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We introduce a concept of emotions that emerge when workers compare their own performance with the performances of co-workers. Assuming heterogeneity among the workers the interplay of emotions and incentives is analyzed within the framework of rank-order tournaments which are frequently used in practice. Tournaments seem to be an appropriate starting point for this concept because the main idea of a tournament is inducing incentives by making workers compare themselves with their opponents. We differentiate between exogenous and endogenous tournament prizes and identify certain conditions under which the employer benefits from emotional workers. In this case, he clearly prefers unfair to fair tournaments. Furthermore, the concept of emotions is used to explain the puzzling findings on the oversupply of effort in experimental tournaments.

Key words: anger, emotions, pride, tournaments.

JEL classification: J3, M5.

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1 Introduction

Emotions are a natural ingredient of human beings. In particular, when evaluating possible consequences of their decisions people take emotions like anger, frustration, joy or pride into account. Hence, an economic decision maker should also incorporate possible emotions into his objective function. Moreover, the experimental findings of Bosman and van Winden (2002) on emotional hazard point out that emotions play an important role in real decision making. However, as Elster (1998) and Loewenstein (2000) complain, economists – with some exceptions[1] – do not pay attention to emotions when modelling economic behavior although introducing emotions may "help us explain behavior for which good explanations seem to be lacking" (Elster 1998, p. 489).

In this paper, emotions are introduced into the theory of rank-order tournaments. In a (rank-order) tournament, at least two workers compete against each other for given prizes. The worker with the best performance receives the winner prize, the second best worker gets the second highest prize and so on. There exist many examples for tournaments in economics.[2] They can be observed between salesmen (e.g., Mantrala et al. 2000), in broiler production (Knoeber and Thurman 1994) and also in hierarchical firms when people compete for job promotion (e.g., Baker et al. 1994, Eriksson 1999, Bognanno 2001). Basically, corporate tournaments will always be created if relative performance evaluation is linked to monetary consequences for the employees. Hence, forced-ranking or forced-distribution systems, in which

[1] See, e.g., Hirshleifer (1987) on emotions as guarantors of threats and promises, Kandel and Lazear (1992) on shame and guilt in the context of peer pressure, Mui (1995) on envy.

[2] For a theoretical analysis of tournaments see Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), Rosen (1986).

supervisors have to rate their subordinates according to a given number of different grades, also belong to the class of tournament incentive schemes (see, for example, Murphy 1992 on forced ranking at Merck). Boyle (2001) reports that about 25 per cent of the so-called Fortune 500 companies utilize forced-ranking systems to tie pay to performance (e.g., Cisco Systems, Intel, General Electric).

In the following, we will consider emotions that will emerge if workers compare their own performance with the performances of co-workers. Typically, workers feel *pride* when outperforming their co-workers, whereas they feel *anger* when falling behind them. Combining such concept of emotions with the concept of rank-order tournaments seems somewhat natural because contestants must compare themselves with their co-workers who compete in the same tournament. Then a worker feels anger when losing against an opponent and pride when winning against him. Similar to the notion of pride, Fershtman, Hvide and Weiss (2003a, 2003b) consider a concept of so-called *competitive preferences* in which a player derives utility from being ahead. They apply their concept to standard individualistic incentive schemes. If we applied this concept to tournaments, the subjective winner prize of each contestant would be larger than the monetary winner prize irrespective of whether workers are homogeneous or heterogeneous. Hence, under that concept standard tournament results will qualitatively remain the same. One would only have to redefine the given tournament prizes as subjective prizes. However, in this paper, we assume that emotions that emerge when comparing one's own performance with the performance of co-workers will depend on the *type of co-worker*. Emotions will be stronger if the workers are *heterogeneous*, since it will be more difficult to beat a more able co-worker than an equally or less talented one. By combining emotions with heterogeneity among workers, we can derive conditions under which an employer prefers heterogeneous departments – and hence *unfair* tournaments – to homogeneous ones – i.e. *fair* tournaments – and vice versa.

The aim of the paper is twofold: First, it will be emphasized that emotions are not always detrimental as pointed out by the experiments on emotional hazard and the model by Mui (1995) on envy. We can show under which conditions emotions are beneficial for a profit maximizing employer. In particular, the employer may even benefit from "negative emotions" of his workers like frustration or anger. Standard tournament results show that unfair tournaments between heterogeneous agents are never optimal. However, when introducing emotions into tournaments this general result no longer holds. On the contrary, equilibrium efforts may even increase in the ability difference of the competitors.

Second, the paper seizes the suggestion made by Elster and utilizes emotions to explain empirical findings that contradict standard economic theory. There exist diverse experimental findings on asymmetric tournaments which are puzzling as they show that players significantly oversupply effort compared to equilibrium effort levels (Bull et al. 1987, Weigelt et al. 1989, Schotter and Weigelt 1992). By using the concept of emotions these results can easily be explained.

The paper is organized as follows. In the next section, the model is introduced. Section 3 introduces anger and pride into tournament theory in order to focus on the central question under which conditions the employer will prefer unfair tournaments to fair ones. Here we also differentiate between situations in which tournament prizes are exogenously given, and situations with endogenously chosen optimal tournament prizes. Section 4 discusses the main findings. Section 5 concludes.

2 The Model

We consider a firm which consists of one risk neutral employer and four risk neutral workers.[3] Each worker's observable (but unverifiable) performance or output can be described by the production function $q_i = e_i + a_i + \varepsilon_i$ $(i = 1, 2, 3, 4)$.[4] e_i denotes endogenous effort which is chosen by worker i, a_i worker i's exogenous ability and ε_i individual noise which is also assumed to be exogenous. The noise variables $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ are identically and independently distributed with density $g(\cdot)$ and cumulative distribution function $G(\cdot)$. Let $f(\cdot)$ denote the density and $F(\cdot)$ the cumulative distribution function of the composed random variable $\varepsilon_j - \varepsilon_i$ of each pair of two workers. It is assumed that $f(\cdot)$ has a unique mode at zero.[5] The employer can only observe realized output q_i but none of its components. Hence, a standard moral hazard problem is considered. Exerting effort entails costs on a worker which are described by the function $c(e_i)$ with $c(0) = 0$, $c'(e_i) > 0$ and $c''(e_i) > 0$. The reservation value of each worker is $\bar{u} \geq 0$.

Two of the workers – the so-called "underdogs" – are characterized by low ability a_U, whereas the two other workers – the "favorites" – have a high ability a_F with $a_F > a_U$.[6] Let $\Delta a := a_F - a_U > 0$ denote the ability difference

[3] Most of the assumptions follow the standard tournament model by Lazear and Rosen (1981). For the optimal design of unfair rent-seeking contests see Feess, Muehlheusser and Walzl (2002).

[4] By assuming an observable but unverifiable performance signal we can exclude standard individualistic incentive schemes like piece rates which would not work in this context whereas tournament incentives will still hold; see Malcomson (1984).

[5] For example, if ε_i and ε_j are uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$ (normally distributed), the convolution $f(\cdot)$ will be a triangular distribution over $[-2\bar{\varepsilon}, 2\bar{\varepsilon}]$ (normal distribution) with mean zero.

[6] Of course, heterogeneity between workers can be modelled in different ways. Here we take the additive model of Meyer and Vickers (1997), Holmström (1999), Höffler and Sliwka

between favorites and underdogs. The respective type U or F of each worker is common knowledge.

It is assumed that the firm consists of two departments and that the employer has to choose the composition of the departments. He can either choose a homogeneous design ($D = HOM$) under which one department contains the two underdogs and the other one the two favorites, or a heterogeneous design ($D = HET$) which is characterized by two heterogeneous departments each consisting of one underdog and one favorite.

In the following, there will be one tournament in each department.[7] In the given context of departmental tournaments, two workers i and j compete for the monetary prizes w_H and w_L with $w_H > w_L$ in each tournament. If $q_i > q_j$, worker i will receive the high winner prize w_H, whereas worker j will get the loser prize w_L. This paper departs from the standard tournament literature by assuming that workers have *perceived prizes* which may differ from the monetary tournament prizes w_H and w_L. In particular, we can imagine that on the one hand a favorite feels anger or shame when losing against an underdog. This would mean that a favorite's subjectively perceived loser prize under $D = HET$ is lower than his monetary one, i.e. he gets $w_L - \delta$ in case of losing with $\delta > 0$, whereas the underdog's perceived loser prize is

(2003), for example. Alternatively, heterogeneity can be introduced via the workers' cost functions (or, very similar, by a multiplicative connection of effort and ability). Concerning the tournament literature, the former modelling used in this paper refers to "unfair" contests, whereas the latter one leads to "uneven" contests in the terminology of O'Keefe, Viscusi and Zeckhauser (1984). This distinction and its implications will be discussed later on in more details.

