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# Bonus Pools, Limited Liability, and Tournaments ‡

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**Abstract**

Tournaments have been objected as resulting from ad hoc restrictions to the contracting problem which are not easily justified. Taking into account that a performance measure might not be verifiable to a third party, however, a restriction to payments which sum up to a constant may be reasonable. The paper analyzes such fixed payment schemes with regard to their optimality and the relation to the special case of tournaments. It emerges that for a group of identical risk-neutral agents, the optimal fixed payment scheme is a tournament.

**Keywords:** bonus pools, relative performance evaluation, subjective performance evaluation, tournaments, verifiability

**JEL classification numbers:** D82, M52, M54

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## 1 Introduction

Tournaments and contests are common instruments for compensation and incentive purposes. For example, they are frequently applied in sales force compensation systems when a salesmen of the year is nominated and rewarded.[1] They are also observed in broiler production (Knoeber and Thurman 1994) or in situations where organizational members compete for a promotion within a hierarchy (Bognanno 2001).

In all of these examples, an agent's compensation mainly or exclusively depends on his rank within a group of competitors, but not (or only to a minor extent) on his absolute level of performance. From an information economics perspective, this provokes the question of why not all available information is used for compensation purposes. In this respect, Mookherjee (1984) applies Holmström's (1979) informativeness result of the standard agency model to show that a tournament contract will be optimal if an agent's rank in output is statistically sufficient for all available information. In a more specific setting, Green and Stokey (1983) show that individual contracts dominate tournaments whenever the agents' outputs admit only idiosyncratic risk.

These results have been derived in an optimal contracting framework, where no further restriction are imposed to the compensation schemes. Tournament contracts, however, have the desirable property that the total sum of payments is constant. This feature has been used to propose tournaments as a general device to overcome the so called unverifiability problem (Malcomson 1984): they can be used for compensation purposes even if the signals applied are not verifiable to a third party. Since the total payment is constant, the principal, contrary to an individual incentive contract, cannot benefit from misreporting these measures.

[1] For details on sales contests, see Kalra and Shi (2001).

But tournaments are not the only compensation form to fulfil the desired property of a constant total for wage payment. As empirical evidence, it can be noted that Japanese firms make extensive use of a special type of relative performance payment in which a constant bonus is distributed to workers of a group proportionate to their contribution to the total output (Kräkel 2003).

The present paper analyzes the role of tournaments within the class of payment schemes which fulfil the proposed property. To that purpose, I distinguish situations in which an outside party is used to hold the total payment constant from payment schemes which distribute the hole amount to a group of agents, irrespective of what outcome is observed. It emerges that under both regimes, a winner-takes-all tournament is optimal if the agents are risk-neutral, identical and of limited wealth. Under more general conditions of risk-averse agents, it is shown that a third party is not needed if the agents are of unlimited wealth, but that it should be engaged as soon as liability constraints become binding and the agents earn rents from this restriction.

The remainder of this paper is organized as follows. Section 2 introduces the basic model, in which section 3 analyzes the use of fixed payment schemes and bonus pools. Section 4 is devoted to the role of tournaments within these classes of payment schemes. Section 5 concludes. All proofs are in the appendix.

## 2 Basic model

I study a moral hazard problem with multiple agents and subjective information about the agents' productive contributions. To be concrete, consider a principal who hires a group of agents $i = 1, \ldots, n$ to perform one-time certain actions $a_i \in [\underline{a}_i, \overline{a}_i] \subset \mathbb{R}$ on his behalf. The agents are assumed to be effort-averse and weakly risk-averse, with additively separable preferences $V_i(s_i, a_i) = U_i(s_i) - d_i(a_i)$, in which $s_i$ is the monetary transfer received from the principal, and $U_i' > 0, U_i'' \leq 0, d_i' > 0, d_i'' > 0$.

