative to the case in which there is no large investors, small investors attack the currency when fundamentals are stronger. Yet, the difference can be small, or null, depending on the relative precision of private information of the small and large investors. Adding signalling makes the influence of the large trader on small traders' behaviour much stronger.

JEL F31, D82

*We are grateful for comments from Richard Clarida, Hélène Rey, Andres Velasco, Shang-Jin Wei and seminar participants at the Bank of Italy, the Board of Governors of the Federal Reserve Bank, Columbia University, Harvard University, the NBER IFM meeting, and the New York Federal Reserve Bank. Corsetti, Dasgupta and Morris: Department of Economics, Yale University, 28 Hillhouse Av., New Haven CT 06520, USA, corsetti@yale.edu, amil.dasgupta@yale.edu, stephen.morris@yale.edu. Shin: Nuffield College, Oxford, OX1 1NF, hyun.shin@nuf.ox.ac.uk

1. Introduction

A commonly encountered view among both seasoned market commentators and less experienced observers of the financial markets is that large traders can exercise a disproportionate influence on the likelihood and severity of a financial crisis by fermenting and orchestrating attacks against weakened currency pegs. The famously acrimonious exchange between the financier George Soros and Dr. Mahathir, the prime minister of Malaysia at the height of the Asian crisis is a prominent example in which such views have been aired and debated. The issues raised by this debate are complex, but they deserve systematic investigation.

At one level, the task is one of dissecting the problem in search of the possible mechanisms (if any) that may be at play in which a large trader may exercise such influence on the market outcome. What is it about the large trader that bestows such influence? Is it merely that this trader can bring to bear larger resources and hence take on larger trading positions? What if the information of the large trader is no better than the small traders in the market? Does the large trader still exercise a disproportionate influence? Finally, does it make a difference to the market outcome as to whether the trading position of the large trader is disclosed publicly to the market? If so, does such "transparency" of the trading position enhance financial stability or undermine it? This last question is especially important given the emphasis placed by policy makers on the public disclosures by the major market participants as a way of forestalling future crises.[1]

We propose to investigate these issues in a model of speculative attacks in which a large trader interacts with a continuum of small traders. The large trader is 'large' by virtue of the size of the speculative position that he can take on as compared to the small traders. The two types of traders face a monetary authority defending a currency peg, and stand to gain if their attack on the peg is successful, but stand to lose if the attack fails to break the peg. Both types of traders are well informed about the underlying fundamentals, but they are not perfectly informed. Moreover, we allow the possibility that the information precision of one type of trader is higher than another. We can examine the case in which the large trader is better informed than the small trader and contrast this with the case in which small trader is relatively better informed.

To anticipate our main conclusions to these questions, we can summarize our findings as follows.

[1] The response of the regulators and official bodies to the financial turbulence of 1998 has been to call for greater public disclosures by banks and hedge funds. The recent document from the Financial Stability Forum (2000) reiterates similar calls by the BIS, IOSCO, and the President's Working Group. In contrast, the private sector is more ambivalent towards the value of public disclosures. See, for instance, Counterparty Risk Management Policy Group (1999).

- As a general rule, the presence of the large trader *does* increase the incidence of attack against a peg. The reason is not so much that the large trader's market power manufactures these crises, but rather that the presence of the large trader makes the small traders more aggressive in their trading strategies. In other words, the large trader injects a degree of strategic fragility to the market.

- However, within this broad general finding, the relative precision of information between the two types of traders matters.

  - When a typical small trader is better informed than the large trader, the influence of the latter on the market is moderate. His presence can make little or no difference on small traders' strategies.

  - But when the large trader is better informed than a typical small trader, his influence is much larger.

- Finally, the influence of the large trader is magnified greatly when the large trader's trading position is revealed to the small traders prior to their trading decisions. Thus, when the large trader moves first, and his position is disclosed publicly to other traders before their trading decisions, the impact of the large trader is that much larger. The reason for this added impact lies in the signalling potential of the large trader's first move. To the extent that a speculative attack is the resolution of a coordination problem among the traders, the enhanced opportunity to orchestrate a coordinated attack helps to resolve this collective action problem.

The technical and modelling innovations necessary to reach our main conclusions deserve some attention by itself, and it is to these that we now turn. The theoretical framework employed in this paper is an extension of the incomplete information game formulation used in Morris and Shin (1998). In this earlier setting, the argument makes heavy use of the fact that the game is *symmetric* - that is, all the speculators are identical. This assumption is clearly not available to us in the current setting. It is not at all obvious that the argument used in Morris and Shin (1998) to prove uniqueness of equilibrium is applicable in asymmetric payoffs settings, and one of the contributions of our current paper is to demonstrate that this argument can be used with some modifications.

There is a more subtle, but important theoretical contribution. The incomplete information game approach of Morris and Shin (1998) is an instance of a more general approach to equilibrium selection pioneered by Carlsson and van Damme (1993), in which the type space underlying the game is generated by adding a small amount of noise in the signals of the players concerning some payoff relevant state. Carlsson and van Damme refer to such games as "global games", and the general class of such games turn out to have a rich and interesting structure. Morris and Shin (2000) discuss some general results and applications. Analysis using global games should be seen as a particular instance of equilibrium selection through perturbations, but it is important to disentangle two distinct sets of results concerning global games. The first question is whether a unique outcome is selected in the game. A second, more subtle, question is whether such a unique outcome depends on the underlying information structure and the structure of the noise in the players' signals. One of the remarkable results for *symmetric* binary action global games is that the answer to the second question is 'no'. In other words, not only is a unique equilibrium selected in the limit as the noise becomes small, but the selected equilibrium is insensitive to the structure of the noise (see Morris and Shin (2000) section 2). However, our second bullet point above points to the fact that, in our model, the structure of the noise *does* make a difference. The equilibrium outcome depends on whether the large trader is relatively better or worse informed as compared to the small traders. Thus, in our *asymmetric* global game, although we have a unique equilibrium being selected, this unique equilibrium depends on the noise structure. It is this latter feature that allows us to draw non-trivial conclusions concerning the economic importance of information. Frankel, Morris and Pauzner (1999) explore the equilibrium selection question in the context of general global games.

Our examination of the sequential move version of the game necessitates a further extension the current state of the art. When moves occur sequentially in which the actions of the early movers are observable to the late movers, herding and signalling effects must be taken into consideration, as well as the usual strategic complementarities. Although a general analysis of sequential move variations of global games is rather intractable, the fact that small traders (individually) are of measure zero in our model allows us to focus attention on the signalling effects of the large trader. This simplifies the analysis sufficiently for us to derive explicit closed form solutions to the game.[2]

The paper is organized as follows. Section 2 lays out the basic framework and establishes two benchmark results in setting the stage for the general analysis. Section 3 characterizes the unique equilibrium in a simultaneous move trading game. Section 4 explores the comparative statics properties of the equilibrium to changes in the traders' information precision. The focus here is on the interaction between the size of the large trader and his information precision. Section 5 investigates the sequential move version of the game. Section 6 concludes.

[2]Dasgupta (1999) has examined some of the issues that arise with many large players and multi-period signalling.

2. The model

The focus of our analysis is on the mechanism by which a fixed exchange parity is abandoned as a result of a speculative attack on the currency. Consider an economy where the central bank pegs the exchange rate. There is a single "large" trader and a continuum of "small" traders. The distinguishing feature of the large trader is that he has access to a sufficiently large line of credit in the domestic currency to take a short position up to the limit of $\lambda < 1$. In contrast, the set of all small traders taken together have a combined trading limit of $1 - \lambda$.

We envisage the short selling as consisting of borrowing the domestic currency and selling it for dollars. There is a cost to engaging in the short selling, denoted by $t > 0$. The cost t can be viewed largely as consisting of the interest rate differential between the domestic currency and dollars, plus transaction costs. This cost is normalized relative to the other payoffs in the game, so that the payoff to a successful attack on the currency is given by 1, and the payoff from refraining from attack is given by 0. Thus, the net payoff to a successful attack on the currency is $1 - t$, while the payoff to an unsuccessful attack is given by $-t$.

