

3-5 MAN MUTUAL ZUGZWANGS IN CHESS

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ABSTRACT

This note reports the work of Wirth and Karrer in twin-sourcing all mutual zugzwang positions, *mzugs*, in 2-5-man endgames. This paper tabulates the *mzug* statistical data, gives examples of maximal *mzugs* and refers to a chess endgame website where further data is to be found.

1. INTRODUCTION

Wirth and Nalimov have generated all 2-to-5-man endgame tables (EGTs) to the Depth-to-Conversion (DTC) and Depth-to-Mate (DTM) metrics respectively (Wirth and Nievergelt, 1999; Nalimov, Haworth and Heinz, 2000). Wirth and Karrer scanned the DTC and DTM EGTs respectively, second-sourcing all the *mzugs*.

A reciprocal or mutual zugzwang, *mzug*, in chess is a position where, ironically, each side could get a better result in theory if it were the other side's turn to move. There are three types of *mzug*:

<i>ww</i>	' = /1-0'	a 'White win' <i>mzug</i> ... the position is a wtm draw and a btm win for White
<i>bw</i>	'0-1/ = '	a 'Black win' <i>mzug</i> ... the position is a wtm win for Black and a btm draw
<i>fp</i>	'0-1/1-0'	a 'full point' <i>mzug</i> ... the side that has to move loses.

They are relatively rare and a running theme in Nunn's endgame trilogy (1992, 1994, 1995). Many endgame studies exploit the *mzug* theme at some point (Roycroft, 1972; Beasley, 1996; Elkies, 1998). Many counts of *mzugs*, principally by Rasmussen (1991-2000), have already been published: they confirm and are confirmed by the data here. This article completes the coverage of the 2-to-5-man domain, providing DTC and DTM data and examples of maximal *mzugs* in these two metrics.

2. THE RESULTS

Karrer (2000) scanned Nalimov's 3-to-5-man EGTs (Hyatt, 2001; Tamplin, 2001b) for *mzug* positions, giving:

- a list of *mzugs* together with statistics about counts and DTM-maximal depths
n.b. for a full-point *mzug*, the *depth* is taken to be the sum of the wtm and btm depths
- a list of the DTM-maximal *mzugs*

The lists were then passed to workers in this field including Elkies, Rasmussen, Roycroft, Tamplin and Wirth. Wirth then computed the 4-1 EGTs to complete his suite of 2-to-5-man EGTs and executed the analogous *mzug* scan on them. Haworth collated the resulting statistics, confirmed full agreement between the results of Karrer and Wirth, and with the DTM data for the EGTs which Rasmussen (2000) had computed. He also identified *mzugs* which were maximal in terms of both the DTC and DTM metrics.

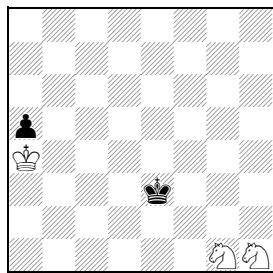
Tamplin (2001a) confirmed that the scans of DTC and DTM tables had yielded exactly the same sets of *mzugs* of each type, valuable confirmation in itself of the integrity of the EGTs. There are 21,677 *ww*, 3,395 *bw* and 33 *fp* 2-to-5-man-*mzugs*. These occur in 59 of the 146 endgames. The full point *mzugs* occur in just six endgames, namely KBPKP, KNP KP, KP KP, KPP KP, KPP KR and KR PKP: each features at least two Pawns. The complete data, statistics and various sets of positions, are available (Tamplin, 2001b). Here, the statistics are in Table 1 and the examples of maximal *mzugs* are in Table 2. Some explanatory notes are appropriate:

- White has at least as many men as Black: the men are listed in the standard K-Q-R-B-N-P order and endgames are listed in alphabetical order. The GBR code *qrn.wb* indicates an endgame with *w* white and *b* black Pawns, *q mod 3* white and $\lfloor q/3 \rfloor$ black Queens. *r*, *c* and *n* similarly define the Rs, Bs and Ns.