I focus on the principal's problem of inducing a certain profile $\mathbf{a}^0 = (a_1^0, \ldots, a_n^0)$ of actions at minimal cost.[2] To that purpose, the principal receives a signal $y_i \in \{y_{i1}, \ldots, y_{im^i}\}$ for the action of each agent $i$. Without loss of generality, let the support of $y_i$ be indexed such that $y_{il} < y_{i,l+1} \, \forall i, l = 1, \ldots m^i - 1$. The signals follow conditional probability functions $p_i(y_i \mid a_i)$ which are assumed to be continuously differentiable in $a_i$, with positive probability for all realizations of $y_i$, regardless of the agent's action (non-moving support). Moreover, the signals are stochastically independent. Thus, from the analysis of Green and Stockey (1983), an individual contract for each agent would dominate a relative payment scheme if no further restrictions were made to the contract. Here, however, I assume that while the probabilities are common knowledge among the principal and all agents, the realization of $\mathbf{y} = (y_1, \ldots, y_n)$ is only observed by the principal. In particular, it cannot be verified to another party and thus is not available for an explicit contract.[3]

Using the signals $y_i$, the principal designs a contract $\mathbf{s}(\mathbf{y}) = (s_1(\mathbf{y}), \ldots, s_n(\mathbf{y}))$ in order to induce the desired action $\mathbf{a}^0$. Since $\mathbf{y}$ is privately observed by the principal, these payments have to be self-enforcing. The next section analyzes conditions under which this is the case. Furthermore, the agents must be willing to sign the contract. To that purpose, their expected utilities under $\mathbf{s}$ and $\mathbf{a}^0$ have to exceed their respective reservation level, which are denoted by $V_i^R$. Finally, the agents may be wealth-constricted, in which case their compensation may not

[2]Referring to the decomposition approach introduced by Grossman an Hart (1983), this is the cost minimization part of the principal's optimization problem.

[3]If the agents also acquired some information on their performance, they might engage in a signaling game as analyzed by McLeod (2003). By assuming that only the principal privately observes $y_i$, I preclude this opportunity and focus on the use of the signals for entirely discretionary bonus payments.

fall short of a certain minimum level $s_i^{min}$.

Two assumptions are adapted from the standard principal-agent model of moral hazard, in order to ensure that the first-order approach is valid to describe the agents' action choices under the contract $\mathbf{s}(\mathbf{y})$:

**Assumption 1** *(MLRP) The signals $y_i$ fulfil the monotone likelihood ratio property: $p_i'(y_i \mid a_i)/p_i(y_i \mid a_i)$ is increasing in $y_i$.*

**Assumption 2** *(CDFC) The signals $y_i$ fulfil the convexity of the distribution function condition:*[4] $\partial^2 F_i(y_i \mid a_i)/\partial a_i^2 \geq 0$.

The logic behind these conditions is analogous to the standard agency model: due to the monotone likelihood ratio property, the optimal compensation of one agent will be nondecreasing in his signal. By the convexity of the distribution function, the agent's expected utility is therefore a concave function of his own effort.[5]

## 3 Fixed payment schemes and bonus pools

If the signals were contractible, the optimal contract would specify payment schemes $s_i(y_i)$ which only depend on the agent's individual performance.[6] Without contractible information, however, the principal has to seek for alternative mechanisms which allow to use the subjective information $y_i$. In a single-agent framework, one such mechanism is a *fixed payment scheme* like it is applied in

[4] $F_i$ denotes the cumulative distribution function of $y_i$.

[5] Considered separately, the MLRP in the present model is not restrictive at all. Since $y_i$ is just a performance measure, MLRP can be seen as the convention that the realizations of $y_i$ are ordered by their likelihood ratios. The CDFC, in contrast, has frequently been objected in the literature as being very restrictive (see Jewitt (1988)). Recently, LiCalzi and Spaeter (2003) provided two rich distribution families which display both MLR and CDF.

[6] Due to the signals' independence, any relative performance evaluation would only add noise to the agent's compensation. See Mookherjee (1984).

McLeod (2003). In such a scheme, the principal divides a fixed amount of money between the agent and a third party, such as a charity. Rajan and Reichelstein (2005) extend this scheme to the multi-agent problem by assuming that a fixed amount is shared by all agents and an outside recipient. Defining $s_i(\mathbf{y})$ as agent $i$'s compensation when the signals $\mathbf{y} = (y_1, \ldots, y_n)$ are observed, a fixed payment scheme in the present model can be defined as follows:

**Definition** A payment scheme $(s_1(\mathbf{y}), \ldots, s_n(\mathbf{y}))$ is called a *fixed payment scheme* if $\sum_{i=1}^n s_i(\mathbf{y}) \leq w$ for all $\mathbf{y}$ and the residual amount $w - \sum_{i=1}^n s_i(\mathbf{y})$ is transferred to an outside party. □

Two remarks are in order here. First, there is no restriction that $s_i$ has to be nonnegative. Negative $s_i$, however, require payments to the principal, which would not be incentive compatible after the signals have been observed. The contract therefore has to be organized as follows: Each agent has to make an up-front payment of $\underline{s}_i = -\min_{\mathbf{y}} s_i(\mathbf{y})$ (or receives a base salary of $\min_{\mathbf{y}} s_i(\mathbf{y})$ if it is nonnegative) when the contract is settled, and the total bonus pool to be paid by the principal is $w + \sum_{i=1}^n \underline{s}_i$. For notational convenience, I neglect this aspect as long as it is of no consequence to the results, and stick to the fiction that negative payments can be stipulated.