Each trader must decide independently, and (for now) simultaneously whether or not to attack the currency. The strength of the economic fundamentals of this economy are indexed by the random variable θ, which has the (improper) uniform prior over the real line.[3]

Whether the current exchange rate parity is viable depends on the strength of the economic fundamentals and the incidence of speculative attack against the peg. The incidence of speculative attack is measured by the mass of traders attacking the currency in the foreign exchange market. Denoting by ℓ the mass of traders attacking the currency, the currency peg fails if and only if

$$\ell \geq \theta \tag{2.1}$$

So, when fundamentals are sufficiently strong (i.e. $\theta > 1$) the currency peg is maintained irrespective of the actions of the speculators. When $\theta \leq 0$, the peg is abandoned even in the absence of a speculative attack. The interesting range is the intermediate case when $0 < \theta \leq 1$. Here, an attack on the currency will bring down the currency provided that the incidence of attack is large enough, but not otherwise. This tripartite classification of fundamentals follows Obstfeld (1996) and Morris and Shin (1998). Although we do not model explicitly the

[3]Improper priors allow us to concentrate on the updated beliefs of the traders conditional on their signals without taking into account the information contained in the prior distribution. In any case, our results with the improper prior can be seen as the limiting case as the information in the prior density goes to zero. See Hartigan (1983) for a discussion of improper priors, and Morris and Shin (2000, section 2) for a discussion of the latter point.

decision of the monetary authorities to relinquish the peg, it may be helpful to keep in mind the example of an economy endowed with a stock of international reserves, where the central bank is willing to defend the exchange rate as long as reserves do not fall below a predetermined critical level. The central bank predetermines this level based on its assessment of the economic fundamentals of the country. The critical level is low when fundamentals are strong (θ is high): the central bank is willing to use a large amount of (non-borrowed and borrowed) reserves in defending the exchange rate. Conversely, the critical level is high when fundamentals are weak (θ is low). Even a mild speculative attack can convince the central bank to abandon the peg.

2.1. Information

Although the traders do not observe the realization of θ, they receive informative private signals about it. The large trader observes the realization of the random variable

$$y = \theta + \tau\eta \tag{2.2}$$

where $\tau > 0$ is a constant and η is a random variable with mean zero, and with smooth symmetric density $g(\cdot)$. We write $G(\cdot)$ for the cumulative distribution function for $g(\cdot)$. Similarly, a typical small trader i observes

$$x_i = \theta + \sigma\varepsilon_i \tag{2.3}$$

where $\sigma > 0$ is a constant and the individual specific noise ε_i is distributed according to smooth symmetric density $f(\cdot)$ (write $F(\cdot)$ for the c.d.f.) with mean zero. We assume that ε_i is i.i.d. across traders, and each is independent of η.

A feature already familiar from the discussion of global games in the literature is that even if σ and τ become very small, the realization of θ will not be common knowledge among the traders. Upon receiving his signal, the representative trader i can guess the value of θ, and the distribution of signals reaching the other traders in the economy, as well as of their estimate of θ. He cannot, however, count on the other traders to know what he knows – and agree with his guesses. The other traders will have to rely exclusively on their own information to form their beliefs. This departure from the assumption of common knowledge of the fundamentals, no matter how small, is key to the results to follow. The relative magnitude of the constants σ and τ indexes the relative precision of the information of the two types of traders.

A trader's strategy is a rule of action which maps each realization of his signal to one of two actions - to attack, or to refrain. We will search for Bayes Nash equilibria of the game in which, conditional on each trader's signal, the action prescribed by this trader's strategy maximizes his conditional expected payoff when all other traders follow their strategies in the equilibrium.

2.2. Two benchmark Cases

Before proceeding to our main task of solving the game outlined above, we present a brief discussion of the coordination problem under two special cases to set a benchmark for our main results. The first is when all traders are small ($\lambda = 0$), the second is when the sole trader is the large trader himself ($\lambda = 1$).

2.2.1. Small traders only

The case when $\lambda = 0$ takes us into the symmetric game case of Morris and Shin (1998). We will conduct the discussion in terms of switching strategies in which traders attack the currency if the signal falls below a critical value x^*. We will show later that this is without loss of generality, and that there are no other equilibria in possibly more complex strategies. The unique equilibrium can be characterized by a critical value θ^* below which the currency will always collapse, and a critical value of the individual signal x^* such that individuals receiving a signal below this value will always attack. To derive these critical values, note first that, if the true state is θ and traders attack only if they observed a signal below x^*, the probability that any particular trader receives a signal below this level is

$$\text{prob}(x_i \leq x^* \mid \theta) = F\left(\frac{x^* - \theta}{\sigma}\right) \tag{2.4}$$

Since the noise terms $\{\varepsilon_i\}$ are i.i.d., the incidence of attack ℓ is equal to this probability. We know that an attack will be successful only if $\ell \geq \theta$. The critical state θ^* is where this holds with equality. Thus, the first equilibrium condition – a "critical mass condition" – is

$$F\left(\frac{x^* - \theta^*}{\sigma}\right) = \theta^*. \tag{2.5}$$

Figure 2.1 overleaf depicts the incidence of attack as the downward sloping curve $F\left(\frac{x^*-\theta}{\sigma}\right)$. Given x^*, any realization of the fundamental $\theta \leq \theta^*$ is associated with a successful speculative attack on the currency.

Second, consider the optimal trigger strategy for an trader receiving a signal x_i, given θ^*. The trader has the conditional probability of a successful attack of

$$\text{prob}(\theta \leq \theta^* \mid x_i) = F\left(\frac{\theta^* - x_i}{\sigma}\right), \tag{2.6}$$

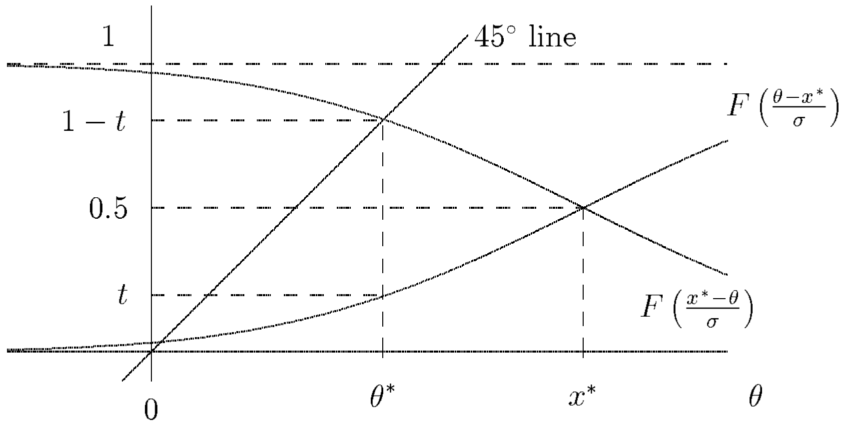

Figure 2.1: Switching equilibrium with small traders only

and hence attacks if and only if his expected gross payoff is at least as high as the cost of attack t. As the expected payoff to attacking for a marginal trader receiving a signal x^* must be 0, the "optimal cutoff" condition for x^* is

$$F\left(\frac{\theta^* - x^*}{\sigma}\right) = t. \tag{2.7}$$

This point x^* is illustrated in figure 2.1. Solving for the equilibrium entails solving the pair of equations above. Equation (2.7) gives $\theta^* = x^* + \sigma F^{-1}(t)$; substituting into (2.5) gives $\theta^* = F(-F^{-1}(t)) = 1 - F(F^{-1}(t)) = 1 - t$. We obtain the following proposition

Proposition 2.1. *If* $\lambda = 0$,

$$\begin{aligned} x^* &= 1 - t - \sigma F^{-1}(t) \\ \theta^* &= 1 - t \end{aligned}$$

The currency will collapse for any realization of the fundamental θ smaller than $1 - t$, while each individual trader will attack the currency for any realization of his signal below $1 - t - \sigma F^{-1}(t)$.[4] Note that this trigger tends to $1 - t$ as $\sigma \to 0$.

2.2.2. A single large trader

We now consider the opposite extreme case of $\lambda = 1$, in which there is a single large trader. This reduces the game to a single person decision problem, and implies a

[4]For $t < 1/2$, $F^{-1}(t)$ is a negative number, so that $x^* > \theta^*$. As $\sigma \to 0$, i.e. letting the private signal become arbitrarily precise, the optimal cutoff point will tend to the fundamental threshold, $x^* \to \theta^*$.

trivial solution to the coordination problem described above. As this single trader controls the market, there is no need of an equilibrium condition equivalent to the "critical mass condition" (2.5). The only condition that is relevant for a single large risk-neutral trader is the "optimal cutoff": he will attack the currency if and only the expected payoff from a speculative position is non-negative, that is when

$$G\left(\frac{1-y}{\tau}\right) \geq t$$

Thus he attacks if and only if $y \leq y^* = 1 - \tau G^{-1}(t)$. Note that the trigger y is smaller than one, but tends to 1 as $\tau \to 0$.

3. Equilibrium with Small and Large Traders

We can now turn to the general case when there are both small and large traders. We will show that there is a unique, dominance solvable equilibrium in this case in which both types of traders follow their respective trigger strategies around the critical points x^* and y^*. The argument will be presented in two steps. We will first confine our attention to solving for an equilibrium in trigger strategies, and then proceed to show that this solution can be obtained by the iterated deletion of strictly interim dominated strategies.