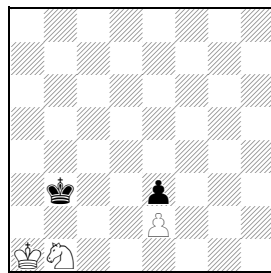
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- with one exception, all positions are essentially unique, i.e. they cannot be transformed into another listed position by board transformation or by switching colours. The exception is that in symmetric endings, only KPKP here, the set of *bw* mzugs is acknowledged even though it is transformed by colour-reversal into the set of *ww* mzugs. The brackets in Table 1 highlight this equivalence.
- depths are in side-to-move moves. In Table 2, *c*, *m* and *cm* denote a DTC-, DTM- and DTC- & DTM-maximal position respectively. *ww*, *bw* and *fp* denote the three types of mzug as above.
- the positions are in *canonical form* in the sense that:
 - the wK is confined to a-d for endgames with Pawns, and to a1-d1-d4 for endgames without Pawns
 - if there are no Pawns and the wK is on a1-d4, the bK is confined to a1-h1-h8
 - if both Kings are on a1-h8, only one of probably two equivalent positions is included in the count.
- a/the DTC-maximal mzug with maximal DTM depth has been cited. A DTM-maximal mzug with maximal DTC depth has also been cited if different, as it is in 16 endgames.
- the following 55 2-to-4-man and 3-2 endgames have no mzugs:

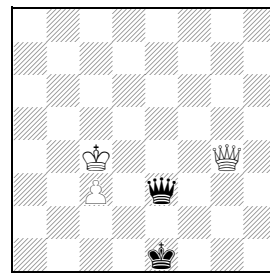
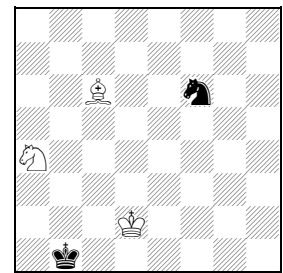
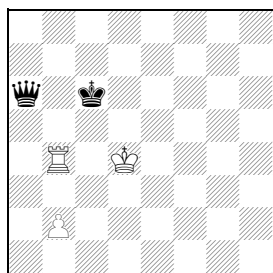
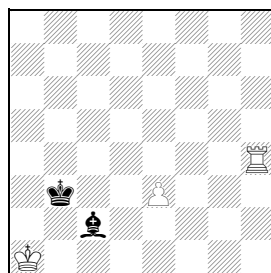
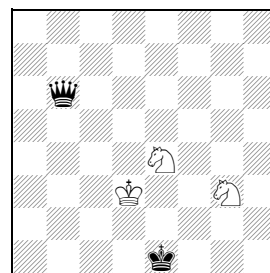
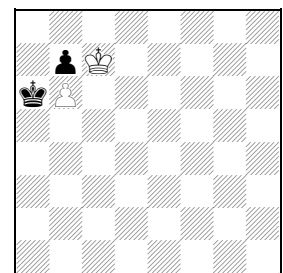
KBBK, KBBKB, KBK, KBKB/N, KBNK, KK, KNK, KNKN, KNNK, KNNKB, KQBK, KQKB/N/P/R, KQK, KQKB/N/P/Q/R, KQNK, KQNK/N/P/R, KQPK, KQPB/N/P, KQQK, KQQKB/N/P/Q/R, KQRK, KQRKB/N/P/R, KRBK, KRBKB/N, KRK, KRKR, KRNK, KRNKB/P, KRPK, KRRK, KRRKN/P/R.
- the only 4-1 endgames with mzugs are KBPPK, KNPPK and KPPPK, echoing the fact that the only 3-1 endgames with mzugs are KBPK, KNPK and KPPK.



MZ1. The deepest mzug



MZ2. The deepest fp mzug

MZ3. btm, $dc = 91$ MZ4. btm, $dc = 67$ MZ5. wtm, $dc = 63$ MZ6. btm, $dc = 58$ MZ7. wtm, $dc = 53$ 

MZ8. Le Trébuchet

3. HIGHLIGHTS

- KNNKP MZ1 max 3-to-5-man mzug: $dc = dm = 105$ moves.
 KNP KP MZ2 max fp mzug: (wtm) $dc = 1$ and $dm = 17$, (btm) $dc = 13$ and $dm = 28$.
 KQPKQ MZ3 max KQPKQ mzug: $dc = 91$ and $dm = 102$.
 KBNKN MZ4 max KBNKN mzug: $dc = 67$ and $dm = 97$.
 KBNKQ 8/8/q7/8/3K4/2N5/8/k1B5: only maximal P-less mzug with both Kings on a1-h8.
 KPPKR only fp mzugs not having Pawns on both sides: one is maxDTC, the other maxDTM.
 KPKP MZ8 the max fp mzug: like most KPKP positions, it has a twin with colours exchanged.
 KPKP provides 15 of the 33 full-point mzugs, all of the well known Le Trébuchet type.
 KRKN 8/8/8/4k3/3R4/2K5/1n6/8: diagonally symmetric mzug.