[7]This assumption rules out the possibility of one centralized tournament between all four workers in order to allow the sorting of workers by the employer. For example, we can assume that workers perform very different tasks in each tournament so that the outputs of the two departments are not comparable.

identical with his monetary one. On the other hand, an underdog might feel joy or pride when winning against a favorite. This would imply that under $D = HET$ an underdog has a higher perceived winner prize $w_H + \gamma$ with $\gamma > 0$ compared to his monetary one whereas the favorite's perceived and monetary winner prizes are the same. These two scenarios catch the typical notion that often the subjective prize of a worker also depends on the strength of his opponent.[8] When winning (losing) against a mighty (weak) opponent a worker realizes an extra utility (disutility) compared to a situation in which he wins or loses against an equally able player. Hence, under $D = HOM$ all subjectively perceived prizes are identical to the monetary prizes.[9]

We assume that each worker wants to maximize expected (subjective) wages minus effort costs. However, the employer's objective function depends on the given situation. We differentiate between a situation in which tournament prizes are exogenously given (e.g., as the outcome of a bargaining process between the union and the employer which is not modelled here) and a situation where the employer endogenously chooses the optimal tournament prizes. In the former case, the employer wants to maximize the sum of the four efforts for given prizes and, therefore, for given labor costs. In the latter case, he maximizes expected net profits, i.e. expected outputs minus prizes.

The timing of the game is as follows: In the situation with *exogenously*

[8]For example, there are parallels to the status motive in competition; see, e.g., Frank and Cook (1996), pp. 112-114.

[9]Note that the pure event of winning (losing) may lead to an extra utility (disutility) for a worker even in the case of $D = HOM$. This will be the case, if workers have *competitive preferences* in the sense of Fershtman et al. (2003a, 2003b). However, then all prizes are subjectively perceived prizes which exceed the monetary ones. In this case, w_H and w_L must be redefined, but the derived results will qualitatively remain the same.

given prizes, we have to solve a two-stage game where, at the first stage, the employer decides on the design of the firm, D. He can either choose two homogeneous departments or *fair* tournaments ($D = HOM$) in which two underdogs and two favorites compete against each other, respectively, or two heterogeneous departments or *unfair* tournaments ($D = HET$) each consisting of an underdog and a favorite.[10] Thereafter the four workers choose their efforts e_i at the second stage. However, there is a three-stage game in the situation with *endogenously* chosen prizes: Again, at the first stage, the employer chooses D. At the second stage he chooses the optimal tournament prizes. At the third stage, for a given design D and given prizes the four workers decide on their efforts.

3 On the Optimality of Unfair Tournaments

We begin the analysis by considering the simple case of fair tournaments. Then we will consider the case in which only the favorite feels anger when losing against an underdog in an unfair tournament. Finally, we will focus on the case of pride within an unfair tournament.

3.1 Fair Tournaments

If the employer chooses $D = HOM$, we will have two fair tournaments in the meaning of O'Keefe, Viscusi and Zeckhauser (1984) in which perceived and monetary prizes are identical. In each of these tournaments the agents i

[10] O'Keefe, Viscusi and Zeckhauser (1984) introduced the notion of an "unfair tournament" in which the favorite has a lead Δa. For optimal seeding in a dynamic context see Rosen (1986) and Groh et al. (2003).

and j $(i, j = t;\ t = U, F)$ want to maximize

$$\begin{aligned} EU_i(e_i) &= w_L + \Delta w \cdot \text{prob}\{q_i > q_j\} - c(e_i) \\ &= w_L + \Delta w \cdot F(e_i - e_j) - c(e_i) \end{aligned}$$

and

$$EU_j(e_j) = w_L + \Delta w \cdot [1 - F(e_i - e_j)] - c(e_j),$$

respectively, with $\Delta w = w_H - w_L$. If an equilibrium in pure strategies exists, it will be described by the following first-order conditions:[11]

$$\Delta w f\,(e_i - e_j) = c'(e_i) \quad \text{and} \quad \Delta w f\,(e_i - e_j) = c'(e_j). \tag{1}$$

Hence, we have a unique symmetric equilibrium $(e_i, e_j) = (e^*, e^*)$ with

$$\Delta w f(0) = c'(e^*). \tag{2}$$

3.2 Anger in Unfair Tournaments

In the case of two unfair tournaments ($D = HET$) in which the favorite feels anger when losing against an underdog whereas the underdog's perceived and monetary prizes are identical, the underdog's first-order condition for his optimal effort e_U^* is given by

$$\Delta w f\,(e_U^* - e_F^* - \Delta a) - c'(e_U^*) = 0, \tag{3}$$

and the favorite's one for e_F^* by

$$\alpha \Delta w f\,(e_U^* - e_F^* - \Delta a) - c'(e_F^*) = 0. \tag{4}$$

[11] To guarantee existence, $f(\cdot)$ has to be sufficiently flat and $c(\cdot)$ sufficiently steep; see Lazear and Rosen (1981), p. 845, Nalebuff and Stiglitz (1983), for example. In the special cases considered below, explicit conditions for existence will be given.

with $\alpha\Delta w \equiv w_H - (w_L - \delta)$ and $\alpha > 1$. A comparison of (3) and (4) immediately shows that a symmetric equilibrium no longer exists. Because of $\alpha > 1$, the favorite always exerts more effort than the underdog in equilibrium: $e_F^* > e_U^*$. Note that standard preferences with $\alpha = 1$ would again lead to a symmetric equilibrium now being described by

$$\Delta w f(-\Delta a) = c'(\hat{e}^*). \tag{5}$$

The resulting effort $\hat{e}^*$ would be smaller than e^* characterized by (2), since $f(\cdot)$ has a unique mode at zero. The more unfair the tournament (i.e., the higher Δa), the smaller would be $f(-\Delta a)$ and, therefore, the effort level $\hat{e}^*$.

However, according to (4) incentives will be (partly) restored for the favorite, if he feels anger from losing against his weaker opponent (i.e., $\alpha > 1$). Because of $e_F^* > e_U^*$ we have $e_U^* - e_F^* - \Delta a < 0$. Hence, equilibrium efforts according to (3) and (4) are determined by using the left-hand tail of the density $f(\cdot)$ with $f'(\cdot) < 0$ because of its unique mode at zero. Considering the system of equations (3) and (4), the general implicit-function rule yields:[12]

$$\frac{\partial e_U^*}{\partial \Delta a} = -\frac{\Delta w \bar{f}' c''(e_F)}{|J|} < 0 \tag{6}$$

$$\frac{\partial e_F^*}{\partial \Delta a} = -\frac{\alpha \Delta w \bar{f}' c''(e_U^*)}{|J|} < 0 \tag{7}$$

$$\frac{\partial e_U^*}{\partial \alpha} = -\frac{\Delta w^2 \bar{f}' \bar{f}}{|J|} < 0 \tag{8}$$

$$\frac{\partial e_F^*}{\partial \alpha} = -\frac{\Delta w \bar{f}}{|J|} \cdot \underbrace{\left(\Delta w \bar{f}' - c''(e_U^*)\right)}_{<\,0 \text{ due to } \mathrm{SOC_U}} > 0 \tag{9}$$

with $\bar{f} := f\left(e_U^* - e_F^* - \Delta a\right)$ and

$$|J| = \underbrace{\left(\Delta w \bar{f}' - c''\left(e_U^*\right)\right)}_{<\,0 \text{ due to } \mathrm{SOC_U}} \underbrace{\left(-\alpha \Delta w \bar{f}' - c''\left(e_F^*\right)\right)}_{<\,0 \text{ due to } \mathrm{SOC_F}} + \alpha \Delta w^2 \left[\bar{f}'\right]^2 > 0$$

[12] "SOC_t" denotes the second-order condition of the worker of type $t \in \{U, F\}$.

as the Jacobian determinant. According to (6) and (7), increasing unfairness in form of Δa leads to decreasing incentives – as under standard preferences. However, the comparison of (8) and (9) shows that $\frac{\partial e_F^*}{\partial \alpha} > \left| \frac{\partial e_U^*}{\partial \alpha} \right|$, i.e. we have a net positive incentive effect from the favorite feeling anger when losing against an underdog. In other words, the employer strictly gains from the favorite's disutility due to anger. Altogether, for given tournament prizes the employer will prefer unfair ($D = HET$) to fair tournaments ($D = HOM$), if $e_U^* + e_F^* > 2e^*$ where e^* is described by (2). Note that e^* is rather large – it is always larger than e_U^* – since the density $f(\cdot)$ has its peak at zero. However, the effort e_F^* may be larger than e^*, if anger is strong enough. The findings can be summarized as follows:

Proposition 1 *Let tournament prizes be exogenously given. If α is sufficiently large and Δa sufficiently small, the employer will prefer $D = HET$ to $D = HOM$.*

The results have shown that the employer benefits from emotions in form of anger when organizing an unfair tournament. If these emotions are strong enough, they will even dominate the incentive loss due to heterogeneity among the workers, and the employer will strictly prefer the design $D = HET$.