3.1. Equilibrium in Trigger Strategies

Thus, as the first step let us suppose that the small traders follow the trigger strategy around x^*. Because there is a continuum of small traders, conditional on θ, there is no aggregate uncertainty about the proportion of small traders attacking the currency. Since $F\left(\frac{x^*-\theta}{\sigma}\right)$ is the proportion of small traders observing a signal lower than x^* and therefore attacking at θ, an attack by small traders alone is sufficient to break the peg at θ if $(1-\lambda) F\left(\frac{x^*-\theta}{\sigma}\right) \geq \theta$. From this, we can define a level of fundamentals below which an attack by the small traders alone is sufficient to break the peg. Let $\underline{\theta}$ be defined by:

$$(1-\lambda) F\left(\frac{x^*-\underline{\theta}}{\sigma}\right) = \underline{\theta} \tag{3.1}$$

Whenever θ is below $\underline{\theta}$, the attack is successful irrespective of the action of the large trader. Figure 3.1 depicts the derivation of this critical level. Note that $\underline{\theta}$ lies between 0 and $1-\lambda$. Clearly $\underline{\theta}$ is a function of x^*.

Next, we can consider the additional speculative pressure brought by the large trader. If the small traders follow the trigger strategy around x^*, the incidence

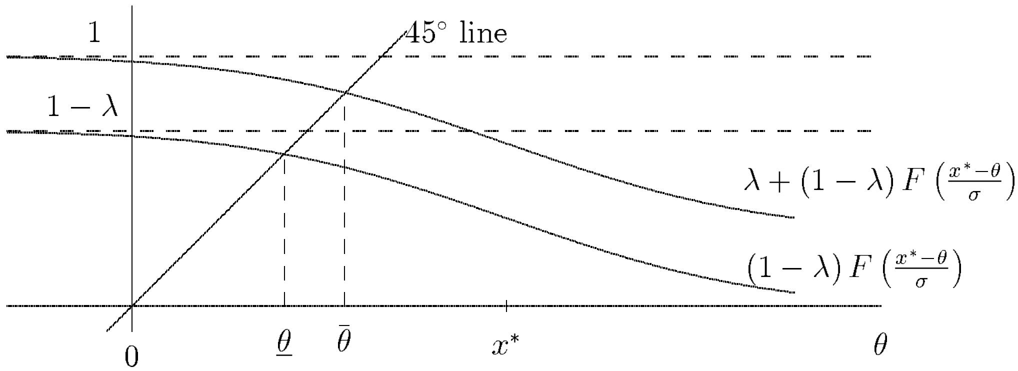

Figure 3.1: Incidence of attack at θ with both types of traders

of attack at θ attributable to the small traders is $(1-\lambda) F\left(\frac{x^*-\theta}{\sigma}\right)$. If the large trader also chooses to attack, then there is an additional λ to this incidence (see figure 3.1). Hence, if the large trader participates in the attack, the peg is broken whenever $\lambda + (1-\lambda) F\left(\frac{x^*-\theta}{\sigma}\right) \geq \theta$. Thus we can define the critical value of the fundamentals at which an attack is successful if and only if the large trader participates in the attack. It is defined by

$$\lambda + (1-\lambda) F\left(\frac{x^* - \overline{\theta}}{\sigma}\right) = \overline{\theta} \tag{3.2}$$

As is evident from figure 3.1, $\bar{\theta}$ lies between $\underline{\theta}$ and 1.

Although our notation does not make it explicit, both $\underline{\theta}$ and $\overline{\theta}$ are functions of the switching point x^*. In turn, x^* will depend on the large trader's switching point y^*. Our task is to solve these two switching points simultaneously from the respective optimization problems of the traders. A large trader observing signal y assigns probability $G\left(\frac{\overline{\theta}-y}{\tau}\right)$ to the event that $\theta \leq \overline{\theta}$. Since his expected payoff to attacking conditional on y is $G\left(\frac{\overline{\theta}-y}{\tau}\right) - t$, his optimal strategy is to attack if and only if $y \leq y^*$, where y^* is defined by:

$$G\left(\frac{\overline{\theta} - y^*}{\tau}\right) = t \tag{3.3}$$

Now consider a small trader. Conditional on signal x, the posterior density over θ for this trader is given by

$$\frac{1}{\sigma} f\left(\frac{\theta - x}{\sigma}\right) \tag{3.4}$$

When $\theta \leq \underline{\theta}$, the strategies of the small traders are sufficient for a successful attack. When $\theta \in \left(\underline{\theta}, \overline{\theta}\right]$ the peg breaks if and only if the large trader attacks, while if $\theta > \overline{\theta}$, the peg withstands the attacks, irrespective of the actions of the traders. Thus, the expected payoff to attack conditional on signal x can be written as

$$\frac{1}{\sigma}\int_{-\infty}^{\underline{\theta}} f\left(\frac{\theta - x}{\sigma}\right) d\theta + \frac{1}{\sigma}\int_{\underline{\theta}}^{\overline{\theta}} f\left(\frac{\theta - x}{\sigma}\right) G\left(\frac{y^* - \theta}{\tau}\right) d\theta \tag{3.5}$$

The first term is the portion of expected payoff attributable to the region of θ where $\theta \leq \underline{\theta}$. The second term is the portion of expected payoff that is attributable to the interval $\left(\underline{\theta}, \overline{\theta}\right]$. Here, one must take into account the fact that the attack is successful if and only if the large trader attacks. The probability that the large trader attacks at θ given his trigger strategy around y^* is given by $G\left(\frac{y^* - \theta}{\tau}\right)$, so that the payoffs are weighted by this value. Beyond $\bar{\theta}$, the attack is never successful, so that the payoff to attack is zero. Since the cost of attack is t, the trigger point x^* for the small trader is defined by the equation:

$$\frac{1}{\sigma}\int_{-\infty}^{\underline{\theta}} f\left(\frac{\theta - x^*}{\sigma}\right) d\theta + \frac{1}{\sigma}\int_{\underline{\theta}}^{\overline{\theta}} f\left(\frac{\theta - x^*}{\sigma}\right) G\left(\frac{y^* - \theta}{\tau}\right) d\theta = t \tag{3.6}$$

There is a unique x^* that solves this equation. To see this, it is helpful to introduce a change of variables in the integrals. Let

$$z \equiv \frac{\theta - x^*}{\sigma} \tag{3.7}$$

and denote

$$\underline{\delta} \equiv \frac{\underline{\theta} - x^*}{\sigma} \quad \text{and} \quad \overline{\delta} \equiv \frac{\overline{\theta} - x^*}{\sigma}. \tag{3.8}$$

Then, the conditional expected payoff to attacking given signal x^* is

$$
\begin{aligned}
&\frac{1}{\sigma}\int_{-\infty}^{\underline{\theta}} f\left(\frac{\theta - x^*}{\sigma}\right) d\theta + \frac{1}{\sigma}\int_{\underline{\theta}}^{\overline{\theta}} f\left(\frac{\theta - x^*}{\sigma}\right) G\left(\frac{y^* - \theta}{\tau}\right) d\theta \\
= &\int_{-\infty}^{\frac{\underline{\theta}-x^*}{\sigma}} f(z)\, dz + \int_{\frac{\underline{\theta}-x^*}{\sigma}}^{\frac{\overline{\theta}-x^*}{\sigma}} f(z)\, G\left(\frac{y^* - \theta}{\tau}\right) dz \\
= &\int_{-\infty}^{\frac{\underline{\theta}-x^*}{\sigma}} f(z)\, dz + \int_{\frac{\underline{\theta}-x^*}{\sigma}}^{\frac{\overline{\theta}-x^*}{\sigma}} f(z)\, G\left(\frac{\overline{\theta} - x^* - \sigma z}{\tau} - G^{-1}(t)\right) dz \\
= &\int_{-\infty}^{\underline{\delta}} f(z)\, dz + \int_{\underline{\delta}}^{\overline{\delta}} f(z)\, G\left(\frac{\sigma}{\tau}\left(\overline{\delta} - z\right) - G^{-1}(t)\right) dz \qquad (3.9)
\end{aligned}
$$

where the third line follows from the fact that

$$
\begin{aligned}
y^* &= \overline{\theta} - \tau G^{-1}(t) \qquad (3.10) \\
&= x^* + \sigma\overline{\delta} - \tau G^{-1}(t).
\end{aligned}
$$

Hence, (3.6) gives:

$$
\int_{-\infty}^{\underline{\delta}} f(z)\, dz + \int_{\underline{\delta}}^{\overline{\delta}} f(z)\, G\left(\frac{\sigma}{\tau}\left(\overline{\delta} - z\right) - G^{-1}(t)\right) dz - t = 0 \qquad (3.11)
$$

However, note that both $\underline{\delta}$ and $\overline{\delta}$ are monotonically decreasing in x^*, since

$$
\begin{aligned}
\frac{d\underline{\delta}}{dx^*} &= -\frac{1}{(1-\lambda) f\left(\underline{\delta}\right) + \sigma} < 0 \\
\frac{d\overline{\delta}}{dx^*} &= -\frac{1}{(1-\lambda) f\left(\overline{\delta}\right) + \sigma} < 0
\end{aligned}
$$

Since the left hand side of (3.11) is strictly increasing in both $\underline{\delta}$ and $\overline{\delta}$, it is strictly decreasing in x^*. For sufficiently small x^*, the left hand side of (3.11) is positive, while for sufficiently large x^*, it is negative. Since the left hand side is continuous in x^*, there is a unique solution to (3.11). Once x^* is determined, the large trader's switching point y^* follows from (3.3).

