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# THE DEATON PARADOX IN A LONG MEMORY CONTEXT WITH STRUCTURAL BREAKS

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## ABSTRACT

This paper contributes to the permanent income hypothesis (PIH) and excess consumption smoothness debate in the context of fractional integration. We show that the excess consumption smoothness result is a consequence of the quarterly data frequency commonly employed in empirical work. In fact, the I(1) hypothesis is rejected for the income process with monthly data in favor of a fractional integration order lower than 1. Moreover, if a structural break is taken into account, we observe a substantial reduction in the degree of consumption smoothness, especially after the break found in 1975.

**Keywords:** Consumption Smoothness, Permanent Income Hypothesis, Long Memory, Structural Breaks, Monthly Frequency

**JEL Classification:** C12, C22, C32, E21, E32

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## 1 Introduction

During the last twenty years, a large amount of macroeconomic research has focused on the relationship between aggregate consumption and aggregate income. Keynes (1936) was the first in postulating such a relationship, introducing the marginal propensity to consume, which was meant to be less than one. Thus, according to Keynes (1936) the volatility of consumption should be lower than the volatility of income. Later, Friedman (1957) and Ando and Modigliani (1963) postulated that consumption depends on permanent and life-cycle income, respectively. Since these two variables are, by definition, smoother than income, this implies that consumption should then be smoother than income. More recently, Hall (1978, 1988) introduced the theoretical formulation of the random walk behavior of consumption, while Flavin (1981) demonstrated that the change in consumption depends not only on its lagged values but also on income innovations (the Random Walk Permanent Income Hypothesis (RWPIH)). According to this theory, if income is highly persistent, as it seems to be the case, the PIH implies that changes in consumption should have a higher volatility than the income innovations. But this clearly contradicts the observed empirical fact, as consumption dynamics are smoother than income innovations. This apparent excess of consumption smoothness relative to the PIH predictions is denominated the "Deaton Paradox".

Several theoretical arguments have been put forward to explain this apparent contradiction. A partial list of these explanations includes liquidity constraints (Hall and Mishkin (1982)), non-constant interest rates (Michener (1984), Hall (1988)), costly adjustments (Attfield, Demery and Duck (1992)), precautionary savings (Zeldes (1989), Caballero (1990), Deaton, 1991), finite planning horizons (Galí (1990)) and imperfect aggregate information (Goodfriend (1992), Pischke (1995), Demery and Duck (2000)).

However, as Diebold and Rudebusch (1991a) note, these theoretical attempts to reconciliate the theory with the observed data ignore the potential failure of the statistical assumptions imposed when testing the PIH model. In a number of papers, Campbell (1987), Deaton (1987) and Campbell and Deaton (1989) found firm evidence in favor of excess smoothness in US consumption. These authors employed ARIMA models to describe income, and found an order of integration of 1 in the underlying stochastic process for income. Diebold and Rudebusch (1991a) extended the ARIMA approach to the fractional case, and given the wide confidence intervals obtained for the fractional differencing parameters, these authors also found strong evidence of excess smoothness in the US. On the contrary, for the UK, using also fractional techniques, Patterson and Sowell (1996) found values for the order of integration strictly smaller than 1, implying that consumption is too volatile compared to the PIH predictions.

In this paper we re-examine the important excess of smoothness issue for the US case using some recent econometric techniques based on fractional integration and allowing one structural break in income dynamics. We focus on real disposable income, with quarterly and monthly data, using both levels and log-transformations. We use these two frequencies since the results about the order of integration can be very sensitive to the data frequency employed (see Gil-Alana and Caporale (2009)). Moreover, using monthly data, in addition to the most frequently employed quarterly frequency, enables our analysis to be less subject to potential small sample bias, which is always convenient in the context of fractional integration. Our results can be summarized as follows: For the quarterly data, the results are consistent with the findings of Diebold and Rudebusch (1991a) and the estimated orders of integration of income are all in the range of 1, implying excess smoothing in consumption. However, with monthly data, most of the estimates are significantly smaller than 1, though still close to 1, reducing the degree of excess smoothing and, in some cases, matching the volatility observed in the data. We argue that the higher degree of dependence observed in the quarterly income data may be related to the higher level of aggregation of the data at this frequency. If a structural break is taken into account, this is found to occur at 1975 and we again show a substantial reduction in the degree of excess consumption smoothing after the break when using monthly data.

The paper is organized as follows. Section 2 describes the PIH, the excess of consumption paradox and our fractional integration framework. Section 3 describes the data and shows the empirical results for all data frequencies and subsamples analyzed. Section 4 concludes.

## 2. The PIH model of consumption and fractional integration

We start from the linear version of the Rational Expectations (RE) PIH as described in Hall (1978) and Flavin (1981). Suppose that $C_t$ is the level of consumption at period t, chosen by an infinitely-lived representative agent in the face of a stream of stochastic future real labour income payments, $Y_{t+I}$, $I$ = 0, 1, …, ∞ . The consumer assumes a constant real interest rate, $r$, over the infinite planning horizon, and possesses an endowment of non-human wealth $W_t$ at the end of period t. Then, according to the PIH, current period consumption can be written as:

$$C_t = \left[\frac{r}{1+r}\right]\left(W_t + \sum_{i=0}^{\infty}\beta^i E_t Y_{t+i}\right), \tag{1}$$

where $E_t$ is the RE operator, conditional on the information available at time $t$, and $\beta$ = $1/(1+r)$. Assuming that the evolution of wealth over time is given by:

$$W_t = (1+r)\left(W_{t-1} + Y_{t-1} - C_{t-1}\right), \tag{2}$$

the first difference of equation (1) can be expressed as:

$$(1-L)C_t = r\sum_{i=0}^{\infty}\beta^i\left[E_tY_{t+i} - E_{t-1}Y_{t+i}\right]. \tag{3}$$

Flavin (1981) showed that changes in consumption are driven by revisions in conditional expectations or income innovations. Under RE, the nature of the income process determines the behavior of consumption. Thus, it is crucial to determine the appropriate stochastic specification of the income process.

