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Spillovers Reconsidered: Analysing Economic Welfare under complementarities in R&D

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Abstract

We analyse economic welfare in R&D intensive industries under varying assumptions on the spillover process. The focus lies on spillover processes with complementary R&D investments such as those modelling absorptive capacity. There spillovers give rise to both negative and positive externalities. We show that the rationale for public policy intervention is strengthened where spillovers also have positive effects. This conclusion is based on the supermodularity of the spillover process and the investment game. We characterise a large class of spillover processes with similar implications for public policy. We show that results of much empirical work on absorptive capacity extend to this class of models.

JEL Classification: L13, O31

Key Words: spillovers, complementarity, absorptive capacity, supermodularity, oligopolistic R&D.

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1 Introduction

Much of the economic literature on innovation casts spillovers in a leading role or provides them an important supporting one. Spillovers may give rise to *appropriability problems* and it is usually assumed that this reduces private incentives for R&D. Meanwhile spillovers have the positive social benefit of spreading the fruits of R&D amongst competing firms, rendering the recipient firms more efficient producers and stronger competitors.

This provides the setting for an 'unpleasant tradeoff'[1] between the attempt to reinforce private incentives for R&D and the efficiency of the industry- or economy-wide R&D process. This tradeoff must be resolved by anybody wishing to design effective public policy for innovation intensive industries. Whilst the tradeoff is carefully explored in Spence (1984) his analysis relies on a *spillover process* in which a linear combination of own R&D and received R&D reduces a firm's marginal costs of production.

In this paper we analyse economic welfare is affected by complementarities between competing firms' research. We compare conventional *spillover processes* with more recent ones embodying complementarities. Such an effort is warranted in the face of a growing number of approaches to the modelling of spillovers[2].

For instance the notion that private incentives for R&D investment are decreasing in the level of involuntary spillovers relies on the assumption that firms have the capacity to absorb knowledge that spills over to them. Cohen and Levinthal (1989) showed that where the capacity to absorb outside knowledge depends on the extent of R&D activity by the receiving firm, private incentives to undertake R&D might increase in the level of spillovers. The spillover process they introduce embodies a complementarity between own and received R&D which arises because own research activity raises the ability to learn from others. The potential reversal of the incentive effects of spillovers this process gives rise to begs the question of the public policy implications that it may have. The existing literature provides only very partial answers to this question as Klette et al. (2000) point out [3].

It might seem intuitive that the need for intervention is diminished if not dissipated where

[1] This rendition of the problem is taken from Spence (1984).

[2] Cohen and Levinthal (1989) first introduced complementarities into the spillover process with their notion of "absorptive capacity". Since then Kamien and Zang (2000) and Grünfeld (2003) have introduced alternative spillover processes based on absorptive capacity. The empirical literature on this topic includes Cockburn and Henderson (1998), Griffith et al. (2000).

[3] Refer to section 5.1 of their paper for more details.

the disincentive effects of spillovers are compensated or reversed[4]. This paper clarifies why such an argument is misleading. We demonstrate below that there is a role for public policy intervention regardless of which way the incentive effects of knowledge spillovers go. This conclusion holds for a wide class of spillover processes of which absorptive capacity is only one. These spillover processes are all characterised by the fact that the ex-post knowledge stock of the receiving firm is supermodular in own and received R&D.

There is a sizeable empirical literature on ”absorptive capacity”. We argue below that to date this has really been a literature on spillover processes embodying complementarities between a firm or industry's own research and that received from outside. We show that all of these processes are characterised by common implications - those which have been tested to date. However this literature commonly assumes that its tests refer to a very tight subset of the possible spillover processes embodying complementarities - those based on learning stories that are implicit in the notion of absorptive capacity. Our research demonstrates that tests of absorptive capacity need to be further refined before they are conclusive about the importance of own R&D for learning. This literature is discussed in greater detail in the section 3.2 below.

We analyse investment in R&D employing the method of monotone comparative statics as far as is possible. This approach has recently been applied to R&D investment by Athey and Schmutzler (2001) and to the modelling of one-way spillovers in Research Joint Ventures by Amir and Wooders (2000). The method has the benefit of providing a simple and intuitive way of classifying the main models of spillover processes. It also allows us to develop a taxonomy of the growing number of variants of absorptive capacity.

The rationale for public policy intervention in firms' R&D investment decisions was first developed by Arrow (1962). He showed that there are two main effects at work, the *undervaluation* effect and the *appropriability* problem. According to his account firms will undervalue the returns to R&D investment because they do not benefit from the social surplus created by such investment. If additionally knowledge created by the firm spills over into the public domain , this may create a negative externality which further dulls the private incentive to undertake R&D. The negative effects of spillovers for the incentive to invest have become a widespread assumption in models of innovation[5].

Building on this work Spence (1984) analysed the ambiguous nature of spillovers. He notes

[4]This intuition seems implicit in the conclusions of Cohen and Levinthal (1989).

[5]For instance the literature on RJVs relies almost exclusively on this spillover process, viz. Amir (2000) and the citations there.

as a puzzle that in some industries such as the electronics industry high spillovers coincide with high levels of R&D investment. As Levin (1988) suggests this industry and others such as computers, communications equipment and aircraft may be characterised by complementarities between firms' research outputs. He bases this suggestion on the cumulative nature of innovation in these industries. Klette et al. (2000) discuss the more recent literature on complementarities between differing firms' R&D activity and raise the question about the policy implications of such complementarities. This paper seeks to answer this question.

The plan of the remainder of this paper is as follows: in the next section we set out the model, section 3 contains the solutions of the model under two classes of spillover process. Section 4 concludes. Finally the propositions set out in section 3 are proved in section 5. The appendix contains the derivation of the main assumptions from the example of linear inverse demand.

2 The Model

This section specifies a model of strategic R&D investment subject to the intervention of a social planner setting a subsidy. We begin by specifying the game played by the social planner and the firms. We go on to set out our main assumptions.

The main focus of this paper is to provide a precise analysis of market failures in the context of spillovers. The previous literature on R&D contains at least two models of the *spillover process*[6]:

i spillovers as an externality under additive R&D;

ii spillovers where the ability to learn depends on own R&D;

Below we show that these two processes are instances of two classes of processes for which our results hold.

Given the spillover process, the social planner seeks to implement a second best optimum through the choice of an R&D subsidy τ. The second best optimum is one in which the social planner is unable to achieve marginal cost pricing by firms. The criterion is a natural one to adopt in a context in which we are interested specifically in the interaction of an R&D subsidy

[6] Amir and Wooders (2000) propose a unidirectional spillover process and analyse it using monotone comparative statics. Katsoulacos and Ulph (1998) propose a process which spans duplicative, additive and complementary spillover processes.

and the spillover process[7]. Henceforth any reference to the social optimum should be taken as a reference to this second best optimum.

The game We posit two firms each producing a single good. Goods may be differentiated. Firms undertake R&D to lower their marginal costs of production.

The model of innovation takes the form of a two stage game in which:

Stage 1 the social planner choose the subsidy rate τ to maximise social welfare, where $\tau \in T = [0, 1]$,

Stage 2 the firms simultaneously choose their investments x, y in innovation, where $x, y \in X = [0, l\bar{C}]$,

Here l represents a limiting factor which will depend on the spillover process. We define it separately for each process below. It ensures that the firms do not drive marginal costs below 0 through undertaking R&D.

We will sometimes denote the firms by the superscripts x, y where x will denote the firm that is choosing an action and y will denote its competitor.

The second stage game is described by the triplet $(S^i, \Pi^i, i \in \{1, 2\})$, where S^i is the strategy space of player i and $\Pi^i : S \rightarrow R$, with $S = X \times X$, is the payoff function of that player. The first stage game is described by the triplet $(S^*, W, 1)$, where S^* is the strategy space of the social planner and $W : S^* \rightarrow R$, with $S^* = T$, is the second best social welfare function.

We apply the solution concept of sub-game perfect Nash equilibrium to the game and solve it by backwards induction.

Main assumptions In this section we set out a number of basic assumptions that apply throughout the paper. This approach is comparable to that employed by Amir and Wooders (2000); Athey and Schmutzler (2001)[8]. In both studies minimal sets of assumptions are derived that underlie the authors' results. Where possible these assumptions are related back to known models of product market competition. This approach is motivated by the application of the method of monotone comparative statics (see Milgrom and Roberts (1994)) in these studies. Our assumptions here are very similar to those adopted in Amir and Wooders (2000). Where there are differences we point these out below.

[7]The criterion has been widely adopted in the literature. Examples include Spence (1984); Suzumura (1992). Leahy and Neary (1997) employ both this second best and a first best welfare criterion.

[8]Refer also to Katsoulacos and Ulph (1998) for further discussion of some of our assumptions.

- To begin with we assume that all the relevant profit and cost functions below are twice continuously differentiable. (A1)

Although the assumption is unnecessarily restrictive in the context of monotone comparative statics, it simplifies the presentation of our results.