In order to check, whether there exist feasible values for α and Δa so that unfair tournaments indeed dominate fair ones from the employer's viewpoint, consider the special case of quadratic costs $c(e_i) = \frac{c}{2}e_i^2$ (with $c > 0$) and noise ε_i being uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$. The resulting convolution $f(x)$

for $\varepsilon_j - \varepsilon_i$ is triangular with[13]

$$f(x) = \begin{cases} \frac{1}{2\bar{\varepsilon}} + \frac{x}{4\bar{\varepsilon}^2} & \text{if } \; -2\bar{\varepsilon} \leq x \leq 0 \\ \frac{1}{2\bar{\varepsilon}} - \frac{x}{4\bar{\varepsilon}^2} & \text{if } \; 0 < x \leq 2\bar{\varepsilon} \\ 0 & \text{otherwise.} \end{cases}$$

as density function and

$$F(x) = \begin{cases} 0 & \text{if } x < -2\bar{\varepsilon} \\ \frac{x}{2\bar{\varepsilon}} + \frac{x^2}{8\bar{\varepsilon}^2} + \frac{1}{2} & \text{if } -2\bar{\varepsilon} \leq x \leq 0 \\ \frac{x}{2\bar{\varepsilon}} - \frac{x^2}{8\bar{\varepsilon}^2} + \frac{1}{2} & \text{if } 0 < x \leq 2\bar{\varepsilon} \\ 1 & \text{if } x > 2\bar{\varepsilon} \end{cases}$$

as corresponding distribution function. As additional assumptions let

$$\Delta w < 4c\bar{\varepsilon}^2 \quad \text{and} \quad \Delta a < 2\bar{\varepsilon}.$$

The first assumption makes the agents' objective functions strictly concave. Without the second assumption, interior pure-strategy solutions cannot exist, because exogenous noise is completely offset by the ability difference. In this case, either the favorite would choose a preemptive effort or there would be an equilibrium in mixed strategies analogously to the case of an all-pay auction with full information. Simple calculations show that

$$e^* = \frac{\Delta w}{2c\bar{\varepsilon}}, \quad \text{and} \tag{10}$$

$$e_U^* = \frac{\Delta w\,(2\bar{\varepsilon} - \Delta a)}{(\alpha - 1)\,\Delta w + 4c\bar{\varepsilon}^2} \quad \text{and} \quad e_F^* = \frac{\alpha \Delta w\,(2\bar{\varepsilon} - \Delta a)}{(\alpha - 1)\,\Delta w + 4c\bar{\varepsilon}^2}. \tag{11}$$

For given tournament prizes, the employer will prefer unfair to fair tournaments, if $2e^* < e_U^* + e_F^*$. By inserting for the three equilibrium efforts according to (10) and (11) we obtain the following result:

[13] For construction of this convolution see analogously Kräkel (2000).

Corollary 1 *Let tournament prizes be exogenously given. For quadratic costs and uniformly distributed noise, the employer will prefer* $D = HET$ *to* $D = HOM$*, iff*

$$\Delta w < 2c\bar{\varepsilon}^2 - \frac{1+\alpha}{\alpha-1}\Delta ac\bar{\varepsilon}. \tag{12}$$

The corollary shows that there are feasible parameter constellations, for which the employer strictly benefits from designing heterogeneous departments. In particular, according to condition (12) this preference is more likely the larger the impact of anger (i.e., the higher α) and the smaller the ability difference Δa.

Now we can analyze the *three-stage game* in which the employer optimally chooses w_H and w_L at the second stage. Here we can differentiate between two subcases. On the one hand, tournament prizes may be chosen by the employer without restriction. In particular, the employer can choose arbitrarily negative loser prizes to extract rents from the workers – in other words, he demands an entrance fee of the workers. On the other hand, workers may be characterized by limited liability so that the loser prize is restricted to nonnegative values ($w_L \geq 0$). The following results can be obtained:

Proposition 2 *Let tournament prizes be endogenously chosen by the employer. (i) Without restriction on* w_L*, the employer strictly prefers* $D = HOM$ *to* $D = HET$*. (ii) If the loser prize is restricted to* $w_L \geq 0$ *(limited liability) and the workers receive positive rents under* $D = HOM$ *in equilibrium, there will exist parameter values for* δ *and* Δa *so that the employer prefers* $D = HET$ *to* $D = HOM$*.*

Proof. See the appendix. ■

If no restrictions are imposed on the loser prize (i.e., we have unlimited liability), the employer is always better off by choosing two fair tournaments ($D = HOM$) (result (i)). Under this design, equilibrium efforts are identical functions of the prize spread so that the employer can implement first-best efforts for both workers by using an appropriate value for Δw. Unlimited liability then ensures that the employer indeed wants to implement this solution, because he can choose an – arbitrarily negative – loser prize w_L in order to extract all rents from the workers. However, under $D = HET$ symmetric equilibria no longer exist at the tournament stage, and the employer is only able to implement first-best effort for at most one worker. Moreover, the worker with the higher expected utility receives a positive rent in equilibrium i.e. full rent extraction is not possible for the employer under $D = HET$. Finally, organizing two unfair tournaments unambiguously leads to a welfare loss amounting to $-\delta$ in each department due to the favorite's anger when losing the tournament. Note that we assumed that the workers' types are common knowledge because otherwise the employer would not be able to choose between $D = HOM$ and $D = HET$. Theoretically the employer could then choose two different pairs of prizes (w_L^t, w_H^t) $(t = U, F)$ in the unfair tournament that depend on the type t of the winner and loser. Now the employer would be able to implement first-best efforts for both workers even under $D = HET$. However, the employer would still prefer $D = HOM$ because of the overall welfare loss -2δ under $D = HET$.[14]

If the loser prize w_L has to be non-negative (limited liability), the compar-

[14] Moreover, the sum of winner and loser prize that are paid after the tournament are typically different depending on whether the underdog or the favorite wins. However, then the employer would always choose the lower sum of prizes ex post which could distort ex ante incentives. In other words, unfair tournaments would lose their important self-commitment properties that have been highlighted by Malcomson (1984).

ison between the two tournament designs may end differently (result (ii)). Given that workers earn positive rents that are sufficiently high and that anger yields an incentive-enhancing effect as in Proposition 1, the employer will prefer unfair tournaments to fair ones. The rents have to be high enough to fully cover both the disutility $-\delta$ of feeling anger and the higher effort costs imposed on the favorite. In this case, more effort is elicited from the workers by the employer but the latter one does not pay for the extra incentives because they only reduce the workers' rents. Note that the lower the workers' reservation utilities the more likely workers will earn positive rents under limited liability and – given positive rents – the higher are these rents. In other words, low reservation utilities support the possible superiority of unfair tournaments with emotional contestants.

To summarize, the results have shown that emotions in form of anger may be beneficial for the employer although they directly lead to a welfare loss. We found out two kinds of situations in which the employer benefits form anger in unfair tournaments. The first situation assumes exogenously given tournament prizes, the second one limited liability and sufficiently high rents for workers. In both situations, the extra incentives induced by anger do not lead to additional costs for the employer.

3.3 Pride in Unfair Tournaments

When considering an unfair tournament with an underdog who feels pride after winning against a favorite, we have to modify the workers' objective functions under $D = HET$. Now the favorite's perceived and monetary prizes are identical, whereas the underdog has a higher perceived winner prize $w_H + \gamma$ with $\gamma > 0$ which leads to a higher perceived prize spread $\beta \Delta w$ with $\beta > 1$ for the underdog. The two workers' first-order conditions for their optimal effort choices are now given by

$$\beta \Delta w f\left(e_U^* - e_F^* - \Delta a\right) - c'(e_U^*) = 0 \tag{13}$$

for the underdog, and

$$\Delta w f\left(e_U^* - e_F^* - \Delta a\right) - c'(e_F^*) = 0 \tag{14}$$

for the favorite. Comparing (13) and (14) shows that again a symmetric equilibrium does not exist. Because of $\beta > 1$, now the underdog always exerts more effort than the favorite in equilibrium. However, now it is no longer clear whether the left-hand side ($e_U^* - e_F^* - \Delta a < 0$) or the right-hand side ($e_U^* - e_F^* - \Delta a > 0$) of the convolution $f(\cdot)$ becomes relevant in equilibrium and, therefore, which type of worker has a higher probability of winning. If the incentive effect outweighs the ability deficit Δa of the underdog (i.e., if $e_U^* > e_F^* + \Delta a$), the underdog will have a higher winning probability than the favorite, otherwise the opposite holds. By applying the implicit-function rule to (13) and (14) we obtain – because of the shape of $f(\cdot)$:[15]

$$\frac{\partial e_U^*}{\partial \Delta a} = -\frac{\beta \Delta w \bar{f}' c''(e_F)}{|J|} \begin{cases} > 0, \text{ if } e_U^* > e_F^* + \Delta a \\ < 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{15}$$

$$\frac{\partial e_F^*}{\partial \Delta a} = -\frac{\Delta w \bar{f}' c''(e_U^*)}{|J|} \begin{cases} > 0, \text{ if } e_U^* > e_F^* + \Delta a \\ < 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{16}$$

$$\frac{\partial e_U^*}{\partial \beta} = -\frac{\Delta w \bar{f}}{|J|} \underbrace{\left(-\Delta w \bar{f}' - c''(e_F^*)\right)}_{< 0 \text{ due to SOC}_\text{F}} > 0 \tag{17}$$

$$\frac{\partial e_F^*}{\partial \beta} = \frac{\Delta w^2 \bar{f}' \bar{f}}{|J|} \begin{cases} < 0, \text{ if } e_U^* > e_F^* + \Delta a \\ > 0, \text{ if } e_U^* < e_F^* + \Delta a \end{cases} \tag{18}$$

[15] Again "SOC$_t$" denotes the second-order condition of the worker of type $t \in \{U, F\}$.

with $\bar{f} := f\left(e_U^* - e_F^* - \Delta a\right)$ and

$$|J| = \underbrace{\left(\beta \Delta w \bar{f}' - c''\left(e_U^*\right)\right)}_{<0 \text{ due to SOC}_\text{U}} \underbrace{\left(-\Delta w \bar{f}' - c''\left(e_F^*\right)\right)}_{<0 \text{ due to SOC}_\text{F}} + \beta \Delta w^2 \left[\bar{f}'\right]^2 > 0$$

as the Jacobian determinant. Hence, for both workers a higher ability difference Δa has a motivating effect at the positive tail and a discouraging effect at the negative tail of $f(\cdot)$. The motivating effect seems to be curious at first sight, because incentives increase in the unfairness of the tournament which is impossible under standard preferences. However, here a large value of β implies an uneven situation $e_U^* > e_F^* + \Delta a$ in favor of the underdog – we are at the positive tail of $f(\cdot)$ – and in this situation an increase of Δa leads back to the mode of $f(\cdot)$ (i.e., it makes the tournament less uneven) where incentives are maximal. Intuitively, here the additional incentives due to β make the underdog exert a very high effort, but by an increase in the ability difference the favorite would get back into the race. $\partial e_U^* / \partial \beta > 0$ shows that the underdog's incentives always increase in the motivating effect of beating a predominant opponent. However, for the favorite the positive incentive effect only holds at the negative tail of $f(\cdot)$. Note that the net effect is always positive since $\frac{\partial e_U^*}{\partial \beta} > \left| \frac{\partial e_F^*}{\partial \beta} \right|$.