Since the seminal work by Nelson and Plosser (1982) many macroeconomic time series have been modelled in terms of stochastic trends or unit roots. Thus, for example, authors such as Deaton (1987) and Campbell (1987) were unable to reject the null hypothesis of a unit root in the income process. Assuming now that income follows an I(1) ARIMA process of the form:

$$\phi(L)\,(1-L)Y_t = \theta(L)\varepsilon_t\,, \tag{4}$$

where

$$\phi(L) = 1 + \phi_1 L + \phi_2 L^2 + \dots + \phi_p L^p,$$
$$\theta(L) = 1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q,$$

and all the roots of $\phi(L)$ and $\theta(L)$ lying outside the unit circle, Flavin (1981), Hansen and Sargent (1981) and others showed that:

$$(1-L)C_t = \tau\varepsilon_t, \tag{5}$$

where:

$$\tau = \frac{\left(1 + \sum_{j=1}^{q}\beta^j\theta_j\right)}{\left(1 + \sum_{j=1}^{p}\beta^j\phi_j\right)}.$$

Writing now equation (4) in its equivalent moving average (MA) form:

$$(1-L)Y_t \;=\; A(L)\varepsilon_t \;=\; \left(1 + \sum_{j=1}^{\infty} a_j L^j\right)\varepsilon_t,$$

we obtain that τ in (5) is equal to:

$$\tau \;=\; 1 \;+\; \sum_{j=1}^{\infty} \beta^j a_j \,. \qquad (6)$$

Thus, the multiplier in (5), which relates income innovations to changes in consumption, is simply the infinite cumulative impulse response, i.e., A(1), adjusted to reflect the discount factor β.

Taking the standard deviation of each side in (5) we have that:

$$\sigma_r[\Delta C_t] \;=\; \tau\,\sigma[\varepsilon_t], \qquad (7)$$

where $\Delta C_t$ =(1-$L$) $C_t$ . Following Patterson and Sowell (1996), $\sigma_r$ is the standard deviation restricted under the PIH, while the unrestricted one omits the subscript:

$$\sigma[\Delta C_t] \;=\; \tau^*\,\sigma[\varepsilon_t], \qquad (8)$$

with $\tau^*$ being the multiplier of the observed changes in consumption. Deaton (1987), Campbell and Deaton (1989) and many others found that, for a variety of ARIMA specifications in (4), τ was substantially above 1, contradicting the empirically observed fact since consumption was found to be smooth relative to income ($\tau^* < 1$).

Diebold and Rudebusch (1989, 1991a), Sowell (1992a), Patterson and Sowell (1996) and others extended the ARIMA model in (4) for the income process to the fractional case, implying that the number of differences in $Y_t$ may not necessarily be an integer value. In fact, it has been shown by many authors that most of the standard unit root testing procedures (e.g., Dickey and Fuller –ADF– (1979), Phillips and Perron –PP– (1988), Kwiatkowski et al. –KPSS– (1992)) have very low power if the alternatives are of a fractional form (Diebold and Rudebusch (1991b), Hassler and Wolters (1995), Lee and Schmidt (1996), etc.). Extending the model in (4) to the fractional case, we have that:

$$\phi(L)\,(1-L)^d\,Y_t \;=\; \theta(L)\,\varepsilon_t\,, \qquad (9)$$

and defining $d^* = d - 1$, we obtain:

$$(1-L)Y_t \;=\; (1-L)^{-d^*}\;\;\phi(L)^{-1}\,\theta(L)\,\varepsilon_t\,, \qquad (10)$$

which is the infinite MA representation of $(1 - L)Y_t$. Models like (9) with non-integer $d$, are called fractionally ARIMA (ARFIMA) models and have been widely employed in recent years to describe the time dependence in macroeconomic series (e.g., Baillie, (1996), Gil-Alana and Robinson (1997), Michelacci and Zaffaroni (2000) Mayoral, (2006) and others).[1] When $d = 0$ in (9), $Y_t$ is I(0) and is said to be *weakly autocorrelated* (short-memory) as opposed to the case of *strongly autocorrelated* (long memory) if $d > 0$. If d belongs to the interval (0, 0.5) the series is covariance stationary but the autocorrelations take longer time to disappear than in the I(0) case. If $d$ is in the interval [0.5, 1) the series is no longer stationary, although it is still mean-reverting so that shocks affecting the series disappear in the long run. Finally, if $d = 1$ or higher than 1, the series is non-stationary and non-mean-reverting. Thus, allowing d to be a real value implies a richer degree of flexibility in the dynamic specification of the series, not achieved in the classical I(0)/I(1) framework.

Campbell and Deaton (1989) formulated a logarithmic version of the PIH. Analogously to equation (3):

$$\frac{(1-L)\,C_{t+1}}{Y_t} = \frac{r}{r-\mu}\sum_{i=1}^{\infty}\rho^i\left[E_{t+1}(1-L)LnY_{t+i} - E_t(1-L)LnY_{t+i}\right], \quad (11)$$

where $\mu$ is the mean rate of growth of $Y_t$ and $\rho = (1+\mu)/(1+r)$. The derivation in (11) requires that $r > \mu$, and that both $r$ and $\mu/r$ be small with the implication that $r/(r- \mu)$ is

[1] See Gil-Alana and Hualde (2009) for an updated revision of fractional integration in macroeconomic time series.

close to 1. Now, if $LnY_t$ follows an ARIMA (ARFIMA) process such that its first difference admits an MA(∞) representation,

$$(1 - L) Ln Y_t = B(L)\varepsilon_t = \left(1 + \sum_{j=1}^{\infty} b_j L^j\right)\varepsilon_t,$$

it can be shown that:

$$\frac{\Delta C_t}{Y_{t-1}} = \kappa \varepsilon_t, \tag{12}$$

with

$$\kappa = 1 + \sum_{j=1}^{\infty} \rho^j b_j. \tag{13}$$

Taking the standard deviation in both sides of (12), we have, under the PIH:

$$\sigma_r\left[\frac{\Delta C_t}{Y_{t-1}}\right] = \kappa \sigma(\varepsilon_t), \tag{14}$$

and we can again define the empirically-observed relationship as:

$$\sigma\left[\frac{\Delta C_t}{Y_{t-1}}\right] = \kappa^* \sigma(\varepsilon_t). \tag{15}$$

Regarding the methodology employed to test the PIH and the excess smoothness debate, Diebold and Rudebusch (1991a) estimated ARFIMA models for the income process using a two-step semi-parametric procedure due to Geweke and Porter-Hudak (1983). With this method, $d$ is first estimated by trying to identify the low ordinates of the series and then the short run (ARMA) components are estimated with $d$ fixed at the first-stage estimate. Using this method, later found to be highly inefficient, the estimates were all in the range of 1, with wide confidence intervals ranging from 0.5 to 1.5. Patterson and Sowell (1996) employed maximum likelihood estimates of $d$ in the time domain (Sowell (1992b)) for the UK labour income series, obtaining values of the fractional differencing parameter in the region of 0.7 and 0.8, and significantly different from 1. Moreover, their conclusions were that, in contrast to the US case, consumption is not too smooth compared with the predictions of the PIH.

