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US Macro Dynamics

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Abstract

This paper identifies structural breaks in the post-World War II joint dynamics of U.S. inflation, unemployment and the short-term interest rate. We derive a structural break-date procedure which allows for long-memory behavior in all three series and perform the analysis for alternative data frequencies. Both long-memory and short-run coefficients are relevant for characterizing the changing patterns of U.S. macroeconomic dynamics. We provide an economic interpretation of those changes by examining the link between macroeconomic events and structural breaks.

JEL Classification: C32, C51, E31, E32, E52

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1 Introduction

The presence of structural breaks in macro time series continues to be an intriguing topic for macroeconomists. The inflationary and recessionary experiences of the post-World War II period have made us aware of the costs entailed by adverse shocks and by wrong responses to those shocks coming from the government, the monetary authority and the private sector. But, what were those turning points in macroeconomic dynamics? When and why did they occur? We take a stab at this important question by introducing a flexible reduced-form model which allows for long-memory (fractional integration) in the dynamics of U.S. inflation, unemployment and the short-term interest rate. In doing so we incorporate as particular cases the two classical VAR structures based on the original $I(0)$ and first differenced $I(1)$ data. We then estimate structural breaks in macro dynamics for alternative data frequencies and connect these breaks with historical macroeconomic events.

This paper presents both methodological and economic contributions. From a methodological viewpoint, the contribution of the paper is two-fold: Firstly, we permit fractional integration in a multivariate context, a more flexible specification than the standard $I(0)/I(1)$ frameworks usually employed in the literature. Fractionally integrated models have been widely employed in univariate contexts to describe the behavior of time series both in macroeconomics (Diebold and Rudebusch (1989), Gil-Alana and Robinson (1997)) and finance (Baillie and Bollerslev (1994) and Ding and Granger (1996)). However, in a multivariate set-up, this is one of the first empirical applications. Secondly, this paper develops a multivariate structural break-date procedure which allows for long-memory structures. The relation between long-memory and structural breaks is a topic that has recently emerged in the time series literature. Thus, for example, Lee and Schmidt (1996) argue that structural breaks may be responsible for the long-memory

in return volatility processes. Other papers by Granger and Hyung (2004), Gouriéroux and Jasiak (2001) and Diebold and Inoue (2001) have also examined the relation between structural breaks and fractional integration. In the present paper, we find that changes in long-memory patterns are related to structural breaks.

From an economic perspective, the paper contributes in several dimensions. First, it compares the structural breaks obtained under stationary standard dynamics with dynamics allowing, additionally, long-memory structures. Second, we estimate the different fractional integration orders across the subsamples split by the breaks. Third, we provide an economic interpretation to the breaks obtained in US post-World War II macro dynamics. Specifically, we examine our estimated structural breaks in the context of monetary policy shifts, oil shocks and technological developments. We also illustrate the structural breaks implications via fractionally integrated impulse response analysis. Fourth, we examine the presence of structural breaks for three different data frequencies: Monthly, quarterly and annual. Most of the literature on structural breaks focuses on quarterly data, the standard business cycle frequency. Yet, monthly and annual data are frequently used in macroeconomic applications. By identifying structural breaks in alternative frequencies, we can provide some insights on the different impacts of macroeconomic changes.

The main empirical results can be summarized as follows: First, most of the estimated structural breaks are scattered around 1973 and 1980 across frequencies. Second, with respect to standard structural break-date tests, such as the Sup-Wald test, our fractionally integrated method identifies additional breaks in different decades at all frequencies. Third, changes in long-memory behavior are more influential in shaping macro dynamics at the quarterly frequency. Fourth, most of the structural breaks are related with shifts in structural supply shocks, whereas monetary policy shifts are only related to the breaks

of the early-80s. Fifth, the quarterly and annual macro variables overall display higher fractional integration orders than the variables at the monthly frequencies, which might be related with their different degrees of persistence.

The procedure employed in the paper is administered on reduced-form dynamics. We believe that this is an advantage of our framework, as we do not have to take a stand on the structural model governing the economy.¹ Given the fact that there is considerable uncertainty surrounding the true structural macro model governing the economy, the approach taken here can be considered quite flexible, as a change in a single structural parameter can have an impact in all of the reduced-form parameters. Moreover, in the spirit of Stock and Watson (2002) and Boivin and Giannoni (2006), we are able to separately identify the changes in structural shocks and propagation through standard recursive identification schemes while allowing for fractional integration. Section 4 also provides some indirect interpretation on the structural sources of the breaks in the economy by linking structural breaks and structural factors.

A limitation in the application of the method described below is that we restrict our attention to the case of a one-and-forever break, reducing to 2 the number of regimes per sample. Nevertheless, we believe that our approach reveals valuable information on the relevant changes existent in macro dynamics for two reasons: First, we examine alternative reduced-form models with and without fractional integration structures. As a result, our multiple specifications yield alternative breaks within each frequency which will shed some light on the potential multiple breaks occurring during the whole sample period. Second, we provide informal evidence of additional breaks in all our model specifications. In the $I(0)$ context, we plot the Sup-Wald test time series over the whole sample period, whereas in the fractional integration framework we study additional candidates for the

¹Moreno (2004), Cogley and Sargent (2005) and Canova (2006) estimate the impact of structural model parameters on macroeconomic performance.

break-dates on the basis of the residuals sum squares values yielded by our method. In fact, the procedure described in section 3 can be easily extended to the case of multiple breaks. Nevertheless, the validity of the estimates of long-memory (fractional integration) models hinges upon a sufficiently large data span in order to detect the dependence of the observations across time. As a result, the inclusion of two or more breaks would result in relatively short subsamples, thereby invalidating the analysis based on fractional integration, especially at quarterly and annual frequencies.

This paper proceeds as follows. Section 2 identifies structural breaks in a classical multivariate framework with U.S. monthly, quarterly and annual data. Section 3 develops a method for detecting breaks in a multivariate framework allowing for the joint estimation of the fractional orders of integration in the variables across sample periods. It then studies the statistical properties of the procedure in finite samples and applies it to our U.S. macroeconomic system across data frequencies. Section 4 provides an economic interpretation to our break-dates by linking them to historical events. Section 5 concludes.

2 Break-Date Tests in Short-Run Dynamics

In this section we identify the break-dates in non-fractional models for U.S. macroeconomic systems including inflation, unemployment and the nominal short-term interest rate. These are three key variables capturing U.S. macro dynamics (see Cogley and Sargent (2005) and Sims and Zha (2006), for instance). We first explain the econometric features of the multivariate test employed in the text and then report the empirical results for alternative vector autoregressive (VAR) orders.

2.1 The Sup-Wald Test

There is a substantial econometric literature on the design of break-date tests for macroeconomic time series. While most of the literature has focused on break-date tests for univariate time series, Bai, Lumsdaine, and Stock (1998) present a break-date test for multivariate systems.² This method allows the researcher to find endogenous break points for all of the parameters in a VAR system. Suppose that a vector of n demeaned macroeconomic time series (Y_t) evolves according to the following system with (possibly) changing coefficients:

$$Y_t = \sum_{i=1}^k \Phi_i^j Y_{t-i} + \varepsilon_t, \quad (1)$$

where the Φ_i^j 's are $n \times n$ squared matrices of coefficients and ε_t is an $n \times 1$ i.i.d. vector. k is the arbitrary VAR lag-length. j is the number of regimes, which, in this framework, is restricted to 2. The break-date test tries to uncover the date at which there was a structural change in the coefficient matrices. The idea is that there was a permanent break in the structure of the economy at a given point in time which altered macro dynamics. Thus, while the stochastic process governing the joint time series is overall non-stationary for the whole time-span, it remains stationary within a given regime.

One of the main advantages of the Bai, Lumsdaine, and Stock (1998) test is that it yields an asymptotically valid confidence interval for each break. Furthermore, this interval can be tightened by the use of several time series which break at the same date. Bekaert, Harvey, and Lumsdaine (2002) show, in the context of the emerging market financial liberalizations, that this test can accurately pin down the break-dates in

²Multivariate CUSUM methods have been proposed by Qiu and Hawkins (2001) and Qiu and Hawkins (2003), and changes in the covariance structure have been examined in Galeano and Peña (2007). Other recent papers are Davis, Lee, and Rodríguez-Yam (2006), who propose a consistent estimator for the location of breaks in multivariate segmented autoregressive models, and Qu and Perron (2007), which consider issues related to the estimation, inference and computation of multiple structural changes occurring at unknown dates in a system of equations.

those markets by using several variables in their multivariate systems. Throughout our analysis, we work with three relevant macroeconomic series: Inflation, unemployment and the short-term interest rate. Given the inter-relations among them laid out in the vast majority of macro models, it is natural to think of a simultaneous structural break.

2.2 Empirical Results

Our dataset comprises the Consumer Price Index for inflation, the unemployment rate and the 3 month Treasury Bill Rate for the short-term interest rate. The dataset was retrieved at a monthly frequency and then we computed arithmetic averages to build the quarterly and annual datasets. The full sample goes from January 1948 to December 2005. All three series were downloaded from the Federal Reserve of St. Louis FRED2 database.

Panels A, B and C of table 1 show the break-dates implied by the Sup-Wald test on monthly, quarterly and annual data. They also show the associated confidence intervals for each break-date. We run the break-date test on VAR systems of orders one up to three in order to highlight potential differences across orders. Higher VAR orders did not reveal additional changes. Figure 1 plots the time series of the Sup-Wald tests across frequencies and VAR orders.³

The results for monthly data reveal that the most likely break occurred around mid-1980. The VAR(1) points at July 1980, whereas April 1980 was the most likely break-date according to the VAR(2) and the VAR(3). Figure 1 shows that there are essentially no additional candidates for the monthly breaks, as the corresponding Sup-Wald values clearly peak over the whole sample.

³We trim the initial and final 15% of the sample when running the Sup-Wald test. As Maddala and Kim (1998) point out, it is customary to do so in order to rule out breaks around the ends.

There is more uncertainty surrounding the break-dates for the quarterly data. Panel B of table 1 reveals important differences across VAR orders, although the break-dates are precisely estimated within each order, only including three quarters. The VAR(1) selects the fourth quarter of 1981, whereas the VAR(2) and the VAR(3) select the second quarter of 1963 and the fourth quarter of 1958, respectively. There is also a difference in the plots of the VAR(1) Sup-Wald series on the one hand, and both the VAR(2) and the VAR(3) on the other hand. While the VAR(1) series resembles that of the monthly data, with a clear peak at the end of 1981, the plots are different for the remaining VAR orders. They show large values for the first quarters of the sample and a steady decline up to the late eighties, when they sharply increase. After 1982, the values of the Sup-Wald start to decline and remain low until the end of the sample. This finding, in itself, suggests that there may have been two periods of increased macroeconomic instability: The end of the 50s-beginning of the 60s and the end of the 70s-beginning of the 80s. Interestingly, the implications of these two sets of plots are respectively consistent with the two most frequent explanations on the increased stability of quarterly macro dynamics. The first one, put forward by Blanchard and Simon (2001) and recently supported by the evidence in Fernandez-Villaverde and Rubio-Ramirez (2007), stresses the idea that there has been a steady increase in macroeconomic stability since the early-60s. The second one, supported by Clarida, Galí, and Gertler (1999), Lubik and Schorfheide (2004) and others, highlights the importance of a monetary policy shift in the early-80s as responsible for a switch to increased stability.

Finally, the annual break-dates are 1981 for both the VAR(1) and the VAR(3) and 1963 for the VAR(2). While the plots for the VAR(1) and VAR(2) Sup-Walds could suggest the existence of the same two breaks commented for quarterly data, the VAR(3) and unreported higher-order VARs clearly pin down 1981 as the break-date. Overall, the

evidence for annual data points at 1981 as the most likely annual break-date.

3 Structural Long-Memory Breaks

In this section, we derive a method which detects structural breaks in the reduced-form macroeconomic systems allowing for both fractional integration and vector autoregressive behavior in the dynamics of all the series. While there exist many structural break statistics in the macroeconomic literature, there are virtually no methods which allow for fractional integration in multivariate models. The first subsection presents the statistical procedure. We then perform a Monte Carlo study which analyzes the behavior of our procedure in finite samples. The third subsection presents an application of the outlined method to the U.S. post-World War II joint macro dynamics, with systems including inflation, unemployment and the interest rate across data frequencies.