These comparative statics are interesting for at least two reasons. First, they give an explanation for the puzzling experimental findings of Weigelt et al. (1989) and Schotter and Weigelt (1992). They conducted several experiments on unfair tournaments and, according to their data, both types of players significantly oversupply effort. Note that their theoretical benchmark is given by $\hat{e}^*$ (see equation (5)), but by the impact of pride as modelled in this paper we obtain $e_U^* > \hat{e}^*$ and $e_F^* > \hat{e}^*$ in the relevant range (i.e., at the negative tail of $f(\cdot)$) due to the stimulating effect of β. Second, we can derive the principal's optimal tournament design at the first stage:

Proposition 3 *Let tournament prizes be exogenously given. If β is sufficiently large and $\Delta a \in [e_U^* - e_F^* - \eta, e_U^* - e_F^* + \eta]$ with $\eta > 0$ being sufficiently small, the employer will prefer $D = HET$ to $D = HOM$. Otherwise, he prefers $D = HOM$ to $D = HET$.*

Proof. The employer will prefer unfair to fair tournaments, if $2e_U^* + 2e_F^* > 4e^*$ where e^* is given by equation (2), whereas the efforts e_U^* and e_F^* are described by (13) and (14), respectively. The comparative statics have shown that $\frac{\partial e_t^*}{\partial \Delta a} > 0$ $(t = U, F)$ for $\Delta a < e_U^* - e_F^*$, and $\frac{\partial e_t^*}{\partial \Delta a} < 0$ for $\Delta a > e_U^* - e_F^*$. In both cases, in the limit $\Delta a \rightarrow (e_U^* - e_F^*)$ implies $\bar{f} \rightarrow f(0)$ and, hence, $e_F^* \rightarrow e^*$ but – because of $\beta > 1$ – $e_U^* > e^*$ (compare (2), (13) and (14)). ■

The proof of the proposition shows that if, in the unfair tournament, the ability difference comes arbitrarily close to the difference of the equilibrium efforts, all three effort levels e^*, e_U^* and e_F^* will be determined by $f(0)$. However, since we have an extra incentive effect in unfair tournaments, the underdogs will exert higher efforts than the competitors in the fair tournaments and, therefore, unfair tournaments dominate fair ones. If, on the other hand, Δa and $e_U^* - e_F^*$ clearly differ, $e_U^* - e_F^* - \Delta a$ will tend to the tails of $f(\cdot)$ so that $\bar{f}$ becomes very small and the employer strictly prefers fair to unfair tournaments.

Of course, the condition of subjectively perceived prizes (i.e., $\beta > 1$) is necessary for unfair tournaments dominating fair ones. However, we can use the framework of Weigelt et al. (1989) and Schotter and Weigelt (1992) – quadratic costs, uniformly distributed noise – in order to show that there are cases in which further restrictions on β are not necessary for the dominance of unfair tournaments. Hence, as an example, consider again the case of quadratic costs $c(e_i) = \frac{c}{2}e_i^2$ (with $c > 0$) and noise ε_i being uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$. The resulting convolution has been already described in Subsection 3.2. To guarantee existence of pure-strategy equilibria I assume that $(\beta - 1)\Delta w < 4c\bar{\varepsilon}^2$ (strict concavity) and $\Delta a < 2\bar{\varepsilon}$. The second assumption ensures that existing noise is not completely offset by the ability difference. Again, symmetric equilibrium efforts in the fair tournament, e^*, are described by equation (10) but, by using (13), (14) and the assumptions concerning the cost and the distribution function, equilibrium efforts in the unfair tournament are now given by

$$e_U^* = \frac{\beta \Delta w \left(2\bar{\varepsilon} + \Delta a\right)}{\left(\beta - 1\right)\Delta w + 4c\bar{\varepsilon}^2} \quad \text{and} \quad e_F^* = \frac{\Delta w \left(2\bar{\varepsilon} + \Delta a\right)}{\left(\beta - 1\right)\Delta w + 4c\bar{\varepsilon}^2} \tag{19}$$

if $e_U^* - e_F^* > \Delta a$, and

$$e_U^* = \frac{\beta \Delta w \left(2\bar{\varepsilon} - \Delta a\right)}{4c\bar{\varepsilon}^2 - (\beta - 1)\Delta w} \quad \text{and} \quad e_F^* = \frac{\Delta w \left(2\bar{\varepsilon} - \Delta a\right)}{4c\bar{\varepsilon}^2 - (\beta - 1)\Delta w} \tag{20}$$

if $e_U^* - e_F^* < \Delta a$. Note that $e_U^* - e_F^* > \Delta a \iff \Delta a < \frac{(\beta-1)\Delta w}{2c\bar{\varepsilon}}$ and $e_U^* - e_F^* < \Delta a \iff \Delta a > \frac{(\beta-1)\Delta w}{2c\bar{\varepsilon}}$. Calculating $2e_U^* + 2e_F^* > 4e^*$ for both cases yields $\Delta a > \frac{\left(\Delta w - 2c\bar{\varepsilon}^2\right)(\beta-1)}{(\beta+1)c\bar{\varepsilon}} =: \Delta\hat{a}_L$ for $e_U^* - e_F^* > \Delta a$, and $\Delta a < \frac{\left(\Delta w + 2c\bar{\varepsilon}^2\right)(\beta-1)}{(\beta+1)c\bar{\varepsilon}} =: \Delta\hat{a}_H$ for $e_U^* - e_F^* < \Delta a$, which do not contradict the two preceding conditions for any $\beta > 1$.[16] Hence, we obtain the following result:

Corollary 2 *Let tournament prizes be exogenously given. For quadratic costs and uniformly distributed noise, the employer will prefer $D = HET$ to $D = HOM$, iff $\Delta a \in [\Delta\hat{a}_L, \Delta\hat{a}_H]$.*

The corollary shows that, the employer will choose two unfair tournaments as long as the ability difference lies inside a certain range. Solving $e_U^* - e_F^* = \Delta a$ for the ability difference Δa, with e_U^* and e_F^* being either given by (19) or (20), leads to the middle of the interval $[\Delta\hat{a}_L, \Delta\hat{a}_H]$, which is given by $(\beta - 1)\Delta w / (2c\bar{\varepsilon})$. Here, the function $e_U^* + e_F^*$ of Δa has its maximum, which confirms the findings of Proposition 3.

[16] Note that in each case $e_U^* - e_F^* - \Delta a \in [-2\bar{\varepsilon}, 2\bar{\varepsilon}]$. In addition, note that $\Delta\hat{a}_H < 2\bar{\varepsilon}$.

In analogy to the case of anger, we can finally consider endogenous tournament prizes that are optimally chosen by the employer within the *three-stage game*. Again we have to differentiate between unlimited liability (i.e., w_L can be arbitrarily negative) and limited liability ($w_L \geq 0$) of the workers. Without restriction on the loser prize, under $D = HOM$ again the employer implements first-best effort for both workers and extracts all rents.[17] Under $D = HET$, as in the anger case, the employer is only able to induce first-best incentives for at most one worker ($e_U^* \neq e_F^*$ according to (13) and (14)), and he has to leave a positive rent to the worker with the higher expected utility. However, there is a crucial difference to the anger case. Under the pride scenario, one of the workers – the underdog – receives an extra utility γ with a certain probability. This expected extra utility relaxes the underdog's participation constraint so that the employer is able to induce higher incentives compared to fair tournaments. We can imagine that there exist specifications for the cost function $c(e_i)$ and the distribution $G(\varepsilon_i)$ for which this incentive effect becomes dominant and the employer prefers $D = HET$ to $D = HOM$ (see the proof of Proposition 4 in the appendix).