As we wish to concentrate on the R&D investment decisions of the firms we do not explicitly model product market competition. We show below that the R&D investment games we consider only have symmetric equilibria. We denote the firms' unit costs in equilibrium as $\widetilde{C}$ and the quantities they produce in equilibrium as $\widetilde{q}$. Then we impose the following assumption on the inverse demand function:

$$p(\widetilde{q}) \qquad \text{where} \quad \left(\frac{\partial p}{\partial \widetilde{q}} + \frac{\partial^2 p}{\partial (\widetilde{q})^2}\widetilde{q}\right) < 0. \tag{A2}$$

Given the symmetry of the game this assumption ensures that symmetric equilibria in quantities exist as solutions of the product market game (see Amir (1996); Vives (1999)). Furthermore the assumption will allow us to undertake comparative statics on the social planner's objective function in such a way that we can separate out producers' and consumers' surplus.

Now we introduce assumptions about the dependence of a firm's profits on own unit costs and those of its competitor. Firms have identical unit costs ex ante and their investments reduce costs with certainty. These assumptions are common in the innovation literature on non-tournament models and we employ them here in order to allow for comparisons between this literature and our results below.

Let the ex-ante unit costs be $\bar{C}$. We assume that firm profits $\pi^x(C^x, C^y)$, fall in own unit costs:

$$\pi^x(C^x, C^y) \qquad \text{where} \quad \frac{\partial \pi^x}{\partial C^x} < 0 \; . \tag{A3}$$

We also rely on conditions characterising effects of competitor's costs on own profits. We distinguish two cases:

$$\frac{\partial \pi^x}{\partial C^y} > 0 \qquad \qquad \frac{\partial^2 \pi}{\partial C^x \partial C^y} \leq 0, \tag{A4(a)}$$

$$\frac{\partial \pi^x}{\partial C^y} < 0 \qquad \qquad \frac{\partial^2 \pi}{\partial C^x \partial C^y} \geq 0 \; . \tag{A4(b)}$$

Case (a) corresponds to competition between firms producing substitutes, whereas (b) captures cases of firms producing complementary products for the same product market. The latter case is not considered by Amir and Wooders (2000). We show in an appendix that it emerges naturally from linear demand for Cournot and differentiated Bertrand models. [9].

We assume that the profit function is either *submodular* (case (a)) or *supermodular* (case (b)) in costs. This assumption implies that the marginal benefit of own cost reductions is reduced/increased where the competing firm also reduces its costs. We show in the appendix how the assumptions are derived from a linear demand function for both quantity and price competition. Athey and Schmutzler (2001) show that assumption $A4$ holds for a range of further models. As Amir and Wooders (2000) note no general conditions on inverse demand that imply $A4$ are known[10].

Finally we assume that the incentive to invest in R&D is sufficiently strong:

- We assume throughout the analysis below that the private marginal benefit from investments in R&D is positive at any level of the involuntary spillover β. (A5)

Turning to the R&D cost function we make the natural assumption that this is nondecreasing in the R&D investment and positive everywhere:

$$\gamma(x) > 0 \quad \text{and} \; \frac{\partial \gamma}{\partial x} > 0 \quad \forall x \geq 0. \tag{A6}$$

As we show in the appendix the most useful expression of social surplus given ex-post unit costs C^x, C^y is the following:

$$S(C^x, C^y) = \left(\int_0^{\widetilde{q}} p(u)du - p(\widetilde{q})\widetilde{q} \right) + \pi^x(C^x, C^y) + \pi^y(C^y, C^x) \; . \tag{1}$$

Here the term in brackets captures (Marshallian) consumers' surplus and the remaining term captures (Marshallian) producers' surplus.

Our main focus is on the process through which spillovers affect a firm's unit costs of production. We assume that these are a decreasing function of the firm's stock of knowledge Z:

$$C^i(Z) = \bar{C} - Z^i \qquad \text{where } i \in \{x, y\} \; . \tag{A7}$$

In general this stock of knowledge will be a function of a firm's own R&D x and the effective spillover from a rival e. We define the stock of knowledge as:

$$Z \equiv f(x, e) \tag{2}$$

[9]The case of complementary products is somewhat neglected in the literature on spillovers. We provide results here as interesting differences with the case of substitute products arise.

[10]Furthermore they note that with hyperbolic demand ($P(Q) = (Q+1)^{-1}$) the condition fails to hold.

This formulation is a generalisation of the spillover processes employed by Spence (1984) or Cohen and Levinthal (1989).

In the sections below we draw out the implications for optimal R&D policy of different assumptions we can make about the functional form of f.

3 Solving the Model

It is demonstrated below that the objective functions of the social planner and the firms may be *smooth supermodular*. Where this is true the second stage games under consideration will be shown to be smooth supermodular.

Given assumption $A1$ Milgrom and Roberts (1990) have shown that the requirements for a game to be smooth supermodular are that:

i S^i be an interval in $\Re^n$;

ii $A1$ holds on S^i;

iii $\frac{\partial^2 \Pi^i}{\partial x \partial y} \geq 0$ where $i \in \{x, y\}$;

iv Π^x is supermodular in x (given y);

Condition **i** is met by the definition of the strategy sets S^i given above, $A1$ holds by assumption and condition **iv** is met trivially here as each firm has only one decision variable (see Vives (1999)). Therefore it remains to verify that condition **iii** holds for the second stage games we analyse.

Below let X_s, Π_s, W_s denote the set of equilibrium investments, the set of equilibrium firm profits and the set of equilibrium second best welfare outcomes, respectively. Wherever they exist we refer to the maximal and minimal points of a set by an upper and a lower bar, e.g. $\bar{x}$ and $\underline{x}$ are the highest and lowest equilibrium R&D investment levels.

The main aim of our analysis is to undertake comparative statics on these equilibrium sets as exogenous parameters, such as the level of spillovers, vary. When we undertake such comparative statics exercises we will predict the direction of change of the *extremal* elements of the equilibrium sets as the parameter of interest varies.

3.1 The benchmark: costless absorption and additive R&D

The most common spillover process in the innovation literature posits that spillovers add knowledge to that produced by the firm through its own R&D efforts. The received way of modelling the interaction of own and outside R&D is to assume that:

$$f(x, y, \beta) = x + \beta y \ . \tag{3}$$

Here f is a linear function of both own and received knowledge. The amount of knowledge that spills over to a firm is determine by the exogenous parameter β which captures the level of spillovers between the two firms. We assume that $\beta \in [0, 1]$.

Notice that the factor l defining the range over which firms choose R&D investments will be $l = \frac{1}{2}$ here.

This approach to the modelling of spillovers is based on the implicit assumption that the firms have chosen to undertake R&D such that the results may be profitably combined by adding them together.

The approach to modelling spillovers captured in (3) can be found in a wide range of theoretical (Spence (1984); Leahy and Neary (1997)) and empirical contributions (Levin and Reiss (1988)). De Bondt (1996) attributes equation (3) to Ruff (1969).

In order to see just how restrictive this approach to the modelling of the spillover process is, notice that it excludes the possibility that firms duplicate research. The problem of duplication of R&D efforts and the associated question regarding the optimal number of laboratories to operate in an industry is raised in Dasgupta and Stiglitz (1980); Dasgupta and Maskin (1987).

The consequences and desirability of duplicative R&D are more easily pursued through stochastic models of innovation. For this reason we exclude duplicative R&D from our analysis in this paper.

The firm's optimisation problem: The firm's objective at stage 2 of the game is to maximise profits, Π^i, by appropriate choice of its investment, x:

$$\max_{x \in X} \Pi^i(x, y, \tau, \beta) = \pi(C^x, C^y) - (1 - \tau)\, \gamma(x) \tag{4}$$

This expression inherits the submodularity of the profit function and therefore the following proposition holds.

Proposition 1

Given assumptions $A1$*-*$A6$ *above and defining* $y' \equiv -y$, $\beta' \equiv \beta$, *at least one symmetric equilibrium of the game* $(S_i, \Pi^i(x, y', \tau, \beta'), i \in \{x, y\})$ *exists. If in addition the R&D cost function is convex the game is smooth supermodular.*

Here the introduction of y', β' serves as a device which reverses the natural order in firm y's strategy set. This device is suggested by Vives (1990, 1999) for duopoly games in which firm actions are strategic substitutes. We show in our proof that a redefinition of the game is useful in deriving the property of supermodularity.

Note that given the symmetry of the firms' equilibrium investments we refer to these as $\widetilde{x} = x = y$ from now on. As a result we re-express the common marginal cost of both firms in equilibrium as $\widetilde{C}(\widetilde{Z})$, where $\widetilde{Z}$ is the equilibrium stock of knowledge at each firm.