3.1 The Statistical Model

Consider the following multivariate fractionally integrated model which allows for a break in all of the model parameters:

$$D^a Y_t = U_t, \quad t = 1, 2, \dots, T_b - 1 \quad (2)$$

$$D^b Y_t = U_t, \quad t = T_b, \dots, T \quad (3)$$

where

$$D^i = \begin{pmatrix} (1-L)^{d_1^i} & 0 & 0 & \dots & 0 \\ 0 & (1-L)^{d_2^i} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & (1-L)^{d_n^i} \end{pmatrix} \quad i = a, b. \quad (4)$$

d_j^i is the fractional order of integration of the j -th variable during the i -th subsample. Y_t is an $n \times 1$ vector of demeaned macro variables and U_t is an $n \times 1$ vector of disturbances, assumed to be $I(0)$.⁴ In this paper, we analyze two alternative processes for the reduced-form errors U_t . First, we let U_t be cross-correlated white noise, so that:

$$U_t = G^a \varepsilon_t, \quad t = 1, \dots, T_b - 1 \quad (5)$$

$$U_t = G^b \varepsilon_t, \quad t = T_b, \dots, T, \quad (6)$$

where G^i ($i = a, b$) is an $n \times n$ lower triangular matrix and ε_t is an $n \times 1$ vector of independent and identically distributed residuals with diagonal variance-covariance matrix Σ^i for each of the two subperiods. We also consider a model where U_t follows a vector autoregressive process of order one for each period:

$$U_t = P^a U_{t-1} + \varepsilon_t, \quad t = 1, \dots, T_b - 1 \quad (7)$$

$$U_t = P^b U_{t-1} + \varepsilon_t, \quad t = T_b, \dots, T, \quad (8)$$

where P^i ($i = a, b$) is an $n \times n$ matrix and ε_t is defined as in the first case.

The procedure introduced in this paper is a natural extension to the multivariate case of the univariate method proposed in Gil-Alana (2007). It allows the researcher to

⁴Imposing diagonalization in D in (4) rules out the possibility of fractional cointegration. Note, however, that even in the context of no breaks, the issue of fractional cointegration in a system-based model has not yet being fully theoretically developed.

jointly estimate the time of the break, T_b , along with all of the model parameters, those in D^a , D^b , G^a , G^b , P^a , P^b , Σ^a and Σ^b . The suggested approach of the test is based on minimizing the Residuals Sum of Squares (RSS) over a grid of values for the fractional orders of integration and recursively for alternative break-dates. More specifically, we first choose a grid of values for the fractionally differencing parameters d_j^a and d_j^b , $j = 1 \dots n$. Then, for a given partition T_b and given the initial values $(d_{10}^a, \dots, d_{n0}^a$ and $d_{10}^b, \dots, d_{n0}^b)$, we compute the sum of square residuals:

$$RSS(T_b; d_{10}^a, \dots, d_{n0}^a) = \sum_{t=1}^{T_b-1} \sum_{j=1}^n [\varepsilon_{jt}]^2 + \sum_{t=T_b}^T \sum_{j=1}^n [\varepsilon_{jt}]^2, \quad (9)$$

where ε_{jt} is the j -th element of the vector ε_t . Let $RSS(T_b; d_{10}^a, \dots, d_{n0}^a, d_{10}^b, \dots, d_{n0}^b)$ be the RSS for the partition T_b and the initial values d_{i0}^a and d_{j0}^b for $i, j = 1, \dots, n$. Minimizing across all values in the grid, we obtain:

$$RSS(T_b) = \arg \min_{i, \dots, k, j, \dots, l} RSS(T_b; d_{1i}^a, \dots, d_{nk}^a, d_{1j}^b, \dots, d_{nl}^b). \quad (10)$$

Then, the estimated break-date, \hat{T}_k is such that:

$$\hat{T}_k = \arg \min_{s=1, \dots, m} RSS(T_s), \quad (11)$$

where the minimization is taken over all partitions T_1, \dots, T_m such that $T_s - T_{s-1} \geq |T\epsilon|$.

The associated least squares estimates for the fractional differencing parameters are then:

$$\hat{d}_i^a = \hat{d}_i^a(\hat{T}_k) \quad (12)$$

$$\hat{d}_j^b = \hat{d}_j^b(\hat{T}_k) \quad i, j = 1, \dots, n. \quad (13)$$

The procedure described above can be extended in several directions. First, we can

include deterministic trending regressors in (2) and (3) such that Y_t can be the vector of errors in a multiple regression system of the form:

$$X_t = B^a Z_t + Y_t \quad t = 1, 2, \dots, T_b - 1 \quad (14)$$

$$X_t = B^b Z_t + Y_t \quad t = T_b, \dots, T, \quad (15)$$

where B^a and B^b are $n \times k$ matrices of coefficients associated with the deterministic regressors Z_t . Moreover, the case of multiple breaks can also be examined. We can consider the model:

$$D^j Y_t = U_t, \quad t = T_{j-1} + 1, \dots, T_j, \quad (16)$$

where D^j is now an $n \times n$ diagonal matrix, with i -th elements of the form: $(1 - L)^{d_i^j}$ and for $j = 1, \dots, m + 1$, $T_0 = 0$ and $T_{m+1} = T$. Here, the parameter m identifies the number of changes. The break-dates (T_1, \dots, T_m) are explicitly treated as unknown and for $i = 1, \dots, m$ we have $\lambda_i = \frac{T_i}{T}$, with $\lambda_1 < \dots < \lambda_m < 1$. Analogously to the previous case, for each j -partition, $\{T_1, \dots, T_j\}$, denoted $\{T_j\}$, an associated RSS is obtained. Substituting them into the new objective function and denoting the sum of squared residuals as $RSS_T(T_1, \dots, T_m)$, the estimated break-dates $(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_m)$ are obtained by applying:

$$\min_{(T_1, T_2, \dots, T_m)} RSS_T(T_1, \dots, T_m) \quad (17)$$

where the minimization is again computed over all partitions (T_1, \dots, T_m) .

The theoretical properties of the resulting estimators are not derived though they should not differ much from those reported in Bai and Perron (1998) since we choose the values in a way such that they minimize the residuals sum squares and, under the appropriate specification, U_t must follow an I(0) vector process (see Gil-Alana (2007)). Another limitation of this procedure is that it does not yield an associated break-date

confidence interval, since it is based on an optimization procedure over a grid of values. However, we derive the statistical properties of the resulting estimates through a set of Monte Carlo experiments in the next subsection, and we are able to show that the procedure performs quite well if the sample size is large enough. In fact, we show that the procedure accurately determines not only the break-dates but also the fractional differencing parameters associated to each subsample.

3.2 Monte Carlo Results

In this section, we consider the data generating process given by equations (2) and (3) and specialize it for the dimensional case treated in the paper ($n = 3$). We assume that the error terms are not autocorrelated and that $(d_1^a, d_2^a, d_3^a, d_1^b, d_2^b, d_3^b) = (0.2, 0.4, 0.2, 0.6, 0.6, 0.4)$ with $T_b = T/2$.⁵ The model is simulated for alternative values of T : 100, 200, 300, 400, 500, 700 and 1000 observations. We generate Gaussian series using the routines of GASDEV and RAN3 of Press, Flannery, Teukolsky, and Vetterling (1986).

In table 2, we report the probabilities of correctly determining both the break-date and the fractional differencing parameters in the model given by (2) and (3), using a grid of values for d_j^x , $\{x = a, b; j = 1, 2, 3\}$ from 0 to 2 with 0.2 intervals, and values for the break-date $T^* = (T/10, T/10 + 2, \dots, (2), \dots, 9T/10 - 2, 9T/10)$. We use 10.000 replications for each case.

The most noticeable finding reported in table 2 is that the procedure accurately determines the break-date in most cases, and we find “almost” 0-probabilities if $T^* < T_b - 4$ or if $T^* > T_b + 4$.⁶ If $T = 100$, the sum of all the probabilities taking place at $T^* = T_b$ is approximately 0.47, and if $T = 200$, it is 0.51. Increasing the sample size to

⁵Allowing for autocorrelated error terms did not significantly change the Monte Carlo results.

⁶By “almost” 0-probabilities we mean cases where the probability is smaller than 0.1%.

700 or 1000 observations, the sum of these probabilities is close to 90%. Thus, in terms of the power properties for the breaks, the probabilities of rejecting a wrong break get closer to 1 as we increase the sample size. With respect to the fractional differencing parameters, we observe that while the probability of correctly determining the break-date and the corresponding d 's is not very high in small samples (15% with $T = 100$ and 27% with $T = 200$), these probabilities substantially increase with T .

Table 3 reports the results of an analogous experiment as the one reported in table 2 but using d -values equal to $(0.8, 1.4, 1.0, 1.0, 1.0, 0.6)$, i.e., with all subseries being in the non-stationary region. Moreover, we set the break-date at $T_b = T/4$. Analogously to the previous case, if T is small, the probabilities are widely disseminated across the grid, most of them being smaller than 0.01. However, when the sample size is increased, the procedure correctly determines the break-date and the fractional differencing parameters in a large percentage of cases. Note that these probabilities are based on the grid employed for the fractional orders of integration and thus, they become smaller as we reduce the value for the increments in the d 's. Alternatively, larger increments for the values of the d 's in the grids would produce higher probabilities of detecting the true values.

A straightforward implication of the Monte Carlo results is that caution should be exercised in interpreting the subsequent empirical results reported with annual data, where only 58 observations are available per variable. The results with monthly and quarterly data are more reliable, since we have 232 and 696 observations per variable respectively. Nevertheless, notice that the Monte Carlo results showed that the highest probability of detecting the break-date corresponded to the true value for all sample sizes. These results also showed that almost the entire probability mass of the break-date was concentrated around the estimated break.

3.3 Empirical Results

In this section we present the break-dates obtained in all our fractional integration models using the method described in section 3.1. Two sets of models are estimated: The first one assumes that the residuals are cross-correlated white noise whereas the second set allows for a VAR(1) structure in the residuals. Therefore, in the former case, the time dependence is exclusively captured through the fractional operator while in the latter case, the fractional polynomial competes with the short-run VAR dynamics in describing such dependence. In both cases we consider $(d_i^a, d_j^b)_{i,j=1,2,3}$ values from -2 to 2 with 0.01 increments, with estimated break-dates equal to $\frac{T}{10}$, $\frac{T}{10} + 1$, $\frac{9T}{10} - 1$ and $\frac{9T}{10}$. Table 4 lists the optimal break-dates for both classes of models across frequencies. Table 5 lists an additional set of candidates for the break-dates, where the values of the d 's were not the optimal ones but were the closest to the minimum RSS. The discussion is again organized along data frequencies.

Panel A in table 4 shows that in the model with white noise residuals the most likely break with monthly data occurred in September, 1972. Panel B in the same table shows that in the model allowing for autocorrelated residuals, August 1973 was the most likely break-date. Therefore, allowing for fractional integration dynamics adds another potential set of monthly structural breaks around 1973. Table 5 shows that July 1980 is another break-date candidate with a slightly higher RSS value for monthly dynamics. Interestingly, despite the methodological differences with respect to the fractional integration break-date test, the VAR(1) model without fractional integration also picked July 1980 as the most likely break-date.

Regarding macro dynamics at a quarterly – business cycle – frequency, our fractional integration method reveals more uncertainty. While the model with white noise residuals

picks the fourth quarter of 1981 as the break-date, the model with autocorrelated residuals selects a quite distant break, the second quarter of 1967. Moreover, table 5 shows that there are two additional candidates with the closest RSS values to the optimum, the first quarter of 1973 and the fourth quarter of 1980. Recall that the break-date selected by sup-Wald test also differed across VAR orders. The break-date picture which emerges from our method thus points to the potential existence of multiple breaks at the important quarterly frequency. Finally, tables 4 and 5 also report the results for the annual frequency. As the Monte Carlo exercises revealed in the previous section, the power of the test can be very low with a very small number of observations. Nevertheless, two dates appear as clear candidates for the break-date: 1973 and 1981. This latter date is the second most likely candidate for the model with VAR(1) residuals and also coincides with the most likely value reported by the sup-Wald test.

4 Factors Behind the Breaks

In the previous sections, we identified a set of potential break-date candidates across data frequencies. This section is an attempt to give some economic interpretation to such breaks in macro dynamics. To this effect, we provide some indirect evidence of the alternative macroeconomic mechanisms behind the breaks. We first show some descriptive statistics of the macro variables second moments before and after the breaks. Then we analyze the relative influence of the changes in shocks and propagation on the advent of the breaks. Finally, we examine the role played by three specific channels which could be held responsible for the breaks: Changes in monetary policy, oil prices and technology shocks.

4.1 Initial Intuition: Descriptive Statistics

Table 6 reports the differences in second moments of our macroeconomic variables across subsamples and data frequencies; it also identifies statistically significant differences. We examine the changes in second moments because they should capture the impact of the estimated structural breaks. The full sample is split in two subsamples according to the different break-dates identified with and without long-memory structures.

Panel A lists the differences for monthly data. It shows relatively large increases in the persistence of inflation around 1973. After this date, the short-term interest rate variance is also much larger, likely reflecting a volatile ex-post interest rate reaction to inflation in the second subperiod. Interestingly, the 1973 break-dates were only detected by the tests allowing for fractional integration. A different picture emerges in the monthly breaks around 1980. The bottom line here is that the variance of inflation greatly and significantly declined after 1980. This phenomenon coincides with a significant increase in the variance of the short-term rate, which, as we show below, plays an important monetary policy role in the period.