Alternatively, other authors claim that fractional integration may be a spurious phenomenon caused by the existence of breaks that have not been taken into account in the data (see, e.g., Cheung (1993), Diebold and Inoue (2001), Giraitis et al. (2001), Mikosch and Starica (2004), Granger and Hyung (2004), etc.). Thus, for instance, Lobato and Savin (1998), argue that structural breaks may be responsible for the long memory in return volatility processes, and several test statistics have been developed in recent years to test for fractional integration versus structural breaks (Beran and Terrin, (1996), Bos et al. (2001), Ohanissian et al. (2008), etc.). Even allowing for fractional integration, some authors have considered the possibility of mean shifts, or more generally, split deterministic trends in the series (Hidalgo and Robinson (1996), Kuan and Hsu (1998), Krämer and Sibbertsen (2002), etc.). These methods have the limitation that they impose the same degree of integration across regimes. In contrast, Gil-Alana (2008) has recently introduced a procedure which allows for different integration orders across regimes and we will employ this method in the empirical section below.

## 3. Data and empirical results

The data analyzed in this section corresponds to seasonally adjusted real disposable personal income (billions of chained 2000 dollars) at the quarterly (1947Q1 – 2008Q2) and monthly (1959M1- 2008M7) frequencies in both levels (DPIC96, DSPIC96) and logarithms (LDPIC96, LDSPIC96). The series were retrieved from the Bureau of Economic Analysis website (http://www.bea.gov).

The first thing we do is to estimate the processes for the four income series under the assumption that there are no breaks in the data. To this end, we consider the following fractionally integrated model, proposed by Diebold and Rudebusch (1989):

$$Y_t = \mu + X_t; \qquad (1 - L)^d X_t = U_t, \qquad t = 1, 2, \dots, \tag{16}$$

where $\mu$ is an intercept, $X_t$ is supposed to be I($d$) and thus, $U_t$ is I(0).[2] We also estimated the model with a time trend for output –as in Mayoral (2006)– but results were essentially the same as the ones reported below. Table 1 displays the estimates of $\mu$ and $d$ in (16) based on the Whittle function in the frequency domain (see Dahlhaus (1989) and Robinson (1994)), under the assumption that the I(0) disturbances $U_t$ follow either a white noise or an AR(1) process.[3] Higher AR orders and other ARMA processes were also considered and the results were very similar in all cases. Note that given the fractional nature of the underlying process, this can be specified in terms of an AR($\infty$) process, and thus the contribution of the short-run ARMA structure disappears relatively fast.

**[Insert Table 1 about here]**

Starting with the results based on white noise $U_t$, we see that for the quarterly series the estimates are 1.031 with the original data, and 0.891 with the log-transformed data, and the unit root null cannot be rejected in any of the two series. However, for the monthly series, the estimates are found to be strictly below 1 (and statistically significantly different from 1), being 0.851 for the original data and 0.821 for its log-transformation. Permitting autocorrelation through the AR(1) model, the values are in

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[2] For the purpose of the present work, an I(0) process is defined as a covariance stationary process with spectral density function which is positive and bounded at the zero frequency. Thus, it includes the stationary ARMA models.

[3] Very similar results were obtained with time domain estimation procedures such as Sowell (1992b) and Tanaka (1999).

all cases slightly higher. The unit root is rejected in favor of $d > 1$ for the quarterly series. The unit root cannot be rejected with the log-transformed quarterly data, but it is again rejected in favor of mean reversion ($d < 1$) for the two monthly series. The intercepts are statistically significant in all cases. Thus, the results seem to be very sensitive to the choice of the data frequency and we obtain evidence of integration orders lower than 1 at the monthly frequency.[4]

Performing Likelihood Ratio (LR) tests on these two specifications (white noise and AR(1) $U_t$), the results indicate that the AR(1) model is more suitable in the four series examined.[5] We next compute the empirically observed multipliers $\tau^*$ and $\kappa^*$ in equations (8) and (15) for the linear and log-linear versions, respectively, using the selected ARFIMA(1, *d*, 0) models in Table 1. The consumption series employed is real personal consumption expenditures, quarterly, monthly and the log-versions (series PCECC96, PCEC96, LPCECC96 and LPCEC96, respectively). We display the results in Table 2.

**[Insert Table 2 about here]**

We see that the coefficients are all smaller than 1, ranging from 0.52 (in case of the monthly data) to 0.72 (log-quarterly series). This is consistent with other studies for the US case which find values for the ratio of the volatility of the change in consumption to the standard deviation of the income innovation in a range centered at 0.64.

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[4] Note, however, that the quarterly data start at 1947Q1 while the monthly start at 1959M1. We performed the analogous analysis with quarterly data starting in 1959Q1 and the results were very similar: for the original series, *d* was estimated to be 0.991 and 1.111 for the white noise and AR(1) cases respectively, whereas we obtained 1.016 and 1.186 for the log-transformed data.

[5] The AR(1) model also outperforms other higher AR order specifications.

As shown in the previous section, the multipliers restricted by the PIH (τ and κ) can be expressed as the infinite cumulative response of income to its innovation adjusted for the real interest rate (equations (6) and (13)). Thus, across Figures 1 - 4 we display the discounted cumulative impulse responses, conditional on four alternative real interest rates **(**0, 1.25, 2.5 and 5%**)** for the same selected ARFIMA (1, *d*, 0) models as in Table 1. The resulting figures with white noise residuals were essentially the same.

**[Insert Figures 1 – 4 about here]**

The results are very similar across the four figures, observing a clear different pattern for the quarterly and the monthly data. Thus, for the unlogged quarterly data (DPIC96), the estimated τ (κ)coefficient is above 1; it is around 1 for the logged data (LDPIC96), and it is strictly below unity for the two monthly series (DSPCI96 and LDSPCI96). At the monthly frequency, consumption does not appear too smooth. Indeed, for the series in levels, the empirical and PIH-restricted multipliers are statistically indistinguishable, since the four figures show that the implied 95% confidence interval for τ ranges between 0.48 and 0.62, whereas $\tau^*$ was estimated to be 0.52, as shown in Table 1.