All full-point mzugs feature at least two pawns: a conjecture is that there is a 6-man fpz with only one pawn. The sets of maxDTC mzugs and maxDTM mzugs are either disjoint, identical or one is contained within the other: further details are given below.

Endgame		— DTC-maximal mzugs —									— DTM-maximal mzugs —									ZC ^ ZM					
		mzug count			counts			max DTC			max-DTC mzugs' max DTM			counts			max DTM			max-DTM mzugs' max-DTC			counts		
Title	w-b	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp	ww	bw	fp
KBBKN	3-2	1	0	0	1	0	0	2	---	---	2	---	---	1	0	0	2	---	---	2	---	---	1	0	0
KBBKP	3-2	1	0	0	1	0	0	2	---	---	2	---	---	1	0	0	2	---	---	2	---	---	1	0	0
KBBKQ	3-2	0	1	0	0	1	0	---	5	---	---	17	---	0	1	0	---	17	---	---	5	---	0	1	0
KBBKR	3-2	3	0	0	3	0	0	1	---	---	19	---	---	1	0	0	19	---	---	1	---	---	1	0	0
KBKP	2-2	0	1	0	0	1	0	---	1	---	---	12	---	0	1	0	---	12	---	---	1	---	0	1	0
KBNKB	3-2	45	0	0	1	0	0	10	---	---	36	---	---	1	0	0	36	---	---	10	---	---	1	0	0
KBNKN	3-2	922	0	0	1	0	0	67	---	---	97	---	---	1	0	0	97	---	---	67	---	---	1	0	0
KBNKP	3-2	61	1	0	1	1	0	13	1	---	36	45	---	4	1	0	37	45	---	8	1	---	0	1	0
KBNKQ	3-2	0	1	0	0	1	0	---	33	---	---	44	---	0	1	0	---	44	---	---	33	---	0	1	0
KBNKR	3-2	6	2	0	1	1	0	5	8	---	26	28	---	1	1	0	32	28	---	1	8	---	0	1	0
KBP K	3-1	6	0	0	1	0	0	14	---	---	20	---	---	1	0	0	20	---	---	14	---	---	1	0	0
KBP KB	3-2	160	0	0	2	0	0	33	---	---	44	---	---	2	0	0	44	---	---	33	---	---	2	0	0
KBP KN	3-2	212	13	0	2	12	0	37	2	---	47	2	---	2	12	0	50	2	---	19	2	---	0	12	0
KBP KP	3-2	403	2	1	1	1	1	20	2	2+8	31	11	16+22	1	1	1	31	14	16+22	20	1	2+8	1	0	1
KBP KQ	3-2	0	16	0	0	2	0	---	20	---	---	28	---	0	1	0	---	28	---	---	20	---	0	1	0
KBP KR	3-2	4	302	0	1	1	0	3	15	---	32	29	---	1	1	0	32	33	---	3	14	---	1	0	0
KBPP K	4-1	6	0	0	1	0	0	6	---	---	12	---	---	1	0	0	16	---	---	1	---	---	0	0	0
KNKP	2-2	22	7	0	6	1	0	6	9	---	6	24	---	6	1	0	6	24	---	6	9	---	6	1	0
KNNKN	3-2	362	0	0	85	0	0	4	---	---	4	---	---	85	0	0	4	---	---	4	---	---	85	0	0
KNNKP	3-2	3124	19	0	1	2	0	105	3	---	105	39	---	1	1	0	105	53	---	105	1	---	1	0	0
KNNKQ	3-2	0	229	0	0	1	0	---	53	---	---	62	---	0	1	0	---	62	---	---	53	---	0	1	0
KNNKR	3-2	0	25	0	0	1	0	---	5	---	---	28	---	0	1	0	---	35	---	---	2	---	0	0	0
KNP K	3-1	75	0	0	1	0	0	14	---	---	19	---	---	2	0	0	19	---	---	14	---	---	1	0	0
KNP KB	3-2	640	2	0	1	1	0	28	2	---	40	2	---	1	1	0	40	2	---	28	2	---	1	1	0
KNP KN	3-2	4128	63	0	3	42	0	40	2	---	56	2	---	1	42	0	56	2	---	40	2	---	1	42	0
KNP KP	3-2	2281	14	8	1	6	1	21	5	1+3	29	24	17+28	1	1	1	33	24	17+28	19	5	1+3	0	1	1
