CP violation in SUSY

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Abstract. CP violation in supersymmetric models is reviewed with focus on explicit CP violation in
the MSSM. The topics covered in particular are CP mixing in the Higgs sector and its measurement
at the LHC, CP-odd observables in the gaugino sector at the ILC, EDM constraints, and the
neutralino relic density.

PACS. 12.60.Jv Supersymmetric models (11.30.Er Charge conjugation, parity, time reversal, and
other discrete symmetries)

1 Introduction

Test of the discrete symmetries, charge conjugation
C, parity P, and time-reversal T, have played an im-
portant role in establishing the structure of Standard
Model (SM). In particular, CP violation has been ob-
erved in the electroweak sector of the SM in the K
and B systems. It is linked to a single phase in the
unitary Cabbibo-Kobayashi-Maskawa (CKM) matrix
(describing transitions between the three genera-
tions of quarks; see e.g. [1] for a detailed review). It is in-
portant to note that this source of CP violation is strictly
non-diagonal.

The strong sector of the SM also allows for CP vio-
lation through a dimension-four term $GG$, which is of
topological origin. Such a term would lead to a vanish-
ing CP violation and hence to electric dipole mo-
mements (EDMs). The current experimental limits on
the EDMs of atomic states and neutrons [2,3,4]

\[ \langle d_{1j} \rangle_{< 9} \quad 10^{-25} \text{ cm} \quad (90\% \text{ C.L.}) \]
\[ \langle d_{3j} \rangle_{< 2} \quad 10^{-28} \text{ cm} \quad (95\% \text{ C.L.}) \]
\[ \langle d_{1j} \rangle_{< 6} \quad 10^{-26} \text{ cm} \quad (90\% \text{ C.L.}) \]

however constrain the strong CP phase to $j < 10^{-9}$.

A comprehensive discussion of this issue can be found
in [5]. While it appears to be extremely tuned, the CKM
contribution to the EDMs is several orders of mag-
nitude below the experimental bounds, e.g.

\[ \langle d_{1j}^{\text{CKM}} \rangle \quad 10^{-32} \text{ cm} \].

Therefore, while providing in principle new con-
temporal constraints, the current EDM bounds still leave ample room for new sources of CP violation beyond the SM.

Such new sources of CP violation are indeed very
interesting in point of view of the observed baryon
asymmetry of the Universe

\[ n_B \quad n \quad (6 \pm 14 \quad 0.25) \quad 10^{10} \]

with $n_B$, $n$ and $n_B$ and $n$ the number densities of baryons,
anti-baryons and photons, respectively; see [6,7] for re-
cent reviews. The necessary ingredients for baryon-
genesis [8] i) baryon number violation, ii) C and CP vio-
lation and iii) departure from equilibrium are in prin-
ciple present in the SM, however not with sufficient
strength. In particular, the amount of CP violation is
not enough. This provides a strong motivation to con-
sider CP violation in extensions of the SM, as reviewed
e.g. in [9].

In general, CP violation in extensions of the SM
may be either explicit or spontaneous. Explicit CP vio-
lation occurs through phases in the Lagrangian, which
cannot be rotated away by field redefinitions. This is the
standard case in the MSSM, on which we will con-
centrate in the following. Spontaneous CP violation,
on the other hand, occurs if an extra Higgs develops a complex vacuum expectation value. This can lead to a vanish-
ing term as well as to a complex CKM matrix. Spontaneous CP violation is a very in-
teresting and elegant idea, but difficult to realize in
SUSY and obviously not possible in the MSSM (where
the Higgs potential conserves CP). There has, however,
been very interesting work on left-right symmetric mod-
els and SUSY GUTs. For instance, models based
on supersymmetric SO (10) may provide a link with the
neutrino seesaw and leptogenesis. I do not follow
this further in this talk but refer to [9] for a review.