In what follows, we assume that the series contain a single break in the data, and use the methodology developed by Gil-Alana (2008), which allows us to estimate alternative fractional differencing parameters across subsamples. The model is now given by:

$$Y_t = \mu_1 + X_t; \quad (1-L)^{d_1} X_t = U_t, \quad t = 1,...,T_b, \qquad (17)$$

and

$$Y_t = \mu_2 + X_t; \quad (1-L)^{d_2} X_t = U_t, \quad t = T_b+1,...,T, \qquad (18)$$

where the $\mu_i$'s are the coefficients corresponding to the intercepts, $d_1$ and $d_2$ can be real values, $U_t$ is I(0) and $T_b$ is the time of the break, which is supposed to be unknown. This method is based on minimizing the residuals squares sum for each subsample, using recursively different break-dates and fractional differencing parameters. The method is briefly described in the Appendix.[6]

**[Insert Tables 3 and 4 and Figure 5 about here]**

Table 3 displays the results under the assumption of white noise $U_t$, while Table 4 refers to the case of AR(1) disturbances. Interestingly, we observe across both tables that the break-date takes place in the first half of 1975 (1975Q2 for the quarterly data, and 1975M5 for the monthly data). In order to draw some intuition about the structural break obtained, Figure 5 graphs the demeaned series of real disposable income before and after the estimated breaks across frequencies. It shows that the slope of the income series became steeper across the second sub-samples, pointing at a clear acceleration in disposable income growth after 1975.

Regarding the results of the PIH and starting with the case of uncorrelated disturbances (in Table 3), we see that for the quarterly data the results are inconclusive, as there is a reduction in the degree of integration after the break with the unlogged data, while there is an increase in the value of d with the logged series. Nevertheless, the confidence intervals include the unit root ($d$ = 1) in all cases, implying lack of mean reversion behavior. However, a different picture emerges again in the case of the monthly data. Here, we observe that for the unlogged data, the estimated $d$ for the first subsample is 1.114 and the unit root null cannot be rejected. After the break, the

[6] Given the sample sizes used in this paper, the inclusion of more than one break would result in relatively short subsamples, thus invalidating the analysis based on fractional integration.

estimate falls to 0.821 and the unit root is now decisively rejected in favor of mean reversion. For the logged series, $d_1$ is equal to 1.137 (the unit root being now rejected in favor of $d_1 > 1$), while $d_2$ is 0.821 (and the unit root is rejected in favor of $d_2 < 1$).

If we focus now on the case of autocorrelated disturbances, in Table 4, the same conclusions emerge. Thus, for the quarterly series, evidence of unit roots is found in practically all cases. The only exception is for the DPIC96 series in the second subsample where the unit root cannot be rejected at the 5% level, though it is rejected at the 1%. For the monthly unlogged data (DSPIC96), $d_1 = 0.882$ and the unit root cannot be rejected, while $d_2 = 0.871$ and the unit root is rejected in favor of mean reversion. In the case of the logged series (LDSPIC96) the evidence in favor of a reduction in the degree of persistence is even stronger ($d_1 = 1.161$ being significantly above unity), while $d_2 = 0.836$ with significant evidence of $d_2 < 1$.

**[Insert Table 5 and Figure 6 about here]**

Figure 6 displays the discounted cumulative impulse responses for the two monthly series in the two subsamples in the context of autocorrelated disturbances with a 5% interest rate (alternative interest rates gave rise to very similar responses). We again note that there is a substantial reduction in the cumulative response function after the break-date with monthly data, so that the consumption excess of smoothness again disappears. Table 5 lists the estimates of the empirically observed multipliers and we observe a substantial reduction in the magnitudes during the second subsamples, which is consistent with the PIH-restricted multipliers implied by the plots in Figure 6.

## 4. Concluding Comments

In this paper we have examined the "Deaton Paradox" and the excess smoothing hypothesis for consumption in the US using some recent techniques based on fractional integration and incorporating a structural break in the income process. We use quarterly and monthly series and the main results can be summarized as follows: Firstly, we observe a different pattern depending on the data frequency used. Thus, if no break is taken into account, evidence of excess consumption smoothing is obtained at a quarterly basis, which is consistent with the results reported in Diebold and Rudebusch (1991a) and others. However, using monthly data, the unit root hypothesis is rejected in the income process in favor of smaller orders of integration, reducing to some extent the excess smoothing in consumption. When we allow for a structural break in the data, found to take place in 1975, we observe a different pattern at the quarterly and monthly frequencies. Using the latter frequency, we observe a substantial reduction in the degree of excess consumption smoothing especially after the structural break.

Thus, in contrast to the widely reported results in the literature obtained with quarterly data, we find evidence consistent with the PIH at the monthly frequency. We show that the key difference in the results obtained with quarterly and monthly data is the different order of integration estimated under the two frequencies. The higher values obtained at the quarterly frequency, where the unit root cannot be rejected, can be related with the higher level of aggregation at this particular frequency, which makes less-than-three-month cyclical variation disappear in the data. Indeed, Rossana and Seater (1995) show that the high frequency dynamic properties of macro variables, such as Industrial Production, disappear at lower frequencies.

This paper can be extended in several directions. Thus, for instance, non-linearities of the form suggested by the Threshold AutoRegressive, TAR, Momentum Threshold AutoRegressive, M-TAR or Smooth Transition Autoregressive, S-TAR-form (see, e.g. Enders and Granger, 1998; Enders and Siklos, 2001; Skalin and Teräsvirta, 2002) can be incorporated for the income process in the context of fractional integration. Moreover, multivariate (long memory) models can also be considered and this may lead to the analysis of (fractional) cointegration in the context of VAR structures, as recently suggested by Johansen and Nielsen (2007), Johansen (2008) and others.

**Appendix**

The model in (17) and (18) can also be written as:

$$(1 - L)^{d_1} y_t \;=\; \alpha_1 \, \tilde{1}_t(d_1) \;+\; \beta_1 \, \tilde{t}_t(d_1) \;+\; u_t, \quad t \;=\; 1, ..., T_b,$$

$$(1 - L)^{d_2} y_t \;=\; \alpha_2 \, \tilde{1}_t(d_2) \;+\; \beta_2 \, \tilde{t}_t(d_2) \;+\; u_t, \quad t \;=\; T_b + 1, ..., T,$$

where $\tilde{1}_t(d_i) = (1 - L)^{d_i} 1$, and $\tilde{t}_t(d_i) = (1 - L)^{d_i} t$, $i = 1, 2$.