3.2. Dominance Solvability

To this point, we have confined our attention to trigger strategies, and have shown that there is a unique equilibrium within this class of strategies. We now show that confining our attention to trigger strategies is without loss of generality. The trigger equilibrium identified above turns out to be the only set of strategies that survive the iterated elimination of strictly interim dominated strategies. The dominance solvability property is by now well understood for symmetric binary action global games (see Morris and Shin (2000) for sufficient conditions for this property). The contribution here is to show that it also applies in our *asymmetric* global game.

Consider the expected payoff to attacking the peg for a small trader conditional on signal x when all other small traders follow the switching strategy around $\hat{x}$ and when the large trader plays his best response against this switching strategy (which is to switch at $y(\hat{x})$, obtained from (3.3)). Denote this expected payoff by $u(x, \hat{x})$. It is given by

$$u(x,\hat{x}) = \frac{1}{\sigma}\int_{-\infty}^{\underline{\theta}(\hat{x})} f\left(\frac{\theta - x}{\sigma}\right) d\theta + \frac{1}{\sigma}\int_{\underline{\theta}(\hat{x})}^{\bar{\theta}(\hat{x})} f\left(\frac{\theta - x}{\sigma}\right) G\left(\frac{y(\hat{x}) - \theta}{\tau}\right) d\theta \qquad (3.12)$$

where $\underline{\theta}(\hat{x})$ indicates the value of $\underline{\theta}$ when small traders follow the $\hat{x}$-switching strategy. $\bar{\theta}(\hat{x})$ is defined analogously. We allow $\hat{x}$ to take the values $-\infty$ and ∞ also, by which we mean that the small traders never and always attack, respectively. Note that $u(\cdot,\cdot)$ is decreasing in its first argument and increasing in its second.

For sufficiently low values of x, attacking the currency is a dominant action for a small trader, irrespective of the actions of the other traders, small or large. Denote by $\underline{x}_0$ the threshold value of x below which it is a dominant action to attack the currency for the small trader. All traders realize this, and rule out any strategy for the small traders which refrain from attacking below $\underline{x}_0$. But then, refraining from attacking cannot be rational for a small trader whenever one's signal is below $\underline{x}_1$ where $\underline{x}_1$ solves

$$u(\underline{x}_1,\ \underline{x}_0) = t \qquad (3.13)$$

This is so, since the switching strategy around $\underline{x}_1$ is the best reply to the switching strategy around $\underline{x}_0$, and even the most cautious small trader (in the sense that he assumes the worst concerning the possibility of a successful attack) believes that the incidence of attack is higher than that implied by the switching strategy around $\underline{x}_0$ and the large trader's best reply $y(\underline{x}_0)$. Since the payoff to attacking is increasing in the incidence of attack by the other traders, any strategy that refrains from attacking for signals lower than $\underline{x}_1$ is dominated. Thus, after *two* rounds of deletion of dominated strategies, any strategy for a small trader that refrains from attack for signals lower than $\underline{x}_1$ is eliminated. Proceeding in this way, one generates the increasing sequence:

$$\underline{x}_0 < \underline{x}_1 < \underline{x}_2 < \cdots < \underline{x}_k < \cdots \tag{3.14}$$

where any strategy that refrains from attacking for signal $x < \underline{x}_k$ does not survive $k+1$ rounds of deletion of dominated strategies. The sequence is increasing since $u(\cdot, \cdot)$ is decreasing in its first argument, and increasing in its second. The smallest solution $\underline{x}$ to the equation $u(x, x) = t$ is the least upper bound of this sequence, and hence its limit. Any strategy that refrains from attacking for signal lower than $\underline{x}$ does not survive iterated dominance.

Conversely, if x is the largest solu;my2;ip2¯;sy2;tyz;sy;oy2;lxoy2;ly23r;oycey2;.yp2i2xplp2m sitdiths we show that $\bar{\delta}$ is bounded below and above so that

$$F^{-1}(t) \leq \overline{\delta} \leq F^{-1}\left(\frac{t}{1-t}\right)$$

Observe that, as $\frac{\sigma}{\tau}\left(\overline{\delta}-z\right) \geq 0$ for $\overline{\delta} \geq z$, the value of $G\left(\frac{\sigma}{\tau}\left(\overline{\delta}-z\right)-G^{-1}\left(t\right)\right)$ in equation (3.11) will lie between $1-t$ (corresponding to the optimal cutoff for the large trader) and 1. Evaluating (3.9) at $G(.)=1$, we obtain $F\left(\overline{\delta}\right)=t$. Conversely, evaluating (3.9) at $G(.)=1-t$, $F(\bar{\delta})$ will be at most $\frac{t}{1-t}$. We conclude that $t \leq F\left(\overline{\delta}\right) \leq \frac{t}{1-t}$.

4.1. Large Trader with Precise Information ($\frac{\sigma}{\tau} \to \infty$)

In the first case of our taxonomy, we allow the large trader's information to become arbitrarily more precise relative to the small traders, i.e. we take the limit of the equilibrium expressions as $\frac{\sigma}{\tau} \to \infty$. Observe that, in this case, the expression $G\left(\frac{\sigma}{\tau}\left(\overline{\delta}-z\right)-G^{-1}\left(t\right)\right)$ in equation (3.11) tends to 1, for all $z<\overline{\delta}$ and $\overline{\delta} \geq F^{-1}(t)$. Equation (3.11) then becomes

$$F\left(\overline{\delta}\right)=t$$

For all practical purposes, small traders treat the signal of the large trader as if it coincided with the true state of the fundamental, *i.e.* they disregard the noise of the large trader's information. For the representative small trader, then, the expected payoff from an attack only depends on the distribution of the fundamental conditional on his own signal.

Substituting the above expression into the other equilibrium conditions, we obtain our first proposition:

Proposition 4.1. *In the limit as* $\frac{\sigma}{\tau} \to \infty$,

$$\overline{\theta} \to \lambda+(1-\lambda)(1-t)$$
$$\frac{\overline{\theta}-x^*}{\sigma} \to F^{-1}(t)$$
$$\frac{\overline{\theta}-y^*}{\tau} \to G^{-1}(t)$$

and $\underline{\theta}$ *tends to the unique solution of*

$$(1-\lambda)\,F\left(\frac{x^*-\underline{\theta}}{\sigma}\right)=\underline{\theta}.$$

The equilibrium expression for the limit of $\overline{\theta}$ has a simple intuitive interpretation. Recall that, conditional on $\bar{\theta}$, the proportion of small traders attacking is $F\left(\frac{x^*-\bar{\theta}}{\sigma}\right) \equiv F(-\bar{\delta})$. Above, we have established that, in the presence of a large trader with superior information, $F(\bar{\delta}) = t$. Thus, the proportion of the population of small traders $(1-\lambda)$ attacking the currency at $\theta = \bar{\theta}$ is $F(-\bar{\delta}) = 1 - F(\bar{\delta}) = 1 - t$. The threshold $\overline{\theta}$ is just the sum of this proportion, and the size of the large trader.

Comparing the equilibrium value of $\overline{\theta}$ with the corresponding expression for θ^* in the benchmark model without a large trader, we see that the former is strictly greater than the latter: with a large trader, the currency collapses at higher values of fundamentals. Moreover, since $\overline{\theta}$ is increasing in λ, the cutoff for small traders, x^*, is also increasing in the size of the large trader. Intuitively, when the information of the large trader is precise, small traders are confident that the large trader's signal will not be far away from their best estimates of the value of the fundamental. Thus, they become more aggressive, in the sense that they find it optimal to attack for higher values of the fundamentals.

Relative to the second benchmark model, where the market coincides with a single large trader, the presence of small traders makes the large trader more cautious. The cutoff point y^* is lower, falling from $1 - \tau G^{-1}(t)$ to $\bar{\theta} - \tau G^{-1}(t)$. This is because the large trader has now to allow for the fact that some proportion of small traders will fail to join him in the attack.[5]

In closing, it is useful to study the behavior of the model for $\frac{\sigma}{\tau} \to \infty$, when we also let the information of both small and large traders become arbitrarily precise $\sigma \to 0$, $\tau \to 0$. The result is summarized by the following corollary.

Corollary 4.2. *As* $\sigma \to 0$, $\tau \to 0$ *and* $\frac{\sigma}{\tau} \to \infty$,

$$\begin{aligned}
x^* &\to \lambda + (1-\lambda)(1-t) \\
y^* &\to \lambda + (1-\lambda)(1-t) \\
\overline{\theta} &\to \lambda + (1-\lambda)(1-t) \\
\underline{\theta} &\to \min\{1-\lambda, \lambda + (1-\lambda)(1-t)\}
\end{aligned}$$

When the large trader is well informed relatively to small traders, in the limit all trigger points tend to the unique value $\lambda + (1-\lambda)(1-t)$. This is strictly greater than the trigger point obtained with small traders only, and its value is increasing in the size of the large trader.