If we restrict the loser prize to non-negative values (limited liability) and the workers earn sufficiently large rents, again $D = HET$ may be beneficial for the employer. The reasoning is the same as for the anger scenario: Pride of the underdog leads to additional incentives for at least one of the workers, and the net incentive effect for both workers is always positive (see equations (17) and (18)). Hence, if the workers receive large rents under $D = HOM$, the employer can induce higher incentives to them under $D = HET$ without paying for the additional effort costs, since they only reduce the workers' rents. Note that such situations are even more likely in the pride case than

[17] See the proof of Proposition 2.

in the anger case since, with pride, the underdog receives the extra utility γ whereas in the anger scenario the favorite suffers from an extra disutility δ. Therefore, the positive rents have to cover δ as well as the additional effort costs of the favorite who feels anger, but in the case in which the underdog feels pride the additional effort costs are partly covered by the expected extra utility $\gamma F\left(e_U^* - e_F^* - \Delta a\right)$. The findings can be summarized in the following proposition:

Proposition 4 *Let tournament prizes be endogenously chosen by the employer. (i) Under unlimited liability of the workers, there exist cost functions* $c\left(e_i\right)$ *and distributions* $G\left(\varepsilon_i\right)$ *for which the employer prefers* $D = HET$ *to* $D = HOM$. *(ii) If, under limited liability, the workers receive sufficiently large rents under* $D = HOM$ *in equilibrium,* $D = HET$ *may dominate* $D = HOM$ *from the employer's viewpoint.*

Proof. See the appendix. ■

4 Discussion

The results above have shown that in unfair tournaments emotions as anger and pride effect both overall welfare and the employer's expected profits. The effects on expected profits have been analyzed in detail: The comparative statics have shown that the net effect of emotions on both workers' efforts is always positive. If emotions create additional incentives compared to fair tournaments and if the employer need not pay for the enhanced incentives – since (1) tournament prizes are exogenous or (2) the underdog's participation constraint is sufficiently relaxed by expected pride or (3) workers receive sufficiently high rents –, the employer will benefit from emotional incentives due to unfair tournaments. Consider, for example, an unfair tournament in which both the underdog and the favorite may feel emotions – the underdog pride and the favorite anger. Then according to equations (4) and (13) the workers' first-order conditions are given by

$$(\Delta w + \gamma) f (e_U^* - e_F^* - \Delta a) - c'(e_U^*) = 0 \quad (21)$$

$$\text{and} \quad (\Delta w + \delta) f (e_U^* - e_F^* - \Delta a) - c'(e_F^*) = 0 \quad (22)$$

with $\gamma, \delta > 0$. If in this situation the employer need not pay for emotional incentives, he will have the following preferences (see Propositions 1 and 3): If $\gamma > \delta$ (i.e., $e_U^* > e_F^*$), then γ and δ should be large and Δa close to $e_U^* - e_F^*$. If $\gamma < \delta$ (i.e., $e_U^* < e_F^*$), then γ and δ should be large and close together, whereas Δa should be close to zero.

However, the welfare effects of emotions are not quite clear. For example, if pride (anger) is extremely important so that the underdog (favorite) realizes a very large extra utility (disutility) γ $(-\delta)$ in case of winning (losing) the unfair tournament, then it will be always (never) efficient to choose $D = HET$ instead of $D = HOM$, since the workers' monetary incomes and the employer's expected profits will only play a marginal role in this situation. If we restrict the welfare analysis to *monetary values* and do not count emotional gains or losses, we will obtain a much clearer result. Recall from equation (A3) from the proof of Proposition 2 that first-best effort e^{FB} which equalizes marginal revenue and marginal costs is implicitly described by

$$1 = c'\left(e^{FB}\right). \quad (23)$$

Hence, monetary welfare is maximized when implementing effort e^{FB} for both workers. The proof of Proposition 2 has shown that under unlimited liability first-best effort is always induced by the employer to both workers in a fair tournament, whereas he cannot implement e^{FB} for both workers in an unfair one if workers feel either anger or pride. However, we can show that even under the most promising circumstances – (a) workers feel anger as well as pride with $\gamma = \delta$ in equations (21) and (22), and (b) workers are characterized by unlimited liability – the employer does not want to implement first-best effort for both workers. Let $EU_t(e_t^*)$ denote the expected utility of the worker of type t $(= U, F)$ in equilibrium. Then we obtain the following result:

Proposition 5 *Let the employer choose prizes endogenously under unlimited liability in an unfair tournament with both anger and pride. If both emotions have the same impact (i.e., $\gamma = \delta$ in (21) and (22)), then we will have a symmetric equilibrium $e_U^* = e_F^* = \tilde{e}^*$ at the tournament stage with*

$$\tilde{e}^* \begin{cases} > e^{FB}, & \text{if } EU_F(\tilde{e}^*) = \bar{u} \\ < e^{FB}, & \text{if } EU_U(\tilde{e}^*) = \bar{u}. \end{cases}$$

Proof. See the appendix. ■

Proposition 5 shows that "symmetric emotions" allow for a symmetric equilibrium at the tournament stage so that the employer is able to implement first-best efforts for both workers in an unfair tournament. However, the employer will never do so. He either induces excessively high efforts so that expected anger leads to a binding participation constraint for the favorite, or he chooses less than efficient effort so that expected pride makes the underdog's participation constraint bind. The intuition for this result is the following: Note that in equilibrium each worker exerts effort according to

$$(\Delta w + \gamma) f(-\Delta a) = c'(\tilde{e}^*).$$

Hence, the lower the ability difference, Δa, and the higher the impact of emotions, γ, the higher will be the effort level $\tilde{e}^*$.[18] In the case of $\tilde{e}^* >$

[18] Recall that the convolution $f(\cdot)$ has a unique mode at zero.

e^{FB}, the underdog's expected utility must exceed the expected utility of the favorite, i.e.

$$\begin{aligned}(\Delta w+\gamma) F(-\Delta a) &> -\gamma+(\Delta w+\gamma)[1-F(-\Delta a)] \Leftrightarrow \\ \gamma &> \left(\frac{1}{2F(-\Delta a)}-1\right)\Delta w.\end{aligned}$$

In other words, for an excessively high effort level the emotional influences have to be sufficiently high and the ability difference sufficiently low.

The tournament considered here is modelled as a one-shot game. In a dynamic setting (e.g., in a career-concerns framework), perhaps alternative interpretations can be given for γ and δ. From a dynamic perspective, both parameters may be interpreted as reputation effects if the labor market is uncertain about the true abilities of the workers. Then if a presumable favorite loses against a presumable underdog, the former one will realize an extra disutility because the labor market adjusts its ability expectations downward whereas the latter one receives an extra utility due to Bayesian updating. Of course, the model considered in this paper is static with abilities being common knowledge and ignores aspects of career concerns, but there are dynamic tournament models which particularly focus on these aspects (see Zabojnik and Bernhardt 2001, Koch and Peyrache 2003).

As mentioned above the distinction between fair and unfair tournaments was introduced in the literature by O'Keefe et al. (1984). We can also apply the concept of emotions and subjectively perceived prizes to "uneven tournaments" in the terminology of O'Keefe et al. In those tournaments, again a favorite competes against an underdog, but now the underdog is characterized by a steeper cost function compared to the favorite. The experimental findings of Bull et al. (1987) and Schotter and Weigelt (1992) on uneven tournaments show that only the underdogs exert significantly more effort than theoretically predicted. The concept introduced in this paper can explain these findings: If pride leads to additional incentives, the underdog will always choose more than the equilibrium effort of a worker with standard preferences. If the impact of pride is (a) larger than that of anger and (b) sufficiently high to compensate for the steeper cost function, the underdog may even choose higher effort than the favorite.

Finally, the concept of emotions can be applied to individualistic incentive schemes like bonuses or piece rates (Kräkel 2004). Under these schemes, workers are compensated independently, but either a worker feels joy/frustration when meeting/non-meeting a certain target, or the pure existence of co-workers and their success may influence the behavior of other workers at the same workplace (peer effects). The findings in Kräkel (2004) show that the tournament results in a similar way also hold for bonuses and piece rates although workers are solely compensated according to individual output. The intuition for the similarity can be explained by the fact that emotions make workers care for their co-workers so that there will be a compensation game between the workers. Interestingly, if anger and pride have a different impact, the employer will never implement first-best incentives under piece rates any longer despite risk neutrality and unlimited liability of the workers. Instead he always prefers to utilize the compensation game between the workers in order to elicit extra effort from them. The field experiments by Falk and Ichino (2003) empirically support the existence of such peer effects: In their experiments, subjects either have to work alone (single treatment) or as pairs consisting of two subjects (pair treatment). Each subject earns a fixed payment. The empirical findings show that the average output in the pair treatment significantly exceeds the output in the single treatment. Hence, observing the performance of co-workers leads to positive peer effects that raise overall productivity.

5 Conclusion

In this paper, we introduce a concept of emotions into the theory of rank-order tournaments. We analyze the impact of emotions on both the workers' incentives and the employer's profits. It can be shown that the net effect of anger and pride on the two workers' efforts is always positive. Furthermore, the employer will benefit from emotional incentives and, hence, from organizing an unfair tournament if he need not directly pay for the enhanced incentives, i.e. if tournament prizes are exogenous or the workers' participation constraints are sufficiently relaxed by expected pride or workers receive sufficiently high rents.

The concept of emotions used in this paper has a special focus. Here, we have concentrated on emotions that emerge when comparing one's own performance with the performance of heterogeneous co-workers. By this, the interplay of emotions and incentives can be analyzed in detail. Moreover, results can be derived concerning the optimal design of departments and the optimality of unfair tournaments from the employer's viewpoint. Finally, the concept is used in order to explain experimental findings on the oversupply of effort in tournaments which contradict standard economic theory.

The analysis of emotions can be extended in several directions. For example, this paper considers the impact of emotions on incentives. Perhaps, there are also matching effects concerning different types of workers with different emotional attitudes. Considering such weak factors like the "chemistry" between co-workers may be important when deciding about the composition of departments and work groups. As another example, it may be interesting to discuss emotions in a dynamic setting. Over time there may be reinforcement effects concerning such emotions like anger or frustration and, hence, the existence of certain threshold levels may be decisive for workers' actions. Furthermore, in a dynamic context evolutionary aspects concerning the emergence or disappearance of certain emotional attitudes in work groups can be analyzed.