The literature on non-tournament models of innovation has usually focused on unique symmetric equilibria. Our assumptions here are not sufficiently restrictive to rule out multiple equilibria but we have shown that no asymmetric equilibria exist. We are able to characterise the welfare properties of the equilibria below. Notice that the extremal equilibria coincide if there is a unique equilibrium.

Given that the extremal equilibria exist they may be characterised by the following first order condition[11]:

$$\frac{\partial \Pi}{\partial x} = \underbrace{\frac{\partial \pi}{\partial C^x} \frac{dC^x}{dZ^x}}_{(+)} 1 + \underbrace{\frac{\partial \pi}{\partial C^y} \frac{dC^y}{dZ^y}}_{(-/+)} \beta - (1 - \tau) \frac{d\gamma}{dx} = 0. \tag{5}$$

Here the first term captures the positive return to innovation for the firm through reduced own costs C^x. The second term captures the externality on own profits arising through the spillover of own R&D to the rival firm. Where firms produce substitutes the externality on own profits is negative and vice versa for complementary products.

Where the externality is negative it is usually referred to as the *appropriability problem*. The case in which it is positive is virtually never discussed. We show below that from the

[11]It is useful to briefly set out the assumptions we would have had to make in order to apply more conventional comparative statics methods. For existence of a global maximum we would have had to assume that the firm's payoff function is concave. This implies that the payoff function is quasi-concave in the firm's own action. Together with our assumptions on the strategy sets above the condition is sufficient for the standard existence theorem to be applied (see Vives (1999)). Under the assumptions of duopoly and $A3$ above this is also sufficient for uniqueness of equilibrium. Although the assumption that payoffs be quasi-concave is common it is unnecessarily restrictive as proposition 1 shows.

perspective of the social planner the positive externality also implies that the firm's private incentives to undertake R&D are too low.

The first order condition captures the well known effects of spillovers and subsidies on R&D investment. In particular it shows that the private marginal benefit from investment in R&D is decreasing (increasing) in the level of the spillover for substitute (complementary) products while the private marginal costs from investment in R&D are decreasing in the level of the R&D subsidy.

Note that by assumption $A5$ the firms will always invest some positive amount in R&D. We are now in a position to derive some useful properties of the extremal equilibria:

Proposition 2

Where the second stage game is supermodular we can show that:

- a *the extremal equilibrium R&D investments $\bar{x}^e$ and $\underline{x}^e$ are decreasing (increasing) in β for substitute (complementary) products and increasing in τ;*
- b *the extremal equilibrium firm profits $\bar{\Pi}^i$ and $\underline{\Pi}^i$ are decreasing (increasing) in β, decreasing (increasing) in the other firm's R&D investments for substitute (complementary) products and increasing in τ.*

Part a of the proposition just confirms our discussion above. Part b clarifies the implications that these results have for equilibrium profits and the ordering of equilibrium profits where there are multiple equilibria.

Propositions 1 and 2 extend the results derived analytically and by simulation in Spence (1984). In particular we extend his results to the case of complementary products and demonstrate the importance of the convexity of the R&D cost function for the comparative statics results that underlie a whole literature on innovation.

Note that most models incorporating R&D spillovers in the literature implicitly assume that spillovers affect the marginal benefit of investment in R&D. We could easily extend our results to a class of models in which R&D spillovers reduced the marginal cost of undertaking R&D.

In such a class of models the R&D cost function would be a function both of own R&D and of spillovers: $\gamma(x, \beta y)$. The important property here would be the submodularity of the R&D cost function, $\gamma_{x\,\beta} < 0$ and $\gamma_{x\,y} < 0$.

This raises the question whether spillovers affecting R&D costs that are not complementary to own R&D, in the sense employed above, can be expected to have any effects. In the case of spillovers affecting the R&D cost function this means that the absolute costs of doing R&D fall as spillovers increase. Where R&D activity represents a fixed cost which acts as an entry barrier to an industry as in Dasgupta and Stiglitz (1980) or Sutton (1998), such a spillover process will clearly have implications for the equilibrium number of firms in an industry. Although we do not pursue this effect here it is clearly an important one where we are interested in R&D policies that affect the level of spillovers directly. To our knowledge the literature on optimal R&D policies has not so far endogenised the number of firms in the industry.

We turn now to the characterisation of the optimal R&D subsidy under additive spillovers. Comparing the optimality conditions of the social planner with those of the private firms will allow us to identify the welfare losses which an R&D subsidy ought to correct. We make no claim here that such a subsidy is implementable, indeed arguments pointing to some of the problems of such subsidies been advanced by Katz and Ordover (1990) and Goolsbee (1998).

The optimal R&D subsidy: In this section we characterise how the social planner chooses the optimal R&D subsidy as the level of involuntary spillovers varies.

The second best optimum level of welfare is achieved through the appropriate choice of the R&D subsidy τ to maximise W:

$$\max_{\tau \in T} W(\tau, \beta) \;=\; \Big[\Big(\int_0^{\widetilde{q}} p(u)du \,-\, p(\widetilde{q})\widetilde{q}\Big) \,+\, \pi^x(\widetilde{C}^x, \widetilde{C}^y) \,+\, \pi^y(\widetilde{C}^y, \widetilde{C}^x)\Big] \,-\, 2\gamma(\widetilde{x}(\tau)) \quad (6)$$

Here welfare is measured as the sum of consumers' surplus and industry profits net of subsidy payments. As is usual in this literature we do not capture the social cost of levying the subsidy here. We can show that the following proposition holds:

Proposition 3

Given assumptions $A1$-$A7$ and the convexity of the R&D cost function, the optimal subsidy is increasing in the level of involuntary spillovers. Furthermore, given the optimal subsidy, social welfare is increasing in the level of spillovers.

This proposition re-confirms some of the results obtained via simulation in Spence (1984). By recasting the R&D investment game proposed there, as a supermodular game, we can show that his results also apply to models with multiple equilibria.

We demonstrate below that in the presence of an optimal subsidy, both consumers' and producers' surplus are increasing in the level of spillovers.

We denote the social planner's equilibrium R&D investment as $\widetilde{x}(\tau)$. Given the existence of the extremal equilibria, these can be characterised by the following first order condition:

$$\frac{\partial W}{\partial \tau} = \underbrace{-\frac{\partial p}{\partial \tilde{q}}\tilde{q}\frac{d\tilde{q}}{d\widetilde{C}}(-1)\frac{d\widetilde{x}}{d\tau}(1+\beta)}_{(i)} + \underset{(ii)}{2}\underbrace{\Big(\frac{\partial \pi^k}{\partial \widetilde{C}^k} + \frac{\partial \pi^k}{\partial \widetilde{C}^{-k}}\Big)(-1)\frac{d\widetilde{x}}{d\tau}(1+\beta)}_{(iii)} - \frac{\partial \gamma}{\partial \widetilde{x}}\frac{d\widetilde{x}}{d\tau} = 0\ , \quad (7)$$

where $k \in \{x, y\}$.

Comparing this expression to the firm's first order condition (equation (5)) above we can identify three effects that lead to underinvestment by the firm:

i The first term in this expression captures the increase in consumers' surplus given the optimal subsidy τ^*. This term is positive and missing from the firm's first order condition (5). It captures the undervaluation effect discussed in the introduction.

ii The second term captures the fact that the social planner takes into account the increase in profits due to innovation in both firms. In contrast each firm will only take into account its own profits. Each firm underinvests due to the *stand alone effect*.

iii The third term captures the change in producers' surplus arising from R&D investment at each firm. To show clearly what effects are at work here we expand the term as follows:

$$\Big(\frac{\partial \pi^i}{\partial \widetilde{C}^i} + \frac{\partial \pi^i}{\partial \widetilde{C}^{-i}}\beta\Big)(-1)\frac{d\widetilde{x}}{d\tau} + \Big[\underbrace{\frac{\partial \pi^i}{\partial \widetilde{C}^i}}_{a}\beta + \underbrace{\frac{\partial \pi^i}{\partial \widetilde{C}^{-i}}}_{b}\Big](-1)\tau\frac{d\widetilde{x}}{d\tau} \quad (8)$$

The terms in the first set of brackets of this expression capture the incentive effects that are at work in the firms' first order conditions (compare equation (5)).

Additionally there are also two further terms, denoted by (a, b) above. Term a captures the effect of own R&D on the other firm's profits. This effect contributes to producers' surplus but is immaterial to the private decision on R&D investment. Term b captures the effect of the firm's own cost reduction on the profits of the other. This effect is negative (positive) where products are substitutes (complements) and is also internalised by the social planner. The net result of this internalisation of spillovers on producers' surplus is positive.

Where products are complements all four terms are positive. The positive externality of the spillover on the other firm's costs raises profits and induces higher investment. However the investing firm does not internalise the positive effects of its investment for the profits of its rival (term a). It also ignores the fact that the rival will benefit from its own lower costs (term b). As a result there is underinvestment (relative to the social optimum) by each firm, even when we abstract from consumers' surplus and the stand alone effect. Here firms will invest more as spillovers increase, but not enough from the social planner's point of view.