The procedure is based on the least squares principle. First we choose a grid for the values of the fractionally differencing parameters $d_1$ and $d_2$, for example, $d_{io} = 0, 0.01, 0.02, \ldots, 1$, $i = 1, 2$. Then, for a given partition $\{T_b\}$ and given initial $d_1$, $d_2$-values, $(d_{1o}^{(1)}, d_{2o}^{(1)})$, we estimate the $\alpha$'s and the $\beta$'s by minimizing the sum of squared residuals,

$$\min_{\text{w.r.t.}\{\alpha_1, \alpha_2, \beta_1, \beta_2\}} \sum_{t=1}^{T_b} \left[ (1-L)^{d_{1o}^{(1)}} y_t - \alpha_1 \, \tilde{1}_t(d_{1o}^{(1)}) - \beta_1 \, \tilde{t}_t(d_{1o}^{(1)}) \right]^2 + \sum_{t=T_b+1}^{T} \left[ (1-L)^{d_{2o}^{(1)}} y_t - \alpha_2 \, \tilde{1}_t(d_{2o}^{(1)}) - \beta_2 \, \tilde{t}_t(d_{2o}^{(1)}) \right]^2$$

Let $\hat{\alpha}(T_b;\, d_{1o}^{(1)}, d_{2o}^{(1)})$ and $\hat{\beta}(T_b;\, d_{1o}^{(1)}, d_{2o}^{(1)})$ denote the resulting estimates for the partition $\{T_b\}$ and initial values $d_{1o}^{(1)}$ and $d_{2o}^{(1)}$. Substituting these estimated values into the objective function, we have $RSS(T_b;\, d_{1o}^{(1)},\, d_{2o}^{(1)})$, and minimizing this expression across all values of $d_{1o}$ and $d_{2o}$ in the grid we obtain $RSS(T_b) = \arg\min_{\{i, j\}} RSS(T_b; d_{1o}^{(i)}, d_{2o}^{(j)})$. Then, the estimated break date, $\hat{T}_k$, is such that $\hat{T}_k = \arg\min_{i=1,\ldots,m} RSS(T_i)$, where the minimization is carried out over all partitions $T_1, T_2, \ldots, T_m$, such that $T_i - T_{i-1} \geq |\varepsilon T|$. Then, the regression parameter estimates are the associated least-squares estimates of the estimated k-partition, i.e., $\hat{\alpha}_i = \hat{\alpha}_i(\{\hat{T}_k\})$, $\hat{\beta}_i = \hat{\beta}_i(\{\hat{T}_k\})$, and their corresponding differencing parameters, $\hat{d}_i = \hat{d}_i(\{\hat{T}_k\})$, for i = 1 and 2.

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**Table 1: Estimates of the parameters in model (16) with no breaks**

| | White noise disturbances | | AR(1) disturbances | | |
|---|---|---|---|---|---|
| Series | Intercept | $d$ - parameter | Intercept | $d$ - parameter | AR(1) |
| DPIC96 (47Q1 – 08Q2) | 1068.462 (19.908) | 1.031 (0.999, 1.075) | 1087.039 (23.316) | 1.150 (1.093, 1.251) | -0.299 |
| LDPIC96 (47Q1 – 08Q2) | 6.878 (99.294) | 0.891 (0.813, 1.057) | 6.967 (195.82) | 1.014 (0.862, 1.242) | -0.096 |
| DSPIC96 (59M1 - 08M7) | 1612.098 (29.363) | **0.851 (0.832, 0.874)** | 1642.124 (32.168) | **0.909 (0.884, 0.941)** | -0.192 |
| LDSPIC96 (59M1 - 08M7) | 7.281 (149.012) | **0.821 (0.800, 0.851)** | 7.314 (169.473) | **0.842 (0.826, 0.883)** | -0.027 |

t-values and confidence intervals in parenthesis. In bold, estimates of $d$ stastistically significantly smaller than 1.

**Table 2: Estimates of the empirically observed multipliers**

| Series | Ratio | Values | $\tau^*$ and $\kappa^*$ |
|---|---|---|---|
| PCEC96 (47Q1 – 08Q2) | $\sigma[\Delta C_t^Q] \ / \ \sigma[\varepsilon_t],$ | 27.20936 / 41.73960 | **0.6518** |
| LPCEC96 (47Q1 – 08Q2) | $\sigma\left[\frac{\Delta C_t^Q}{Y_{t-1}}\right] \ / \ \sigma(\varepsilon_t).$ | 0.00746 / 0.01034 | **0.7221** |
| PCECC96 (59M1 - 08M7) | $\sigma[\Delta C_t^M] \ / \ \sigma[\varepsilon_t],$ | 22.6890 / 43.4267 | **0.5224** |
| LPCECC96 (59M1 - 08M7) | $\sigma\left[\frac{\Delta C_t^M}{Y_{t-1}}\right] \ / \ \sigma(\varepsilon_t).$ | 0.00498 / 0.00760 | **0.6559** |

**Table 3: Estimates of model in (17) and (18) in the context of white noise disturbances**

| Series | Break date | First subsample | | Second subsample | |
|---|---|---|---|---|---|
| | | $\mu_1$ | $d_1$ | $\mu_2$ | $d_2$ |
| DPIC96 (47Q1 – 08Q2) | 1975Q2 | 1080.816 (33.653) | 1.079 (0.998, 1.259) | 3213.689 (32.650) | 0.954 (0.910, 1.020) |
| LDPIC96 (47Q1 – 08Q2) | 1975Q2 | 6.814 (58.727) | 0.903 (0.803, 1.147) | 8.019 (103.516) | 0.979 (0.879, 1.077) |
| DSPIC96 (59M1 - 08M7) | 1975M5 | 1670.175 (75.427) | 1.114 (0.955, 1.452) | 3173.404 (39.781) | **0.808 (0.783, 0.839)** |
| LDSPIC96 (59M1 - 08M7) | 1975M5 | 7.415 (382.457) | 1.137 (1.043, 1.345) | 7.912 (115.968) | **0.821 (0.801, 0.842)** |

t-values and confidence intervals in parenthesis. In bold, estimates of *d* stastistically significantly smaller than 1.