Appendix

Proof of Proposition 2:

(i) In the case of two fair tournaments ($D = HOM$), for each department the employer chooses tournament prizes in order to maximize

$$\pi = 2e^* (\Delta w) + 2a_t - \Delta w - 2w_L \qquad (t = U, F) \tag{A1}$$

subject to the workers' individual participation constraint

$$\frac{\Delta w + 2w_L}{2} - c\left(e^* (\Delta w)\right) \geq \bar{u} \tag{A2}$$

with $e^* (\Delta w)$ being described by the incentive constraint (2). Note that first-best effort e^{FB} is defined by

$$e^{FB} = \arg \max_{e_t} \{q_t - c(e_t)\} \qquad (t = U, F),$$

which leads to

$$1 = c' \left(e^{FB}\right). \tag{A3}$$

Since the loser prize w_L decreases the employer's objective function, he chooses w_L so that (A2) is binding, i.e. the employer extracts all rents from the workers and wants to maximize overall welfare by implementing first-best efforts. Hence, the employer chooses

$$w_H = c\left(e^{FB}\right) + \bar{u} + \frac{1}{2f(0)} \quad \text{and} \quad w_L = c\left(e^{FB}\right) + \bar{u} - \frac{1}{2f(0)}.$$

In an unfair tournament ($D = HET$), the employer wants to maximize

$$\pi = e_U^* (\Delta w) + e_F^* (\Delta w) + a_U + a_F - \Delta w - 2w_L \tag{A4}$$

subject to the workers' participation constraints

$$w_L + \Delta w F\left(e_U^* (\Delta w) - e_F^* (\Delta w) - \Delta a\right) - c\left(e_U^* (\Delta w)\right) \geq \bar{u} \tag{A5}$$

$$w_L - \delta + (\Delta w + \delta)\left[1 - F\left(e_U^* (\Delta w) - e_F^* (\Delta w) - \Delta a\right)\right] - c\left(e_F^* (\Delta w)\right) \geq \bar{u} \tag{A6}$$

with $e_U^*(\Delta w)$ and $e_F^*(\Delta w)$ being implicitly defined by (3) and (4). To save labor costs, the employer chooses w_L to make the participation constraint of the worker with the lower expected utility just bind, whereas the other worker receives a positive rent. However, recall that $e_F^*(\Delta w) > e_U^*(\Delta w)$ which implies $F(e_U^*(\Delta w) - e_F^*(\Delta w) - \Delta a) < 0.5$ but also $c(e_F^*(\Delta w)) > c(e_U^*(\Delta w))$. Hence without further specifying the distribution and the cost function it is not clear whether the left-hand side of (A5) is larger than the left-hand side of (A6) or vice versa. Anyway, since the incentive-enhancing effect of δ is irrelevant here – incentives can be continuously adjusted by appropriately choosing Δw, whereas w_L solely serves for transferring wealth between the employer and the workers –, disutility δ yields a welfare loss, and the employer cannot implement e^{FB} for both workers, $D = HOM$ unambiguously dominates $D = HET$ from the employer's viewpoint.

(ii) As a starting point look at the participation constraint (A2) under $D = HOM$ and let (A2) be non-binding in equilibrium, i.e. workers earn positive rents. If we now switch to $D = HET$ with Δa being arbitrarily close to zero and with δ fulfilling $e_U^* + e_F^* > 2e^*$ for given tournament prizes according to Proposition 1, then the employer may prefer $D = HET$ to $D = HOM$: Overall efforts are higher in the unfair tournament but the employer does not have to pay for the large effort costs, $c(e_F^*(\Delta w))$, which only reduce agent F's rent. Of course, according to (A6) the workers' rents have to be sufficiently large so that they are still positive after the switch to $D = HET$ despite the additional disutility δ and the higher effort costs $c(e_F^*(\Delta w))$.

In order to illustrate that such scenarios indeed exist for feasible values of δ and Δa, consider the following example: Let again ε_i $(i = A, B)$ be uniformly distributed over $[-\bar{\varepsilon}, \bar{\varepsilon}]$. Effort costs are described by $c(e_i) = \frac{c}{3} e_i^3$.

Let, for simplicity, $c = \bar{\varepsilon} = \delta = 1$, $\Delta a = 0.1$ and $\bar{u} = 0$. Hence, we can use the triangular convolution above with range $[-2, 2]$ and $f(0) = \frac{1}{2\bar{\varepsilon}} = \frac{1}{2}$. According to (A3), first-best effort is given by $e^{FB} = 1$, and the optimal loser prize w_L for implementing e^{FB} under $D = HOM$ by $w_L = \frac{1}{3}(1)^3 - \frac{2\cdot 1}{2} = \frac{1}{3} - 1 < 0$, which is not feasible under limited liability. The optimal solution under $D = HOM$ can be calculated as follows: The workers' incentive constraint (2) simplifies to

$$e^* = \sqrt{\frac{\Delta w}{2}}.$$

Hence, the employer wants to maximize

$$\pi_{HOM} = 2\sqrt{\frac{\Delta w}{2}} + 2a_t - \Delta w - 2w_L \qquad (t = U, F)$$

subject to

$$\frac{\Delta w + 2w_L}{2} - \frac{1}{3}\left(\sqrt{\frac{\Delta w}{2}}\right)^3 \geq 0 \quad \text{and} \quad w_L \geq 0.$$

The employer optimally chooses $\Delta w^* = \frac{1}{2}$ and $w_L^* = 0$ which yields overall profits $2\pi_{HOM}^* = 1 + 2a_L + 2a_H$ from both fair tournaments, whereas each worker receives a positive rent $\frac{5}{24} = 0.20833$.

Under $D = HET$, we know from (3) and (4) and the left-hand side of the triangular convolution that workers behave according to

$$\Delta w \left(\frac{1}{2} + \frac{e_U^* - e_F^* - 0.1}{4}\right) = e_U^{*2} \tag{A7}$$

$$\text{and} \quad (\Delta w + \delta)\left(\frac{1}{2} + \frac{e_U^* - e_F^* - 0.1}{4}\right) = e_F^{*2} \tag{A8}$$

which implies

$$e_F^* = \sqrt{\frac{\Delta w + \delta}{\Delta w}} e_U^*.$$

Inserting into the first-order condition (A7) and solving for e_U^* gives

$$\begin{aligned} e_U^* &= \frac{1}{40}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + (25\delta + 760)\,\Delta w}\right) \\ &= \frac{1}{40}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + 785\Delta w}\right) \\ e_F^* &= \frac{1}{40}\sqrt{\frac{\Delta w + 1}{\Delta w}}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + 785\Delta w}\right) \end{aligned}$$

with $\Omega := \left(1 - \sqrt{\frac{\Delta w+\delta}{\Delta w}}\right) = \left(1 - \sqrt{\frac{\Delta w+1}{\Delta w}}\right)$. The employer's expected profits for organizing an unfair tournament are

$$\begin{aligned} \pi_{HET} &= e_U^* + e_F^* + a_L + a_H - \Delta w - 2w_L \\ &= \left(1 + \sqrt{\frac{\Delta w + 1}{\Delta w}}\right) e_U^* + a_L + a_H - \Delta w - 2w_L \\ &= \left(1 + \sqrt{\frac{\Delta w + 1}{\Delta w}}\right) \frac{1}{40}\left(5\Delta w\Omega + \sqrt{50\Delta w^2\Omega + 785\Delta w}\right) \\ &\quad + a_L + a_H - \Delta w - 2w_L. \end{aligned}$$

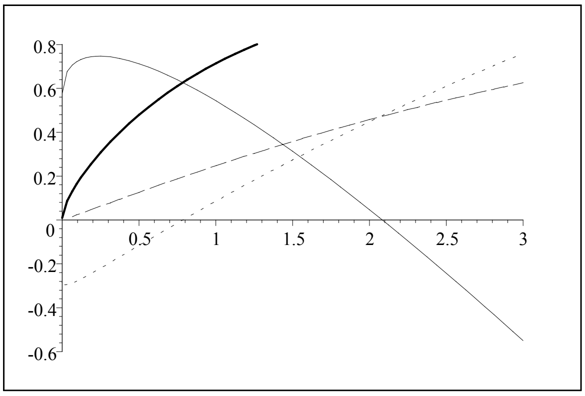
The workers' expected utilities can be written as

$$EU_U\left(e_U^*\right) = w_L + \Delta w\left(\frac{\left(\Omega e_U^* - 0.1\right)}{2} + \frac{\left(\Omega e_U^* - 0.1\right)^2}{8} + \frac{1}{2}\right) - \frac{1}{3}e_U^{*3}$$

and

$$\begin{aligned} EU_F\left(e_F^*\right) &= (\Delta w + 1)\left(1 - \left(\frac{\left(\Omega e_U^* - 0.1\right)}{2} + \frac{\left(\Omega e_U^* - 0.1\right)^2}{8} + \frac{1}{2}\right)\right) \\ &\quad + w_L - 1 - \frac{1}{3}\left(\sqrt{\frac{\Delta w + 1}{\Delta w}}e_U^*\right)^3. \end{aligned}$$