2 CP violation in the MSSM

In the general MSSM, the gaugino mass parameters
$M_i$ (i = 1, 2; 3), the higgsino mass parameter, and
the trilinear couplings $A_{\xi}$ can be complex,

\[ M_i = M_i j e^{i \alpha_i} = j e^{i \alpha_i} \quad A_{\xi} = A_{\xi} j e^{i \epsilon} \quad (3) \]

(assuming B to be real by convention) thus induc-
ing explicit CP violation in the model. Not all of the
phases in eq. (3) are, however, physical. The physical combinations indeed are $\operatorname{Arg}(M_1)$ and $\operatorname{Arg}(A_\beta)$. They can

1. act particle masses and couplings through their mixing,
2. induce CP mixing in the Higgs sector through radiative corrections,
3. influence CP-even observables like cross sections and branching ratios,
4. lead to interesting CP-odd asymmetries at colliders.

Non-trivial phases, although constrained by EDMs, can significantly influence the collider phenomenology of Higgs and SUSY particles, and as we will see also the properties of neutralino dark matter.

Let me note here that CP violation in the MSSM alone is a large eld with a vast amount of literature; it is essentially possible to give a complete review in 25 min. I will hence not try a tour de force but rather present some selected examples, and I apologize to those whose work is not mentioned here. This said, let us begin with the MSSM Higgs sector:

2.1 Higgs-sector CP mixing

The neutral Higgs sector of the MSSM consists in principle of two CP-even states, $H^0$ and $A^0$, and one CP-odd state, $A^0$. Complex parameters, eq. (3), here have a dramatic effect, inducing a mixing between the three neutral states through loop corrections \cite{11, 12}. The resulting mass eigenstates $H_1, H_2, H_3$ (with $m_{H_1} < m_{H_2} < m_{H_3}$ by convention) are no longer eigenstates of CP.\footnote{The large top Yukawa coupling, the largest tree couplings from stop loops, with the size of the CP mixing proportional to \cite{13}}

\[
\frac{3}{16} = m(A_\beta^0) = \frac{m_1^2}{m_{\tilde c}^2} \quad \text{or} \quad \frac{3}{16} \leq m(A_\beta^0) = \frac{m_1^2}{m_{\tilde c}^2}.
\]

CP mixing in the Higgs sector can change the collider phenomenology quite substantially. For example, it is possible for the lightest Higgs boson to develop a significant CP-odd component such that its coupling to a pair of vector bosons becomes vanishingly small. This also considerably weakens the LEP bound on the lightest Higgs boson mass \cite{14}, as illustrated in Fig. 1, which shows the LEP exclusions at 95\% CL (in light-grey or light-green) and 99.7\% CL (dark-grey or dark-green) for the CPX scenario with maximal phases; the top mass is taken to be $m_t = 174.3$ GeV. The CPX scenario \cite{15} is the default benchmark scenario for studying CP-violating Higgs-mixing phenomena. It is defined as

\[
\begin{aligned}
M_{\tilde c}, M_{\tilde \tau}, M_{\tilde \nu}, M_{c, \tilde c}, M_{\tilde \nu}, \equiv M_{\text{ SUSY}}; \\
j = 4 M_{\text{ SUSY}}, \quad j = 2 M_{\text{ SUSY}}, \quad \beta = 1 \text{ TeV}.
\end{aligned}
\]

The free parameters are $\tan \beta$, the charged Higgs-boson pole mass $M_{H^\pm}$, the common SUSY scale $M_{\text{ SUSY}}$, and the CP phases. Typically one chooses $\beta = 0$, which leaves $\tan \beta$ and $\beta$ as the relevant ones. The ATLAS discovery potential \cite{16} for Higgs bosons in the CPX scenario with $\beta = 0$ is shown in Fig. 2. As can be seen, also here there remains an uncovered region at small $\tan \beta$ and small Higgs mass, comparable to the holes at small $m_{H_2}$ in Fig. 1.