KNP KQ	3-2	0	52	0	0	1	0	---	29	---	---	45	---	0	1	0	---	45	---	---	29	---	0	1	0
KNP KR	3-2	23	158	0	2	1	0	4	34	---	19	58	---	1	1	0	28	58	---	3	34	---	0	1	0
KNPP K	4-1	93	0	0	6	0	0	5	---	---	11	---	---	1	0	0	16	---	---	1	---	---	0	0	0
KP K	2-1	80	0	0	1	0	0	18	---	---	27	---	---	1	0	0	27	---	---	18	---	---	1	0	0
KP KP	2-2	106	106	15	1	1	15	10	10	1+1	24	24	19+12	2	2	1	27	27	19+12	6	6	1+1	0	0	1
KP P K	3-1	43	0	0	6	0	0	11	---	---	26	---	---	1	0	0	26	---	---	11	---	---	1	0	0
KP P KB	3-2	211	1	0	2	1	0	14	2	---	25	2	---	1	1	0	28	2	---	12	2	---	0	1	0
KP P KN	3-2	920	157	0	1	11	0	22	12	---	36	12	---	2	15	0	36	12	---	22	12	---	1	11	0
KP P KP	3-2	4179	52	6	3	2	1	24	7	6+3	38	18	15+19	1	2	1	86	23	16+20	10	1	4+3	0	0	0
KP P KQ	3-2	0	2	0	0	1	0	---	8	---	---	13	---	0	1	0	---	13	---	---	8	---	0	1	0
KP P KR	3-2	18	99	2	1	1	1	11	21	1+4	39	36	18+17	1	1	1	39	36	15+21	11	21	3+1	1	1	0
KP P P K	4-1	11	0	0	2	0	0	7	---	---	17	---	---	1	0	0	17	---	---	7	---	---	1	0	0
KQBKQ	3-2	25	0	0	1	0	0	9	---	---	11	---	---	1	0	0	14	---	---	6	---	---	0	0	0
KQNKQ	3-2	38	0	0	1	0	0	24	---	---	30	---	---	1	0	0	30	---	---	24	---	---	1	0	0
KQPKQ	3-2	640	0	0	1	0	0	91	---	---	102	---	---	1	0	0	102	---	---	91	---	---	1	0	0
KQPKR	3-2	1	0	0	1	0	0	2	---	---	11	---	---	1	0	0	11	---	---	2	---	---	1	0	0
KQRKQ	3-2	1	0	0	1	0	0	11	---	---	18	---	---	1	0	0	18	---	---	11	---	---	1	0	0
KRBKP	3-2	1	0	0	1	0	0	1	---	---	8	---	---	1	0	0	8	---	---	1	---	---	1	0	0
KRBKQ	3-2	0	372	0	0	1	0	---	38	---	---	67	---	0	1	0	---	67	---	---	38	---	0	1	0
KRBKR	3-2	17	0	0	1	0	0	49	---	---	55	---	---	1	0	0	55	---	---	49	---	---	1	0	0
KRKB	2-2	5	0	0	1	0	0	14	---	---	25	---	---	1	0	0	25	---	---	14	---	---	1	0	0
KRKN	2-2	18	0	0	2	0	0	10	---	---	24	---	---	2	0	0	24	---	---	10	---	---	2	0	0
KRKP	2-2	12	0	0	6	0	0	6	---	---	20	---	---	4	0	0	20	---	---	6	---	---	4	0	0
KRNKN	3-2	3	0	0	1	0	0	19	---	---	24	---	---	2	0	0	24	---	---	19	---	---	1	0	0
KRNKQ	3-2	0	455	0	0	1	0	---	42	---	---	63	---	0	1	0	---	63	---	---	42	---	0	1	0
KRNKR	3-2	10	0	0	2	0	0	22	---	---	23	---	---	2	0	0	23	---	---	22	---	---	2	0	0
KRP KB	3-2	225	0	0	1	0	0	58	---	---	67	---	---	1	0	0	67	---	---	58	---	---	1	0	0
KRP KN	3-2	413	0	0	4	0	0	37	---	---	47	---	---	1	0	0	47	---	---	37	---	---	1	0	0
KRP KP	3-2	0	2	1	0	2	1	---	1	1+4	---	15	15+15	0	1	1	---	15	15+15	---	1	1+4	0	1	1
KRP KQ	3-2	2	241	0	2	1	0	2	63	---	12	91	---	1	1	0	12	91	---	2	63	---	1	1	0
KRP KR	3-2	209	0	0	1	0	0	43	---	---	54	---	---	2	0	0	54	---	---	43	---	---	1	0	0
KRRKB	3-2	1	0	0	1	0	0	8	---	---	16	---	---	1	0	0	16	---	---	8	---	---	1	0	0
KRRKQ	3-2	10	0	0	1	0	0	7	---	---	20	---	---	1	0	0	20	---	---	7	---	---	1	0	0