Where products are substitutes the social planner takes into account that one firm's benefit from R&D comes at the cost of foregone profits at the other firm. By assumption $A5$ the sum of the two effects is always positive. Thus the first term above always outweighs the effects of term b and term a always outweighs the negative externality of the spillover for the investing firm. Therefore abstracting from consumers' surplus and the stand alone effect the social planner will prefer greater investment the higher are spillovers. Here private incentives go in the opposite direction.

Returning to equation (7) we know now that the sign of term (iii) of that equation is always positive. This bracket is multiplied by a term increasing in the level of spillovers. This shows that *ceteris paribus* producers' surplus is increasing in the level of spillovers.

The most important point to take away from this analysis is the difference between the investment problem from the perspective of the firms and from the perspective of the social planner. Since the firms maximise only their own profits their investment problem may be submodular in their own investment and the level of involuntary spillovers as we noted above. In contrast even if the social planner were only concerned with maximising producers' surplus, their objective would be supermodular in the level of subsidy and the level of involuntary spillovers. As we show in the following section this difference underpins our main result.

Now we turn to the analysis of an alternative class of spillover processes. These are characterised by complementarities between the firms' R&D investments.

3.2 Complementarities in the spillover process

In this section we analyse spillover processes characterised by complementarities between firms' R&D investments. These spillover processes are often discussed under the heading of "absorptive capacity" in the literature[12]. They have the implication that R&D is undertaken for its own sake *and* to enhance the ability to *learn* from others through spillovers. Such learning stories have interesting policy implications as Cohen and Levinthal (1989) show. In particular enhancing the ease of learning from basic research or other fields will raise the incentives

[12]While Cohen and Levinthal (1989) introduce the term "absorptive capacity" a similar idea can already be found in Jaffe (1986).

to undertake R&D. We argue below that the relevance of these stories has not been properly established by the existing literature.

Our main aim is to determine whether complementarities in spillover processes introduce effects that reduce the need for R&D subsidies. A secondary aim is to provide a taxonomy of the growing number of formulations for absorptive capacity[13]. Finally we demonstrate that spillover processes that encompass learning stories and those that do not can look very similar and are difficult to distinguish empirically.

We begin with a short discussion of the literature on absorptive capacity. Recent research suggests that independently of the level of knowledge that spills over into the public domain, recipient firms' ability to use such knowledge may vary. Levin and Reiss (1988) argue that the productivity of a firm's research for its competitors may vary from industry to industry. However their attempts to test this hypothesis remain inconclusive.

Similarly Cohen and Levinthal (1989) suggest that the ability of a firm to learn from its competitors may be endogenous. They introduce the notion of a firm's *absorptive capacity*, which is dependent on its own R&D efforts. They also provide empirical support for the positive incentive effects of spillovers on R&D investments that are associated with absorptive capacity. As we show below these effects derive from the complementarity between own R&D and others' R&D and are not specific to models of absorptive capacity.

Cockburn and Henderson (1998) show that firms value their research staff both for their ability to produce knowledge and for their linkages to institutions providing basic research. They show that firms with stronger linkages are more productive but are cautious to infer a causal relationship. Recently Griffith et al. (2000) have introduced the idea of absorptive capacity into the empirical literature on productivity convergence[14]. They find that own R&D contributes to an industry's ability to absorb knowledge from the technological frontier. In their paper the empirical importance of absorptive capacity is established through an interaction term between intensity of industry R&D and intensity of R&D at the frontier. The significance of such interaction terms provides evidence of complementarities in spillover processes.

Finally there is evidence of direct complementarities between differing firms' research outputs. For instance Maurseth (2001) shows that some patents receive citations across many different technology fields. He shows that the value of these patents for their owners is enhanced

[13]Beside the original description of absorptive capacity by Cohen and Levinthal (1989) we are aware of alternative formulations by Kamien and Zang (2000) and Grünfeld (2003).

[14]Compare also their shorter paper Griffith et al. (2003).

by such citations. This is in contrast to citations to older patents coming from new patents in the same patent class. These tend to reduce the value of the older patents for their owners.

Citations between patents are frequently interpreted as signs of knowledge spillovers. The evidence in Maurseth (2001) shows that both the negative effects attributed to spillovers in the models based on additive spillover processes and the positive effects deriving from complementarities between firms' research efforts exist.

Now we introduce a general characterisation of complementarities in the spillover process. On this basis we characterise specific examples of such processes. A general spillover process capturing the complementarity of firm R&D investments can be defined as follows:

$$Z^x = f_a^x(x, y, \beta) \text{ where } \quad \frac{\partial f_a^x}{\partial x} \geq \frac{\partial f_a^x}{\partial y} > 0, \; \frac{\partial f_a^x}{\partial \beta} > 0, \quad \frac{\partial^2 f_a^x}{\partial x \partial y} > 0, \; \frac{\partial^2 f_a^x}{\partial x \partial \beta} > 0, \; \frac{\partial^2 f_a^x}{\partial y \partial \beta} > 0 \; . \tag{C}$$

The most important feature of this spillover process is that each firm's knowledge stock ex-post (Z^x), is a supermodular function of both firms' R&D investments and the level of involuntary spillovers.

This general spillover process captures the most important characteristics of the following specific spillover processes:

$$[i] \; f_A(x, y, \nu, \beta) = x + \beta\eta(x, \nu)y \; . \qquad [ii] \; f_C(x, y, \beta) = xy^\beta \tag{9}$$

$$[iii] \; f_{KZ} = x + \beta x^\delta y^{1-\delta} \tag{10}$$

The first process was introduced by Cohen and Levinthal (1989). It extends the additive spillover process discussed in the previous section by introducing an absorption function which we denote $\eta(x, \nu)$. This function depends on a firm's own R&D effort as well as the exogenous cost of learning ν and captures the firm's ability to make use of knowledge that has spilled over into the public domain. They make the following assumptions about the absorption function:

$$\eta(x, \nu) \in Z \qquad \eta(x, 0) \equiv 1 \qquad \frac{\partial \eta}{\partial x} > 0, \qquad \frac{\partial \eta}{\partial \nu} < 0, \qquad \frac{\partial^2 \eta}{\partial x \partial \nu} > 0 \; , \tag{ABS}$$

i.e. a firm's ability to absorb increases with its own R&D investment, it falls as learning becomes costlier for the firm and finally the importance of own R&D for the ability to learn rises as the cost of absorption rises.

Here the cost of learning captures the difficulty which a firm has in making use of knowledge that it finds in the public domain. Cohen and Levinthal (1989) suggest that a firm will need to undertake more own R&D as the specificity of outside R&D for the firm's purposes falls (ν increases) and it becomes more difficult for the firm to absorb knowledge from others[15].

The second spillover process captures a simple complementarity between two firms' R&D investments. The third process is a simplified version of Kamien and Zang (2000)'s spillover process. All of these spillover processes are captured by the general process introduced above.

There are important differences between these three spillover processes. The first and third process both imply that firms need to learn by undertaking research in order to assimilate outside knowledge. The second process does not carry this implication. Distinguishing between these specific instances empirically is important as the policy implications derived by Cohen and Levinthal (1989) will not apply to the second process.

While Cohen and Levinthal (1989) show that the appropriability problem is mitigated by positive spillover effects in their data this is only a test for the presence of complementarities in the spillover process. They cannot distinguish between learning stories such as specifications i and iii above and strict complementarities as in ii above. Similarly Griffith et al. (2000) adopt the term "absorptive capacity" which is suggestive of a learning story. Their empirical specification does not allow us to establish whether learning matters and how much it matters relative to complementarities between R&D at and behind the frontier.

Finally note that Grünfeld (2003) introduces a spillover process that also contains complementarities and captures a learning story:

$$f_G = x + \frac{\beta + sx}{1 + sx} y \qquad \text{where } s \geq 0$$

His process does not fall under the general process (f_a^x) defined above. The difference is that in his model the knowledge stock is submodular in own R&D and the level of spillovers: $\frac{\partial^2 Z^x}{\partial x \partial \beta} = \frac{-s}{(1+sx)^2} y < 0$. This difference captures the idea that the marginal rate of absorptive capacity is decreasing in a firm's R&D investments. He argues that fostering the ability to learn from others may reduce R&D incentives.

Distinguishing between all these spillover processes is clearly important if correct policy conclusions are to be drawn. In what follows we build on the general spillover process f_a^x as much as possible. We discuss the policy implications of Grünfeld (2003)'s variant of absorptive capacity where they differ from those derived below.

[15]Notice that Cohen and Levinthal (1989) assume it is the firm's investment in R&D which increases its ability to learn from others. Hammerschmidt (1999) has shown that learning may become a substitute to research where these are both independent activities which firms invest in, to lower their marginal costs. Compare also footnote 52 in Klette et al. (2000). We do not believe that these two approaches are strict alternatives.