[5]For $t < 1/2$, both x^* and y^* are above the threshold $\bar{\theta}$. As $\frac{\sigma}{\tau} \to \infty$, y^* will be below x^*, and closer to $\bar{\theta}$.

4.2. Relatively Uninformed Large Trader ($\frac{\sigma}{\tau} \to 0$)

What if the information of the large trader is relatively less precise than that of small traders? Observe that, when $\frac{\sigma}{\tau} \to 0$, the expression $G\left(\frac{\sigma}{\tau}\left(\overline{\delta} - z\right) - G^{-1}(t)\right)$ in equation (3.11) tends to $1-t$, for all z and $\overline{\delta} \leq F^{-1}(\frac{t}{1-t})$. So equation (3.11) becomes

$$F(\underline{\delta}) + (1-t)\left[F\left(\overline{\delta}\right) - F(\underline{\delta})\right] = t \tag{4.1}$$

In contrast with the previous case, the representative small trader will now consider his own signal (as opposed to the signal of the large trader) as approximately equal to the fundamental. At the same time, he will be concerned with the poor precision of the large trader information — recall that the behavior of the large trader is crucial for the small traders' payoff when the fundamental is between $\underline{\theta}$ and $\bar{\theta}$.

To understand (4.1), note that, if a small trader believes the fundamental to be equal to $\bar{\theta}$, he can calculate the exact probability that the large trader signal be below y^*. Using (3.3), this probability is $G\left(\frac{y^*-\bar{\theta}}{\tau}\right) = 1-t$. What can a small trader conclude, however, if his signal x_i tells him that θ is different from $\bar{\theta}$? The key is that, as the signal of the large trader is quite noisy, the probability $G\left(\frac{y^*-\theta}{\tau}\right)$ will remain approximately constant for values of θ around $\bar{\theta}$. Specifically, it will be approximately constant and equal to $1-t$ for fundamentals in the interval between $\underline{\theta}$ and $\bar{\theta}$. Thus, the small traders will think that the probability of an attack by a large uninformed trader is constant over the relevant range of the fundamental.

In light of this consideration, the two terms on the left-hand side of the optimal cutoff condition (4.1) have a simple intuitive explanation. The first term is the expected gross payoff for values of the fundamental below $\underline{\theta}$. The second term is the expected gross payoff for θ in the interval between $\underline{\theta}$ and $\bar{\theta}$, allowing for a constant probability $(1-t)$ of an attack by a large trader.[6]

From equations (3.1) and (3.2), we also have

$$\lambda = (1-\lambda)\left(F\left(\overline{\delta}\right) - F(\underline{\delta})\right) + \sigma\left(\overline{\delta} - \underline{\delta}\right). \tag{4.2}$$

Writing $\overline{\delta}$ and $\underline{\delta}$ for the unique solutions to the pair of equations (4.1) (4.2), we state our second proposition.

[6]Note that eq. (4.1) becomes $F\left(\overline{\delta}\right) = t$ as $\underline{\delta} \to \overline{\delta}$, and $F\left(\overline{\delta}\right) = \frac{t}{1-t}$ as $\underline{\delta} \to -\infty$.

Proposition 4.3. *In the limit as* $\frac{\sigma}{\tau} \to 0$,

$$\begin{aligned}\overline{\theta} &\to \lambda + (1-\lambda)\left(1 - F\left(\overline{\delta}\right)\right)\\ \frac{\overline{\theta} - x^*}{\sigma} &\to \overline{\delta}\\ \frac{\overline{\theta} - y^*}{\tau} &\to G^{-1}(t)\\ \underline{\theta} &\to (1-\lambda)\left(1 - F\left(\underline{\delta}\right)\right)\end{aligned}$$

What do we know about $\overline{\delta}$ and $\underline{\delta}$? To address this question, we adopt the following strategy. First, we combine the pair of equations in (4.1) and (4.2) as to obtain an explicit expression for the value of σ as a function of t, λ, $\bar{\delta}$ and $\underline{\delta}$:[7]

$$\sigma = \widetilde{\sigma}\left(t, \lambda, \overline{\delta}\right) = \frac{\lambda - (1-\lambda)\left[\frac{1}{t}F\left(\overline{\delta}\right) - 1\right]}{\overline{\delta} - F^{-1}\left(1 - \left(\frac{1-t}{t}\right)F\left(\overline{\delta}\right)\right)}. \tag{4.3}$$

We have already established that $\overline{\delta}$ varies between the two boundaries $F^{-1}(t)$ and $F^{-1}\left(\frac{t}{1-t}\right)$. We can therefore analyze the behavior of the function $\widetilde{\sigma}\left(t, \lambda, \overline{\delta}\right)$ as $\overline{\delta}$ varies between these two boundaries. Second, we use the results from this analysis to draw conclusions about the behavior of $\bar{\delta}$ and $\underline{\delta}$ as $\sigma \to 0$ for a finite τ.

Before proceeding further, note that the numerator of (4.3) is zero for $\bar{\delta} = F^{-1}\left(\frac{t}{1-\lambda}\right)$. Given that $\bar{\delta}$ is bounded from above by $F^{-1}\left(\frac{t}{1-t}\right)$, it is easy to verify that this is a relevant solution only if $\lambda \leq t$, that is to say, if the large trader is not "too large". Thus, in studying the function $\widetilde{\sigma}\left(t, \lambda, \overline{\delta}\right)$ we need to consider two cases.

For a large trader with $\lambda \geq t$, $\widetilde{\sigma}\left(t, \lambda, \overline{\delta}\right)$ tends to infinity as $\overline{\delta} \to F^{-1}(t)$, it is strictly positive for all $\overline{\delta} \in \left(F^{-1}(t), F^{-1}\left(\frac{t}{1-t}\right)\right)$, and tends to zero as $\overline{\delta} \to F^{-1}\left(\frac{t}{1-t}\right)$. Thus we conclude that

$$\lim_{\sigma \to 0} \overline{\delta} = F^{-1}\left(\frac{t}{1-t}\right) \tag{4.4}$$

implying that $\lim\limits_{\sigma \to 0} \underline{\delta} = -\infty$.

If, instead, the size of the large trader is such that $\lambda < t$, then $\widetilde{\sigma}\left(t, \lambda, \overline{\delta}\right)$ tends to infinity as $\overline{\delta} \to F^{-1}(t)$, is strictly positive for all $\overline{\delta} \in \left(F^{-1}(t), F^{-1}\left(\frac{t}{1-\lambda}\right)\right)$, and equals zero when $\overline{\delta} = F^{-1}\left(\frac{t}{1-\lambda}\right)$, which is below the upper boundary for $\overline{\delta}$.[8] Thus,

[7]From equation (4.2), we obtain $\underline{\delta} = F^{-1}\left(1 - \left(\frac{1-t}{t}\right)F\left(\overline{\delta}\right)\right)$. Substituting into equation (4.2) and re-arranging gives the expression in the text.

[8]We also note that $\widetilde{\sigma}\left(t, \lambda, \overline{\delta}\right)$ is strictly negative for all $\overline{\delta} \in \left(F^{-1}\left(\frac{t}{1-\lambda}\right), F^{-1}\left(\frac{t}{1-t}\right)\right)$ and tends to zero as $\overline{\delta} \to F^{-1}\left(\frac{t}{1-t}\right)$, but these values are not relevant for our analysis.

we conclude that

$$\lim_{\sigma \to 0} \overline{\delta} = F^{-1} \left(\frac{t}{1-\lambda} \right). \tag{4.5}$$

This implies $\lim_{\sigma \to 0} \underline{\delta} = F^{-1}\left(1 - \left(\frac{1-t}{t}\right)\frac{t}{1-\lambda}\right) = F^{-1}\left(\frac{t-\lambda}{1-\lambda}\right)$. We summarize the relevant findings with the following corollary:

Corollary 4.4. *If $\lambda \geq t$, then as $\sigma \to 0$, $\tau \to 0$ and $\frac{\sigma}{\tau} \to 0$,*

$$\begin{aligned} x^* &\to \lambda + (1-\lambda)\left(1 - \frac{t}{1-t}\right), \\ y^* &\to \lambda + (1-\lambda)\left(1 - \frac{t}{1-t}\right), \\ \overline{\theta} &\to \lambda + (1-\lambda)\left(1 - \frac{t}{1-t}\right), \\ \text{and } \underline{\theta} &\to 1-\lambda; \end{aligned}$$

while if $\lambda \leq t$, then as $\sigma \to 0$, $\tau \to 0$ and $\frac{\sigma}{\tau} \to 0$,

$$\begin{aligned} x^* &\to 1-t, \quad & y^* \to 1-t, \\ \overline{\theta} &\to 1-t, \quad & \text{and } \underline{\theta} \to 1-t. \end{aligned}$$

When $\lambda \geq t$, the proportion of small traders attacking the currency when the fundamental is at the threshold $\bar{\theta}$ is equal to $1 - F(\bar{\delta}) = \left(1 - \frac{t}{1-t}\right)$. The proportion of traders attacking the currency at $\theta = \bar{\theta}$ is therefore substantially lower than the case discussed in the previous subsection. Despite the low precision of the large trader information, however, its size still affects all the relevant thresholds. The larger is λ, the higher is the probability of a collapse.