An overview of the implications for Higgs searches at different colliders is given in \cite{17}, and a review of MSSM Higgs physics at higher orders, for both CP-conserving and CP-violating cases, is in \cite{18}. For an extensive discussion of Higgs-sector CP violation, see the CPN@N report \cite{19}.

A question that naturally arises is whether and how the CP properties of the Higgs boson(s) can be determined at the LHC. (At the ILC, which is a high-precision machine in particular for Higgs physics, this can be done quite well, see \cite{20} and references therein).
A very promising channel is $H \to ZZ \to 4\text{ leptons}$; cf. the contributions by Godbole et al. [19]. Were here follow Godbole et al. [19]: The $ZZ$ coupling can be written as in the general form

$$g_{ZZ} \left( a \cos \theta_1 + b \sin \theta_1 \right) + c \sin \theta_1 \sin \theta_2 ;$$

up to a factor $g=(m_2 \cos \psi)$, where $k_1$ and $k_2$ the four-momenta of the two $Z$ bosons. The terms associated with $a$ and $b$ are CP-even, while that associated with $c$ is CP-odd. Not totally antisymmetric with $a_{123} = 1$. CP violation is realized if at least one of the CP-even terms is present (i.e. either $a$, $b$, or $c$ is non-zero). This can be tested through polar and azimuthal angular distributions in $H \to ZZ \to (f_1 f_1)(f_2 f_2)$, etc. Fig. 4. Denoting the polar angles of the fermions $f_1, f_2$ in the rest frame 1 and 2, we have e.g.

$$\cos \theta_1 = \frac{p_{f_1}}{|p_{f_1}|} \frac{p_{f_1}}{|p_{f_1}|} \frac{p_{f_2}}{|p_{f_2}|} \frac{p_{f_2}}{|p_{f_2}|}$$

where $p_1$ are the three-vectors of the corresponding fermion with $p_{f_1}$ and $p_{f_2}$ in their parent $Z$’s rest frame but $p_{f_1}$ and $p_{f_2}$ in the Higgs rest frame, see Fig. 3. The angular distribution in $\cos \theta_1$ for a CP-odd state is $(1 + \cos^2 \theta_1)$, corresponding to transversely polarized $Z$ bosons, which is very distinct from the purely CP-even distribution proportional to $\sin^2 \theta_1$ for longitudinally polarized $Z$ bosons in the large Higgs mass limit. $m > m (c \geq 0)$ will introduce a term linear in $\cos \theta_1$, leading to a forward-backward asymmetry. The distributions for $\cos \theta_1$ are shown in Fig. 4 for a Higgs mass of 200 GeV and a purely scalar, purely pseudoscalar, and CP-mixed scenario. The asymmetry is absent if CP is conserved for both CP-odd and CP-even states but is non-zero $m > m (c \geq 0)$ while simultaneously $a > 0$. A further probe of CP violation is the azimuthal angular distribution $d \cos \theta_1$ with the angle between the planes of the fermion pairs, see Fig. 3. For a detailed discussion of various distributions and asymmetries sensitive to CP violation in $H \to ZZ \to 4\text{ leptons}$, see [21].

Another possibility to test Higgs CP mixing at the LHC are correlations arising in the production process. Here the azimuthal angle correlations between the two additional jets in $H \to jj$ events have emerged as a promising tool [22]. Higgs boson production in association with two tagging jets, analyzed in detail in [23], is mediated by electroweak vector boson fusion and by gluon fusion. The latter proceeds through top-quark loops, which induce an effective Hgg vertex. Writing the $H tt$ Yukawa coupling as $\lambda t = \lambda_3 H t + \lambda_4 A t^2$, where $H$ and $A$ denote scalar and pseudoscalar Higgs fields, the tensor structure of the effective Hgg vertex has the form [24, 22]