The break-date tests presented in the previous sections identified as many as six potential break-dates at the quarterly frequency, in 1958, 1963, 1967, 1973, 1980 and 1981. During the second subsamples associated with the first four breaks, there was a large increase in the first order autocorrelation of inflation, although not statistically significant. By and large, the behavior of all the macro variables worsens in the associated four second subsamples. Regarding the breaks of the early-80s, the most important fact is that the variance of inflation greatly and significantly declined during the second subsamples. Panel C shows the results with annual data. While after both 1963 and 1973 all of the second moments in our macro variables worsened, most of these moments

greatly improved after 1980.

To sum up, the structural breaks are clearly associated with changes in the inflation and short-term interest rate second moments. Thus, the sources of the changes in inflation and interest rate dynamics appear highly relevant in explaining the changes in overall macro dynamics. We turn now to an analysis of these sources.

4.2 Shocks or Propagation

One advantage of our empirical approach is that we can decompose the changes of macro dynamics into different sources, shocks and propagation. The most general framework in our analysis is described by equations (2), (3) (7) and (8). Clearly, the coefficients in D^a, D^b, P^a and P^b capture the internal macroeconomic propagation mechanism. In order to identify the structural macro shocks, we assume a recursive identification scheme. The macroeconomic system can be rewritten as:

$$A^i D^i Y_t = \eta_t \tag{18}$$

$$\eta_t = \Gamma^i \eta_{t-1} + \xi_t, \quad i = a, b \quad t = 1, 2, \dots, T, \tag{19}$$

where A^i is lower triangular, η_t is the vector of autocorrelated structural shocks, Γ^i is the matrix of VAR coefficients in period i and ξ_t is the vector of structural innovations. The recursive assumption, popularized by Christiano, Eichenbaum, and Evans (1999), allows to exactly identify the diagonal variance matrix of ξ_t , as shown by Gil-Alana and Moreno (2006) in a related fractional integration context. In line with standard identification schemes, we assume that inflation is not affected contemporaneously by either unemployment or the short-rate, whereas unemployment is not affected by the short-rate contemporaneously.

Tables 7, 8 and 9 show the propagation and shock parameter estimates across pairs of subsamples and data frequencies. The propagation coefficients can be classified into long-memory parameters (the d 's) and the short-run ones, which we represent by the three eigenvalues of the matrices Γ^a and Γ^b . The standard deviations of the structural shocks are also identified. These shocks can be labelled as the supply, demand and monetary policy shocks.

Panels A and B of table 7 show the propagation and shock coefficients for the monthly breaks around 1973. Panel A is our model featuring only long-memory persistence whereas Panel B also allows for short-run dynamics. The increase in overall macroeconomic persistence of the post-1973 periods in Panel A is reflected in the larger fractional integration parameter of inflation in the second subsample. However, the more general model in Panel B –where both long and short-run persistence compete– reveals that the key factor behind the increase in persistence is the more autocorrelated short-term behavior. Supply shocks decreased after 1973 whereas monetary policy shocks increased. Panel C lists the analogous parameters associated with the break in July, 1980. It shows a decline in long-memory for all three series but more persistent short-run dynamics. Structural supply shocks have also declined in the last 25 years.

Panels A and D of table 8 list the parameter sets for the subsamples generated by the quarterly breaks in the early-80s. In both cases there is a significant decline in the fractional integration order of inflation, which stays below 0.5 during the second subsamples. Both panels show that shocks were overall milder in the post-1981 era. While McConnell and Quirós (2000) and Cogley and Sargent (2005) emphasize the importance of a decrease in the demand shocks in the post-1980s, our results show a more significative decrease in the supply shocks. Another important result reported in Panels A and D is that we obtain high orders of fractional integration for the unemployment variable in the

post-1980s (1.64 and 1.57, respectively), rendering unemployment clearly non-stationary. Panel D also shows that short-term persistence declined during this period, but this decline seems to be less influential than the drop of long-memory in the decrease of inflation persistence and volatility. The results for the breaks in 1967 and 1973 are reported in Panels B and C. They present the following features: An increase in the fractional integration order of inflation during the second subsamples but a decrease in the long-memory of unemployment and the short-term rate. Short-term persistence increased after both 1967 and 1973 whereas inflation shocks declined during these second subsamples.

Table 9 shows the results with annual data. Panels A and B describe the parameter changes occurring after 1973. The increase in macro persistence for all three variables is captured by higher degrees of fractional integration in the model with only long-memory persistence, whereas Panel B shows that short-run dynamics were more important than long-memory in the case of inflation and the interest rate. The structural shocks were by and large more volatile in the post-73 period. Panel C shows the results for the 1980 structural break. Recall that the most important feature of the post-80 period is the decline in inflation variance. Our results point to the lower shocks and less persistent short-run dynamics as the key factors behind the lower inflation volatility. All structural shocks were milder during this period, whereas the long-memory behavior of the unemployment rate increased.

Taken together, our results highlight the importance of both long-memory and short-run persistence structures in the dynamics of the macro series. For instance, we find that while the drop in the long-memory of inflation has contributed to reducing monthly and quarterly inflation persistence and volatility since the early-80s, changes in short-run dynamics are more relevant at the annual frequency. Another important feature of

our results is that supply shocks seem to have steadily declined in the post-World War II (especially at the monthly and quarterly frequencies) as the shocks are consistently smaller across second subsamples. This result is related to the findings of Sims and Zha (2006), who estimate regime-switching VARs and find that the model which best characterizes U.S. post-World War II macro dynamics only includes shifts in the standard deviations of the structural shocks. Finally, it is noteworthy that the orders of integration at quarterly and annual frequencies are higher overall than at the monthly frequency for a given subsample. An interpretation of this result is that the more frequent changing dynamic patterns of macro data at higher frequencies induce a reduced time dependence in these series relative to lower frequencies.

4.3 Economics Behind the Breaks

This section examines the potential coincidences between the break-dates in macro dynamics and different macroeconomic events. To this end, we propose three alternative candidates which can be related with the break-dates: Oil price shocks, changes in monetary policy behavior and technology shocks. While other explanations, such as fiscal changes, can also be important, we believe that the events analyzed here are probably the most obvious candidates based on both historical developments and relevant strands of the macroeconomic literature.

4.3.1 Oil Price Shocks

The oil shocks of the 1970s caused recession across industrialized countries. As is well-known, the drop in the world production of oil increased the marginal costs of firms generating profit losses, unemployment and inflation. Figure 2 plots a series of oil shocks

against the structural breaks obtained throughout the paper across data frequencies. The oil shock series is constructed according to the method proposed by Hamilton (1996), who stresses the asymmetric effects of oil shocks.⁷ The measure of the oil shock (O_t) is then constructed as

$$O_t = \max \begin{cases} 0 \\ X_t \end{cases} \quad t = 1, 2, \dots$$

where X_t is the percentage point difference between the current oil price and the maximum price during the previous year.

Figure 2 shows that the monthly breaks, scattered around 1973 and 1980, are aligned with the most important oil shocks. In September 1973, the Arab-Israeli War leads to a drop of a 7.8% in the world production of oil, whereas in July 1980, the Iran-Iraq War leads to a 7.2% decline. The oil shock measure indicates that oil shocks were high between March 1973 and February 1974 and between September 1977 and February 1981. This means that our monthly structural breaks, found in October 1972, August 1973, April 1980 and July 1980, coincide with these periods of oil price turbulence. It is interesting to note that oil shocks tend to affect structural disturbances in macroeconomic systems, such as the VARs. This idea squares well with the shock-propagation evidence described in the previous sections, which highlighted the changes in supply shocks as a key factor in triggering structural breaks at the monthly frequency.

The middle graph of figure 2 plots the quarterly oil shocks against the quarterly structural breaks. Out of the six breaks, two are related to the aforementioned oil shocks: The first quarter of 1973 and the fourth quarter of 1980. The shock-propagation VAR analysis of the previous section also showed an important decline in supply shocks after

⁷The data for the oil shock series was retrieved from the Mark Watson's dataset (http://www.princeton.edu/~mwatson/sw/SW2e_data.html).

the structural break of the fourth quarter of 1980. Finally, the bottom graph of figure 2 shows that the 1973 and 1981 structural breaks coincide with high values for the oil shocks at the annual frequency. Overall, many of our structural breaks coincide with oil shocks, especially at the monthly and annual frequencies, where changes in structural shocks seem to have a greater importance on macroeconomic volatility.

4.3.2 Changes in Monetary Policy

As pointed out by Lubik and Schorfheide (2004) and Boivin and Giannoni (2006) among others, an alternative source of structural change is a shift in the systematic monetary policy response of the U.S. Federal Reserve to inflation fluctuations. In order to examine the role of monetary policy on the structural breaks, we describe the monetary policy behavior of the U.S. Fed as a standard Taylor rule with partial adjustment:

$$i_t = \rho i_{t-1} + (1 - \rho)(\bar{i} + \beta(\pi_t - \bar{\pi}) + \gamma u_t), \quad (20)$$

where i_t is the nominal interest rate, \bar{i} is the natural nominal interest rate, π_t is the inflation rate, $\bar{\pi}$ is the inflation target and u_t is the unemployment rate. β is the long-run interest rate response to inflation and thus captures the stance of the U.S. Fed against inflation: The larger the β , the more aggressively the Fed fights the inflation deviations from its target. We then estimate rolling-regressions of the Taylor rule starting with 10 observations and then adding the subsequent data points. By examining the resulting vector of β 's, we can assess the marginal contribution of a given period to the overall monetary policy stance. This is what we do in figures 3, 4 and 5, where we plot the β 's against all the structural breaks found in the previous sections across data frequencies.⁸

⁸For ease of visual interpretation, we bind the space of β between 0 and 5. That is, whenever the estimated β was below 0 or above 5, we assign it a value of 0 and 5, respectively.

Figure 3 plots the β 's estimated with monthly data against the structural breaks. Up to 1982, the series displays a changing behavior, which is consistent with the nature of monthly data. Interestingly, the series becomes much more stable since then. The breaks in April and July of 1980 are related to instability in the interest rate response to inflation, coinciding with the beginning of the announced Volcker disinflation. The September 1972 break coincides with a low interest rate response to inflation and thus seems unrelated to monetary policy. Finally, the August 1973 break is related to a period of volatile interest rate dynamics.

We obtained up to six potential structural breaks with quarterly data. They are plotted in figure 4 against the β 's obtained with quarterly data. The four breaks occurring in 1958, 1963, 1967 and 1973 do not coincide with relevant shifts in monetary policy behavior. However, the peaks in the fourth quarters of 1980 and 1981 seem intimately related with the more aggressive shift in monetary policy, as they coincide with the two most important peaks in the β -series since the mid-70s. A similar picture emerges with annual data in figure 5, where the break in 1981 occurs at a high value of β whereas the 1963 and 1973 breaks occur at low and medium values of β .

It is noteworthy that the interest rate response to inflation is economically significantly larger after 1982 across data frequencies, in agreement with the findings of Clarida, Galí, and Gertler (1999), who focus on quarterly data. The shift is reflected in both a sharp peak at the beginning of the 1980s and in a higher response to inflation in the aftermath of the Volcker disinflation than in the previous decades. Figure 6 shows that our estimated fractionally integrated VAR is consistent with this evidence. It compares the dynamic interest rate impulse response to a supply shock across subsamples and data frequencies. The subsamples are those associated with the breaks in the early-80s. The structural supply shock is identified recursively, as outlined in section 4.2. The

second period responses are conditional on the first period shocks so that changes in the responses capture, exclusively, changes in the propagation of the economy. While the macroeconomic propagation mechanism includes other factors apart from the monetary policy stance, figure 6 clearly shows that the interest rate responses were much larger in the post-1980s periods across data frequencies, suggesting a very aggressive stance on behalf of the monetary policy authority.

Finally, the more aggressive monetary policy since the early-80s should also translate into a shift in the propagation coefficients of the inflation reduced-form dynamics. This is exactly what we observe in Panel C of table 7 for monthly data and in Panels A and D of table 8 for quarterly data, where the inflation long-memory coefficients experience a large drop in the second subsamples. In the case of annual data, Panel C of table 9 shows that the short-run parameters capture the more flexible propagation mechanism associated with the post-1980 macro dynamics.