The second remark refers to the structure of the payment scheme. Note that fixed payment schemes according to the definition also include those contracts composed of $n$ separate schemes $s_i(y_i)$ with $s_i(y_i) \leq w_i$, where the balance $\sum_{i=1}^n (w_i - s_i(y_i))$ is transferred to an outsider. An obvious question is whether the principal can improve such schemes by relative performance evaluation, using (part of) $w_i - s_i$ for compensating the other agents instead of "burning" the money. As an extreme, Rajan and Reichelstein (2006) consider *discretionary* *bonus pools* in which the whole amount is used internally for contracting:

**Definition** A payment scheme $(s_1(\mathbf{y}), \ldots, s_n(\mathbf{y}))$ is called a *bonus pool scheme* if $\sum_{i=1}^{n} s_i(\mathbf{y}) = w$ for all $\mathbf{y}$. $\square$

A bonus pool scheme divides a fixed amount $w$ among the agents, without any residual transferred to a third party. Any variation in one agent's compensation inevitably affects the compensation of at least one other agent. Thus, bonus pools in any case represent a nontrivial form of relative performance evaluation, entailing the negative effect that additional risk it put on an agent by making his compensation depend on signals which are noisy, but contain no information with respect to his action. Despite this drawback, Rajan and Reichelstein (2006, proposition 1) prove that for two agents with unlimited liability, the optimal fixed payment scheme to induce a certain profile $\mathbf{a}^0$ of actions is a bonus pool. The result is easily generalized to the present setting of $n$ agents:

**Proposition 1** *(Rajan and Reichelstein 2006) If the first-order approach is valid and agents are of unlimited wealth, the optimal fixed payment scheme is a bonus pool scheme.*

Technically, proposition 1 is due to the fact that all $y_i$ are stochastically independent. Under this assumption, if the principal's budget constraint were slack for some $\hat{\mathbf{y}}$, the optimization with respect to $s_i(\hat{\mathbf{y}})$ could be separated from the optimization with respect to all other $s_j$. This transfers the slackness to all realizations of $\mathbf{y}$, which of course cannot be optimal.

The economic reasoning behind this – at first glance surprising – result is that no agent suffers from the additional risk associated with the distribution of the balance because these payments will always be nonnegative. To see this, consider the separate fixed payment schemes $s_i(y_i)$. Since the signals are stochastically independent, a distribution of the balances $w_i - s_i(y_i)$ to the other agents' payments can be made without affecting the incentives of the respective agents. To that purpose, the principal simply raises agent $j$'s compensation by the same amount for all possible realizations of $y_j$, provided $y_i$ displays the realization for which the residual $w_i - s_i(y_i)$ is sought to be distributed. By this procedure, the agent $j$'s expected utility is increased. Thus, in a second step, $w_j$ can be reduced, resulting in a smaller total bonus pool $w$.

Such a reduction, however, in general can only be done if compensations are not bounded from below. Limited liability on side of the agents may therefore obscure the application of discretionary bonus pools. This can be illustrated by the following example:

**Example** Let there be $n = 2$ identical, risk neutral agents with utility function $V_i = s_i - 2a_i^2$ who can take action $a_i \in [0, 1]$ and have a reservation utility of 0.5. The signals $y_i \in \{0, 1\}$ are Bernoulli distributed with $p_i(1 \mid a_i) = a_i$. The principal seeks to induce $a_1^0 = a_2^0 = 0.5$.

First consider separate fixed sum payments for each agents. The optimal contract in the absence of liability constraints stipulates $s_i(1) = 2$ and $s_i(0) = 0$. Thus, the total payment to one agent is $w_i = 2$, and a balance of $w_i - s_i(0) = 2$ is transferred to a third party in case that $y_i = 0$.