**Table 4: Estimates of model in (17) and (18) in the context of AR(1) disturbances**

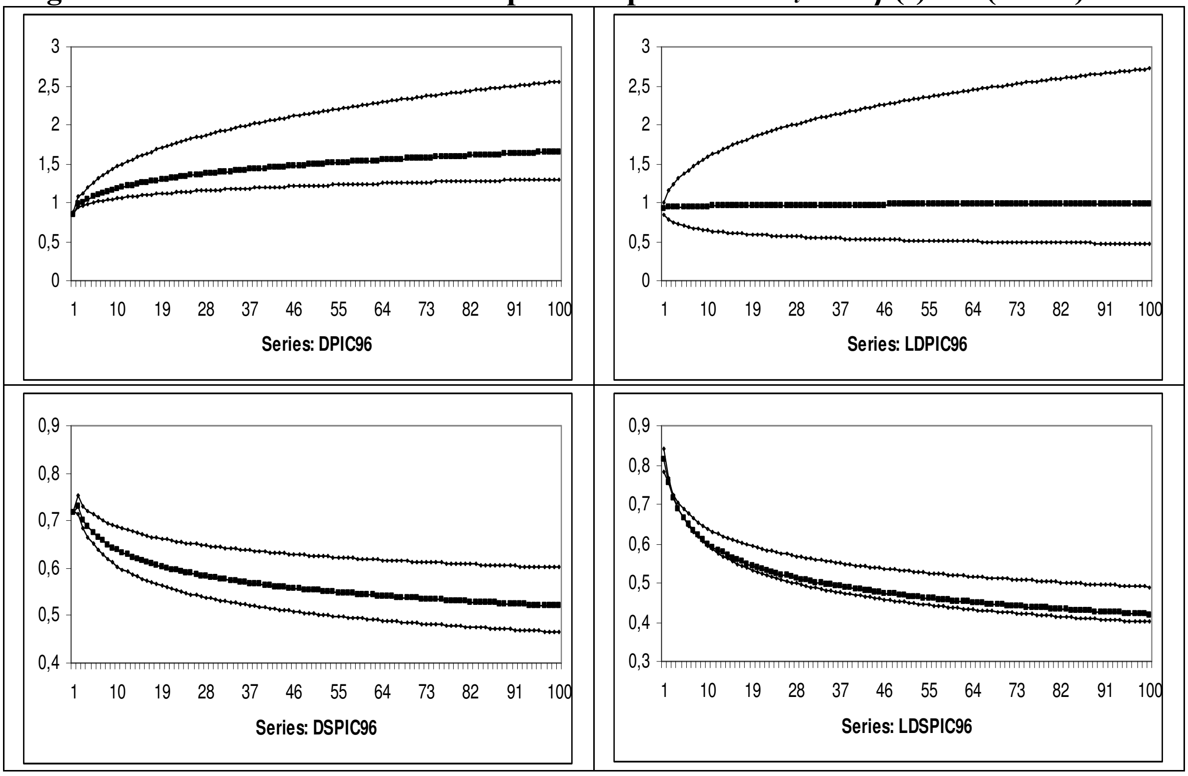
| Series | Break date | First subsample | | | Second subsample | | |
|---|---|---|---|---|---|---|---|
| | | $\mu_1$ | $d_1$ | $\rho_1$ | $\mu_2$ | $d_2$ | $\rho_2$ |
| DPIC96 (47Q1 – 08Q2) | 1975Q2 | 1062.49 (26.337) | 0.984 (0.920, 1.097) | 0.191 | 3271.76 (47.637) | 1.092 (1.003, 1.359) | -0.311 |
| LDPIC96 (47Q1 – 08Q2) | 1975Q2 | 6.900 (81.080) | 0.967 (0.895, 1.124) | -0.084 | 8.086 (250.05) | 1.141 (0.966, 1.457) | -0.270 |
| DSPIC96 (59M1 - 08M7) | 1975M5 | 1627.00 (43.473) | 0.882 (0.835, 1.043) | 0.244 | 3251.10 (48.030) | **0.871 (0.839, 0.913)** | -0.192 |
| LDSPIC96 (59M1 - 08M7) | 1975M5 | 7.417 (436.08) | 1.161 (1.046, 1.473) | -0.025 | 7.944 (126.70) | **0.836 (0.818, 0.880)** | -0.069 |

t-values and confidence intervals in parenthesis. In bold, estimates of *d* stastistically significantly smaller than 1.

**Table 5: Estimates of the empirically observed multipliers**

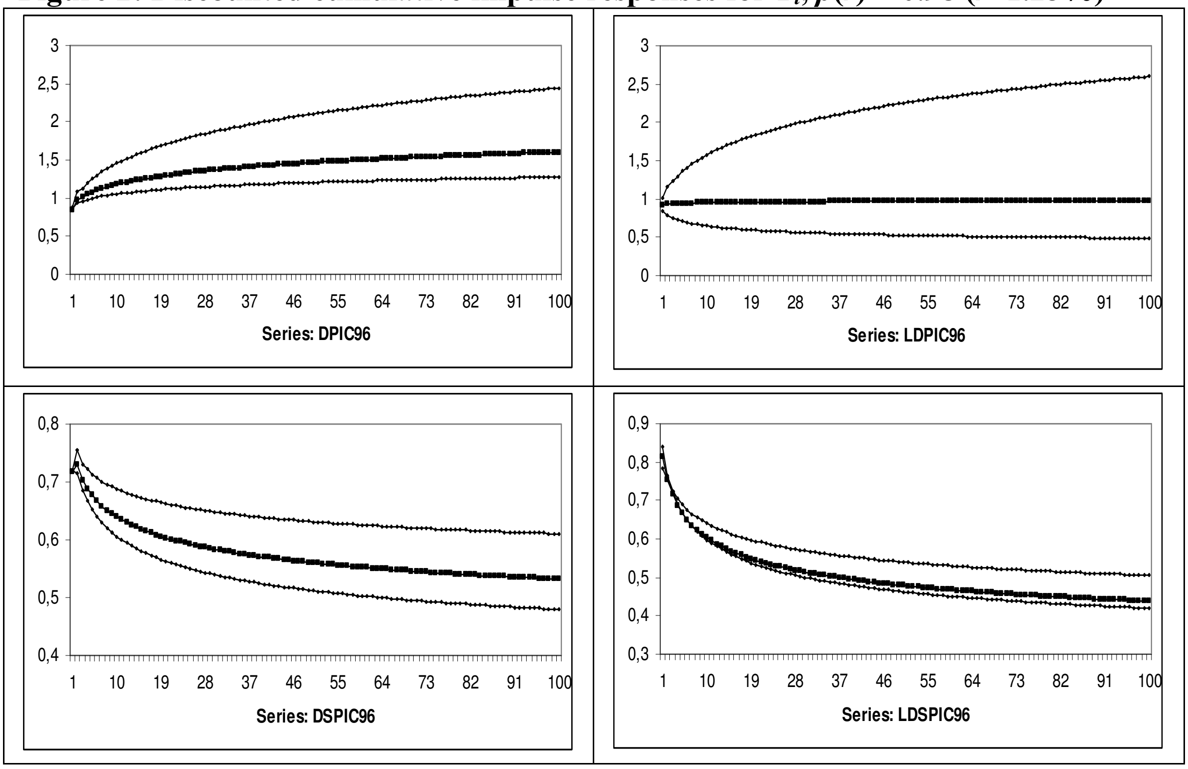
| i) First subsamples (ending in 1975) | | | |
|---|---|---|---|
| Series | Ratio | Values | $\tau^*$ ( and $\kappa^*$) |
| PCEC96 (47Q1 – 08Q2) | $\sigma[\Delta C_t^Q]$ / $\sigma[\varepsilon_t]$, | 18.05688 / 24.84831 | **0.72668** |
| LPCEC96 (47Q1 – 08Q2) | $\sigma\left[\frac{\Delta C_t^Q}{Y_{t-1}}\right]$ / $\sigma(\varepsilon_t)$. | 0.009419 / 0.01185 | **0.79485** |
| PCECC96 (59M1 - 08M7) | $\sigma[\Delta C_t^M]$ / $\sigma[\varepsilon_t]$, | 13.93689 / 20.12889 | **0.69238** |
| LPCECC96 (59M1 - 08M7) | $\sigma\left[\frac{\Delta C_t^M}{Y_{t-1}}\right]$ / $\sigma(\varepsilon_t)$. | 0.005594 / 0.00691 | **0.80906** |
| ii) Second subsamples (starting in 1975) | | | |
| Series | Ratio | Values | $\tau^*$ ( and $\kappa^*$) |
| PCEC96 (47Q1 – 08Q2) | $\sigma[\Delta C_t^Q]$ / $\sigma[\varepsilon_t]$, | 28.42585 / 50.78695 | **0.55970** |
| LPCEC96 (47Q1 – 08Q2) | $\sigma\left[\frac{\Delta C_t^Q}{Y_{t-1}}\right]$ / $\sigma(\varepsilon_t)$. | 0.00521 / 0.00845 | **0.61751** |
| PCECC96 (59M1 - 08M7) | $\sigma[\Delta C_t^M]$ / $\sigma[\varepsilon_t]$, | 25.67652 / 50.31933 | **0.51027** |
| LPCECC96 (59M1 - 08M7) | $\sigma\left[\frac{\Delta C_t^M}{Y_{t-1}}\right]$ / $\sigma(\varepsilon_t)$. | 0.00465 / 0.00763 | **0.60971** |