Plotting π_{HET}, EU_U and EU_F as functions of Δw with $a_L = a_H = 0$ gives the following figure:



(independent variable at the abscissa: Δw; solid thin line: π_{HET} under $w_L = 0$; dashed line: EU_U under $w_L = 0$; dotted line: EU_F under $w_L = 0$; solid bold line: π_{HET} under $EU_F = 0$)

Note that all but the solid bold line hold for $w_L = 0$. Since the objective functions (function) of both workers (the employer) strictly increase (decreases) in w_L, only values between the maximum of the π_{HET} graph (= solid thin line) and the intersection between the EU_F graph (= dotted line) and the abscissa are relevant for the optimal Δw. Note also that $EU_U(e_U^*) > EU_F(e_F^*)$ in the relevant parameter range for Δw. Hence, the employer chooses Δw and w_L to maximize π_{HET} subject to $EU_F \geq 0$ and $w_L \geq 0$. Since π_{HET} strictly decreases in w_L but both restrictions, $EU_F \geq 0$ and $w_L \geq 0$, relax with increasing w_L, at least one of the two constraints is binding in equilibrium. In the figure above with $w_L = 0$, the employer would choose Δw so that EU_F just intersects the abscissa. This happens at $\Delta w = 0.78525$ where the employer receives profits $\pi_{HET} = 0.62644 + a_L + a_H$. If otherwise $w_L > 0$, the employer would choose w_L to make the favorite's participation constraint just bind which implies

$$\begin{aligned}
w_L^* &= 1+\frac{1}{3}\left(\sqrt{\frac{\Delta w+1}{\Delta w}}e_U^*\right)^3 \\
&\quad -(\Delta w+1)\left(1-\left(\frac{(\Omega e_U^*-0.1)}{2}+\frac{(\Omega e_U^*-0.1)^2}{8}+\frac{1}{2}\right)\right).
\end{aligned}$$

When inserting into π_{HET} we obtain

$$\begin{aligned}
\pi_{HET} &= \left(1+\sqrt{\frac{\Delta w+1}{\Delta w}}\right)e_U^*-\frac{2}{3}\left(\sqrt{\frac{\Delta w+1}{\Delta w}}e_U^*\right)^3 \\
&\quad +a_L+a_H-1-2(\Delta w+1)\left(\frac{(\Omega e_U^*-0.1)}{2}+\frac{(\Omega e_U^*-0.1)^2}{8}\right).
\end{aligned}$$

which is described by the solid bold line in the figure above. We can easily see that in this case the employer would choose the corner solution $\Delta w = 0.78525$. Altogether, when organizing two unfair tournaments the employer's overall profits are $2\pi_{HET}^* = 1.2529+2a_L+2a_H > 2\pi_{HOM}^* = 1+2a_L+2a_H$.

Proof of Proposition 4:

Since result (ii) proceeds analogously to result (ii) of Proposition 2, it remains to show that under unlimited liability of the workers there exist cost functions $c(e_i)$ and distributions $G(\varepsilon_i)$ for which the employer prefers $D = HET$ *to* $D = HOM$. The employer's optimization problem can be characterized by the Lagrangian

$$\begin{aligned}
L \quad & (e_U, e_F, \Delta w, w_L) = e_U+e_F+a_U+a_F-\Delta w-2w_L \\
& +\lambda_1\cdot[(\Delta w+\gamma)f(e_U-e_F-\Delta a)-c'(e_U)] \\
& +\lambda_2\cdot[\Delta w f(e_U-e_F-\Delta a)-c'(e_F)] \\
& +\lambda_3\cdot[w_L+(\Delta w+\gamma)F(e_U-e_F-\Delta a)-c(e_U)-\bar{u}] \\
& +\lambda_4\cdot[w_L+\Delta w[1-F(e_U-e_F-\Delta a)]-c(e_F)-\bar{u}]
\end{aligned}$$

with $\lambda_1, \lambda_2 \geq 0$ as multipliers for the workers' incentive constraints (13) and (14), and $\lambda_3, \lambda_4 \geq 0$ as multipliers for the workers' participation constraints. In optimum, we must have that

$$\begin{aligned}\frac{\partial L}{\partial e_U} &= 1 + \lambda_1 \left[(\Delta w + \gamma) \bar{f}' - c''(e_U) \right] + \lambda_2 \Delta w \bar{f}' \\ &\quad + \lambda_3 \left[(\Delta w + \gamma) \bar{f} - c'(e_U) \right] - \lambda_4 \Delta w \bar{f} = 0\end{aligned} \tag{A9}$$

$$\begin{aligned}\frac{\partial L}{\partial e_F} &= 1 - \lambda_1 (\Delta w + \gamma) \bar{f}' + \lambda_2 \left[-\Delta w \bar{f}' - c''(e_F) \right] \\ &\quad - \lambda_3 (\Delta w + \gamma) \bar{f} + \lambda_4 \left[\Delta w \bar{f} - c'(e_F) \right] = 0\end{aligned} \tag{A10}$$

$$\frac{\partial L}{\partial \Delta w} = -1 + (\lambda_1 + \lambda_2) \bar{f} + (\lambda_3 - \lambda_4) \bar{F} + \lambda_4 = 0 \tag{A11}$$

$$\frac{\partial L}{\partial w_L} = -2 + \lambda_3 + \lambda_4 = 0 \tag{A12}$$

with $\bar{f} := f(e_U - e_F - \Delta a)$ and $\bar{F} := F(e_U - e_F - \Delta a)$. Condition (A12) shows that at least one participation constraint is binding in equilibrium. Typically, exactly one participation constraint will be binding: Since the loser prize w_L only serves to transfer wealth between the employer and the workers and because this prize can be arbitrarily negative, the employer chooses it so that the worker with the lower expected utility just receives $\bar{u}$ in expected terms. Combining (A9) and (A10) gives

$$2 - \lambda_1 c''(e_U) - \lambda_2 c''(e_F) - \lambda_3 c'(e_U) - \lambda_4 c'(e_F) = 0. \tag{A13}$$

The two incentive constraints together yield

$$\frac{c'(e_U)}{\Delta w + \gamma} = \frac{c'(e_F)}{\Delta w}. \tag{A14}$$

Of course, without further specifying the cost function and the probability distribution no clear results can be derived. Hence, Proposition 4(i) only claims that for certain specifications the employer prefers $D = HET$ to

$D = HOM$. Consider, for example, the case of quadratic costs $c(e_i) = \frac{c}{2}e_i^2$ and uniformly distributed noise $\varepsilon_i \in [-\bar{\varepsilon}, \bar{\varepsilon}]$ so that $\varepsilon_j - \varepsilon_i$ is triangularly distributed – as in the Corollaries 1 and 2. In order to guarantee a strictly concave objective function for both workers and the existence of pure-strategy equilibria, let

$$\Delta w + \gamma < 4c\bar{\varepsilon}^2 \tag{A15}$$

and

$$\Delta a < 2\bar{\varepsilon}. \tag{A16}$$

Furthermore, let the favorite's participation constraint be binding so that we have $\lambda_3 = 0$ and $\lambda_4 = 2$ (see (A12)). In this case, (A13) can be rewritten as

$$\lambda_1 + \lambda_2 = \frac{2}{c} - 2e_F.$$

Inserting into (A11) (together with $\lambda_3 = 0$ and $\lambda_4 = 2$) leads to

$$\left(\frac{2}{c} - 2e_F\right) f(e_U - e_F - \Delta a) - 2F(e_U - e_F - \Delta a) + 1 = 0.$$

By substituting for the triangular distribution and assuming $e_U - e_F - \Delta a < 0$ (hence, later on we have to check whether this condition indeed holds) we can rearrange the last condition to

$$\left(\frac{4\bar{\varepsilon}}{c} - 4\bar{\varepsilon}e_F\right) + \left(\frac{2}{c} - 2e_F - 4\varepsilon\right)(e_U - e_F - \Delta a) - (e_U - e_F - \Delta a)^2 = 0. \tag{A17}$$

For quadratic costs, (A14) simplifies to

$$\frac{e_U}{(\Delta w + \gamma)} = \frac{e_F}{\Delta w} \tag{A18}$$

and the favorite's participation constraint to

$$\Delta w \left(\frac{1}{2\bar{\varepsilon}} + \frac{e_U - e_F - \Delta a}{4\bar{\varepsilon}^2}\right) = ce_F. \tag{A19}$$

Solving the system of equations (A17)–(A19) for e_U, e_F and Δw yields

$$e_U^* = \frac{\gamma^2 + 4c\bar{\varepsilon}^2 \left(c\Delta a \left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 2\gamma c\Delta a\left(4\bar{\varepsilon} - \Delta a\right)}{2c\left(4c\bar{\varepsilon}^2 - \gamma\right)\left(2\bar{\varepsilon} - \Delta a\right)}$$

$$e_F^* = \frac{\gamma^2 + 4c\bar{\varepsilon}^2 \left(c\Delta a \left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{2c\left(4c\bar{\varepsilon}^2 - \gamma\right)\left(2\bar{\varepsilon} - \Delta a\right)}$$

$$\Delta w^* = \frac{\gamma^2 + 4c\bar{\varepsilon}^2 \left(c\Delta a \left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{2c\left(2\bar{\varepsilon} - \Delta a\right)^2}.$$