Using a joint fixed sum payment, this balance could be transferred to the second agent, rendering payments $s_i(y_i, y_j)$ as follows: $s_i(0, 0) = s_i(1, 1) = 2$, $s_i(1, 0) = 4$ and $s_i(0, 1) = 0$. Incentives are unchanged, but the scheme leaves each agent with a utility of 1.5. Consequently, all payments can be reduced by 1, which in fact yields an optimal fixed sum contract $s_i(0, 0) = s_i(1, 1) = 1$, $s_i(1, 0) = 3$ and $s_i(0, 1) = -1$.

Now let the agents be wealth-constrained such that $s_i \geq 0$: no payments can be made to the principal. Obviously, the separate schemes remain valid under this restriction, whereas the proposed joint payment scheme requires a payments of the agents. Increasing all payments by the missing amount, however, leaves the agents with a positive rent, while the principal's net benefit is identical to that under the separate contract considered before. In fact, this is one optimal bonus pool arrangement[7] with a total cost of 4.

The question on hand, however, is whether the principal can do better with a joint fixed payment scheme even though the agents are wealth-constrained. In fact he can, but this requires payments to the outside party. An optimal fixed payment scheme determines $s_i(0,0) = s_i(0,1) = 0, s_i(1,1) = 4/3$ and $s_(1,0) = 8/3$. The total cost is $w = 8/3$, which is paid to a third party if $(y_1, y_2) = (0,0)$. $\square$

The example clarifies the inherent problem of bonus pool schemes: Even if all agents produce the lowest outcome, the bonus $w$ has to be distributed to them. If the agents earn a rent from limited liability, incentives can therefore be improved by transferring part of $w$ to an outside party. To see this, consider the agent's incentive compatibility constraint under the first-order approach:

$$\frac{\partial V_i}{\partial a_i} = \sum_{\mathbf{y}} U_i(s_i(\mathbf{y}) \frac{\partial}{\partial a_i} p(\mathbf{y} \mid \mathbf{a}^0) - d'(a_i) = 0 \tag{1}$$

Due to the assumption that the signals are stochastically independent, the marginal

[7] The solution is not unique because the principal may choose how to distribute the rent among the agents.

probability in (1) can be written as

$$\frac{\partial}{\partial a_i} p(\mathbf{y} \mid \mathbf{a}^0) = p_i'(y_i \mid a_i^0) \prod_{j \neq i} p_j(y_j \mid a_j^0) = \frac{p_i'(y_i \mid a_i^0)}{p_i(y_i \mid a_i^0)} \prod_{j=1}^{n} p_j(y_j \mid a_j^0), \qquad (2)$$

which allows to make use of the properties of the likelihood ratio. Due to the MLRP and the fact that the expected value of the likelihood ratio is zero for a non-moving support, the likelihood-ratio for the poorest outcome will be negative. From this, however, it is obvious that incentives can be improved by third-party contracting whenever a payment $s_i(y_{i1}, \mathbf{y}_{-i}) > s_i^{min}$ is stipulated or, more generally, if $s_i(y_i, \mathbf{y}_{-i}) > s_i^{min}$ and $p_i'(y_i \mid a_i^0)/p(y_i \mid a_i^0) < 0$ for some $(y_i, \mathbf{y}_{-i})$. The following proposition makes use of this effect to derive conditions under which a bonus pool scheme cannot be optimal.

**Proposition 2** *If in the optimal fixed payment scheme to implement* $\mathbf{a}^0$ *all agents earn a positive rent from limited liability, this scheme is* **not** *a bonus pool.*

The simple idea behind the proof of proposition 2 is that if effort levels $a_i^0 > \underline{a}_i$ are implemented by a bonus pool scheme, the total payment $w$ has to exceed the sum of minimum payments $s_i^{min}$ – otherwise only flat wages $s_i = s_i^{min}$ were possible. Therefore, at least one agent (say $i$) receives an extra bonus even if the poorest overall outcome $\mathbf{y}_1 = (y_{11}, \ldots, y_{n1})$, for which all likelihood ratios are negative, is realized. As the incentive constraint (1) in conjunction with (2) shows, agent $i$'s incentives are increased by reducing $s_i(\mathbf{y}_1)$, transferring the balance to an outside recipient. With regard to the implementation of $\mathbf{a}^0$, this can be used to reduce the overall wage payment $w$.

At first sight, the proposition seems to conflict a further result by Rajan and Reichelstein (2006, corollary 1), who prove that the first-best solution can be obtained by a bonus pool arrangement even under limited liability, if the signals

$y_i$ provide perfect information on $a_i$. Contrary to proposition 2, however, the liability constraint will never be binding in the situation they analyze because they assume that the agents have a nontrivial outside option in the sense that $U_i^R > U_i(s_i^{min}) - d(a_i^0)$. Therefore, a positive penalty for disobedient behavior is possible even under limited liability. Since only a minimal penalty is necessary to enforce a certain action under perfect information, this suffices to induce the first-best action, which is also without an additional cost because the punishment will never occur in the equilibrium.