$$T_a = a_2 (q_1, q_2) + a_3 (q_1, q_2);$$

with

$$a_2 = \frac{\lambda_3}{\sqrt{\lambda_3^2 + \lambda_4^2}} \frac{s}{\cos \phi_1},$$

$$a_3 = \frac{\lambda_4}{\sqrt{\lambda_3^2 + \lambda_4^2}} \frac{s}{\cos \phi_2}.$$
having an important impact on the masses and couplings of these particles. This is particularly interesting for the precision measurement envisaged at the ILC. The effects of CP phases in measurements of neutralinos, charginos, and sfermions at the ILC have been studied in great detail by various groups; see below as well as references in [9,20]. They fall into two different classes. On the one hand, there are CP-even observables: spin dependent observables, cross sections, branching ratios, etc. If measured precisely enough, they allow for a parametric determination of the CP phases; see below as well as references in [9,20]. They fall into two different classes. On the other hand, there are CP-odd (or T-odd) observables, e.g. rate asymmetries or triple-product asymmetries, which are a direct signal of CP violation. Indeed, the measurement of CP-odd effects is necessary to prove that CP is violated, and to determine the CP odd parameters, including phases, in an unambiguous way.

An example for a rate asymmetry is the chargino decay into a neutralino and a W boson, \( \tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^- W^+ \). Here, non-zero phases can induce an asymmetry between the decay rates of \( \tilde{\chi}_1^- \) and \( \tilde{\chi}_1^+ \),

\[
A_{CP} = \frac{(\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^- W^+) - (\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^+ W^+)}{(\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^- W^+) + (\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^+ W^+)} \quad (10)
\]

through absorptive parts in the one-loop corrections [30,31,32].

Figure 6 shows the dependence of \( A_{CP} \) on \( \lambda \) for \( M_1 = 500 \text{ GeV} \), \( M_{\tilde{\chi}_1} = 600 \text{ GeV} \), \( M_{\tilde{\chi}_2} = 400 \text{ GeV} \), and various \( M_{\tilde{\chi}_1} \). \( A_{CP} \) has its maximum at \( j = 2 \) and is larger at large negative values of the phase of \( M_1 \). The obvious advantage of such a rate asymmetry is that it can be measured in a "simple" counting experiment. A analogous asymmetry have been computed for Higgs particles [31,32,33]. Ref. [34] also discusses CP-violating forward-backward asymmetries.

Triple-product asymmetries rely on spin correlations between particle production and decay processes. They have been computed for neutralino [44,45,46,47,48] and chargino [49,50,51] production in \( e^+e^- \) followed by two- or three-body decays. Let me take the most recent work [49] on chargino-pair production with subsequent three-body decay as an illustrative example. The processes considered are \( e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \) at a linear collider with longitudinal beam polarizations, followed by three-body decays of the \( \tilde{\chi}_1^- \),

\[
\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^- \chi_1^+ W^+ \quad \text{or} \quad \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^- \chi_1^0 s c \; (11)
\]

where \( \chi_1^0 \). It is assumed that the m om enta \( p_{\tilde{\chi}_1} \), \( p_{\chi_1^+} \), \( p_{\chi_1^0} \), and \( p_{\chi_1^0} \) of the associated particles can be measured or reconstructed. The relevant triple products are:

\[
T_r = p_{\tilde{\chi}_1} \cdot (p_{\chi_1^+} p_{\chi_1^0}) \quad (12)
\]

\[
T_q = p_{\chi_1^+} (p_{\chi_1^0} p_{\chi_1^0}) \quad (13)
\]