4.3.3 Technology Shocks

An important strand of the macroeconomic literature has stressed the importance of technology shocks in driving business cycle fluctuations (Kydland and Prescott (1982)). The underlying idea is that a positive productivity shock increases labor demand, employment and income. In figure 7, we plot the labor productivity shocks estimated in the context of a bivariate VAR identified under the restriction that hours worked do not affect productivity in the long-run, as in Galí (1999). The top graph corresponds to quarterly data whereas the bottom graph corresponds to annual data.⁹ The top panel

⁹We do not consider monthly technology shocks for two reasons. First, data availability: The productivity variables are typically not sampled at higher frequencies than quarterly. Second, and most importantly, the macro literature has mainly focused on technology shocks at business cycle frequencies. Since our goal is to link structural breaks with relevant macroeconomic events studied in the literature, we restrict our attention to quarterly and annual data.

shows that technology shocks became less volatile after the mid-70s (the shock standard deviation went down to 0.80 from 1.16), but do not coincide with any of our estimated structural breaks. The 1967 and 1973 breaks, in turn, coincide with times of negative technology shocks. As for the annual shocks plotted in the bottom graph, they are clearly more autocorrelated after the 1973 structural break (the first order autocorrelation goes up to 0.40 from -0.28), coinciding with a large negative productivity shock.

While the real business-cycle (RBC) literature has attracted a lot of attention during more than two decades, it has received several criticisms recently. Galí (1999) and Neville and Ramey (2005) have shown that the productivity shocks identified in standard VARs account for a negligible part of the business cycle variation and that hours worked fall after a technology shock, in contrast to the main implications of the RBC theory. Fisher (2006), however, has recently introduced an interesting spin on the technology-business cycle relation by distinguishing between the neutral technology shocks of the RBC literature and the investment specific technology shocks. Proxying these latter shocks with the real investment price growth series, he has shown that a positive investment specific technology shock induces an increase in the number of hours worked and explains a sizeable part of the business cycle. Figure 8 plots this series at quarterly and annual frequencies against the estimated structural breaks. The real investment price series was computed as the ratio between the equipment investment and the personal expenditures price indexes, both obtained from the NIPA databases. The quarterly series displays a volatile pattern whereas its annual counterpart exhibits a clear secular downward trend, i.e. a pronounced increase in annual productivity growth. The quarterly series presents an important peak – a negative technology shock – in the first quarter of 1975. The closest structural break is on the first quarter of 1973, which, as mentioned earlier, seems to be closely related to an oil shock. Similarly to the neutral technology case, the investment

specific technology shock series experiences a sharp drop in volatility after the mid-70s (it goes from 0.009 before 1976 to 0.005 since then), not coinciding with any of our structural breaks.

5 Conclusions

One contribution of the article is to derive a new method which detects structural breaks of macro dynamics in systems which allow for a flexible fractional integration framework. The empirical application of our method, with U.S. data, reveals the important role of long-memory structures in shaping macro dynamics. Specifically, changes in the fractional order integration of inflation are clearly related to the variations in quarterly multivariate macro dynamics. We show that monetary policy shifts are intimately related with the change in the inflation long-memory of the early-80s. Changes in the structural supply shocks identified in our fractionally integrated macroeconomic system are also closely related to the structural breaks across frequencies.

In future research, we intend to pursue the estimation of breaks in the intercepts of our fractional integration systems. While this is a straightforward extension in the case of $I(0)$ or $I(1)$ break-date tests such as the Sup-Wald, fractionally integrated systems introduce a particular time-varying structure in the system intercepts. Designing econometric models which allow for breaks in both intercepts and fractional integration orders is in our research agenda, as they are competing sources of changing dynamics. Additionally, as pointed out by Banerjee and Urga (2005), studying the relationship between regime switching and fractional integration is an important topic for future research. Diebold and Inoue (2001), for instance, show that it is easy to confuse these two statistical models in a univariate framework. It would be desirable to determine the relative importance of

regime switching and fractional integration in the context of multivariate macroeconomic systems.

Finally, this paper highlights once again the presence of long-memory in macroeconomic time series. It becomes then an important task to rationalize the presence of fractional integration in the context of standard macro models, such as dynamic stochastic general equilibrium frameworks. One alternative which we intend to pursue in future work is modeling the exogenous shocks of such models displaying long-memory. This extension is indeed trivial when $0 < d < 0.5$, where stationarity of the shocks -and of the macro variables- is retained. If $d \geq 0.5$, then first differences and economic arguments should be applied to the macro series in order to work in a stationary framework. Alternatively, the emerging study of fractional cointegration can shed some light in this environment. In any case, the presence of macroeconomic shocks with long-memory seems a potentially fruitful avenue in order to endow macro models with richer and more realistic dynamic patterns.

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Table 1: **Sup-Wald break-dates**

Panel A: Monthly Data				
<i>Sample Period</i>	<i>VAR</i>	<i>Sup-Wald</i>	<i>Break-date</i>	<i>90% Confidence Interval</i>
1948:1M-2005:12M	1	75.11	1980:7M	1980:6M-1980:8M
1948:1M-2005:12M	2	102.98	1980:4M	1980:3M-1980:5M
1948:1M-2005:12M	3	130.47	1980:4M	1980:3M-1980:5M

Panel B: Quarterly Data				
<i>Sample Period</i>	<i>VAR</i>	<i>Sup-Wald</i>	<i>Break-date</i>	<i>90% Confidence Interval</i>
1948:1Q-2005:4Q	1	67.76	1981:4Q	1981:3Q-1982:1Q
1948:1Q-2005:4Q	2	72.11	1963:2Q	1963:1Q-1963:3Q
1948:1Q-2005:4Q	3	137.96	1958:2Q	1958:1Q-1958:3Q

Panel C: Annual Data				
<i>Sample Period</i>	<i>VAR</i>	<i>Sup-Wald</i>	<i>Break-date</i>	<i>90% Confidence Interval</i>
1948-2005	1	48.36	1981	1980-1982
1948-2005	2	89.51	1963	1962-1964
1948-2005	3	118.67	1981	1980-1982

Note: This table lists the Sup-Wald values of the break-date test derived by Bai, Lumsdaine, and Stock (1998) with U.S. monthly, quarterly and annual data. The test detects the most likely break-date in all of the parameters of unconstrained VARs of orders 1 to 3. The table shows the results of the test using CPI inflation, the unemployment rate and the 3 month-T Bill Rate.

Table 2: Probabilities of Detecting the True Break ($T_b = \frac{T}{2}$) across Fractional Integration Parameters

T_b	d_1^a	d_2^a	d_3^a	d_1^b	d_2^b	d_3^b	$T = 100$	$T = 200$	$T = 300$	$T = 400$	$T = 500$	$T = 700$	$T = 1000$
$T_b - 4$	0.2	0.2	0.2	0.6	0.6	0.4	0.01	–	–	0.00	0.01	–	–
	0.2	0.4	0.2	0.6	0.4	0.6	0.01	–	0.02	–	–	–	–
	0.2	0.4	0.2	0.6	0.6	0.2	0.01	–	0.01	–	–	–	–
	0.2	0.4	0.2	0.6	0.6	0.4	0.05	–	0.09	0.10	0.05	0.02	–
	0.2	0.4	0.2	0.6	0.6	0.6	0.01	–	–	–	–	–	–
	0.2	0.4	0.4	0.6	0.6	0.4	0.01	0.06	0.01	–	–	–	–
	0.2	0.4	0.4	0.6	0.6	0.6	0.01	0.01	–	–	–	–	–
	0.2	0.6	0.2	0.6	0.6	0.4	0.01	0.01	–	–	–	–	–
	0.2	0.6	0.4	0.6	0.6	0.6	–	0.01	–	–	–	–	–
	0.4	0.4	0.2	0.6	0.6	0.4	0.01	–	–	–	–	–	–
$T_b - 2$	0.2	0.2	0.2	0.6	0.6	0.4	0.01	0.02	–	–	–	–	–
	0.2	0.4	0.2	0.4	0.6	0.4	0.01	–	0.01	0.01	–	–	–
	0.2	0.4	0.2	0.6	0.4	0.4	0.02	–	0.01	0.01	0.01	0.01	–
	0.2	0.4	0.2	0.6	0.4	0.6	0.01	–	–	–	–	–	–
	0.2	0.4	0.2	0.6	0.6	0.2	0.01	0.01	0.01	–	–	–	–
	0.2	0.4	0.2	0.6	0.6	0.4	0.04	0.08	0.09	0.06	0.11	0.02	–
	0.2	0.4	0.4	0.6	0.4	0.4	0.01	–	–	–	–	–	–
	0.2	0.4	0.4	0.6	0.6	0.6	0.01	0.01	–	–	–	–	–
	0.4	0.4	0.2	0.6	0.6	0.4	0.01	–	–	–	–	–	–
	T_b	0.2	0.2	0.2	0.4	0.6	0.4	0.01	0.01	0.01	0.02	–	0.01
0.2		0.2	0.2	0.6	0.6	0.4	0.03	0.04	0.02	–	–	–	–
0.2		0.2	0.2	0.6	0.6	0.6	0.02	–	0.03	–	–	–	–
0.2		0.2	0.2	0.6	0.6	0.4	0.01	0.01	0.03	0.02	0.01	0.01	0.01
0.2		0.4	0.2	0.4	0.6	0.4	0.01	0.03	0.02	–	–	–	–
0.2		0.4	0.2	0.4	0.6	0.6	0.07	0.02	0.01	–	–	–	–
0.2		0.4	0.2	0.6	0.4	0.2	0.01	0.01	0.02	0.01	–	–	–
0.2		0.4	0.2	0.6	0.4	0.4	0.03	0.02	0.01	–	–	–	–
0.2		0.4	0.2	0.6	0.6	0.2	0.05	0.03	0.01	–	–	–	–
0.2		0.4	0.2	0.6	0.6	0.4	0.15	0.27	0.29	0.38	0.52	0.77	0.84
0.2		0.4	0.2	0.6	0.6	0.6	0.02	0.03	0.01	–	–	–	–
0.2		0.4	0.4	0.6	0.6	0.4	0.01	0.01	0.01	–	–	–	–
0.2		0.6	0.2	0.6	0.4	0.4	0.02	0.01	0.01	0.01	–	–	–
0.2		0.6	0.2	0.6	0.6	0.4	0.01	–	–	0.02	0.01	0.01	0.01
0.4		0.4	0.2	0.4	0.6	0.4	–	–	–	0.01	–	–	–
0.4	0.4	0.2	0.6	0.6	0.4	0.01	0.01	0.01	0.01	0.01	–	–	
$T_b + 2$	0.2	0.2	0.2	0.6	0.6	0.2	–	–	0.01	–	–	–	–
	0.2	0.2	0.2	0.6	0.6	0.4	0.01	0.01	0.01	0.01	–	–	–
	0.2	0.4	0.4	0.6	0.4	0.4	0.01	–	–	–	–	–	–
	0.2	0.2	0.2	0.6	0.6	0.6	0.02	–	–	–	–	–	–
	0.2	0.4	0.2	0.4	0.6	0.4	0.02	–	–	0.01	0.01	0.01	–
	0.2	0.4	0.2	0.4	0.6	0.6	0.01	0.01	–	–	–	–	–
	0.2	0.4	0.2	0.6	0.4	0.4	0.01	0.01	0.01	–	–	0.01	–
	0.2	0.4	0.2	0.6	0.6	0.2	0.03	–	0.01	0.01	0.01	–	–
	0.2	0.4	0.2	0.6	0.6	0.4	0.03	0.09	0.11	0.19	0.10	0.04	–
	0.2	0.4	0.2	0.6	0.6	0.6	0.01	0.01	–	–	–	–	–
$T_b + 4$	0.2	0.2	0.2	0.6	0.6	0.2	0.01	–	–	–	–	–	–
	0.2	0.2	0.2	0.6	0.6	0.4	0.01	–	–	0.01	0.01	–	–
	0.2	0.4	0.2	0.4	0.6	0.4	0.01	0.01	–	–	–	–	–
	0.2	0.4	0.2	0.6	0.4	0.4	0.01	–	0.01	–	–	–	–
	0.2	0.4	0.2	0.6	0.6	0.4	0.04	0.05	0.08	0.10	0.15	0.02	–
	0.2	0.4	0.4	0.6	0.6	0.4	0.01	–	–	0.01	–	–	–
	0.4	0.4	0.2	0.6	0.6	0.4	0.01	0.01	–	–	–	–	–

Note: This table lists the probabilities of detecting the true structural break and the fractional integration parameters for alternative sample sizes when the structural break T_b equals $\frac{T}{2}$, with T being the sample size. The true d 's and associated probabilities appear in bold.