Table 1. Mutual zugzwangs: the statistics.

As is usual, the statistics of Table 1 may be affected by the presence of unreachable positions in which the side-not-to-move is not in check. Elkies (2000) and van der Heijden (2000) have both pointed out such positions:

KBPKP: 8/8/8/1p6/1k6/1P6/BK6

KBPPK: kB6/8/1PK5/1P6/8/8/8/8, kB6/8/KP6/1P6/8/8/8/8 ... leaving 4 of the 6

KNPPK: kN6/8/KP6/1P6/8/8/8/8, kN6/8/1PK5/1P6/8/8/8/8, K1k5/P1PN4/8/8/8/8/8 ... leaving 90 of 93.

Endgame	type	dc	dm	a maximal mzug	Endgame	type	dc	dm	a maximal mzug
KBBKN 0023	cm ww	2	2	8/8/8/8/6n12K4B/kB6	KNPPK 0001.20	c ww	5	11	K7/P Ik5/P7/8/8/8/N7
KBBKP 0020.01	cm ww	2	2	B Ik5/lpB5/3K4/8/8/8/8	m ww	1	16	kN6/8/1P K5/1P 6/8/8/8/8	
KBBKQ 3020	cm bw	5	17	8/8/8/8/q7/2BB4/IK6/3k4	KP K 0000.10	cm ww	18	27	8/8/8/8/k6/8/IK6/6P 1/8
KBBKR 0320	cm ww	1	19	8/8/8/B7/8/3k4/2r5/KB6	KP KP 0000.11	c ww	10	24	8/k7/6p1K5P 1/8/8/8/8
KBKP 0010.01	cm bw	1	12	8/8/8/8/8/lpK5/kB6	m ww	6	27	8/8/k7/8/K5p1/8/5P 2/8	
KBNKB 0041	cm ww	10	36	8/8/8/8/lb6/8/2K5/kIBN4	c bw	10	24	8/8/8/8/k5p1/6P 1/K7/8	
KBNKN 0044	cm ww	67	97	8/8/2B2n2/8/N7/8/3K4/IK6	m bw	6	27	8/5p2/8/k5P 1/8/K7/8/8	
KBNKP 0011.01	c ww	13	36	8/8/2K5/8/2k5/5p2/8/2N1B3	cm fp	1+1	19+12	8/lpK5/kP 6/8/8/8/8/8	
	m ww	8	37	8/8/8/8/8/B7/p3N3/k2K4	KP P K 0000.20	cm ww	11	26	8/8/8/8/k6/1P 6/1P K5/8
	cm bw	1	45	8/8/8/8/IN6/3K4/B7/5p2/k7	KP P KB 0030.20	c ww	14	25	lb6/8/3P 4/1P 3/8/8/8/3KIk2
KBNKQ 3011	cm bw	33	44	8/8/q7/8/3K4/2N5/8/kIB5	m ww	12	28	8/8/5b1P 8/IK6/1P 6/IK6/8	
KBNKR 0311	c ww	5	26	3r4/8/2B5/8/IN6/8/8/kIK5	cm bw	2	2	8/8/8/8/8/b2k4/P 2P 4/IK6	
	m ww	1	32	8/8/8/8/B7/lr6/N Ik5/K7	KP P KN 0003.20	cm ww	22	36	8/5n2/8/K6P 3/P 4/k7/8/8
	cm bw	8	28	8/r7/8/B7/8/8/N Ik5/K7	cm bw	12	12	K7/2k5/8/P 7/1P 7/5n2/8/8	
KBPK 0010.10	cm ww	14	20	IB IK4/8/8/k7/8/P 7/8/8	KP P KP 0000.21	c ww	24	38	IK3k2/8/7p/8/8/7P 6P 1/8
KBP KB 0040.10	cm ww	33	44	8/8/8/2P 5/4b3/IB 6/8/kIK5	m ww	10	86	8/2p5/8/8/8/2k1P 3/P 7/3K4	
KBP KN 0013.10	c ww	37	47	8/8/8/8/K5n1B5P 1/k7	c bw	7	18	IKk5/8/7p/P 7/2P 5/8/8/8	
	m ww	19	50	k7/8/Kn1B3P 3/8/8/8/8	m bw	1	23	8/2p5/8/8/1P IP 4/2k5/8/2K5	
	cm bw	2	2	K7/P Ik5/8/8/8/IB6/ln6/8	c fp	6+3	15+9	8/8/8/2k5/Klp5/P 3P 3/8/8	
KBP KP 0010.11	cm ww	20	31	2K5/8/3k4/8/7p/5P 2/8/7B	m fp	4+3	16+20	8/8/8/8/5k2/3Klp2/3P 3P 8	
	c bw	2	11	8/8/8/8/8/IK6/1P lp4/KB6	KP P KQ 3000.20	cm bw	8	13	8/2KP 3q/8/2P 3k1/8/8/8/8
	m bw	1	14	8/8/8/8/8/2p5/2P 5/kBK5	KP P KR 0300.20	cm ww	11	39	2k5/K6P 6/P r/8/8/8/8/8
	cm fp	2+8	16+22	8/8/8/8/8/klp5/2P 5/IBK5	cm bw	21	36	8/8/8/8/2P 5/2KIP 3/4r3/2k5	