**Figure 1: Discounted cumulative impulse responses for $Y_t$ with $\beta(r) = 1$ ($r$=0%)**



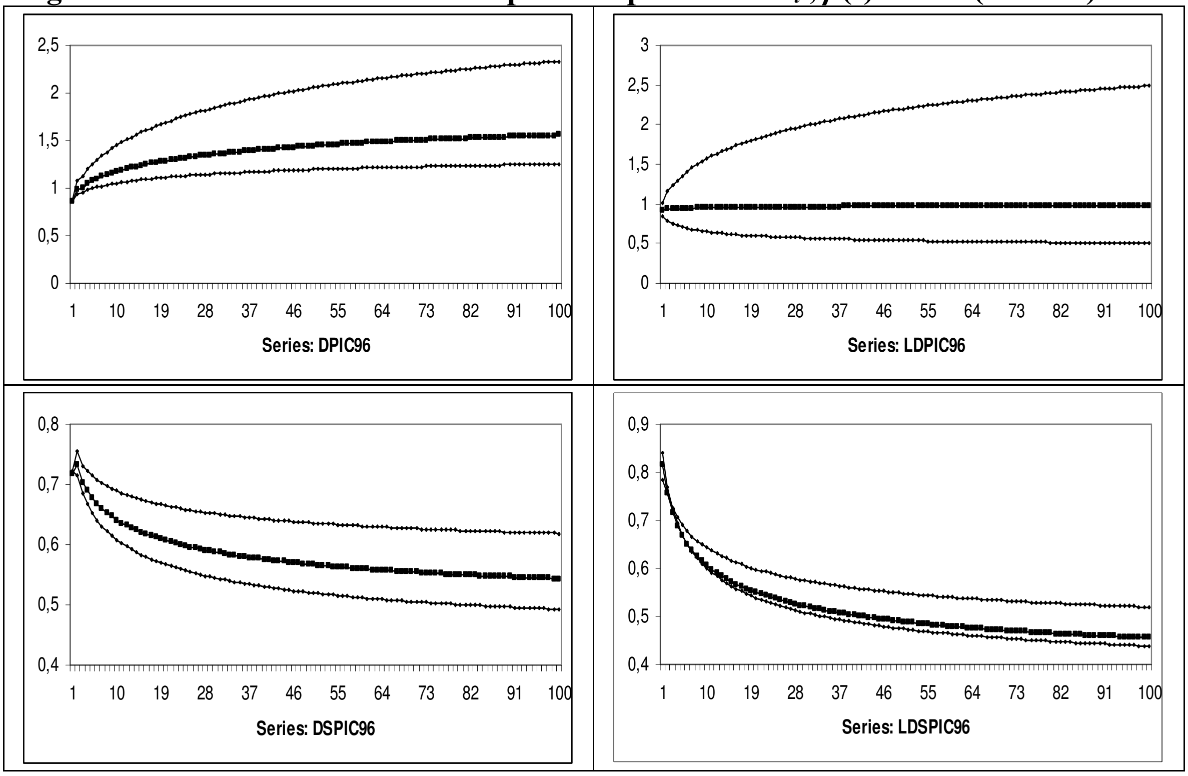

The thin lines refer to the 95% confidence band.