Note that the distance between $\bar{\theta}$ and $\underline{\theta}$ is equal to $\lambda - (1-\lambda)\left(\frac{t}{1-t}\right)$, so that the difference between $\bar{\theta}$ and $\underline{\theta}$ tends zero as the size of the large trader λ approaches t. The second part of our corollary establishes that this difference is identically equal to zero for any λ below t.[9] Most crucially, when $\lambda \leq t$, varying the size of the large trader does not affect the probability of the collapse. All optimal cutoff points are independent of λ.

The reason underlying these results reveals an important feature of the equilibrium. A feature of the limit is that, at the switching point x^*, a small trader believes that the proportion of small traders attacking the currency is a uniformly

[9] In this case, $x^* \geq \bar{\theta}$ only for $t \in (\frac{1}{3}, \frac{1}{2})$. The inequality reverses for smaller t. Instead, $y^* \geq \bar{\theta}$ for $t \leq \frac{1}{2}$.

distributed random variable with support over the unit interval $[0, 1]$. Morris and Shin (2000) dub such beliefs as being "Laplacian" and show that this is quite a general property of binary action global games. Denote this random variable by $\widetilde{\alpha}$. Since a large uninformed trader is perceived to attack the currency with probability $(1-t)$, while the small traders' information is arbitrarily precise, the expected gross payoff from attacking the currency for a small trader receiving a signal x (which, in the limit, coincides with θ) can be written as

$$z(x) = (1-t)\ \text{prob}\left(\lambda + (1-\lambda)\,\widetilde{\alpha} \geq \theta\right)\ +\ t\ \left(\text{prob}\left(1-\lambda\right)\widetilde{\alpha} \geq \theta\right)$$

Assuming that both λ and t are below $1/2$, we can calculate the expected gross payoff for different value of the signal as follows:

$$z(x) = \begin{cases} 1 & \text{if } \ \theta \leq 0 \\ (1-t) + t\frac{1-\lambda-\theta}{1-\lambda} & \text{if } \ 0 \leq \theta \leq \lambda \\ (1-t)\,\frac{1-\theta}{1-\lambda} + t\frac{1-\lambda-\theta}{1-\lambda} & \text{if } \ \lambda \leq \theta \leq 1-\lambda \\ (1-t)\,\frac{1-\theta}{1-\lambda} & \text{if } \ 1-\lambda \leq \theta \leq 1 \\ 0 & \text{if } \ 1 \leq \theta \end{cases}$$

Consider the case in which λ is small relative to t. In this case, the cutoff point x^*, satisfying $z(x^*) = t$, falls in the interval$(\lambda, 1-\lambda)$. Small traders know that their own mass is enough to cause a collapse of the peg, while the large trader is behaving noisily. For a given x^*, any change in the size of the large trader is compensated by a change in the mass of small traders attacking the currency. As mentioned above, the two thresholds $\bar{\theta}$ and $\underline{\theta}$ will coincide. If "Soros" is large relative to t, instead, the optimal cutoff point will fall in the interval $(1-\lambda, 1)$. Then, differences in the size of the large trader can no longer be compensated by changes in the mass of small traders attacking.

4.3. A comparison

We close this section by gathering together the various strands in our taxonomy of cases, and providing a synthesis of what we have accomplished so far. We note the following points.

The presence of the large trader induces the small traders to become more aggressive sellers. When $\lambda = 0$, the trigger point for the small traders is given by $x^* = 1-t$ in the limit. When $\lambda > 0$, this trigger point can be higher, irrespective of the relative informational quality of the large trader. In particular, when the large trader is well informed relative to the small traders (given by Corollary 4.2), the small trader's trigger point tends to

$$x^* = \lambda + (1-\lambda)\,(1-t)\,. \tag{4.6}$$

This is strictly larger than $1-t$, and is increasing in the size of the large trader.

However, when the large trader is less well informed relative to the small traders (given by Corollary 4.4), the impact of the large trader on the trading strategies of the small traders is less pronounced. When $\lambda < t$, the large trader has no impact on the trading strategies of the small traders (since $x^* = 1-t$). When $\lambda > t$, the trigger point tends to

$$\begin{aligned} x^* &= \lambda + (1-\lambda)\left(1 - \frac{t}{1-t}\right) \\ &= \lambda + (1-\lambda)(1-t) - \frac{t^2(1-\lambda)}{1-t} \end{aligned}$$

which is lower than (4.6). Hence, the small traders are less aggressive sellers.

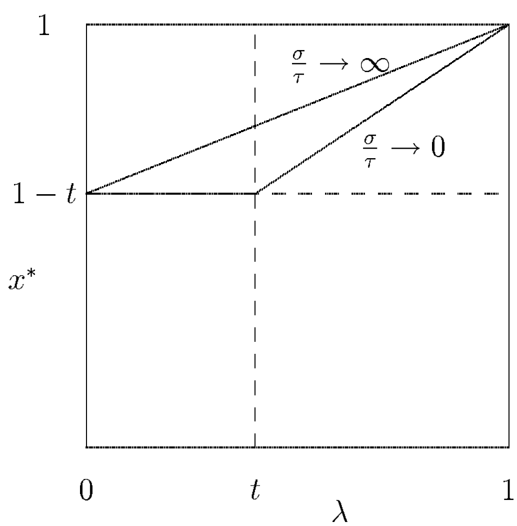

Figure 4.1: Trigger point x^* as a function of λ

The table below and figure 4.1 summarize the behaviour of the trigger point x^* as a function of λ. As the size of the large trader increases, the small traders become more aggressive sellers.

Table
Limiting properties of the equilibrium:
Equilibrium value of the trigger x^* of small traders
by size and relative precision of the large trader

	Size:	$\lambda > t$	$t > \lambda > 0$	$\lambda = 0$
Information precision				
$\frac{\sigma}{\tau} \to \infty$		$1 - t + \lambda t$	$1 - t + \lambda t$	$1 - t$
$\frac{\sigma}{\tau} \to 0$		$1 - t + \lambda t - t^2 \frac{1-\lambda}{1-t}$	$1 - t$	$1 - t$

We also observe that everything converges nicely to the two benchmarks cases discussed in section 3. Regardless of the relative precision of information (i.e. regardless of whether $\frac{\sigma}{\tau} \to \infty$ or $\frac{\sigma}{\tau} \to 0$), we have x^*, y^*, $\overline{\theta}$ and $\underline{\theta}$ all converging to $1-t$ as $\sigma \to 0$, $\tau \to 0$ and $\lambda \to 0$; and all converging to 1 as $\sigma \to 0$, $\tau \to 0$ and $\lambda \to 1$.

Furthermore, if the large speculator's signal is informative relative to the continuum speculators' signals (i.e., if $\sigma \to 0$, $\tau \to 0$ and $\frac{\sigma}{\tau} \to \infty$) then the convergence is very smooth, and in particular x^*, y^* and $\overline{\theta}$ are all linear in λ. If, instead, the large speculator's signal is uninformative relative to the continuum speculators' signals (i.e., if $\sigma \to 0$, $\tau \to 0$ and $\frac{\sigma}{\tau} \to 0$) then the existence of the large speculator has no impact until his size crosses the threshold λ.

5. Sequential Move Game

An important feature of large traders is their visibility in the market - a feature that is only captured to a limited extent by our framework so far. Market participants know the degree of precision of the large trader information, but have no prior information about the exact speculative position of the large trader. In this section, we explore the predictions of our model under a more general assumption regarding observability of actions. Specifically, we let the speculative position taken by any market participant to be observable by the rest of the market. We will see that in equilibrium the large trader will have an incentive to move before the others, so as to maximize his influence.

The analytical framework adopted in this section has essentially the same features of the model presented in Section 2. The main difference is that, instead of analyzing a simultaneous move by all traders, we now allow traders to take a speculative position in either of two periods, 1 and 2, preceding the government decision on the exchange rate. At the beginning of each period, each trader gets a chance to choose an action. However, once he has attacked the currency, he may not do so again and may not reverse his position. So, each trader can choose when, if at all, to attack the currency.