Table 3: Probabilities of Detecting the True Break ($T_b = \frac{T}{4}$) across Fractional Integration Parameters

T_b	d_1^a	d_2^a	d_3^a	d_1^b	d_2^b	d_3^b	$T = 100$	$T = 200$	$T = 300$	$T = 400$	$T = 500$	$T = 700$	$T = 1000$	
T_{b-2}	0.8	1.2	0.8	1.0	0.8	0.4	0.01	0.01	–	–	–	–	–	
	0.8	1.4	0.8	1.0	1.0	0.6	0.01	0.01	–	–	–	–	–	
T_b	0.6	1.2	1.0	1.0	1.0	0.6	0.01	0.01	0.02	–	–	–	–	
	0.6	1.2	1.2	1.0	1.0	0.6	0.01	0.01	0.01	0.01	–	–	–	
	0.6	1.4	1.0	0.8	1.0	0.6	0.01	0.02	0.01	0.01	–	–	–	
	0.6	1.4	1.0	0.8	0.8	0.6	0.01	–	0.01	–	–	–	–	
	0.6	1.4	1.0	1.0	1.0	0.4	0.01	0.01	0.01	–	–	–	–	
	0.6	1.4	1.0	1.0	1.0	0.6	0.01	0.02	0.09	0.02	0.03	0.02	0.01	
	0.6	1.4	1.2	1.0	1.0	0.6	0.01	0.01	0.03	–	–	–	–	
	0.6	1.6	1.0	1.0	0.8	0.4	0.01	–	0.02	–	–	–	–	
	0.6	1.6	1.0	1.0	0.8	0.6	0.01	0.01	0.01	–	0.01	–	–	
	0.8	1.2	0.8	1.0	1.0	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	
	0.8	1.2	0.8	1.0	1.0	0.8	0.01	–	0.01	–	–	–	–	
	0.8	1.2	1.0	1.0	0.8	0.4	0.01	0.01	0.02	–	–	–	–	
	0.8	1.2	1.0	1.0	1.0	0.4	0.01	0.01	0.01	0.01	–	–	–	
	0.8	1.2	1.0	1.0	1.0	0.6	0.01	0.01	0.02	0.05	0.02	0.01	–	
	0.8	1.2	1.0	1.0	1.2	0.6	0.01	0.01	–	–	0.01	–	–	
	0.8	1.2	1.0	1.2	1.0	0.8	0.01	0.01	–	0.02	–	–	–	
	0.8	1.4	0.8	1.0	0.8	0.4	0.01	0.03	0.01	–	–	–	–	
	0.8	1.4	0.8	1.0	1.0	0.6	0.01	0.02	0.03	0.01	0.04	0.01	0.01	
	0.8	1.4	0.8	1.0	1.2	0.6	0.01	–	–	0.01	–	–	–	
	0.8	1.4	0.8	1.2	1.0	0.6	0.01	0.01	0.01	–	–	–	–	
	0.8	1.4	1.0	0.8	0.8	0.6	0.01	0.03	0.02	–	–	–	–	
	0.8	1.4	1.0	0.8	0.8	0.8	0.01	–	0.01	–	–	–	–	
	0.8	1.4	1.0	0.8	1.0	0.6	0.01	0.02	0.04	0.03	0.03	0.02	–	
	0.8	1.4	1.0	0.8	1.0	0.8	0.01	–	–	0.03	–	–	–	
	0.8	1.4	1.0	1.0	0.8	0.4	0.01	0.01	0.01	0.02	0.01	–	–	
	0.8	1.4	1.0	1.0	0.8	0.6	–	–	0.01	0.04	0.03	0.03	–	
	0.8	1.4	1.0	1.0	0.8	0.8	–	–	0.02	0.02	–	–	–	
	0.8	1.4	1.0	1.0	1.0	0.4	0.02	0.02	0.02	0.04	0.02	0.01	0.01	
	0.8	0.8	1.4	1.0	1.0	1.0	0.6	0.04	0.06	0.15	0.38	0.59	0.71	0.83
	0.8	1.4	1.0	1.0	1.0	0.8	0.01	0.01	0.02	0.01	0.05	–	–	
	0.8	1.4	1.0	1.0	1.2	0.6	0.01	0.01	0.02	0.01	–	–	–	
	0.8	1.4	1.0	1.2	1.0	0.6	0.01	–	0.01	0.02	–	–	–	
	0.8	1.4	1.0	1.2	0.8	0.6	0.01	–	–	–	0.01	–	–	
	0.8	1.4	1.2	0.8	1.0	0.6	0.01	0.01	0.03	0.01	0.05	–	–	
	0.8	1.4	1.2	1.0	0.8	0.8	0.01	0.01	0.03	–	0.01	–	–	
	0.8	1.4	1.2	1.0	1.0	0.4	0.01	0.01	–	0.01	–	–	–	
	0.8	1.4	1.2	1.0	1.0	0.6	0.01	–	–	0.04	0.01	0.01	–	
	0.8	1.6	1.0	0.8	0.8	0.4	0.01	0.01	0.01	–	–	–	–	
	0.8	1.6	1.0	0.8	1.0	0.4	0.01	0.01	0.01	–	–	–	–	
	0.8	1.6	1.0	1.0	1.0	0.4	0.01	0.01	0.02	–	–	–	–	
	0.8	1.6	1.0	1.0	1.0	0.6	0.01	0.03	0.03	0.04	0.05	0.02	0.01	
	0.8	1.6	1.0	1.0	1.0	0.8	0.01	0.01	0.01	0.02	–	–	–	
0.8	1.6	1.2	1.0	1.0	0.6	0.01	–	–	0.01	0.01	–	–		
1.0	1.2	1.0	1.0	1.0	0.6	0.01	0.01	0.02	–	–	–	–		
1.0	1.4	1.0	1.0	0.8	0.6	0.01	0.01	0.02	–	–	–	–		
1.0	1.4	1.0	1.0	1.0	0.6	0.01	0.01	0.04	0.02	0.03	0.02	–		
1.0	1.4	1.0	1.0	1.0	0.8	0.01	0.01	0.02	0.01	–	–	–		
T_{b+2}	0.8	1.4	1.0	1.0	1.0	0.6	0.01	–	–	–	0.01	0.01	–	
	0.8	1.4	1.0	1.0	0.8	0.6	0.01	0.01	–	–	–	–	–	

Note: This table lists the probabilities of detecting the true structural break and the fractional integration parameters for alternative sample sizes when the structural break T_b equals $\frac{T}{4}$, with T being the sample size. The true d 's and associated probabilities appear in bold.

Table 4: **Break-Dates in Fractional Integration Systems**

Panel A: White Noise Residuals			
<i>Data Frequency</i>	<i>Sample Period</i>	<i>Value of the Test</i>	<i>Break-date</i>
Monthly	1948:1M-2005:12M	7028.37	1972:9M
Quarterly	1948:1Q-2005:4Q	1676.53	1981:4Q
Annual	1948-2005	356.71	1973
Panel B: VAR(1) Residuals			
<i>Data Frequency</i>	<i>Sample Period</i>	<i>Value of the Test</i>	<i>Break-date</i>
Monthly	1948:1M-2005:12M	6506.89	1973:8M
Quarterly	1948:1Q-2005:4Q	1266.56	1967:2Q
Annual	1948-2005	514.34	1973

Note: This table lists the most likely break-date values of the fractional integration break-date test derived in this paper with U.S. monthly, quarterly and annual data. The test detects the most likely break-date in the parameters of the system: Fractional integration parameters, variances of the structural shocks and those in the autocorrelation of the stationary residuals. The table shows the results of the test using CPI inflation, the unemployment rate and the 3 Month T-Bill Rate.

Table 5: **Alternative Break-Dates in Fractional Integration Systems**

VAR(1) Residuals			
<i>Data Frequency</i>	<i>Sample Period</i>	<i>Value of the Test</i>	<i>Break-date</i>
Monthly	1948:1M-2005:12M	6532.60	1980:7M
Quarterly	1948:1Q-2005:4Q	1286.44	1973:1Q
Quarterly	1948:1Q-2005:4Q	1369.54	1980:4Q
Annual	1948-2005	562.36	1981

Note: This table lists a set of break-dates and their associated values estimated by the fractional integration derived in this paper with U.S. monthly, quarterly and annual data. These dates are the second and third most likely break-dates, following those reported in table 4. The table shows the results of the test using CPI inflation, the unemployment rate and the 3 Month T-Bill Rate.

Table 6: **Second Moments, Before and After the Breaks**

Panel A: Monthly Data

	72:9M			73:8M			80:4M			80:7M		
	π	u	r	π	u	r	π	u	r	π	u	r
$\sigma_2^2 - \sigma_1^2$	0.4	0.6	6.3	-1.6	-1.0	6.2	-13.1	0.2	3.6	-13.6	0.2	3.6
$\rho_2 - \rho_1$	0.3	0.1	0.1	0.3	0.2	-0.2	-0.01	0.01	-0.01	-0.1	0.01	0.0

Panel B: Quarterly Data

	58:2Q			63:2Q			67:2Q		
	π	u	r	π	u	r	π	u	r
$\sigma_2^2 - \sigma_1^2$	-1.2	0.6	-2.8	-1.2	0.6	-2.8	1	0.6	6.6
$\rho_2 - \rho_1$	0.4	0.07	0.3	0.5	0.07	0.3	0.5	0.06	-0.01

Panel B: Quarterly Data (Continued)

	73:1Q			80:4Q			81:4Q		
	π	u	r	π	u	r	π	u	r
$\sigma_2^2 - \sigma_1^2$	3	0.6	6.2	-11.3	0.2	2.9	-13.3	0.1	-2.8
$\rho_2 - \rho_1$	0.4	0.05	-0.01	-0.4	0.03	0.03	-0.7	0.02	-0.06

Table 6 (Continued): Panel C: Annual Data

	1963			1973			1980		
	π	u	r	π	u	r	π	u	r
$\sigma_2^2 - \sigma_1^2$	3.4	0.6	6.4	5.8	0.7	6.0	-8.9	0.3	3.7
$\rho_2 - \rho_1$	0.6	0.5	0.3	0.4	0.3	0.03	-0.3	0.3	-0.1

Note: This table presents the difference between the variances and first order autocorrelations of the three macro variables across the two different subsamples generated by the break-date tests. The second moment statistics were computed via the three following moments in a GMM set-up:

$$\begin{aligned}
 e_{1t} &= x_t - \bar{x} \\
 e_{2t} &= (x_t - \bar{x})^2 - \sigma_x^2 \\
 e_{3t} &= (\pi_t - \bar{\pi})(\pi_{t-1} - \bar{\pi}) - \rho_x(x_{t-1} - \bar{x})^2
 \end{aligned}$$

where \bar{x} is the sample mean of variable x . e_{1t} , e_{2t} and e_{3t} are the disturbances so that $e_t = \{e_{1t}, e_{2t}, e_{3t}\}^T$ and $E[e_t] = 0$. There are three parameters to be estimated and three orthogonality conditions, so that the system is exactly identified. The statistically significant differences (at the 5% level) appear in bold. The Wald statistic for parameter differences used is: $W = (\theta_p^1 - \theta_p^2)'(V_p^1 + V_p^2)^{-1}(\theta_p^1 - \theta_p^2)$. Andrews and Fair (1988), show that it is distributed as a chi-square with p degrees of freedom under the null of parameter stability.

Table 7: Shocks or Propagation?: Monthly Data

Panel A: Cross-Correlated White Noise Residuals: 1972:9M

d_{1st}	d_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.31$	$d_{2,\pi} = 0.42$	$\sigma_{1,\pi} = 3.52$	$\sigma_{2,\pi} = 2.82$
$d_{1,u} = 1.12$	$d_{2,u} = 1.01$	$\sigma_{1,u} = 0.29$	$\sigma_{2,u} = 0.18$
$d_{1,r} = 1.17$	$d_{2,r} = 1.14$	$\sigma_{1,r} = 0.31$	$\sigma_{2,r} = 0.53$

Panel B: VAR(1) Residuals: 1973:8M

d_{1st}	d_{2nd}	eig_{1st}	eig_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.37$	$d_{2,\pi} = 0.32$	$eig_{1,1} = 0.10$	$eig_{2,1} = 0.05$	$\sigma_{1,\pi} = 3.49$	$\sigma_{2,\pi} = 2.58$
$d_{1,u} = 0.41$	$d_{2,u} = 0.01$	$eig_{1,2} = 0.44$	$eig_{2,2} = 0.80$	$\sigma_{1,u} = 0.17$	$\sigma_{2,u} = 0.18$
$d_{1,r} = 0.78$	$d_{2,r} = 0.43$	$eig_{1,3} = 0.85$	$eig_{2,3} = 0.99$	$\sigma_{1,r} = 0.18$	$\sigma_{2,r} = 0.51$

Panel C: VAR(1) Residuals, Candidate: 1980:7M

d_{1st}	d_{2nd}	eig_{1st}	eig_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.47$	$d_{2,\pi} = -0.03$	$eig_{1,1} = -0.22$	$eig_{2,1} = 0.31$	$\sigma_{1,\pi} = 3.42$	$\sigma_{2,\pi} = 2.40$
$d_{1,u} = 0.47$	$d_{2,u} = 0.33$	$eig_{1,2} = 0.63$	$eig_{2,2} = 0.91$	$\sigma_{1,u} = 0.19$	$\sigma_{2,u} = 0.17$
$d_{1,r} = 0.59$	$d_{2,r} = 0.39$	$eig_{1,3} = 0.82$	$eig_{2,3} = 0.93$	$\sigma_{1,r} = 0.39$	$\sigma_{2,r} = 0.41$

Note: This table compares the fractional integration orders, eigenvalues of the VAR(1) and standard deviations of the structural shocks with monthly data across subsamples. $d_{i,\pi}$, $d_{i,u}$ and $d_{i,r}$ are the fractional orders of integration of inflation, the unemployment rate and the interest rate, respectively, for a given period i . The columns eig_{1st} and eig_{2nd} list the eigenvalues of the VAR(1) system followed by the residuals in the 1st and 2nd subsamples, respectively. $\sigma_{i,\pi}$, $\sigma_{i,u}$ and $\sigma_{i,r}$ are the standard deviations of the structural shocks in the inflation, unemployment rate and interest rate equations, respectively, for a given period i .