## 4 Symmetric agents and tournaments

To analyze the role of tournaments among fixed payment schemes, I will now focus on environments in which agents are symmetric. Thus, assume that all action spaces, reservation utilities and liability levels are identical and all performance measures $y_i$ have the same support and follow the same probability distributions. The principal wishes to implement a symmetric Nash equilibrium of actions $a_i^0 = a^0$ for all $i$. To that purpose, he offers a *symmetric* fixed payment scheme to the agents:

**Definition** A payment scheme is called symmetric if

$$s_i(y_{il}, y_{jk}, \mathbf{y}_{-ij}) = s_j(y_{ik}, y_{jl}, \mathbf{y}_{-ij}) \tag{3}$$

for all $i, j \in N$ and all $\mathbf{y}_{-ij}$. □

Condition (3) is an anonymity property stating that an agent's payment should not depend on his identity, but only on his performance. A similar condition is well-known from the literature on contests, where it serves as part of an axiomatic approach to *contest success functions* (see Skaperdas 1996).

With regard to bonus pool schemes, probably the most prominent symmetric payment scheme is a tournament, where an agent's compensation is entirely determined by the rank of his performance within the group of all agents. But also more general and smoother payment schemes are included, like those applied in Japanese firms, where payments $s_i = wy_i / \sum_j y_j$ are proportionate to outputs (see Kräkel 2003). The latter may also be generalized to payments $s_i = wf(y_i)/\sum_j f(y_j)$ for some increasing function $f$, like it is known from contest success functions. An extreme case of such contest success functions, in turn, is again a tournament, which is obtained for $f(y) = \lim_{x\to\infty} y^x$. In this case, only the best performing agent(s) receive(s) a prize. Since under certain conditions, such winner-takes-all tournaments are cost-minimizing tournament schemes,[8] they are of particular interest for the following analysis. In the present model, they can be described as follows:

**Definition** A symmetric fixed payment scheme is called a winner-takes-all tournament if

$$s_i(y_{il}, s_{jk}, \mathbf{y}_{-ij}) = s^{min}$$

for all $i \in N$ and $j \in N/\{i\}$ such that $y_{il} < y_{jk}$. □

This is a very general definition of a tournament. It only requires that an agent is not rewarded if there is a better performing competitor. Among the best-performing agent(s), the division of the prize directly follows from the symmetry assumption: in case of a tie, it is shared equally among the best competitors. The size of this prize, however, is not specified by the definition. In particular,

[8]See Budde (2006) or, for specific distributions, Kalra and Shi (2001).

the principal may transfer money to a third party, thereby reducing the prize paid to the winner(s) of the tournament. With regard to bonus pool schemes, I will first abandon this opportunity and assume that the prize is constant for all realizations of $\mathbf{y}$. Later, I will return to the more general payment scheme.

**Definition** A tournament is called fixed-prize if its prize is identical for all signal realizations. □

Obviously, a fixed-prize winner-takes-all-tournament is a bonus pool scheme. Within this class, it can be proven as the cost-minimizing payment scheme to induce a certain action:

**Proposition 3** *If agents are risk-neutral and identical and earn a rent from limited liability, the optimal symmetric bonus pool scheme is a fixed-prize winner-takes-all-tournament.*

Like proposition 2, the proof of proposition 3 makes use of the fact that in the agent incentive compatibility constraint (1), marginal utilities are weighted by likelihood ratios. Since marginal utilities are constant for risk-neutral agents, this can be used to show that each scheme that devotes a positive payment to an inferior agent can be improved by giving the premium to (one of) the most successful agents instead.

To see this, consider two agents $i$ and $j$ and a symmetric bonus pool scheme $\mathbf{s}(y_i, y_j)$ for which there exists $\mathbf{y}^* = (y_{il^*}, y_{jk^*})$ such that $y_{il^*} < y_{jk^*}$ and $s_i(\mathbf{y}^*) > s^{min}$. Compare this scheme to $\hat{\mathbf{s}}(\mathbf{y})$ which is identical to $s$ except that $\hat{s}_i(\mathbf{y}^*) = s_i(\mathbf{y}^*) - \Delta$ and $\hat{s}_j(\mathbf{y}^*) = s_j(\mathbf{y}^*) + \Delta$ for some $\Delta \in (0, s_i(\mathbf{y}^*) - s^{min}]$. By the symmetry of the scheme, this implies that also the compensation for the permutation $\mathbf{y}^0 = (y_{ik*}, y_{jl^*})$ is changed. For agent $i$ this means that $\hat{s}_i(\mathbf{y}^0) = s_i(\mathbf{y}^0) + \Delta$.