Traders receive their private signal (x_i and y) at the beginning of period 1. In addition, traders are now also able to observe at the beginning of period 2, the action choices of other traders in period 1. Thus, traders can learn from the actions of other market participants, and also use their own actions to signal to other traders. We assume that individual small traders ignore the signalling effect of their actions.[10] Payoffs are the same as in section 2, and are realized at the end of period 2. Payoffs do not depend on the timing of traders' actions, i.e., there are no costs of waiting.[11]

5.1. Equilibrium

We begin by making two simple observations about timing incentives in the sequential move game. Small traders will always have an incentive to postpone any action until period 2. Each trader perceives no benefit to signalling, because he believes that he has no power to influence the actions by others by attacking early. On the other hand, he will learn something by waiting to attack: he will find out the large trader's action and he may learn more about the state of the world. There are no costs of waiting, but there is a weak informational benefit to doing so. So it is a dominant strategy for each small trader to wait to period 2 before deciding whether to attack or not. But if small traders wait until period 2, the large trader knows that in equilibrium he can never learn from the actions of the small traders. On the other hand, he knows that if he attacks in period 1, he will send a signal to the small traders, and thereby influence their actions. In particular, since the large trader is concerned with coordinating his actions with those of the continuum of small traders, he benefits from signalling to the small traders. Thus the large trader has a weak incentive to attack in period 1, if he is ever going to attack. Given that small traders wait until period 2, it is a dominant strategy for the larger trader to move early. For these reasons, we assume in the analysis that follows that the large trader moves in period 1 and the small traders moves in period 2.[12]

We first characterize trigger equilibria in this game. Suppose that the large trader, acting first, chooses to attack only if his signal is lower than y^*. If he

[10] Levine and Pesendorfer (1995) and others have provided formal limiting justifications for this standard assumption in continuum player games.

[11] Our two period game is best interpreted as a discrete depiction of a continuous time setting, in which the difference between the time periods is very small and represents the time it takes traders to observe and respond to others' actions.

[12] See Dasgupta (2000) for a formal analysis of the endogenous timing decision in this game.

does not attack in period 1, small traders that receive a low enough signal may nonetheless attack the currency, thinking that they can bring the currency down alone. This will define a threshold $\underline{x}^*$ for the signal of small traders, below which these would attack in period 2 even if the large trader has not attacked in period 1. But if the large trader does attack the currency in period 1, then of course this sends a signal to the small traders that (based upon his information) the large trader believes the economy to be weak enough to risk speculating. When the large trader attacks in period 1, small traders would therefore be inclined to attack for a larger range of signals they might receive. This defines a different threshold $\overline{x}^*$ for their signal, where in equilibrium $\underline{x}^* \leq \overline{x}^*$. We should note here that these thresholds need not be finite. As shown below, there are situations in which the move by the large trader in period 1 will completely determine the behavior of small traders.

Since traders' signals are correlated with fundamentals, corresponding to these triggers are critical mass conditions, i.e. threshold levels for the fundamentals below which there will be always a successful attack. As before, we can derive two conditions, depending on whether the large trader participates in the attack, $(\bar{\theta})$, or not $(\underline{\theta})$.

A trigger equilibrium is then a 5-tuple $\left(y^*, \underline{x}^*, \overline{x}^*, \underline{\theta}, \bar{\theta}\right)$. The equilibrium conditions described above now become:

- y^* solves the equation

$$\Pr\left(\theta \leq \bar{\theta} \mid y = y^*\right) = t \tag{5.1}$$

- $\underline{x}^*$ solves the equation

$$\Pr\left(\theta \leq \underline{\theta} \mid y > y^* \text{ and } x_i = \underline{x}^*\right) = t \tag{5.2}$$

  if a solution exists. If the LHS is strictly larger than the RHS for all x_i, $\underline{x}^* = \infty$. Conversely, if the LHS is strictly smaller than the RHS for all x_i, $\underline{x}^* = -\infty$.

- $\overline{x}^*$ solves the equation

$$\Pr\left(\theta \leq \bar{\theta} \mid y \leq y^* \text{ and } x_i = \bar{x}^*\right) = t \tag{5.3}$$

  if a solution exists. If the LHS is strictly larger than the RHS for all x_i, $\overline{x}^* = \infty$. Conversely, if the LHS is strictly smaller than the RHS for all x_i, $\overline{x}^* = -\infty$.

- $\underline{\theta}$ solves the equation

$$(1-\lambda)\Pr(x_i \leq \underline{x}^* \mid \theta = \underline{\theta}) = \underline{\theta} \tag{5.4}$$

- $\bar{\theta}$ solves the equation

$$\lambda + (1-\lambda)\Pr(x_i \leq \overline{x}^* \mid \theta = \bar{\theta}) = \bar{\theta} \tag{5.5}$$

To solve the model, recall that, in our setting, the information system and the definition of the large trader's signal implies

$$\begin{aligned} y &= x_i + \tau\eta - \sigma\varepsilon_i \\ y^* &= \bar{\theta} - \tau G^{-1}(t) \end{aligned}$$

Now, consider a small trader's posterior probability assessment of a successful attack conditional upon observing the large trader attack in period 1 and the signal x_i. Using the above expressions, such probability can be expressed as

$$\begin{aligned} & \Pr\left(\theta \leq \bar{\theta} \mid y \leq y^*\right) \\ = & \Pr\left(\varepsilon_i \geq \frac{x_i - \bar{\theta}}{\sigma} \;\middle|\; \tau\eta - \sigma\varepsilon_i \leq \bar{\theta} - x_i - \tau G^{-1}(t)\right) \end{aligned}$$

We can thus derive $\bar{x}^*$ by solving the following equation

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\bar{x}^* - \bar{\theta}}{\sigma}\ ,\ \tau\eta - \sigma\varepsilon_i \leq \bar{\theta} - \bar{x}^* - \tau G^{-1}(t)\right)}{\Pr\left(\tau\eta - \sigma\varepsilon_i \leq \bar{\theta} - \bar{x}^* - \tau G^{-1}(t)\right)} = t \tag{5.6}$$

By the same token, $\underline{x}^*$ can be derived by the analogous condition for the case in which the large trader has not attacked the currency in period 1:

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\underline{x}^* - \bar{\theta}}{\sigma}\ ,\ \tau\eta - \sigma\varepsilon_i \leq \bar{\theta} - \underline{x}^* - \tau G^{-1}(t)\right)}{\Pr\left(\tau\eta - \sigma\varepsilon_i \leq \bar{\theta} - \underline{x}^* - \tau G^{-1}(t)\right)} = t \tag{5.7}$$

It is apparent that neither of these equations can be solved in closed form in the general case, without making further parametric assumptions on the distribution functions of the error terms. Using our specification, we can however proceed to analyze limiting results for the different relative precision of the large trader information relative to the rest of the market.

5.2. The role of signalling

Adopting the format of Section 3, we now discuss the limiting properties of the model allowing for differences in the information precision across traders of different size. We consider first the case of a large trader who is arbitrarily better informed than small traders. The following proposition summarizes our result.

Proposition 5.1. *As $\frac{\sigma}{\tau} \to \infty$, there is a unique trigger equilibrium in Γ, with*

$$\begin{aligned} \frac{1-y^*}{\tau} &\to G^{-1}(t) \\ \bar{x}^* &\to \infty \\ \underline{x}^* &\to -\infty \\ \bar{\theta} &\to 1 \\ \underline{\theta} &\to 0 \end{aligned}$$

Proof: We first rewrite equation (5.6) as

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\bar{x}^*-\bar{\theta}}{\sigma}\ ,\ \frac{\tau}{\sigma}\eta - \varepsilon_i \leq \frac{\bar{\theta}-\bar{x}^*}{\sigma} - \frac{\tau}{\sigma}G^{-1}(t)\right)}{\Pr\left(\frac{\tau}{\sigma}\eta - \varepsilon_i \leq \frac{\bar{\theta}-\bar{x}^*}{\sigma} - \frac{\tau}{\sigma}G^{-1}(t)\right)} = t$$

Taking the limit as $\frac{\tau}{\sigma} \to 0$, the LHS tends to

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\bar{x}^*-\bar{\theta}}{\sigma}\ ,\ -\varepsilon_i \leq \frac{\bar{\theta}-\bar{x}^*}{\sigma}\right)}{\Pr\left(-\varepsilon_i \leq \frac{\bar{\theta}-\bar{x}^*}{\sigma}\right)}$$

which is equal to 1. Thus, in the limit there is no solution to the above equation. Since $t < 1$, we use the definition of $\bar{x}^*$ to set $\bar{x}^* = \infty$. We can then substitute $\bar{x}^*$ into equation (5.5) to derive $\bar{\theta} = 1$. Symmetric arguments establish that $\underline{x}^* = -\infty$ and $\underline{\theta} = 0$. Thus using the definition of y^*, we get $y^* = 1 - \tau G^{-1}(t)$.

In words, this result says that, when the large trader is arbitrarily better informed than the small traders, they follow him blindly, and therefore, he completely internalizes the payoff externality in the currency market. This type of equilibrium corresponds to the strong herding equilibrium in Dasgupta (1999), where all the followers ignore their information completely.

This result implies that, when actions are observable, a relatively well-informed large trader can (but not always will) make small traders either extremely aggressive in selling a currency, or not at all aggressive. His influence in this case is much larger (as should be true, intuitively), in comparison to the case of a simultaneous move game, analyzed in the previous section.