Table 8: Shocks or Propagation?: Quarterly Data

Panel A: Cross-Correlated White Noise Residuals: 1981:4Q

d_{1st}	d_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.48$	$d_{2,\pi} = 0.15$	$\sigma_{1,\pi} = 2.87$	$\sigma_{2,\pi} = 1.93$
$d_{1,u} = 1.61$	$d_{2,u} = 1.64$	$\sigma_{1,u} = 0.43$	$\sigma_{2,u} = 0.35$
$d_{1,r} = 1.01$	$d_{2,r} = 1.27$	$\sigma_{1,r} = 0.81$	$\sigma_{2,r} = 0.91$

Panel B: VAR(1) Residuals: 1967:2Q

d_{1st}	d_{2nd}	eig_{1st}	eig_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.29$	$d_{2,\pi} = 0.66$	$eig_{1,1} = -0.24$	$eig_{2,1} = -0.10$	$\sigma_{1,\pi} = 2.63$	$\sigma_{2,\pi} = 1.94$
$d_{1,u} = 1.05$	$d_{2,u} = 0.50$	$eig_{1,2} = 0.21$	$eig_{2,2} = -0.26$	$\sigma_{1,u} = 0.23$	$\sigma_{2,u} = 0.26$
$d_{1,r} = 1.36$	$d_{2,r} = -0.07$	$eig_{1,3} = 0.21$	$eig_{2,3} = 0.85$	$\sigma_{1,r} = 0.23$	$\sigma_{2,r} = 0.26$

Panel C: VAR(1) Residuals, Candidate: 1973:1Q

d_{1st}	d_{2nd}	eig_{1st}	eig_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.32$	$d_{2,\pi} = 0.66$	$eig_{1,1} = 0.18$	$eig_{2,1} = 0.20$	$\sigma_{1,\pi} = 2.43$	$\sigma_{2,\pi} = 2.04$
$d_{1,u} = 1.04$	$d_{2,u} = 0.37$	$eig_{1,2} = 0.21$	$eig_{2,2} = 0.20$	$\sigma_{1,u} = 0.24$	$\sigma_{2,u} = 0.27$
$d_{1,r} = 1.31$	$d_{2,r} = 1.14$	$eig_{1,3} = 0.62$	$eig_{2,3} = 0.91$	$\sigma_{1,r} = 0.28$	$\sigma_{2,r} = 0.70$

Panel D: VAR(1) Residuals, Candidate: 1980:4Q

d_{1st}	d_{2nd}	eig_{1st}	eig_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.56$	$d_{2,\pi} = 0.25$	$eig_{1,1} = 0.19$	$eig_{2,1} = 0.15$	$\sigma_{1,\pi} = 2.57$	$\sigma_{2,\pi} = 1.75$
$d_{1,u} = 1.15$	$d_{2,u} = 1.57$	$eig_{1,2} = 0.46$	$eig_{2,2} = 0.33$	$\sigma_{1,u} = 0.28$	$\sigma_{2,u} = 0.23$
$d_{1,r} = 0.72$	$d_{2,r} = 0.69$	$eig_{1,3} = 0.46$	$eig_{2,3} = 0.49$	$\sigma_{1,r} = 0.51$	$\sigma_{2,r} = 0.57$

Note: This table compares the fractional integration orders, eigenvalues of the VAR(1) and residual standard deviations of the structural shocks with quarterly data across subsamples. $d_{i,\pi}$, $d_{i,u}$ and $d_{i,r}$ are the fractional orders of integration of inflation, the unemployment rate and the interest rate, respectively, for a given period i . The columns eig_{1st} and eig_{2nd} list the eigenvalues of the VAR(1) system followed by the residuals in the 1st and 2nd subsamples, respectively. $\sigma_{i,\pi}$, $\sigma_{i,u}$ and $\sigma_{i,r}$ are the standard deviations of the structural shocks in the inflation, unemployment rate and interest rate equations, respectively, for a given period i . 40

Table 9: **Shocks or Propagation?: Annual Data**

Panel A: Cross-Correlated White Noise Residuals: 1973

d_{1st}	d_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.42$	$d_{2,\pi} = 0.78$	$\sigma_{1,\pi} = 1.93$	$\sigma_{2,\pi} = 2.12$
$d_{1,u} = 0.54$	$d_{2,u} = 0.95$	$\sigma_{1,u} = 1.07$	$\sigma_{2,u} = 0.94$
$d_{1,r} = 0.75$	$d_{2,r} = 0.45$	$\sigma_{1,r} = 1.08$	$\sigma_{2,r} = 1.65$

Panel B: VAR(1) Residuals: 1973

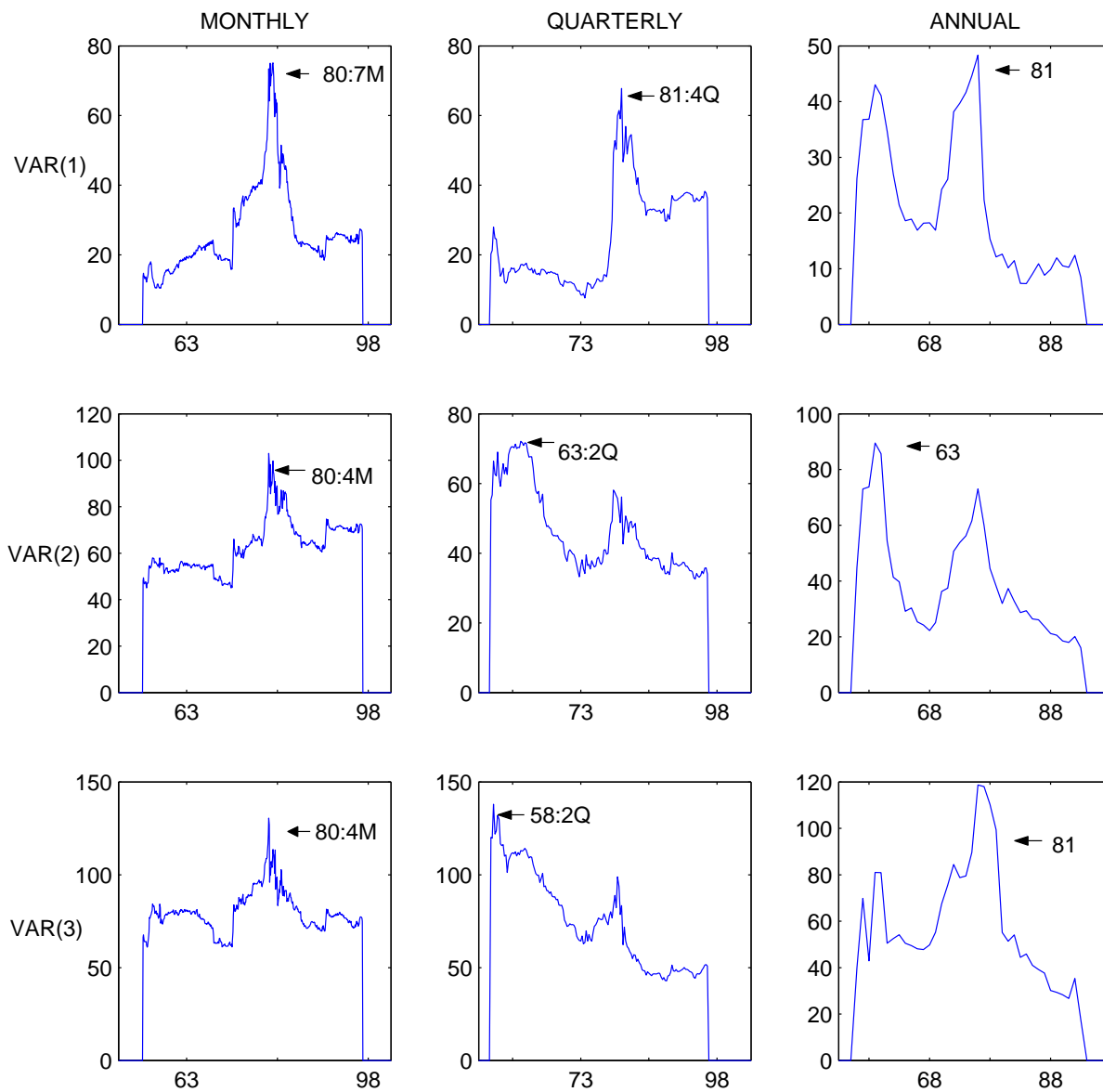
d_{1st}	d_{2nd}	eig_{1st}	eig_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = 0.33$	$d_{2,\pi} = -0.24$	$eig_{1,1} = 0.03$	$eig_{2,1} = 0.32$	$\sigma_{1,\pi} = 1.39$	$\sigma_{2,\pi} = 1.59$
$d_{1,u} = 0.61$	$d_{2,u} = 0.90$	$eig_{1,2} = 0.31$	$eig_{2,2} = 0.83$	$\sigma_{1,u} = 0.67$	$\sigma_{2,u} = 0.80$
$d_{1,r} = 1.36$	$d_{2,r} = -0.07$	$eig_{1,3} = 0.31$	$eig_{2,3} = 0.83$	$\sigma_{1,r} = 0.48$	$\sigma_{2,r} = 0.92$

Panel C: VAR(1) Residuals, Candidate: 1981

d_{1st}	d_{2nd}	eig_{1st}	eig_{2nd}	σ_{1st}	σ_{2nd}
$d_{1,\pi} = -0.28$	$d_{2,\pi} = 0.52$	$eig_{1,1} = 0.31$	$eig_{2,1} = 0.21$	$\sigma_{1,\pi} = 1.57$	$\sigma_{2,\pi} = 1.02$
$d_{1,u} = 0.15$	$d_{2,u} = 1.26$	$eig_{1,2} = 0.31$	$eig_{2,2} = 0.21$	$\sigma_{1,u} = 0.73$	$\sigma_{2,u} = 0.62$
$d_{1,r} = 1.63$	$d_{2,r} = -0.20$	$eig_{1,3} = 0.97$	$eig_{2,3} = 0.88$	$\sigma_{1,r} = 0.76$	$\sigma_{2,r} = 0.69$