KBP KQ 3010.10	cm bw	20	28	5k2/1P IK4/lqB5/8/8/8/8/8	c fp	1+4	18+17	lrIk4/1P 6/1P K5/8/8/8/8/8	
KBP KR 0310.10	cm ww	3	32	K7/lrB IP 3/k7/8/8/8/8/8	m fp	3+1	15+21	8/8/8/8/k7/r1P 5/1K6/5/8	
	c bw	15	29	8/8/4r3/8/2k5/K6P 5/B2/8	KP P P K 0000.30	cm ww	7	17	8/8/8/8/IK IP 4/8/P K6/P 7/8
	m bw	14	33	8/8/8/8/k7/r1P 5/IK6/B7	KQB KQ 4010	c ww	9	11	8/3K4/3B4/8/k7/3Q4/8/2q5
KBPPK 0010.20	c ww	6	12	8/B Ik5/K7/P 7/P 7/8/8/8/8	m ww	6	14	lq6/8/2Q5/B7/8/IK6/8/IK6	
	m ww	1	16	kB6/8/1P K5/1P 6/8/8/8/8	KQN KQ 4001	cm ww	24	30	8/3q4/1QIN4/8/k7/8/3K4/8
KNKP 0001.01	cm ww	6	6	8/8/8/8/8/p7/k2N4/2K5	KQP KQ 4000.10	cm ww	91	102	2k5/K6P 6/P r/8/8/8/8/8
	cm bw	9	24	8/K7/N Ik5/p7/8/8/8/8/8	KQP KR 1300.10	cm ww	2	11	k7/8/KQ lr4/P 7/8/8/8/8/8
KNNKN 0005	cm ww	4	4	8/8/8/8/n7/8/2KN4/kN6	KQR KQ 4100	cm ww	11	18	8/8/8/8/IR6/k4q2/8/IK2Q3
KNNKP 0002.01	cm ww	105	105	8/8/8/p7/k7/4k3/8/6NN	KRB KP 0110.01	cm ww	1	8	IkIK4/7R/8/8/8/8/6p1/7B
	c bw	3	39	7N/8/KIN5/8/8/lpk5/8/8	KRB KP 3110	cm bw	38	67	lq6/8/IB3R2/8/k7/8/8/IK6
	m bw	1	53	IN6/8/8/N7/K7/8/kp6/8	KRB KR 0410	cm ww	49	55	5R2/8/8/8/8/3K4/5Br1/2k5
KNNKQ 3002	cm bw	53	62	8/8/lq6/8/4N3/3K2N1/8/4k3	KRKB 0130	cm ww	14	25	8/8/lb6/5R2/8/3K4/8/2k5
KNNKR 0302	c bw	5	28	6rN/5N2/8/8/8/2k5/8/IK6	KRKN 0103	cm ww	10	24	8/8/8/6n1/3K4/4R3/3k4/8
	m bw	2	35	5N2/IN6/8/3r4/8/2k5/8/2K5	KRP K 0100.01	cm ww	6	20	8/K7/8/k7/lp6/8/8/IR6
KNP K 0001.10	cm ww	14	19	8/8/8/8/8/IK IP 4/8/KN6	KRNKN 0104	cm ww	19	24	8/8/8/8/8/3n4/N2k4/RK6
KNP KB 0031.10	cm ww	28	40	8/6b1/8/8/IN5P 8/2K5/k7	KRNKQ 3101	cm bw	42	63	INk5/8/8/8/8/IR6/6q1/2K5
	cm bw	2	2	K5b1P Ik2N2/8/8/8/8/8/8	KRNKR 0401	cm ww	22	23	8/8/8/8/8/2KRN3/8/2k1r3
KNP KN 0004.10	cm ww	40	56	2n5/8/8/IN6/8/6P 1/8/kIK5	KRP KB 0130.10	cm ww	58	67	8/8/8/8/7R/IK2P 3/2b5/K7
	cm bw	2	2	K7/P 2n4/IK6/N7/8/8/8/8	KRP KN 0103.10	cm ww	37	47	K7/IR6/2n5/4k3/8/4P 3/8/8
KNP KP 0001.11	c ww	21	29	8/8/8/8/8/2k2p2/P 7/IKN5	KRP KP 0100.11	cm bw	1	15	8/8/8/8/8/lp6/kP 6/IRK5
	m ww	19	33	K7/N6p/k7/8/8/8/7P 8	cm fp	1+4	15+15	8/8/8/8/8/2p5/1kP 5/2Rk4	
	cm bw	5	24	8/8/8/8/6P 1/3k2p1/8/2KN4	KRP KQ 3100.10	cm ww	2	12	7k/2K2P lq/8/8/8/8/5R2/8
	cm fp	1+3	17+28	8/8/8/8/8/IK2p3/4P 3/KN6	cm bw	63	91	8/8/q Ik5/8/IR IK4/8/1P 6/8	
KNP KQ 3001.10	cm bw	29	45	4N3/3P 4/2K5/q7/IK6/8/8/8	KRP KR 0400.10	cm ww	43	54	8/8/8/8/8/2RP r3/8/2KIk3
KNP KR 0301.10	c ww	4	19	k6r/5P 2/K7/3N4/8/8/8/8	KRR KB 0230	cm ww	8	16	8/8/8/8/8/b Ik5/IR6/IRK5
	m ww	3	28	3k4/rIN5/2KP 4/8/8/8/8/8	KRR KQ 3200	cm ww	7	20	6R1/8/8/8/6R1/7q/IK5k/8
	cm bw	34	58	8/8/8/5N2/K7/2k5/P 2r4/8					