Thus, with regard to his incentive compatibility constraint (1), the effect of a change from $\mathbf{s}$ to $\hat{\mathbf{s}}$ is

$$\begin{aligned} D &\equiv \frac{\partial \mathrm{EV}_i}{\partial a_i}(\hat{s}_i, a_i^0) - \frac{\partial \mathrm{EV}_i}{\partial a_i}(s_i, a_i^0) \\ &= \Delta \frac{p_i'(y_{ik^*} \mid a_i^0)}{p_i(y_{ik^*} \mid a_i^0)} p(\mathbf{y}^0 \mid \mathbf{a}^0) - \Delta \frac{p_i'(y_{il^*} \mid a_i^0)}{p_i(y_{il^*} \mid a_i^0)} p(\mathbf{y}^* \mid \mathbf{a}^0) \end{aligned} \tag{4}$$

For a symmetric equilibrium, $p(\mathbf{y}^* \mid \mathbf{a}^0) = p(\mathbf{y}^0 \mid \mathbf{a}^0)$. Thus, under the MLRP the sign of $D$ is positive because $y_{ik^*} > y_{il^*}$, and incentives are higher for $\hat{\mathbf{s}}$.

The economics behind this result are related to those of a standard agency model (cf. Demougin and Fluet 1998): under risk neutrality and limited liability it is most effective to reward only the top performance, the probability of which reacts most sensitive to the agent's action if the monotone likelihood ratio property is fulfilled. For a symmetric bonus pool scheme, this implies that a bonus should only be awarded to the best performing agent(s).

This result was most easily shown for bonus pool schemes. As proposition 2 has shown, however, these schemes are not the optimal fixed payment schemes if agents earn a rent from their limited liability. Thus, without further restrictions to the payment scheme, namely the exclusion of third-party contracting, only little insight can be gained from proposition 3.

As an inspection of the above reasoning shows, however, the same arguments as in the proof of proposition 3 can be applied to schemes in which part of the bonus $w$ is transferred to an outside recipient. The only requirement to apply the improvement is that the same amount is paid under $\mathbf{y}^*$ and all its permutations, which is guaranteed for any symmetric payment scheme. Proposition 3 can therefore be generalized to symmetric fixed payment schemes:

**Corrollary 1** *If agents are risk-neutral and identical and earn a rent from lim-*

*ited liability, the optimal symmetric fixed payment scheme is a winner-takes-all tournament.*

The optimal fixed payment scheme devotes a prize only to the best performing agent(s). This prize, however, may vary with the absolute level of performance. In the above example, for instance, no prize is paid if both agents fail to deliver the high outcome of the signal. In all other cases, a total amount of 8/3 is given to the best performing agent(s). If $(y_1, y_2) = (0, 0)$, this amount is transferred to the outside recipient. Consequently, not only the agent's rank, but also his absolute performance affects his compensation. By this means, the problem of tacit collusion which is inherent in fixed-prize tournaments can be mitigated.

## 5 Conclusion

This paper has analyzed fixed payment schemes under limited liability. As long as the liability constraints are not binding, the optimal fixed payment scheme is a bonus pool arrangement. As soon as agents earn a rent from limited liability, however, it becomes advantageous to restrict the payments to the agents, transferring the balance to an outside party, such as a charity. If agents are symmetric and risk-neutral, the optimal fixed payment scheme in both situations is a winner-takes-all tournament. Without liability constraints, the prize of this tournament is fixed, whereas under limited liability, the prize may be reduced for poor overall performance of the agents. This complies with procedures in corporate practice, where a bonus pool frequently is of variable size (Rajan and Reichelstein 2006, fn. 2). In most instances, however, the size of the bonus pool is not left to the principal's discretion, but is contingent to some verifiable signal, such as revenues or accounting income. In view of the fact that both fixed and variable bonus pools are observed in practice, it may therefore be an avenue for future research to analyze under which conditions one of the two alternatives prevails.