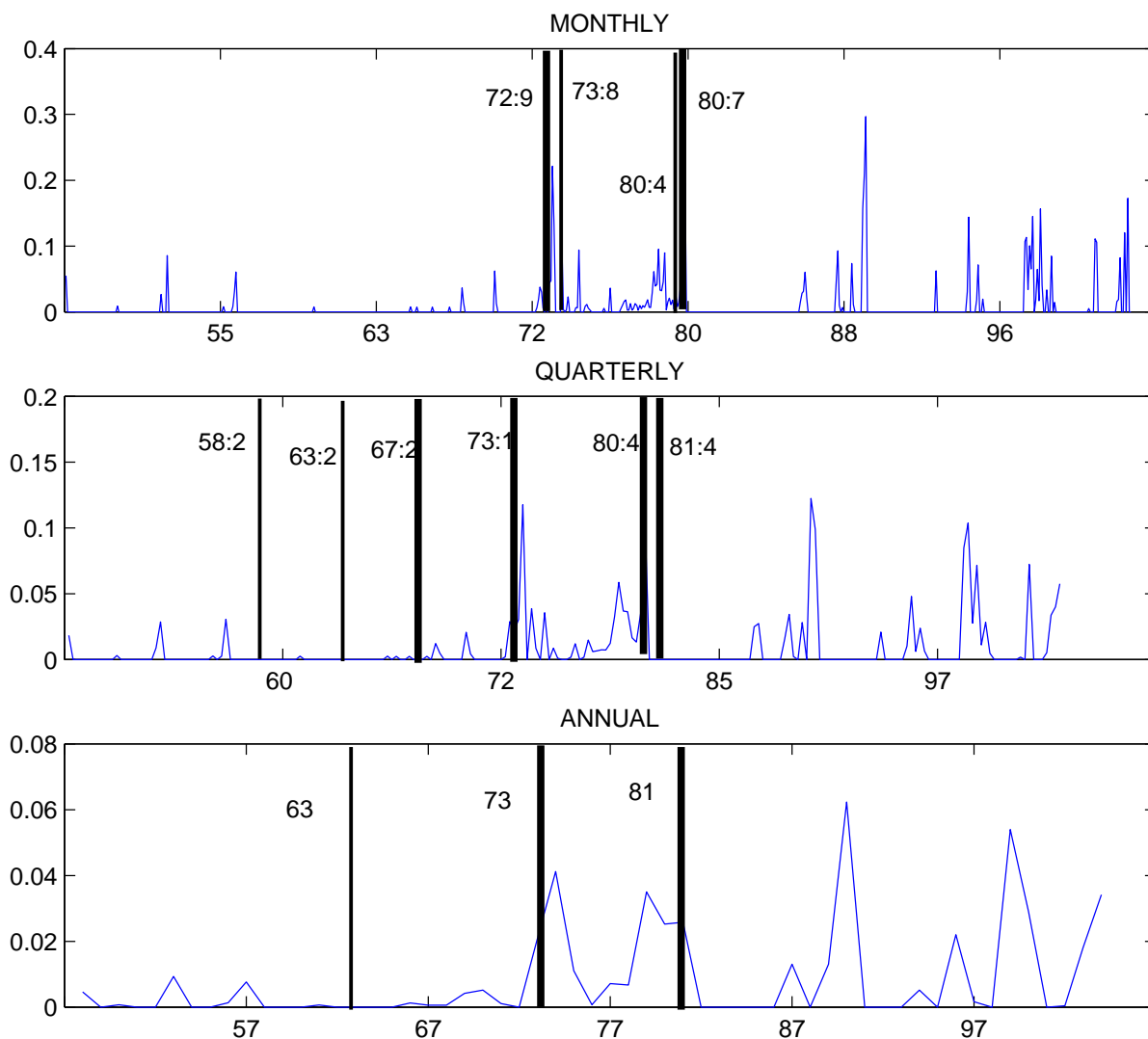
Note: This table compares the fractional integration orders, eigenvalues of the VAR(1) and the residual standard deviations of the structural shocks with annual data across subsamples. $d_{i,\pi}$, $d_{i,u}$ and $d_{i,r}$ are the fractional orders of integration of inflation, the unemployment rate and the interest rate, respectively, for a given period i . The columns eig_{1st} and eig_{2nd} list the eigenvalues of the VAR(1) system followed by the residuals in the 1st and 2nd subsamples, respectively. $\sigma_{i,\pi}$, $\sigma_{i,u}$ and $\sigma_{i,r}$ are the standard deviations of the structural shocks in the inflation, unemployment rate and interest rate equations, respectively, for a given period i .

Figure 1: Sup-Wald Tests for Parameter Stability



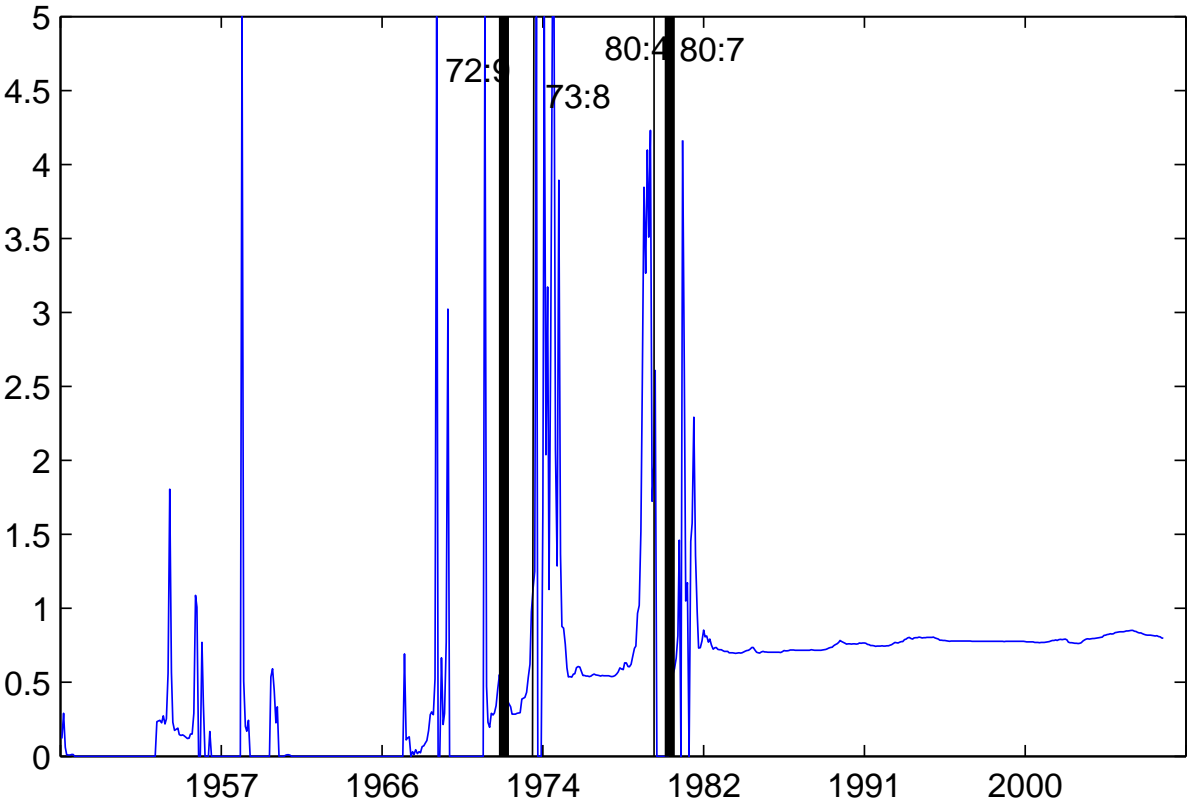
Note: This figure plots the time series of the sup-Wald statistics for parameter stability of the joint macro system with inflation, unemployment and short-term interest rates. The sup-Wald is the most likely date for a break in all of the parameters of the vector autoregressions (VAR). The figure displays the sup-Walds for alternative data frequencies and VAR orders.

Figure 2: Oil Shocks Against Structural Breaks



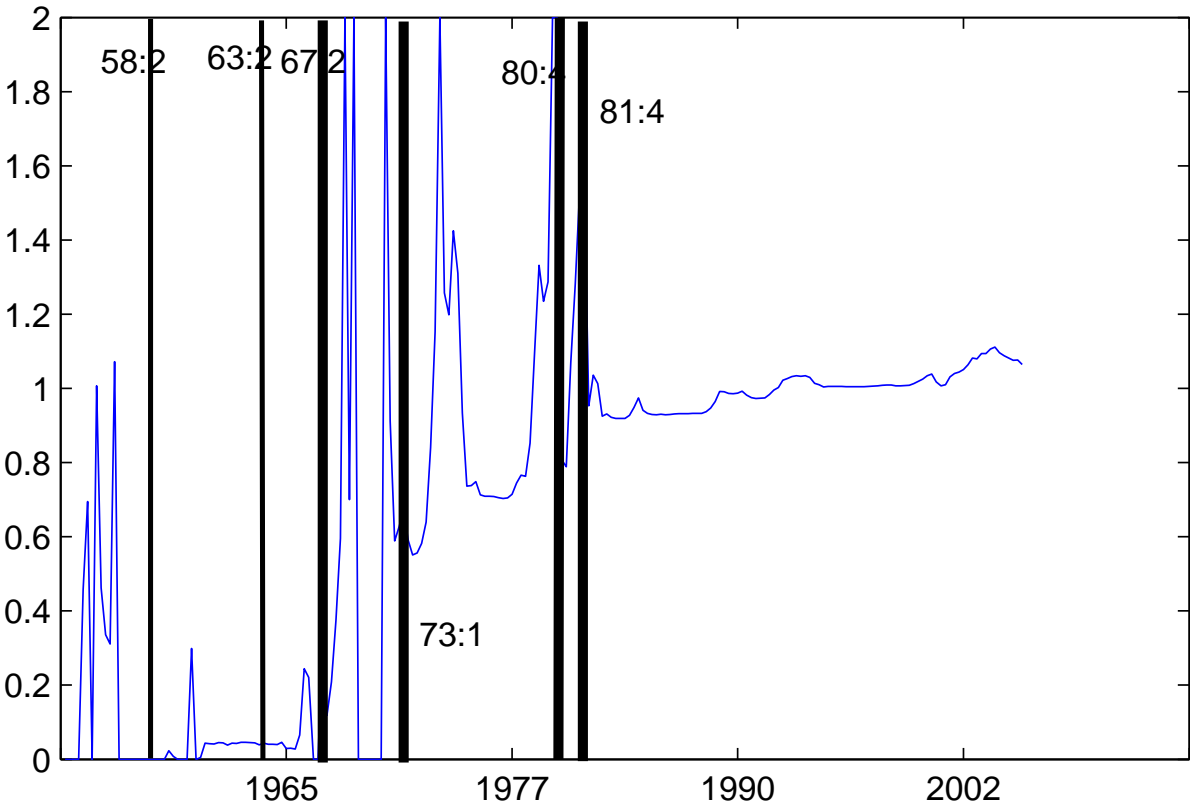
Note: This figure plots the historical series of oil shocks and compares it with the structural breaks across sample frequencies. The thick vertical lines correspond to the breaks estimated in the model with fractional integration whereas the thin lines are the breaks obtained with the Sup-Wald test. The oil shock time series is obtained through the methodology in Hamilton (1996). It equals the greater of zero and the percentage point difference between the current oil price and the maximum price during the past year. The quarterly and annual series are the corresponding arithmetic averages of the monthly shocks.

Figure 3: **Long-Run Inflation Coefficients in the Taylor Rule Against Structural Breaks: Monthly Frequency**



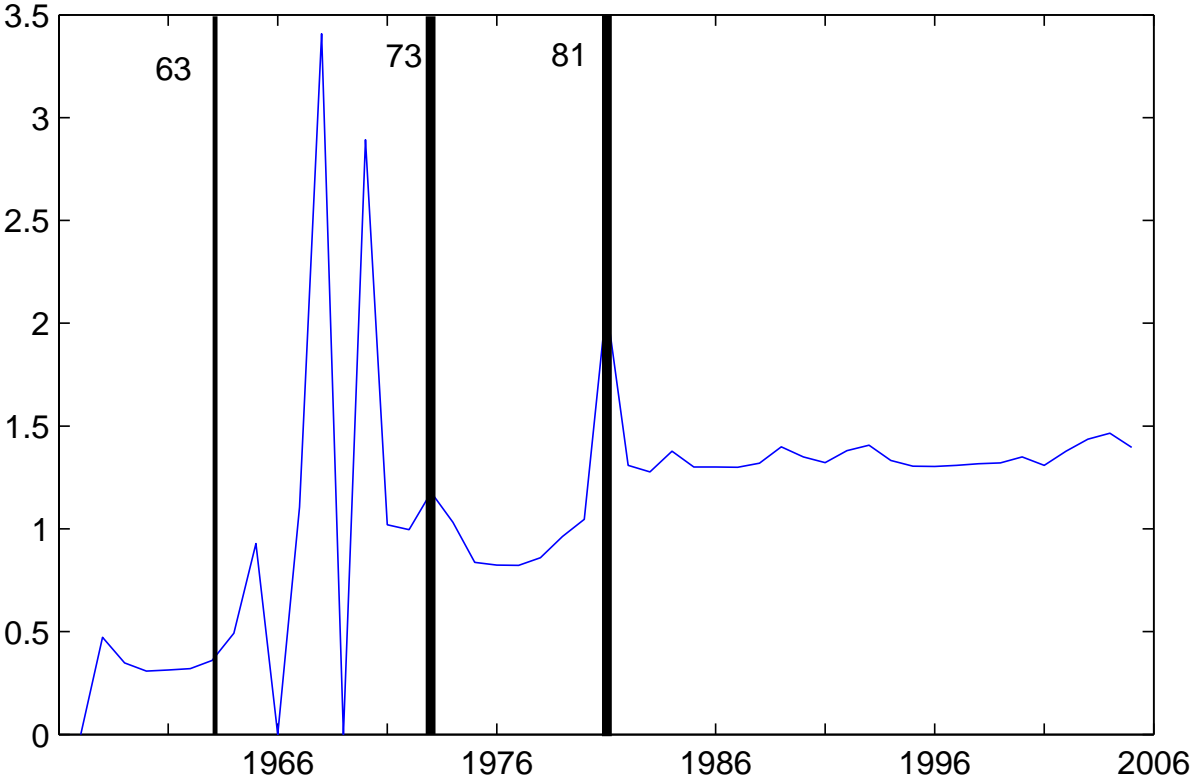
Note: This figure plots the long-run interest rate response to inflation against the break-dates obtained with monthly data. The thick vertical lines correspond to the model with fractional integration whereas the thin lines are the breaks obtained with the Sup-Wald test.