Note that \( T_r \), relates \( p_{\tilde{\chi}_1} \) of initial, intermediate and final particles, whereas \( T_q \), uses only \( p_{\chi_1^+} \) from the initial and final states. Therefore, both triple products depend in a different way on the production and decay processes. From \( T_{rR} \) one can define T-odd asymmetries

\[
A_T(T_{rR}) = \frac{N[< \lambda >] - N[> \lambda >]}{N[< \lambda >] + N[> \lambda >]} \quad (14)
\]

where \( N[< \lambda >] \) (\( N[> \lambda >] \) ) is the number of events for which \( T_{rR} > ( < ) \). A genuine signal of CP violation is obtained by combining \( A_T(T_{rR}) \) with the corresponding asymmetry for the charge-conjugated processes:

\[
A_{CP}(T_{rR}) = \frac{A_T(T_{rR}) + A_T(T_{rR})}{2} \quad (15)
\]

Figure 7 shows the phase dependence of \( A_{CP}(T_{rR}) \) for \( e^+e^- \rightarrow \tilde{\chi}_1^- \tilde{\chi}_1^- \) followed by \( \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^- \chi_1^+ W^+ \quad \text{or} \quad \tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^- \chi_1^0 s c \).
A\text{CP}(T_j) \text{ in } \% \\
\phi_{M_1}/\pi \\
\text{(a)}

A\text{CP}(T_j) \text{ in } \% \\
\phi_{M_1}/\pi \\
\text{(b)}

Fig. 7. CP asymmetry $A_{\text{CP}}(T_j)$ for $e^+e^- \rightarrow \gamma \gamma + \nu\bar{\nu}$ with subsequent decay $\gamma \rightarrow \mu^+\mu^-$. The phase $\theta_1$ is scanned from $0$ to $\frac{\pi}{2}$ for $\phi_0 = 280$ GeV and beam polarizations $(P_x; P_y) = (0.0; 0.1)$ (solid), $(P_x; P_y) = (0.0; 0.1)$ (dashed); the parameter are $M_1 = 280$ GeV, $j_j = 200$ GeV, $\tan \theta = 5$, with $\theta = 0$ in (a) and $\theta = 0$ in (b); from [44].

500 GeV and polarized $e$ beam s. The authors conclude that $A_{\text{CP}}(T_j)$ can be probed at the 5 level in a large region of the MSSM parameter space, while $A_{\text{CP}}(T_j)$ has a somewhat lower sensitivity.

2.3 EDM constraints

Let us next discuss the EDM constraints in some more detail. The constraints $\theta_1$, especially the one on $d_{13}$, translate into a tight bound on the electron EDM,

$$\theta_1 < 1.5 \times 10^{-27} \text{ cm}.$$  \hspace{1cm} (16)

Setting all soft breaking parameters in the selectron and gaugino sector equal to $M_{\text{SU3}}$, one can derive a simplified formula for the one-loop contributions [45]

$$d_S = f_S \left[ \frac{g_0^2}{24} + \frac{g_1^2}{24} \sin[\text{Arg}(M_2)] \tan \frac{\phi}{\pi} \right] + \frac{g_1^2}{12} \sin[\text{Arg}(M_1; A_\ell)] \right);$$  \hspace{1cm} (17)

where $f_S = (m_0) = (16 \text{ GeV})^2$, and $\text{Arg}(M_2) = 0$ by convention. Note the small enhancement of the first term. It is the main reason why the phase $\theta_1$ is severely constrained than the phases of the $A$ parameters. The phases of the third generation, $\theta_{13}$ only enter the EDM at the two-loop level. However, there can be a similar small enhancement for these two-loop contributions [46], so they have to be taken into account as well.

Indeed, the EDM constraints pose a serious problem in the general MSSM: for O (100) GeV masses and O (1) phases, the EDMs are typically three(!) orders of magnitude too large [47,48,49,50]. Some other mechanisms are needed to satisfy the experimental bounds. The possibilities include

\begin{itemize}
  \item all phases,
  \item heavy sparticles [51,52,53,54],
  \item accidental cancellations [55,56,57,58,59,60,61],
  \item avour diagonal CP phases [62].
\end{itemize}

Detailed analyses of the EDM constraints have recently been performed e.g. in [56,63]. As example that large phases can be in agreement with the current EDM limits, Fig. 8 shows the results for the CM SSM benchmark point D of [65], which has $(m_{1,2}; m_0; \tan \theta) = (525; 130; 10)$. The strongest constraint comes from the EDM of $T_1$; that of $H_1$ is not shown because it is satisfied over the whole plane. As can be seen, for this benchmark point there is no limit to $\theta_1$, while $j_j = 0.065$. 