The firm's optimisation problem The firm's objective at stage 2 of this game is to maximise the redefined profits, Π_a, by appropriate choice of its investment, x_a:

$$\max_{x \in X} \Pi_a^i(x, y, \tau, \beta) = \pi(C^x, C^y) - (1 - \tau)\,\gamma(x) \tag{11}$$

Given this problem we can show that the following proposition holds:

Proposition 4

Given the spillover process characterised by assumption (C) *an equilibrium of the second stage game exists. The second stage game is not generally supermodular under this class of spillover processes.*

Note that just as before the symmetry of the firms' equilibrium investments implies that we may refer to these as $\widetilde{x} = x = y$ from now on. As a result we re-express the common marginal cost of both firms in equilibrium as $\widetilde{C}(\widetilde{Z})$, where $\widetilde{Z}$ is the equilibrium stock of knowledge at each firm.

Where firms produce substitutes we embed a supermodular function ($f_a(x, y, \beta)$) in a submodular function $\pi(C^x, C^y)$ here. We show in the appendix 6.2 that together with assumption $A2$ this implies that the firm's objective function is neither supermodular nor submodular. By implication we cannot apply the method of monotone comparative statics to this game.

This means that the effects of changes in spillovers on R&D investment levels and firms' profits are no longer monotone. As Cohen and Levinthal (1989) find comparative statics results for such models are difficult to derive.

In contrast where firms produce complementary products we embed a supermodular function ($f_a(x, y, \beta)$) within a supermodular function $\pi(C^x, C^y)$ and the game is supermodular. We show this in the proof of Proposition 4.

The existence of an equilibrium of the second stage game is not affected by this result. However it does mean that general comparative statics results, such as those in Proposition 2, are obtainable for the second stage game only where products are complements. We derive the following Corollary to Proposition 2:

Corollary 1

Where products are complements and the spillover process is supermodular we can show that:

- a *the extremal equilibrium R&D investments $\bar{x}^e$ and $\underline{x}^e$ are increasing in β and increasing in τ;*
- b *the extremal equilibrium firm profits $\bar{\Pi}^i$ and $\underline{\Pi}^i$ are increasing in β, increasing in the other firm's R&D investments and increasing in τ.*

This corollary follows directly from the supermodularity of the game and the definition of f_a^x. It does not apply to the version of absorptive capacity suggested by Grünfeld (2003) on account of the submodularity of the knowledge stock in R&D investments and spillovers.

The first order condition that characterises a firm's equilibrium investment under a spillover process with complementarities is:

$$\frac{\partial \Pi}{\partial x} = \underbrace{\frac{\partial \pi}{\partial C^x}(-1)\frac{\partial f_a^x}{\partial x}}_{(+)} + \underbrace{\frac{\partial \pi}{\partial C^y}(-1)\frac{\partial f_a^y}{\partial x}}_{(-)} - (1-\tau)\frac{d\gamma}{dx} = 0. \tag{12}$$

To help with the interpretation of this expression notice that in case of the example spillover process [ii] introduced above we can show that $\frac{\partial f_C^x}{\partial x} = y^\beta$ and $\frac{\partial f_C^y}{\partial x} = \beta y x^{\beta-1}$.

Just as in the case of additive R&D (compare equation (5) above) equation (12) shows that the marginal return to R&D investment is reduced by higher spillovers of knowledge. In contrast to equation (5) the first (positive) element of equation (12) is also a function of the spillover. This captures the effect of the complementarity between the two firms' R&D efforts that is enhanced the greater is the degree of spillovers.

The positive effect of spillovers due to investment complementarities has led some to suggest that the case for R&D subsidies may be weakened where such complementarities affect R&D investment. The intuition this is based on relies on the fact that the negative externality which spillovers impose is being compensated and is therefore partially or wholly reduced.

This ignores the fact that spillovers have very different effects on the firms' and the social planner's problems as we argued in the previous section. We show next that the positive externality due to spillovers identified here adds a further market failure, rather than mitigating an existing one.

The optimal R&D subsidy In this section we characterise the social planner's choice of an optimal subsidy given the complementarity of firms' R&D investments.

The second best optimum level of welfare is achieved through the appropriate choice of the R&D subsidy τ to maximise W_a:

$$\max_{\tau \in Z} W_a(\tau, \beta) = \Big[\Big(\int_0^{\widetilde{q}} p(u)du - p(\widetilde{q})\widetilde{q}\Big) + \pi^x(C^x, C^y) + \pi^y(C^y, C^x)\Big] - 2\gamma(x(\tau)) \tag{13}$$

This problem differs from the social planner's problem under additive spillovers only in as far as it depends on our definition of f_a. As we demonstrated in the previous section the switch from f to f_a had strong implications for our solutions of the private firm's problem. However we also showed previously that the social planner's problem is supermodular in the subsidy and spillovers. We contrasted this with the firm's problem which is submodular in own investment and spillovers when products are substitutes.

As a result when we switch to the spillover process with complementarities here, we are embedding a supermodular function (f_a^x) in a supermodular function: the social planner's objective function. Furthermore it is important to note that producers' surplus and consumers' surplus are both increasing in the subsidy and the spillover by Proposition 3. Therefore we are embedding a supermodular function increasing in both its arguments in the same type of function. We demonstrate in appendix 6.2 that this preserves the supermodularity of the function.

We can show that the following proposition holds:

Proposition 5

Given assumptions $A1$ to $A7$, as well as the assumptions on the spillover process in (C)*, the optimal subsidy is increasing in the level of involuntary spillovers. Furthermore given the optimal subsidy, social welfare is increasing in the level of spillovers.*

Proposition 5 demonstrates that as the complementarity of firms' R&D efforts grows due to rising spillover levels, the need for a subsidy to R&D activity increases. Although this tells us nothing about the development of the absolute level of welfare losses, we can conclude that the rationale for social planner intervention becomes stronger under spillover processes involving complementarities.

This proposition does not apply to the model of Grünfeld (2003) for the same reason given above.

We now show in detail where the additional welfare loss arises. Denote the social planner's equilibrium investment as $x(\tau)$. Given that we know the extremal equilibria exist, they can be characterised by the following first order condition:

$$\frac{\partial W_a}{\partial \tau} = \underbrace{-\frac{\partial p}{\partial \widetilde{q}}\widetilde{q}\left(\frac{\partial \widetilde{q}}{\partial \widetilde{C}}(-1)\right)\frac{\partial f_a}{\partial \widetilde{x}}\frac{d\widetilde{x}}{d\tau}}_{(i)} + \underset{(ii)}{2}\underbrace{\left(\frac{\partial \pi^i}{\partial C^i} + \frac{\partial \pi^i}{\partial C^{-i}}\right)(-1)\frac{\partial f_a}{\partial \widetilde{x}}\frac{d\widetilde{x}}{d\tau}}_{(iii)} - \frac{\partial \gamma}{\partial \widetilde{x}}\frac{d\widetilde{x}}{d\tau} = 0. \quad (14)$$

Just as above we separate the effects on producers' and consumers' surplus. We begin by analysing the effects of investments in absorptive capacity on consumers' surplus.

i The first term above captures underinvestment in Consumers' surplus. Here it is multiplied by a factor reflecting the complementarity in firms' R&D investments. This captures the fact that the firms will undervalue the additional consumers' surplus returns to investment arising from the complementarity, just as they undervalue the consumers' surplus gains from investment in R&D in general. The social planner will seek to counteract this investment to capture the additional benefits from the complementarity and increase consumers' surplus.

Here we identify an additional reason for underinvestment which arises only when there are complementarities in the spillover process.

ii The factor 2 in the expression for producers' surplus shows that the *stand alone effect* contributes to underinvestment by the firms here. We discussed this effect above. It is independent of the spillover process.

iii Turn now to the contribution to producers' surplus of one of the firms given by term (iii). We expand this term here to allow for better interpretation:

$$\Big(\frac{\partial \pi}{\partial C^i}\frac{\partial f_a^i}{\partial \widetilde{x}^i}+\frac{\partial \pi}{\partial C^{-i}}\frac{\partial f_a^{-i}}{\partial \widetilde{x}^i}\Big)(-1)\frac{d\widetilde{x}}{d\tau}+\Big[\frac{\partial \pi}{\partial C^i}\frac{\partial f_a^i}{\partial \widetilde{x}^{-i}}+\frac{\partial \pi}{\partial C^{-i}}\frac{\partial f_a^{-i}}{\partial \widetilde{x}^{-i}}\Big](-1)\frac{d\widetilde{x}}{d\tau} \tag{15}$$

The term in round brackets captures all the effects present in the first order condition of the firm (equation (12)). The term in square brackets captures internalisation by the social planner of the effects of a firm's own investment on its competitor's position. Just as in our previous discussion we can show that by assumption $A5$ the net effect of this is positive, i.e. *ceteris paribus* higher spillovers raise total producers' surplus.