Figure 4: Long-Run Inflation Coefficients in the Taylor Rule Against Structural Breaks: Quarterly Frequency



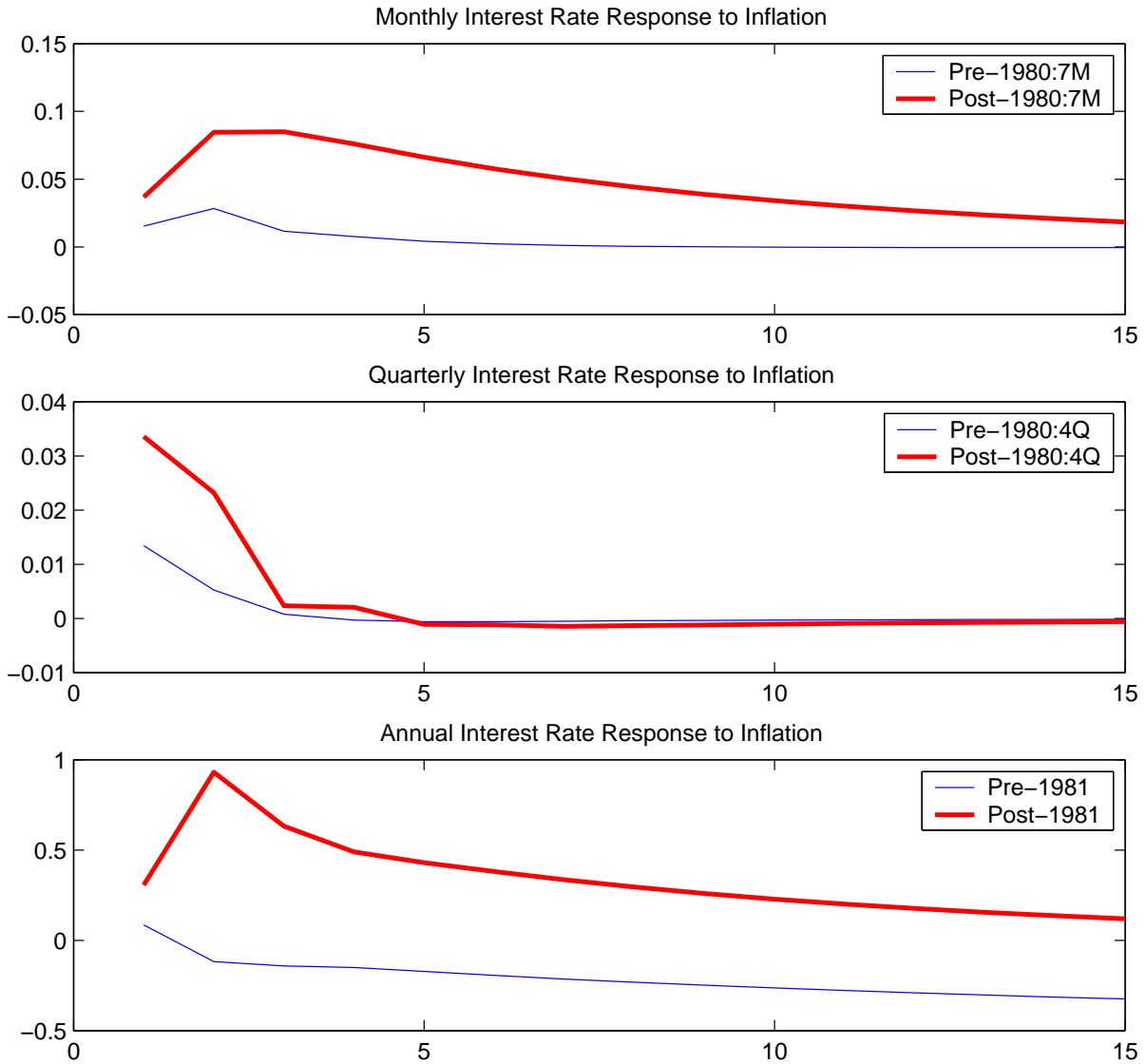
Note: This figure plots the long-run interest rate response to inflation against the break-dates obtained with quarterly data. The thick vertical lines correspond to the model with fractional integration whereas the thin lines are the breaks obtained with the Sup-Wald test.

Figure 5: Long-Run Inflation Coefficients in the Taylor Rule Against Structural Breaks: Annual Frequency



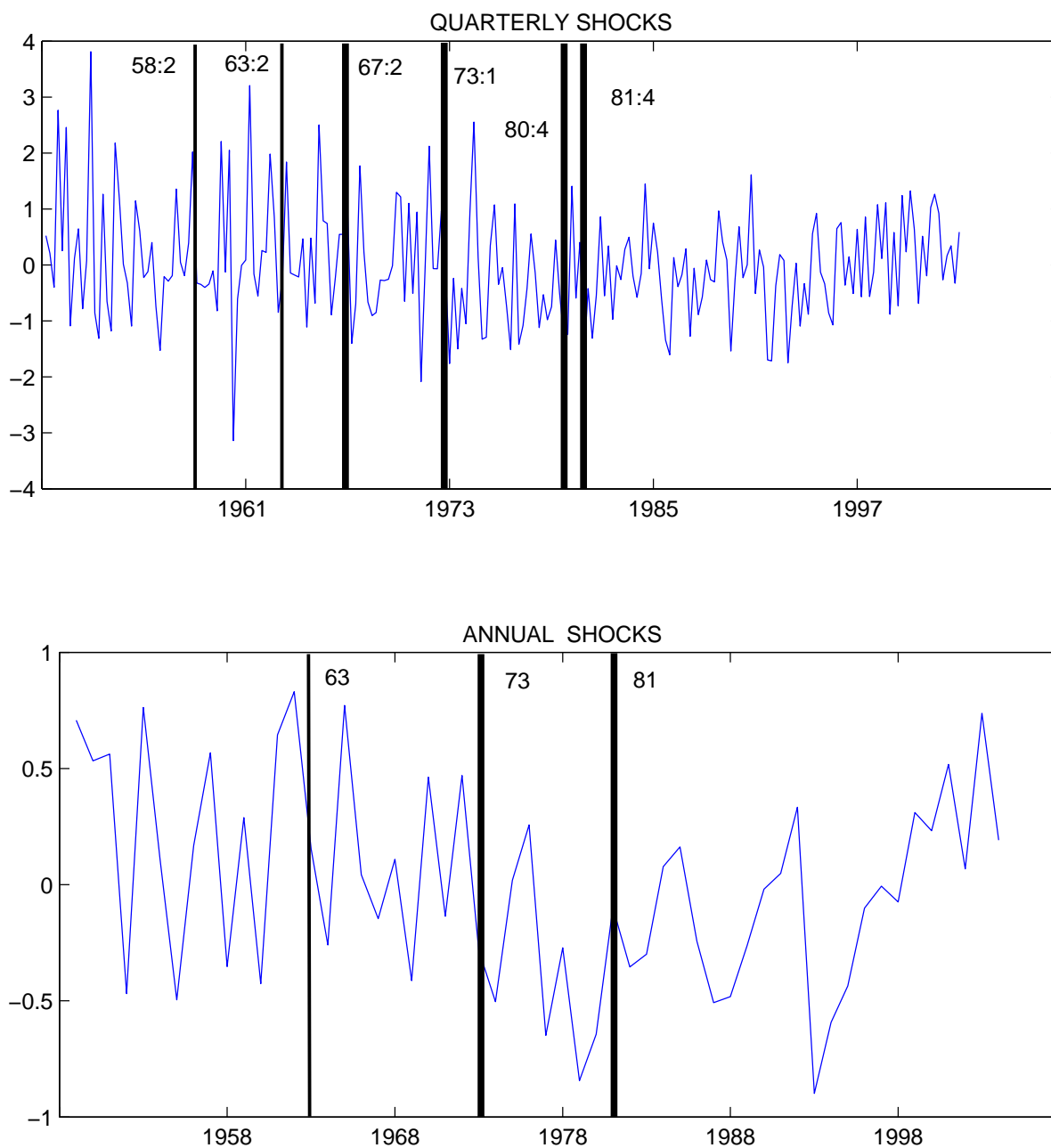
Note: This figure plots the long-run interest rate response to inflation against the break-dates obtained with annual data. The thick vertical lines correspond to the model with fractional integration whereas the thin lines are the breaks obtained with the Sup-Wald test.

Figure 6: Interest Rate Response Function to Supply Shock



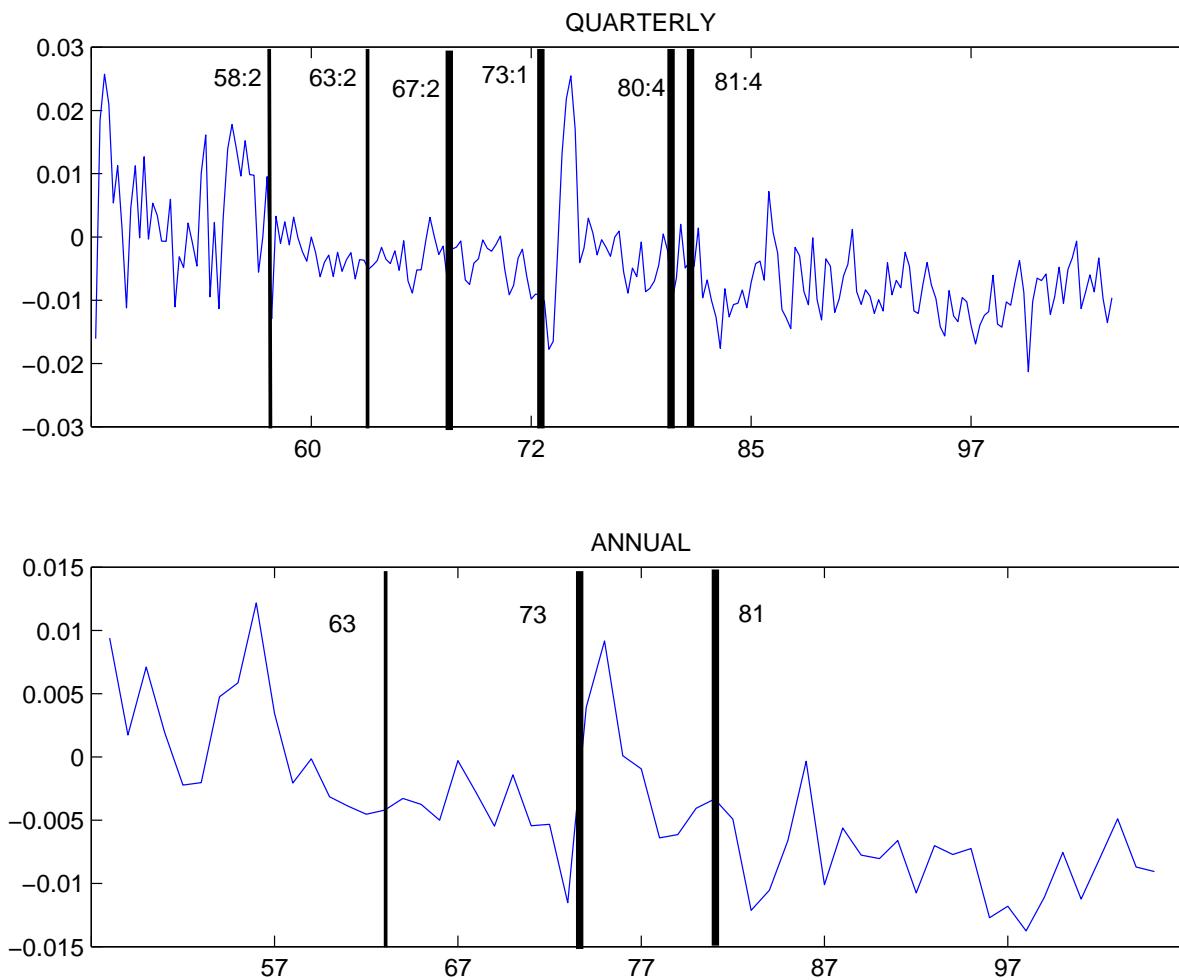
Note: This figure shows the responses of the short-term interest rate to a structural supply shock implied by our fractionally integrated model with VAR(1) residuals. It compares the responses before and after the structural breaks found in the early-80s across data frequencies. The second period responses are conditional on the structural shocks of the first periods.

Figure 7: Labor Productivity Shocks



Note: This figure shows the labor productivity shocks of an identified structural bivariate VAR with labor productivity and hours worked. The productivity shock is identified assuming that there is no long-run effect of hours worked on labor productivity. The VAR is estimated with quarterly data; the annual shocks in the bottom graph are averages of the quarterly shocks. The thick vertical lines correspond to the model with fractional integration whereas the thin lines are the breaks obtained with the Sup-Wald test.

Figure 8: Real Investment Price Growth



Note: This figure shows the quarterly and annual series of the real investment price growth. The real investment price is the ratio of the price index for equipment and the price index for personal consumption expenditures; the annual shocks in the bottom graph are averages of the quarterly shocks. The thick vertical lines correspond to the model with fractional integration whereas the thin lines are the breaks obtained with the Sup-Wald test.