Fig. 8. The EDM constraints on the EDMs relative to their respective experimental limits in the $\phi_1$ plane for benchmark point D of [65]. Inside the shaded regions, the EDMs are less than or equal to their experimental bounds. Each of the EDMs vanishes along the black contour within the shaded region; from [65].
2.4 Neutralino relic density

If the \( \tilde{\chi}^0 \) is the lightest supersymmetric particle (LSP) and stable, it is a very good cold dark matter candidate. In the framework of them all freeze-out, its relic density is \( h^2 \) \( 1/\Omega_\chi v \), where \( h \) and \( v \) is the thermally averaged annihilation cross section sum and over all contributing channels. These channels are: annihilation of a bino LSP into fermion pairs through t-channel fermion exchange in case of very light particles; annihilation of a mixed bino-Higgsino or bino-Wino LSP into gauge boson pairs through t-channel chargino and neutralino exchange, and into top-quark pairs through s-channel Z exchange; and annihilation near a Higgs resonance (the so-called Higgs funnel); and finally coannihilation processes with sparticles that are close in mass with the LSP. Since the neutralino couplings to other (sparticles) sensitively depend on CP phases, the same can be expected for \( h \) and hence \( h^2 \).

The effect of CP phases on the neutralino relic density was considered in [54, 60, 66, 69, 70, 71, 72, 73], although only for specific cases. The most general analysis, (i) including all annihilation and coannihilation processes and (ii) separating the phase dependence of the couplings from pure kinematic effects, was done in [74].

It was found that modulations in the couplings due to non-trivial CP phases can lead to variations in the neutralino relic density of up to an order of magnitude. This is true not only for the Higgs funnel but also for other scenarios, like for instance the case of a mixed bino-Higgsino LSP. Even in scenarios which feature a modest phase dependence once the kinematic effects are singled out, the variations in \( h^2 \) are of parable to (and often much larger than) the \( 10\% \) range in \( h^2 \) of the WMAP bound. Therefore, when aiming at a precise prediction of the neutralino relic density from collider measurements, it is clear that one does not only need precise sparticle spectroscopy but one also has to precisely measure the relevant couplings, including possible CP phases.

This is illustrated in Fig. 9, which shows the regions where the relic density is in agreement with the 2 WMAP bound, \( 0.945 < \Omega_c h^2 < 0.287 \), for the case of a mixed bino-Higgsino LSP. When all phases are zero, only the narrow blue (dark grey) band is allowed. When allowing all phases to vary arbitrarily, while still satisfying the EDM constraints, the allowed band increases to the green (light grey) region. In the \( \tilde{\chi}^0 \) plane (left panel), the allowed range for \( h^2 \) increases roughly from 10 GeV to 50 GeV for a given \( \tilde{\chi}_1 \). In terms of relative mass differences (right panel) this means that in the CP-violating case much smaller \( -\frac{\cos \phi}{\sin \phi} \) mass differences can be in agreement with the WMAP bound than in the CP-conserving case.

3 Conclusions

The observed baryon asymmetry of the Universe necessitates new sources of CP violation beyond those of the SM. In this talk, I have discussed effects of such new CP phases, focusing on the case of the MSSM. The topics covered include CP mixing in the Higgs sector and its measurement at the LHC, CP-odd observables in the gaugino sector at the ILC, EDM constraints, and the neutralino relic density. Each topic was discussed by means of som e recent examples from the literature. For a more extensive discussion, in particular of topics that could not be covered here, I refer the reader to the recent review by Ibrahim and Nath [8].
References


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