Where products are complements each firm fails to take into account the positive effects of its own R&D on the other firm's profits. Just as under an additive spillover process the firm underinvests in R&D because of this.

Where products are substitutes the negative externality associated with spillovers is partially or wholly compensated by a positive one. The social planner internalizes the negative and the positive externality. From their point of view both spillover effects are positive for social welfare. Once more there are private incentives which go in the wrong direction here. These will reduce investment even if they are partly compensated by positive effects.

This discussion shows that where spillover processes incorporate complementarities firms will tend to underinvest in the benefits of these complementarities just as they would have underinvested in the benefits of cost reduction under an additive spillover process.

We conclude that industries in which there are complementarities between firms' research activities conforming to our definition of (f_a^x), at least as much intervention will be required as in those industries where there are no such complementarities. We also note that some models of absorptive capacity (e.g. Grünfeld (2003)) fall outside of our definition. Our conclusions do not apply to these models. Our analysis has the advantage of making precise why our conclusions fail to apply there.

4 Conclusion

This paper analyses the two important models of the process through which spillovers affect firms' R&D investment decisions. It shows that these processes are instances of two classes of spillover process that have differing implications for private R&D incentives.

The main focus of the paper is the analysis of public policy in the context of these spillover processes. We show that contrary to an intuition based on the inspection of private R&D incentives, the need for public policy intervention will increase where complementarities arise in the spillover process.

The paper shows that these conclusions are based mainly on the sub/supermodularity of firms' profit functions and the nature of the spillover process itself. By employing the method of monotone comparative statics we are able to derive general implications of the two main classes of spillover process for public policy.

In particular we show that under additive R&D the widely accepted effects that suggest a role for public policy intervention (undervaluation of returns and appropriability problems) apply even in settings in which the firms' payoff functions are not quasi-concave and where there are multiple equilibria. This is because we can show that this spillover process induces a supermodular game in R&D investment.

We go on to show that a social planner setting an optimal subsidy will always raise it where the level of involuntary spillovers increases. We show that this increases social welfare. These results are shown to arise because the social planner's decision problem is supermodular in subsidies and spillovers.

We then contrast these results with a class of spillover process that involves complementarities between own and rival R&D. We show that the firms' R&D investment game is no longer supermodular where products are substitutes. We also demonstrate that the social planner still faces a supermodular decision problem in this case. Most importantly we show that the rationale for social planner intervention under spillover processes with complementarities between firms' R&D investments is strengthened.

We show that this class of models shares both the private and the social characteristics of the model introduced by Cohen and Levinthal (1989). This will have implications for empirical papers which seek to show that the complementarities inherent in the model of learning through R&D are present in the data. Where these papers identify complementarities between different actors' R&D efforts they provide support for the importance for the class of spillover processes we have identified here. To identify the particular process introduced by Cohen and Levinthal (1989) and to support their policy conclusions it would be important to show that learning is the carrier of the complementarity.

5 Proofs

This section contains the proofs of all propositions introduced above.

Proposition 1 The second stage game here is symmetric and the players' strategy spaces are one-dimensional. In addition the assumption that the payoff functions are twice continuously differentiable ($A1$) implies that the firms' best replies do not jump downwards. Therefore there will be at least one equilibrium of the game. In addition because of the symmetry of the game (meaning that the game is exchangeable against a permutation of the players) the equilibria are symmetric. While this argument is enough to show existence of equilibria we now present an alternative proof which is the foundation for the comparative statics results of Proposition 2.

Here we show that under a suitable redefinition of the second stage game it is *smooth supermodular*. This implies that at least one equilibrium exists. Furthermore the symmetry of the game implies that all equilibria are symmetrical.

We begin by showing why a redefinition of the game can be useful here. Notice that given assumptions $A1 - A6$, the second stage game does not look supermodular, where firms produce substitute products (assumption $A4(a)$). The cross-partial derivative of profits w.r.t. the competing firm's R&D investment makes this clear:

$$\frac{\partial^2 \Pi^x}{\partial x \partial y} = \underbrace{\frac{\partial^2 \pi^x}{\partial C^x \partial C^y}(-1)^2\left(1+\beta^2\right)}_{(-)} + \underbrace{\left(\frac{\partial^2 \pi^x}{\partial (C^x)^2} + \frac{\partial^2 \pi^x}{\partial (C^y)^2}\right)(-1)^2\beta}_{(+)}$$

The sign of the expression under assumption $A4(a)$ depends on the relative strength of the negative and positive contributions. The expression above shows only that the game is submodular in the absence of spillovers.

Where firms produce complementary products (assumption $A4(b)$) the game is clearly *smooth supermodular* as the first term has a positive sign.

A simple redefinition of the firms' objective functions allows us to derive conditions under which the game is supermodular. Following Amir (1996) we redefine the firms' objective function such that firm x chooses its total cost reduction Z^x. Furthermore we introduce a change of variables, let: $y' \equiv -y$ and $\beta' \equiv -\beta$. For additive spillovers this implies that $x = Z^x - y'\beta'$[16].

Therefore it is firm x's objective to maximise profits, Π^x, by appropriate choice of its total cost reduction Z^x, given the investment y' by its competitor and the spillover level β':

$$\max_{Z^x \geq y'\beta'} \Pi^x(Z^x, y', \tau, \beta') = \pi(C^x(Z^x), C^y(y', Z^x, \beta')) - [1-\tau]\,\gamma(Z^x - y'\beta')$$

The cross partial of this redefined game is:

$$\frac{\partial^2 \pi}{\partial Z^x \partial y'} = \underbrace{\frac{\partial^2 \pi^x}{\partial C^x \partial C^y}(-1)^2\left(-\left[1-\beta'^2\right]\right)}_{(+)} + \underbrace{\frac{\partial^2 \pi}{\partial (C^y)^2}(-1)^2(-\beta')\left(-\left[1-\beta'^2\right]\right)}_{(+)} - [1-\tau]\,\frac{\partial^2 \gamma}{\partial (x)^2}(-\beta') > 0$$

Where the R&D cost function is convex the redefined game is always *smooth supermodular*. For concave R&D cost functions it will depend on the degree of concavity whether or not the game is supermodular. ■

Proposition 2 This proposition provides comparative statics results for the equilibria of the firms' second stage game w.r.t. the level of involuntary spillovers β'. A general discussion of comparative statics in supermodular games can be found in Vives (1999).

Just as in the previous proof we rely on the transformed version of the second stage game.

Part (a) of Proposition 2 can be shown to hold for substitute products where the firms' objective function is characterised by a strictly positive cross-partial w.r.t. β':

$$\frac{\partial^2 \Pi^x}{\partial Z^x \partial \beta'} = \underbrace{\frac{\partial^2 \pi}{\partial (C^y)^2}(-1)^2\left[-\left(Z^x - 2y'\beta'\right)\right](-\beta')}_{(+)} + \underbrace{\frac{\partial^2 \pi}{\partial C^x \partial C^y}(-1)^2\left[-\left(Z^x - 2y'\beta'\right)\right]}_{(+)} + \underbrace{\frac{\partial \pi}{\partial C^y}(-1)^2}_{(+)} - [1-\tau]\,\frac{\partial^2 \gamma}{\partial (x)^2}(-y') > 0$$

Where the second stage game in (Z^x, y') is supermodular this expression is strictly positive. This implies that the extremal equilibrium R&D investments are strictly increasing in β' and therefore strictly decreasing in β.

[16]This change of variables is admissible for two player games only; see Vives (1999)

For complementary products the cross-derivative of the firms' objective function w.r.t. the level of spillovers is positive:

$$\frac{\partial^2 \Pi^x}{\partial x \partial \beta} = \frac{\partial^2 \pi}{\partial (C^x)^2}(-1)^2 y + \frac{\partial^2 \pi}{\partial C^x \partial C^y}(-1)^2 x + \frac{\partial^2 \pi}{\partial (C^y)^2}(-1)^2 \beta x + \frac{\partial^2 \pi}{\partial C^y \partial C^x}(-1)\beta y + \frac{\partial \pi}{\partial C^y}(-1) > 0 \ . \quad (16)$$

This implies that the extremal equilibrium R&D investments are strictly increasing in β.

Turning to the cross partial w.r.t. the optimal R&D subsidy we can show that this is also positive:

$$\frac{\partial^2 \Pi^x}{\partial Z^x \partial \tau} = \tau \frac{\partial \gamma}{\partial x} > 0 \ .$$

Therefore the extremal R&D investments are strictly increasing in τ. ■

Part (b) of Proposition 2 relies on the use of the Monotone Maximum Theorem (Theorem 5 below) due to Topkis (1978) (see also Carter (2001)). To show that firm profits are decreasing(increasing) in the level of spillovers(the subsidy) we need to show that payoffs decrease(increase) in spillovers (the subsidy).