Table 2. Maximal mutual zugs: examples.

For a given endgame and type of mzug, the sets ZC and ZM are the sets of DTC-maximal and DTM-maximal mzugs respectively. ZC & ZM are either disjoint, identical or one is a subset of the other. There is no endgame for which $ZC - ZM \neq \emptyset$ and $ZM - ZC \neq \emptyset$.

$ZC \cap ZM = \emptyset$	KBNKP ww, KBNKR ww, KBPKN ww, KBPKP bw, KBPKR bw, KNNKP bw, KNNKR bw, KNPKP ww, KNPKR ww, KPKP ww (and therefore bw), KPPKB ww, KPPKP ww, bw and fp, KPPKR fp and KQBKQ ww.
$ZC \subset ZM$	KNPK ww (1-2), KPPKN ww (1-2) and bw (11-15), KRNKN ww (1-2) and KRPKR ww (1-2)
$ZC \supset ZM$	KBBKR ww (3-1), KBPKQ bw (2-1), KNPKN ww (3-1), KNPKP bw (6-1), KPKP fp (15-1), KPPK ww (6-1), KRKP ww (6-4), KRPKN ww (4-1), KRPKP bw (2-1), KRPKQ ww (2-1)

4 SUMMARY

A complete set of data about 3-to-5-man mzugs in chess has been compiled, second-sourced, summarised here and made available via the web. This includes maximal mzug and depth statistics to both the DTC and DTM metrics.

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