Notably, the size of the large trader never appears in the expressions that define the unique trigger equilibrium. The distinctive feature of a large trader is that he does not ignore the signalling effect of his actions. What emerges from our result is that, when he is significantly better informed than the small traders, his absolute size is irrelevant.

The following proposition states our results corresponding to the case in which the large trader is less precisely informed than the rest of the market.

Proposition 5.2. *As $\frac{\sigma}{\tau} \to 0$ there is a unique trigger equilibrium, with*

$$\begin{aligned}
\frac{\lambda + (1-\lambda)(1-t) - y^*}{\tau} &\to G^{-1}(t) \\
\frac{\lambda + (1-\lambda)(1-t) - \bar{x}^*}{\sigma} &\to F^{-1}(t) \\
\frac{(1-\lambda)(1-t) - \underline{x}^*}{\sigma} &\to F^{-1}(t) \\
\bar{\theta} &\to \lambda + (1-\lambda)(1-t) \\
\underline{\theta} &\to (1-\lambda)(1-t)
\end{aligned}$$

Proof: Rewrite equation (5.6) and taking limits as $\frac{\sigma}{\tau} \to 0$, we get

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\bar{x}^* - \bar{\theta}}{\sigma} \text{ , } \eta \leq \frac{\bar{\theta} - \bar{x}^*}{\tau} - G^{-1}(t)\right)}{\Pr\left(\eta \leq \frac{\bar{\theta} - \bar{x}^*}{\tau} - G^{-1}(t)\right)} = t$$

which, given independence of ε_i and τ implies that

$$\bar{x}^* = \bar{\theta} - \sigma F^{-1}(t)$$

Combining with equation (5.5) we get

$$\bar{\theta} \to \lambda + (1-\lambda)(1-t)$$

Thus

$$\bar{x}^* \to \lambda + (1-\lambda)(1-t) - \sigma F^{-1}(t)$$

The remaining quantities are then uniquely defined.

In words, this proposition means that even a relatively uninformed large trader attempts to influence the market. However, since he does not have any informational signalling ability, his actions affect the equilibrium outcome of the game only inasmuch as his size is relevant. Intuitively, as his signal is quite noisy, he cannot reduce the small traders' uncertainty about the fundamental. By moving first, however, he can eliminate uncertainty about his action. If, in addition, we suppose that $\sigma \to 0$, then

$$\bar{x}^* \to 1 - t + \lambda t$$

Observe that as $\lambda \to 0$, the equilibrium triggers converge exactly to the case in which the large trader does not exist.

5.3. A synthesis of our results

We are now in the position to offer a complete overview of our results, and reach some conclusions about the role of a large trader in a currency crisis. As explained in the introduction, there are three main elements in our theory: size, information precision and signalling.

Focusing on the limiting properties of our equilibria, the following table presents the equilibrium value of the trigger for small traders in the different cases discussed above.

Table
Limiting properties of equilibria
Equilibrium trigger for small traders by relative precision of information

Large trader is:	informed	uninformed
	$(\frac{\tau}{\sigma} \to 0,\ \sigma \to 0)$	$(\frac{\tau}{\sigma} \to \infty,\ \sigma \to 0)$
Actions are:		
unobservable	$x^* = 1 - t - \lambda t$	$x^* = 1 - t - \lambda t - t^2 \frac{1-\lambda}{1-t}$ if $\lambda > t$ $x^* = 1 - t$ if $\lambda \leq t$
observable	$\bar{x}^* = \infty$ $\underline{x}^* = -\infty$	$\bar{x}^* = 1 - t + \lambda t$ $\underline{x}^* = (1-t)(1-\lambda)$

In each column of the table, the two thresholds $\bar{x}^*$ and $\underline{x}^*$ in the game where actions are observable are higher and lower, respectively, than the corresponding threshold x^* derived in our game with unobservable action. In other words, regardless of the relative precision of information, a large trader can have a much larger influence in the market if he is able to signal to small traders.

As discussed above, the size of the large trader is irrelevant in the sequential move game when the large trader is relatively well informed – this case corresponds to the bottom left cell of the table. What matter here is not the size per se, but the signalling ability associated with size. Conversely, size matters in all other cases.

Reading the entries on the main diagonal of the table, observe that the critical signal (x^*) in the unobservable action, information larger trader case is equal to critical signal contingent on the larger trader have attacked ($\bar{x}^*$) in the observable action, uninformed large trader case. This equality provides an interesting link across the two games. When actions are not observable, small traders do not expect a better informed trader to "add noise" to the game. Their problem is to estimate the fundamental as well as possible, given their own signal. When actions are observable, the potential noise added to the game by a relatively uninformed large trader is eliminated by his moving first. So, also in this case, the problem of the smaller traders is the same as above, i.e. to estimate the fundamental as well as possible given their own information.

6. Concluding Remarks

Economists and policy makers have long debated whether speculation, especially speculation by large traders, is destabilizing. In our model, a large trader in the market may exacerbate a crisis, and render small traders more aggressive. Figure 4.1 illustrates this well. The small traders' trading strategies as defined by the switching point x^* become more aggressive as the size of the large trader increases. However, the relative precision of the information available to the traders affects this conclusion. If the large trader is less well informed than the small traders, this effect may be quite small. Finally, the influence of the large trader is magnified greatly if the large trader's trading position is publicly revealed to the other traders, although this result also must be qualified by the relative precision of information of the two types of traders.

Crucial to our conclusion is the assumption that the large trader stands to gain in the event of the devaluation. This may not be an assumption that is widely accepted. If the large trader is an investor with a substantial holding of assets denominated in the currency under attack (say, a U.S. pension fund with equity holdings in the target country), he may prefer that an attack not occur, even though, if he thinks the attack is sufficiently likely he will join the attack. In such a case, the presence of a large trader will have the opposite effect, making attacks less likely. This points to the importance of understanding the initial portfolio positions of the traders in such instances.

Our analysis also abstracted from a large trader's incentive to take a position discreetly in order to avoid adverse price movements. If this effect were important, a trader would have an incentive to delay announcing his position until it is fully established. But even once a trader has established his position, he may prefer to avoid public disclosures when he is holding a highly leveraged portfolio in possibly illiquid instruments. One of the motivations for the call for greater public disclosures by banks and hedge funds (see Financial Stability Forum (2000)) is the idea that if leveraged institutions know that their trading positions are to be revealed publicly, they would be wary of taking on large speculative positions. The recent decisions by several well known fund managers (Mr. Soros being one of them) to discontinue their 'macro hedge fund' activities raise deeper questions concerning the trade-off between the sorts of mechanisms outlined in our model against the diseconomies of scale that arise due to the illiquidity of certain markets. It is perhaps not a coincidence that the closure of such macro hedge funds comes at a time when many governments have stopped pursuing currency pegs and other asset price stabilization policies.

References

[1] Carlsson, H. and E. van Damme (1993) "Global Games and Equilibrium Selection," *Econometrica* **61**, 989-1018.

[2] Corsetti, G., P. Pesenti and N. Roubini (1998) "What Caused the Asian Financial Crisis?" NBER Working Paper 6833-6834.

[3] Counterparty Risk Management Policy Group (1999) *Improving Risk Management Practices,* http://www.crmpolicygroup.org/crmpolicygroup/

[4] Dasgupta A. (1999) "Social Learning with Payoff Complementarities", Yale University, http://www.econ.yale.edu/~amil/research.html

[5] Dasgupta A. (2000) "Brain and Brawn: How Large Traders can Trigger Currency Crises", Yale University, http://www.econ.yale.edu/~amil/research.html

[6] Diamond, D. and P. Dybvig (1983) "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* **91**, 401-419.

[7] Financial Stability Forum (2000) *Report of the Working Group on Highly Leveraged Institutions,* http://www.fsforum.org/Reports/RepHLI.html

[8] Frankel, D., S. Morris and A. Pauzner (1999) "Equilibrium Selection in Global Games with Strategic Complementarities" http://www.econ.yale.edu/~smorris/fmp.pdf

[9] Hartigan, J. (1983) *Bayes Theory.* Springer-Verlag.

[10] International Monetary Fund (1999) *IMF-Supported Programs in Indonesia, Korea, and Thailand: A Preliminary Assessment*, January

[11] Levine, D. and W. Pesendorfer (1995) "When Are Agents Negligible?" *American Economic Review* **85**, 1160-1170.

[12] Morris, S. and H. S. Shin (1998) "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks" *American Economic Review* **88**, 587-597.

[13] Morris, S. and H. S. Shin (2000) "Global Games: Theory and Applications", invited paper for the Eighth World Congress of the Econometric Society, Seattle, 2000, http://www.econ.yale.edu/~smorris/seattle.pdf

[14] Obstfeld, M. (1996) "Models of Currency Crises with Self-Fulfilling Features," *European Economic Review* **40**, 1037-1047.