At last, the favorite's binding participation constraint

$$w_L + \Delta w\left[1 - F\left(e_U^* - e_F^* - \Delta a\right)\right] - \frac{c}{2}e_F^{*2} = \bar{u}$$

leads to the optimal loser prize

$$\begin{aligned} w_L^* &= \bar{u} - \left(2\gamma\left(2\bar{\varepsilon} - \Delta a\right) + 3\gamma\left(\gamma - 8c\bar{\varepsilon}^2\right) + 4c\bar{\varepsilon}^2\left(8c\bar{\varepsilon}^2 + c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(\Delta a - 2\bar{\varepsilon}\right)\right)\right) \times \\ &\quad \frac{\gamma^2 + 4c\bar{\varepsilon}^2 \left(c\Delta a \left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{8c\left(4c\bar{\varepsilon}^2 - \gamma\right)^2\left(2\bar{\varepsilon} - \Delta a\right)^2}. \end{aligned}$$

The employer's expected profits from organizing an unfair tournament are, therefore,

$$\begin{aligned} \pi_{HET} &= e_U^* + e_F^* + a_U + a_F - \Delta w^* - 2w_L^* \\ &= a_U + a_F - 2w_L^* \\ &\quad + \frac{\gamma\left(8c\bar{\varepsilon}^2 - \gamma\right) - 2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right)\left(\left(c\bar{\varepsilon} - 1\right)2\bar{\varepsilon} + \Delta a\right)}{2c\left(2\bar{\varepsilon} - \Delta a\right)^2} \\ &= a_U + a_F - 2\bar{u} \\ &\quad + \frac{\gamma\left(8c\bar{\varepsilon}^2 - \gamma\right) - 2\left(c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right)\left(\left(c\bar{\varepsilon} - 1\right)2\bar{\varepsilon} + \Delta a\right)}{2c\left(2\bar{\varepsilon} - \Delta a\right)^2} \\ &\quad + \left(2\gamma\left(2\bar{\varepsilon} - \Delta a\right) + 3\gamma\left(\gamma - 8c\bar{\varepsilon}^2\right) + 4c\bar{\varepsilon}^2\left(8c\bar{\varepsilon}^2 + c\Delta a\left(4\bar{\varepsilon} - \Delta a\right) + 2\left(\Delta a - 2\bar{\varepsilon}\right)\right)\right) \times \\ &\quad \frac{\gamma^2 + 4c\bar{\varepsilon}^2 \left(c\Delta a \left(4\bar{\varepsilon} - \Delta a\right) + 2\left(2\bar{\varepsilon} - \Delta a\right)\right) - 2\gamma\left(2\bar{\varepsilon} - \Delta a\right) - 8c\bar{\varepsilon}^2\gamma}{4c\left(4c\bar{\varepsilon}^2 - \gamma\right)^2\left(2\bar{\varepsilon} - \Delta a\right)^2} \end{aligned}$$

However, when organizing a fair tournament the employer's expected profits amount to

$$\begin{aligned}
\pi_{HOM} &= 2e^{FB} - 2c\left(e^{FB}\right) + a_U + a_F - 2\bar{u} \\
&= \frac{2}{c} - 2\frac{c}{2}\left(\frac{1}{c}\right)^2 + a_U + a_F - 2\bar{u} \\
&= \frac{1}{c} + a_U + a_F - 2\bar{u}.
\end{aligned}$$

The comparison

$$\begin{aligned}
&\frac{\gamma\left(8c\bar{\varepsilon}^2-\gamma\right)-2\left(c\Delta a\left(4\bar{\varepsilon}-\Delta a\right)+2\left(2\bar{\varepsilon}-\Delta a\right)\right)\left(\left(c\bar{\varepsilon}-1\right)2\bar{\varepsilon}+\Delta a\right)}{2c\left(2\bar{\varepsilon}-\Delta a\right)^2} \\
&+\left(2\gamma\left(2\bar{\varepsilon}-\Delta a\right)+3\gamma\left(\gamma-8c\bar{\varepsilon}^2\right)+4c\bar{\varepsilon}^2\left(8c\bar{\varepsilon}^2+c\Delta a\left(4\bar{\varepsilon}-\Delta a\right)+2\left(\Delta a-2\bar{\varepsilon}\right)\right)\right)\times \\
&\frac{\gamma^2+4c\bar{\varepsilon}^2\left(c\Delta a\left(4\bar{\varepsilon}-\Delta a\right)+2\left(2\bar{\varepsilon}-\Delta a\right)\right)-2\gamma\left(2\bar{\varepsilon}-\Delta a\right)-8c\bar{\varepsilon}^2\gamma}{4c\left(4c\bar{\varepsilon}^2-\gamma\right)^2\left(2\bar{\varepsilon}-\Delta a\right)^2} > \frac{1}{c}.
\end{aligned}$$

can be simplified to

$$\begin{aligned}
&\gamma^4-4\gamma^3\left(2\bar{\varepsilon}\left(2c\bar{\varepsilon}+1\right)-\Delta a\right)+4c\gamma^2\left(8\bar{\varepsilon}^2+4\Delta a\bar{\varepsilon}-\Delta a^2\right)\left(2\bar{\varepsilon}\left(c\bar{\varepsilon}+1\right)-\Delta a\right) \\
&-32\bar{\varepsilon}^2\gamma c^2\left(2\Delta a\bar{\varepsilon}^2\left(4c\bar{\varepsilon}+3\right)-2\bar{\varepsilon}\Delta a^2\left(3+c\bar{\varepsilon}\right)+\Delta a^3+4\bar{\varepsilon}^3\right) \\
&+16\Delta a c^3\bar{\varepsilon}^4\left(4\bar{\varepsilon}-\Delta a\right)\left(4\bar{\varepsilon}\left(c\Delta a+2\right)-\Delta a\left(c\Delta a+4\right)\right) > 0. \qquad \text{(A20)}
\end{aligned}$$

According to Proposition 4(i), we have only to show that inequality (A20) holds for at least one feasible parameter constellation. It can easily be checked that $c = \bar{\varepsilon} = 1$ and $\Delta a = \gamma = 0.5$ satisfy (A20). Moreover, we obtain

$$\begin{aligned}
e_F^* &= 1.3095 > 1 = e^{FB} \\
e_U^* &= 1.5238 > 1 = e^{FB}
\end{aligned}$$

so that (A15), (A16) and $e_U^* - e_F^* - \Delta a < 0$ hold. Hence, under the given specifications it is optimal for the employer to induce higher than first-best efforts to both workers.

Proof of Proposition 5:

Suppose we have $\gamma = \delta$ in (21) and (22). This yields a symmetric solution for the tournament stage, $\tilde{e}^* = \tilde{e}^*(\Delta w)$, implicitly defined by

$$(\Delta w + \gamma)\, f(-\Delta a) = c'(\tilde{e}^*).$$

The employer's Lagrangian at the second stage of the three-stage game (with $D = HET$ at the first stage) can be written as

$$\begin{aligned} L(\Delta w, w_L) &= 2\tilde{e}^*(\Delta w) + a_U + a_F - \Delta w - 2w_L \\ &\quad + \lambda_1[w_L + (\Delta w + \gamma)\, F(-\Delta a) - c(\tilde{e}^*(\Delta w)) - \bar{u}] \\ &\quad + \lambda_2[w_L - \gamma + (\Delta w + \gamma)[1 - F(-\Delta a)] - c(\tilde{e}^*(\Delta w)) - \bar{u}] \end{aligned}$$

with $\lambda_1, \lambda_2 \geq 0$ as multipliers. In optimum, we must have

$$\begin{aligned} \frac{\partial L}{\partial \Delta w} &= 2\frac{\partial \tilde{e}^*}{\partial \Delta w} - 1 + \lambda_1 F(-\Delta a) - \lambda_1 c'(\tilde{e}^*)\frac{\partial \tilde{e}^*}{\partial \Delta w} \\ &\quad + \lambda_2[1 - F(-\Delta a)] - \lambda_2 c'(\tilde{e}^*)\frac{\partial \tilde{e}^*}{\partial \Delta w} = 0 \end{aligned} \tag{A21}$$

and

$$\frac{\partial L}{\partial w_L} = -2 + \lambda_1 + \lambda_2 = 0. \tag{A22}$$

Hence, according to (A22) at least one participation constraint is binding in equilibrium. In general, we have $(\Delta w + \gamma)\, F(-\Delta a) \neq -\gamma + (\Delta w + \gamma)[1 - F(-\Delta a)]$ and the employer chooses w_L to make the participation constraint of the worker with the lower expected utility bind. If, therefore, $\lambda_1 = 0$ and $\lambda_2 = 2$, equation (A21) yields

$$2\frac{\partial \tilde{e}^*}{\partial \Delta w}(1 - c'(\tilde{e}^*)) + 1 - 2F(-\Delta a) = 0.$$

Since $F(-\Delta a) < \frac{1}{2}$ because of the symmetry of the convolution, we must have $c'(\tilde{e}^*) > 1$. Comparing with (A3) gives $\tilde{e}^* > e^{FB}$ because marginal costs are strictly increasing due to the convexity of the cost function. If, however, $\lambda_1 = 2$ and $\lambda_2 = 0$, equation (A21) leads to

$$2\frac{\partial \tilde{e}^*}{\partial \Delta w}\left(1 - c'\left(\tilde{e}^*\right)\right) - 1 + 2F\left(-\Delta a\right) = 0$$

and, hence, to $\tilde{e}^* < e^{FB}$.

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