We begin by investigating how profits vary with β' for substitute products:

$$\frac{\partial \Pi^x}{\partial \beta'} = \frac{\partial \pi^x}{\partial C^y}(-1)\left[-\left(Z^x - 2y'\beta\right)\right] - \left[1-\tau\right]\frac{\partial \gamma}{\partial x}(-y') > 0.$$

It is easy to see that profits are increasing in $beta$ for complementary products.

Therefore by the Monotone Maximum Theorem equilibrium profits are decreasing(increasing) in β for substitute(complementary) products. Similarly we can show that profits are increasing in the level of the R&D subsidy:

$$\frac{\partial \Pi^x}{\partial \tau} = \tau\gamma(Z^x - y\beta) > 0 \ . \qquad ■$$

Proposition 3 This proposition rests on the application of the Monotone Maximum Theorem to the social planner's decision problem. For the theorem to be applicable we need to show that the cross partial of social welfare w.r.t. the subsidy and the spillover is positive:

$$\frac{\partial^2 W}{\partial \tau \partial \beta} = -\frac{\partial p}{\partial \widetilde{q}}\widetilde{q}\frac{d\widetilde{q}}{d\widetilde{C}}(-1)\frac{dx}{d\tau} - \Big(\frac{\partial p}{\partial \widetilde{q}} + \frac{\partial^2 p}{\partial (\widetilde{q})^2}\widetilde{q}\Big)(\frac{d\widetilde{q}}{d\widetilde{C}})^2(-1)^2\frac{d\widetilde{x}}{d\tau}\left(1+\beta\right)\widetilde{x} + 2\Big(\frac{\partial \pi^i}{\partial \widetilde{C}^i} + \frac{\partial \pi^i}{\partial \widetilde{C}^{-i}}\Big)(-1)\frac{d\widetilde{x}}{d\tau} + 2\Big(\frac{\partial^2 \pi^i}{\partial (\widetilde{C}^i)^2} + \frac{\partial^2 \pi^i}{\partial (\widetilde{C}^{-i})^2} + 2\frac{\partial^2 \pi^i}{\partial \widetilde{C}^i \partial \widetilde{C}^{-i}}\Big)(\frac{d\widetilde{x}}{d\tau})^2\left(1+\beta\right)\widetilde{x} > 0 \quad (17)$$

We have now shown that the optimal subsidy is increasing in the level of involuntary spillovers. To show that social welfare, given the optimal subsidy will also increase in the level of spillovers the following derivative must be positive:

$$\frac{\partial W}{\partial \beta} = -\frac{\partial p}{\partial \widetilde{q}}\widetilde{q}\frac{\partial \widetilde{q}}{\partial \widetilde{C}}(-1)\widetilde{x} + 2\Big(\frac{\partial \pi^i}{\partial C^i} + \frac{\partial \pi^i}{\partial C^{-i}}\Big)(-1)\widetilde{x} > 0 \ .$$

Therefore equilibrium social welfare is increasing in spillovers. Both statements in the proposition have now been shown to hold. ■

Proposition 4 The method of proof here is analogous to that employed in Proposition 1. The existence of a symmetric equilibrium here follows from the argument set out there.

Notice however that the redefinition of the game which we employed there is not possible here, given the generality of our definition of f_a.

We can show that the game is supermodular only where products are complements:

$$\frac{\partial^2 \Pi}{\partial x \partial y} = \frac{\partial^2 \pi}{\partial (C^x)^2}(-1)^2\left(\frac{\partial f_a^x}{\partial x}\right)^2 + \frac{\partial \pi}{\partial C^x}(-1)\frac{\partial^2 f_a^x}{\partial x \partial y} + 2\frac{\partial^2 \pi}{\partial C^x \partial C^y}(-1)^2\frac{\partial f_a^x}{\partial x}\frac{\partial f_a^y}{\partial x} + \frac{\partial^2 \pi}{\partial (C^y)^2}(-1)^2\left(\frac{\partial f_a^y}{\partial x}\right)^2 + \frac{\partial \pi}{\partial C^y}(-1)\frac{\partial^2 f_a^y}{\partial x \partial y} > 0. \quad (18)$$

This expression is positive only under assumption $A4(b)$ as the third and fifth terms are negative under assumption $A(a)$.

Where products are substitutes we demonstrate that for the spillover process defined as f_C above the resulting game is no longer supermodular. This provides us with the necessary counterexample to show that the class of spillover processes captured in the definition of f_a does not generally admit supermodularity of the second stage game.

Given our definition of f_C above, we introduce the variables y' and β' just as in Proposition 1. This implies that $x = Z^x(-y')^{\beta'}$ and $Z^y = (-y')(z^x)^{(-\beta')}$.

We can now show that even where firm x maximises, Π^x, by appropriate choice of Z^x, give n y' and β' the resulting game is not supermodular:

$$\frac{\partial^2 \Pi_a^x}{\partial Z^x \partial y'} = \underbrace{\frac{\partial \pi}{\partial C^y}(-1)^2(-\beta')(Z^x)^{(-\beta'-1)}}_{(-)} + \underbrace{\frac{\partial^2 \pi}{\partial (C^y)^2}(-1)^2(-\beta')(-y')(-Z^x)^{(-2\beta'-1)}}_{(-)} - \underbrace{[1-\tau]\frac{\partial \gamma}{\partial x}(\beta')(-y')^{(\beta'-1)}}_{(-)} - [1-\tau]\frac{\partial^2 \gamma}{\partial (x)^2}\beta' Z^x(-y')^{(2\beta'-1)}$$

Irrespective of the convexity or concavity of the R&D cost function this expression is not generally positive or negative. This shows that even the transformed game is not supermodular. ■

Proposition 5 Analogously to Proposition 3 we show that we can apply the Monotone Maximum Theorem here:

$$\frac{\partial^2 W}{\partial\tau\partial\beta} = -\frac{\partial p}{\partial\widetilde{q}}\widetilde{q}\frac{d\widetilde{q}}{d\widetilde{C}}(-1)\frac{\partial^2 f_a}{\partial\widetilde{x}\partial\beta}\frac{d\widetilde{x}}{d\tau} - \Big(\frac{\partial p}{\partial\widetilde{q}} + \frac{\partial^2 p}{\partial(\widetilde{q})^2}\widetilde{q}\Big)(\frac{d\widetilde{q}}{d\widetilde{C}})^2(-1)^2\frac{\partial^2 f_a}{\partial\widetilde{x}\partial\beta}\frac{d\widetilde{x}}{d\tau}$$
$$+2\Big(\frac{\partial\pi^i}{\partial\widetilde{C}^i} + \frac{\partial\pi^i}{\partial\widetilde{C}^{-i}}\Big)(-1)\frac{\partial^2 f_a}{\partial\widetilde{x}\partial\beta}\frac{d\widetilde{x}}{d\tau} + 2\Big(\frac{\partial^2\pi^i}{\partial(\widetilde{C}^i)^2} + \frac{\partial^2\pi^i}{\partial(\widetilde{C}^{-i})^2} + 2\frac{\partial^2\pi^i}{\partial\widetilde{C}^i\partial\widetilde{C}^{-i}}\Big)\frac{\partial^2 f_a}{\partial\widetilde{x}\partial\beta}\left(\frac{d\widetilde{x}}{d\tau}\right)^2 > 0$$

This shows that the optimal subsidy is increasing in the level of involuntary spillovers. To show that social welfare, given the optimal subsidy will also increase in the level of spillovers the following derivative must be positive:

$$\frac{\partial W_a}{\partial\beta} = -\frac{\partial p}{\partial\widetilde{q}}\widetilde{q}\frac{\partial\widetilde{q}}{\partial\widetilde{C}}(-1)\frac{\partial f_a}{\partial\beta} + 2\Big(\frac{\partial\pi^i}{\partial C^i} + \frac{\partial\pi^i}{\partial C^{-i}}\Big)(-1)\frac{\partial f_a}{\partial\beta} > 0\ .$$

Therefore equilibrium social welfare is increasing in spillovers. Both statements in the proposition have now been shown to hold. ■

6 Appendix

6.1 Derivation of assumptions from linear demand

Suppose that the inverse demand functions are given by $p = a - q^x - \lambda q^y$, where q^x is firm x's output and $\lambda \in [-1, 1]$ captures the degree of product market substitution. Define $\delta \in [0, 1]$ as the conjectural variations parameter. Then $\delta = 0$ represents quantity competition and $\delta = 1$ price competition. The first order conditions determining the firms' equilibrium outputs may be written as follows:

$$\begin{bmatrix} a - C^x \\ a - C^y \end{bmatrix} = \begin{bmatrix} v & w \\ w & v \end{bmatrix} \begin{bmatrix} q^x \\ q^y \end{bmatrix}.$$

where we define $v \equiv (2 - \lambda\delta)$ and $w \equiv \lambda$. The corresponding output levels are:

$$\begin{bmatrix} q^x \\ q^y \end{bmatrix} = d \begin{bmatrix} v & -w \\ -w & v \end{bmatrix} \begin{bmatrix} a - C^x \\ a - C^y \end{bmatrix},$$

where we define $d \equiv \frac{1}{v^2 - w^2}$.