## A Proofs

### A.1 Proof of proposition 1

The proofs replicates the arguments of Rajan and Reichelstein (2006) for $n$ agents. The principal's optimization problem is given by

$$\begin{aligned}
\min \quad & w && (5)\\
\text{s.t.} \quad & w \geq \sum_{i=1}^{n} s_i(\mathbf{y}) \quad \forall \mathbf{y}, && (6)\\
& \sum_{\mathbf{y}} U_i(s_i(\mathbf{y}))p(\mathbf{y} \mid \mathbf{a}^0) - d(a_i^0) \geq U_i^R \quad i = 1, \ldots, n && (7)\\
& \sum_{\mathbf{y}} U_i(s_i(\mathbf{y}))p_{a_i}(\mathbf{y} \mid \mathbf{a}^0) - d'(a_i^0) = 0 \quad i = 1, \ldots, n, && (8)
\end{aligned}$$

where $p_{a_i}(\mathbf{y} \mid \mathbf{a}^0) = \partial p(\mathbf{y} \mid \mathbf{a}^0)/\partial a_i$ denotes the marginal probability of $\mathbf{y}$ with respect to $a_i$. Denoting the multipliers of the restrictions by $\lambda(\mathbf{y}), \mu_i$ and $\nu_i$, respectively, the optimal solution has to fulfil the following conditions:

$$\begin{aligned}
\frac{\partial L}{\partial w} &= -1 + \sum_{\mathbf{y}} \lambda(\mathbf{y}) = 0 \quad \forall\, \mathbf{y} && (9)\\
\frac{\partial L}{\partial s_i(\mathbf{y})} &= -\lambda(\mathbf{y}) + \mu_i U_i'(s_i(\mathbf{y}))p(\mathbf{y} \mid \mathbf{a}^0) \\
&+ \nu_i U_i'(s_i(\mathbf{y}))p_{a_i}(\mathbf{y} \mid \mathbf{a}^0) = 0 \quad \forall\, \mathbf{y}, i && (10)
\end{aligned}$$

Now consider a fixed payment scheme $s$ which is not a bonus pool: there exist $\hat{\mathbf{y}}$ such that $w > \sum_{i=1}^{n} s_i(\hat{\mathbf{y}})$. Consequently, $\lambda(\hat{\mathbf{y}}) = 0$ and thus (10) yields

$$-\frac{\mu_i(\hat{\mathbf{y}})}{\nu_i(\hat{\mathbf{y}})} = \frac{p_{a_i}(\hat{\mathbf{y}} \mid \mathbf{a}^0)}{p(\hat{\mathbf{y}} \mid \mathbf{a}^0)} \quad \forall i \tag{11}$$

Due to the independence of the $y_i$s, the term $p_{a_i}(\mathbf{y} \mid \mathbf{a}^0)/p(\mathbf{y} \mid \mathbf{a}^0) = p_i'(y_i \mid a_i^0)/p_i(y_i \mid a_i^0)$ only depends on $y_i$. Thus, (11) holds for all $(\hat{y}_i, \mathbf{y}_{-i})$. Since (11) is valid for all $i$, it therefore holds for all $\mathbf{y}$. But then $\lambda(\mathbf{y}) = 0$ for all $\mathbf{y}$, which contradicts (9). Hence, $s$ cannot be optimal. □

### A.2 Proof of proposition 2

The principal's optimization problem under limited liability is given by (5) – (8), with the additional constraints

$$s_i(\mathbf{y}) \geq s_i^{min} \qquad \forall\, i, \mathbf{y}. \tag{12}$$

Let $\eta_i(\mathbf{y})$ denote the Lagrange multipliers of constraints (12). The first-order condition with respect to $s_i(\mathbf{y})$ becomes

$$\begin{aligned} \frac{\partial L}{\partial s_i(\mathbf{y})} &= -\lambda(\mathbf{y}) + \mu_i U_i'(s_i(\mathbf{y}))p(\mathbf{y} \mid \mathbf{a}^0) \\ &+ \nu_i U_i'(s_i(\mathbf{y}))p_{a_i}(\mathbf{y} \mid \mathbf{a}^0) + \eta_i(\mathbf{y}) = 0 \quad \forall\, \mathbf{y}, i. \end{aligned} \tag{13}$$

Now consider the situation that all agents earn rents from limited liability in the optimal fixed payment scheme. I show that this scheme cannot be a bonus pool. To that purpose, note that $\mu_i = 0 \,\forall\, i$ when all agents earn rents. Moreover, if $\mathbf{s}(\mathbf{y})$ is a bonus pool, for each $\mathbf{y}$ at least one agent's compensation has to exceed his liability level because otherwise, only flat wages $s_i(\mathbf{y}) = s_i^{min} \,\forall \mathbf{y}, i$ are possible.