Given $C(Z)$ we can then show that:

$$\pi^x(C^x, C^y) = (3 - v)\, d^2 \Big([a - C^x(Z^x)]\, v - [a - C^y(Z^y)]\, w \Big)^2$$

Given this expressions we can show that assumptions $A2 - A4$ hold for the model:

$$\frac{\partial \pi^x}{\partial C^x} = -2v\,(3-v)\,d^2 \Big([a - C^x(Z^x)]\,v - [a - C^y(Z^y)]\,w \Big) < 0 \,,$$
$$\frac{\partial \pi^x}{\partial C^y} = 2w\,(3-v)\,d^2 \Big([a - C^x(Z^x)]\,v - [a - C^y(Z^y)]\,w \Big) \,,$$
$$\frac{\partial^2 \pi^x}{\partial C^x \partial C^y} = -2vw\,(3-v)\,d^2 \,.$$

Here the sign of $\frac{\partial \pi^x}{\partial C^y}$ clearly depends on the sign of w. The sign of the cross derivative $\frac{\partial^2 \pi^x}{\partial C^x \partial C^y}$ also depends on this parameter. This fact underlies the distinction we make in assumption $A4$.

The social surplus corresponding to the linear inverse demand function chosen above is given by $S_{ij} = \left(a - \widetilde{C}\right) 2\widetilde{q} - \widetilde{q}^2\,(1+\lambda)$, where we employ the assumption that the firms are symmetric and their equilibrium output is $\widetilde{q}$. In the example above we can show that $\widetilde{q}(\widetilde{C}) = d\left(a - \widetilde{C}\right)(v - w)$.

Above we revert to the more general expression for the Marshallian surplus. This allows us to separate out the contributions of consumers' and producers' surplus more clearly than the specific expression above:

$$S = \left[\int_0^{\widetilde{q}} p(z)dz - p(\widetilde{q})\widetilde{q} \right] + 2\pi^i(\widetilde{C}, \widetilde{C}) \,,$$

where the first term captures consumers' surplus and the second producers' surplus.

6.2 Embedding supermodular and submodular functions in one another

We demonstrate briefly here the effect of embedding a supermodular function in a submodular function.

Let $g(a,b)$ be a submodular function where $g_a < 0$ and $g_b > 0$. Let $h(x,y)$ be a supermodular function where $h_x, h_y > 0$. Finally assume that $a(h)$ and $b(h)$ where $a_h, b_h < 0$. This captures the private firm's problem where g represents profits π and h is f.

Then we can show that the sign of $g_{x\,y}$ will be indeterminate:

$$g_{x\,y} = 2\underset{(-)}{g_{a\,b}}\underset{(-)}{b_h}\,\underset{(-)}{a_h}\,\underset{(+)}{h_x}\,\underset{(+)}{h_y} + \left(\underset{(-)}{g_a}\,\underset{(-)}{a_h} + \underset{(+)}{g_b}\,\underset{(-)}{b_h} \right) \underset{(+)}{h_{x\,y}} \,. \tag{19}$$

The expression shows that it is the embedding of the two types of function along with the sign restriction on g_a and assumption A6 which leads to the sign indeterminacy of the cross-partial.

To understand the social planner's problem observe that here g is supermodular and that $g_a, g_b < 0$. Here g captures W which is decreasing in either firm's costs and h is f_a. Then we can show that the sign of $g_{x\,y}$ is determinate:

$$g_{x\,y} = 2\underset{(+)}{g_{a\,b}}\underset{(-)}{b_h}\underset{(-)}{a_h}\underset{(+)}{h_x}\underset{(+)}{h_y} + \left(\underset{(-)}{g_a}\underset{(-)}{a_h} + \underset{(-)}{g_b}\underset{(-)}{b_h}\right)\underset{(+)}{h_{x\,y}} > 0 \tag{20}$$

References

Amir, R. (1996). Cournot oligopoly and the theory of supermodular games. *Games and Economic Behaviour*, 15:132–148.

Amir, R. (2000). Modelling imperfectly appropriable R&D via spillovers. *International Journal of Industrial Organization*, 18:1013–1032.

Amir, R. and Wooders, J. (2000). One-way spillovers, endogenous innovator/imitator roles, and research joint ventures. *Games and Economic Behaviour*, 31:1–25.

Arrow, K. J. (1962). Economic welfare and the allocation of resources for invention. In Nelson, R. R., editor, *The Rate and Direction of Inventive Activity*. Princeton University Press, Princeton.

Athey, S. and Schmutzler, A. (2001). Investment and market dominance. *RAND Journal of Economics*, 32(1):1–26.

Carter, M. (2001). *Foundations of Mathematical Economics*. MIT Press, Cambridge, Massachusetts and London, England.

Cockburn, I. M. and Henderson, R. (1998). Absorptive capacity, coauthoring behaviour and the organization of research in drug discovery. *The Journal of Industrial Economics*, XLVI(2):157–182.

Cohen, W. M. and Levinthal, D. A. (1989). Innovation and learning: The two faces of R&D. *The Economic Journal*, 99:569–596.

Dasgupta, P. and Maskin, E. (1987). The simple economics of research portfolios. *The Economic Journal*, (97):581–595.

Dasgupta, P. and Stiglitz, J. (1980). Uncertainty, industrial structure and the speed of R&D. *Bell Journal of economics*, pages 1–28.

De Bondt, R. (1996). Spillovers and innovative activities. *International Journal of Industrial Organization*, 15:1–28.

Goolsbee, A. (1998). Does government r&d policy mainly benefit scientists and engineers? *American Economic Review*, 88(2):298–302.

Griffith, R., Redding, S., and Reenen, J. V. (2000). Mapping the two faces of R&D: Productivity growth in a panel of oecd industries. Technical Report 2457, CEPR.

Griffith, R., Redding, S., and Reenen, J. V. (2003). R&D and absorptive capacity: Theory and empirical evidence. *Scandinavian Journal of Economics*, 105(1):99–118.

Grünfeld, L. A. (2003). Meet me halfway but don't rush: absorptive capacity and strategic R&D investment revisited. *International Journal of Industrial Organization*, 21(8):1091–1110.

Hammerschmidt, A. (1999). R&D spillovers and absorptive capacity. Mimeo.

Jaffe, A. (1986). Technological opportunity and spillovers of r&d: Evidence form firms patents, profits and market value. *American Economic Review*, 76(5):984–1001.

Kamien, M. and Zang, I. (2000). Meet me halfway: research joint ventures and absorptive capacity. *International Journal of Industrial Organization*, 18:995–1012.

Katsoulacos, Y. and Ulph, D. (1998). Endogenous spillovers and the performance of research joint ventures. *Journal of Industrial Economics*, 46:333–357.

Katz, M. L. and Ordover, J. A. (1990). R&D cooperation and competition. In *Brookings Papers: Microeconomics*, pages 137–203.

Klette, T. J., Moen, J., and Griliches, Z. (2000). Do subsidies to commercial R&D reduce market failures? *Research Policy*, 29:471–495.

Leahy, D. and Neary, P. (1997). Public policy towards R&D in oligopolistic industries. *American Economic Review*, 87(4):642–662.

Levin, R. C. (1988). Appropriability, R&D spending, and technological performance. *American Economic Review*, 78(2):424–428.

Levin, R. C. and Reiss, P. C. (1988). Cost-reducing and demand creating R&D with spillovers. *RAND Journal of Economics*, 19(4):538–556.

Maurseth, P. B. (2001). Lovely but dangerous: The impact of patent citations on patent duration. Working Paper 614, Norwegian Institute for International Affairs.

Milgrom, P. and Roberts, J. (1990). Rationalizability, learning and equilibrium in games with strategic complementarities. *Econometrica*, 58(6):1255–1278.

Milgrom, P. and Roberts, J. (1994). Comparing equilibria. *American Economic Review*, 84(3):441–459.

Ruff, L. (1969). Research and technological progress in a cournot economy. *Journal of Economic Theory*, 1:397–415.

Spence, M. (1984). Cost reduction, competition, and industry performance. *Econometrica*, 52(1):101–121.

Sutton, J. (1998). *Technology and Market Structure*. The MIT Press, Cambridge, Massachusetts.

Suzumura, K. (1992). Cooperative and noncooperative R&D in an oligopoly with spillovers. *American Economic Review*, 82(5):1307–1320.

Topkis, D. (1978). Minimizing a submodular function on a lattice. *Operations Research*, 26:305–321.

Vives, X. (1990). Nash equilibrium with strategic complementarities. *Journal of Mathematical Economics*, 19(3):305–321.

Vives, X. (1999). *Oligopoly Pricing, Old Ideas and New Tools*. MIT Press, Cambridge, Massachusetts and London, England.