Let $i$ be the agent who receives a wage $s_i(\mathbf{y}_1) > s_i^{min}$ for the poorest overall outcome $\mathbf{y}_1 = (y_{11}, \ldots, y_{n1})$. The first-order condition w.r.t. $s_i(\mathbf{y}_1)$ becomes

$$\frac{\partial L}{\partial s_i(\mathbf{y}_1)} = -\lambda(\mathbf{y}_1) + \nu_i U_i'(s_i(\mathbf{y}))p_{a_i}(\mathbf{y} \mid \mathbf{a}^0) = 0 \tag{14}$$

because $\mu_i = 0$ and $\eta_i(\mathbf{y}_1) = 0$. Since due to the assumption of a non-moving support it holds that $\sum_{l=1}^{m^i} p'(y_{il} \mid a_i) = 0$, MLRP implies that $p_{a_i}(\mathbf{y} \mid \mathbf{a}^0) = p'(y_{i1} \mid a_i^0) \prod_{j \neq i} p(y_{j1} \mid a_j^0) < 0$. Since $\lambda(\mathbf{y}_1) > 0$ by proposition 1, it follows that $\nu_i < 0$. This, in turn, by substitution in (12) implies that $\eta_i(\mathbf{y}) > 0$ for all $\mathbf{y}$ for which $p'(y_i \mid a_i^0) \geq$, i.e. only the minimum wage $s_i^{min}$ is paid for high outcomes of $y_i$. Then, however, the incentive compatibility constraint (8) cannot be fulfilled because $\sum_{l=1}^{m^i} p'(y_{il} \mid a_i) = 0$ and $s_i(\mathbf{y}_1) > s_i^{min}$. □

### A.3 Proof of proposition 3

The proof is by contradiction. So consider a payment scheme $s_i(\mathbf{y})$ for which there exist an output $\mathbf{y}^*$ such that $y_{il}^* \equiv y_{il^*} < y_{jk}^* \equiv y_{jk^*}$ and

$$s_i(\mathbf{y}^*) \equiv s_i(y_{il^*}, s_{jk^*}, \mathbf{y}_{-ij}^*) > s^{min}. \tag{15}$$

for some $i, j, l^*, k^*$. If agents earn a rent from limited liability, the first-order conditions with respect to $s_i$ and $s_j$ are given by

$$\frac{\partial L}{\partial s_i(\mathbf{y}^*)} = -\lambda(\mathbf{y}^*) + \nu_i \frac{\partial}{\partial a_i} p(\mathbf{y} \mid \mathbf{a}^0) + \eta_i(\mathbf{y}^*) = 0 \tag{16}$$

and

$$\frac{\partial L}{\partial s_j(\mathbf{y}^*)} = -\lambda(\mathbf{y}^*) + \nu_i \frac{\partial}{\partial a_j} p(\mathbf{y} \mid \mathbf{a}^0) + \eta_j(\mathbf{y}^*) = 0. \tag{17}$$

For identical agents, a symmetric payment scheme and a symmetric equilibrium $\mathbf{a}^0$, it holds that $\nu_i = \nu_j$. Furthermore, $\eta_i = 0$ if $s_i(\mathbf{y}^*) > s^{min}$. Since

$$\frac{\partial}{\partial a_i} p(\mathbf{y} \mid \mathbf{a}^0) = \frac{p_i'(y_{il^*} \mid a_i^0)}{p_i(y_{il^*} \mid a_i^0)} p(\mathbf{y}^* \mid \mathbf{a}^0) < \frac{\partial}{\partial a_j} p(\mathbf{y} \mid \mathbf{a}^0) = \frac{p_j'(y_j \mid a_j^0)}{p_j(y_j \mid a_i^0)} p(\mathbf{y}^* \mid \mathbf{a}^0)$$

by MLRP and the fact that $y_{il^*} < y_{jk^*}$, it follows that $\nu_i < 0$. This, however, cannot be true for $a^0 > \underline{a}$ (see proof of proposition 2). □

### A.4 Proof of corollary 1

The corollary can be proven analogously to proposition 3. The only difference is that $\lambda(\mathbf{y}^*)$ may be zero, which does not change the line of arguments. □

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