**Figure 2: Discounted cumulative impulse responses for $Y_t$, $\beta(r) = 0.98$ ($r$=1.25%)**



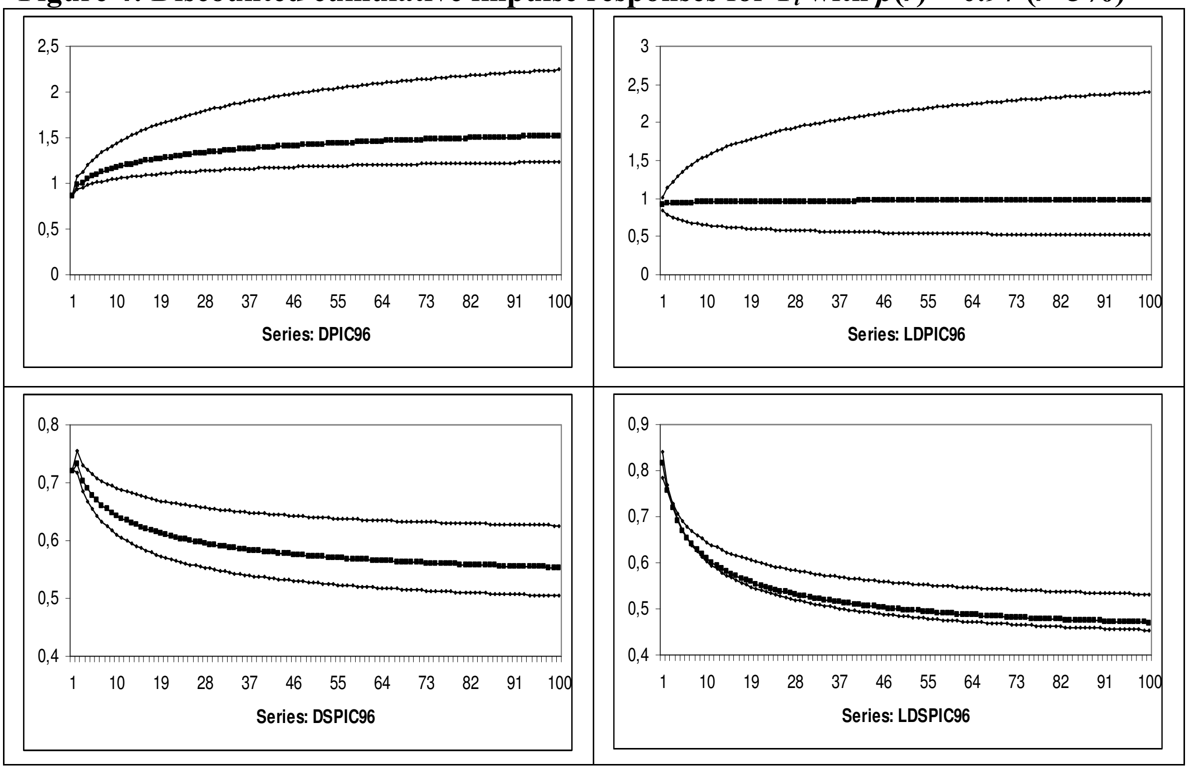

The thin lines refer to the 95% confidence band.

**Figure 3: Discounted cumulative impulse responses for $Y_t$ , $\beta(r)$ = 0.97 ($r$=2.5%)**




The thin lines refer to the 95% confidence band.

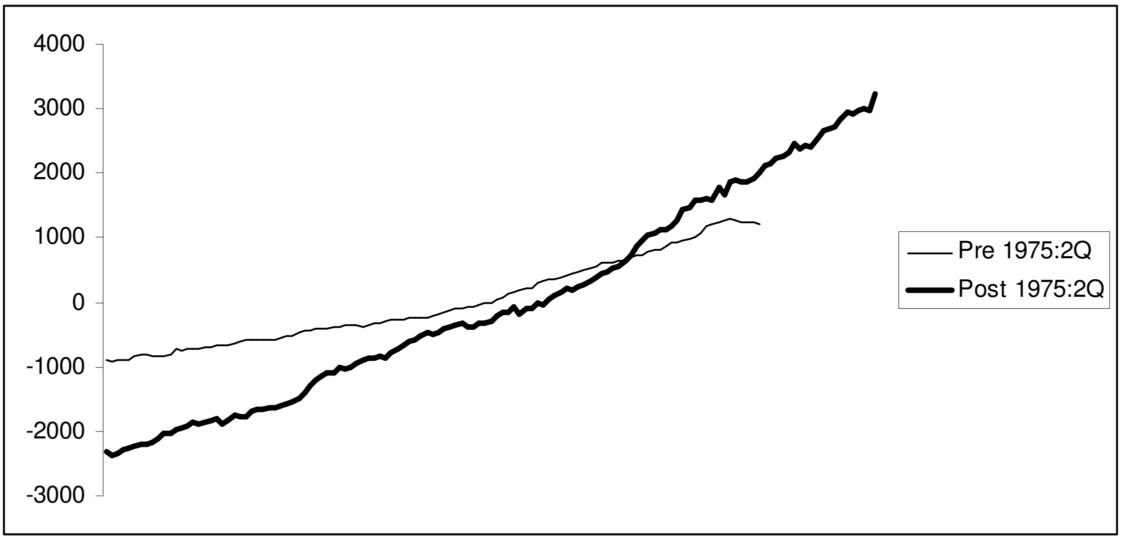
**Figure 4: Discounted cumulative impulse responses for $Y_t$ with $\beta(r)$ = 0.97 ($r$=5%)**



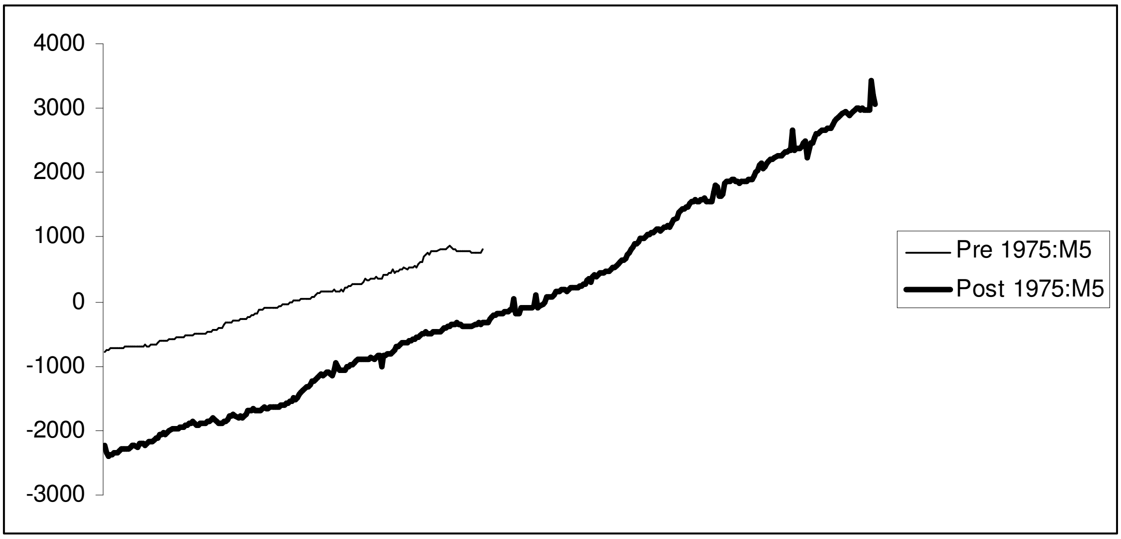

The thin lines refer to the 95% confidence band.

**Figure 5: Demeaned Real Disposable Income per Subsamples**

**Quarterly**

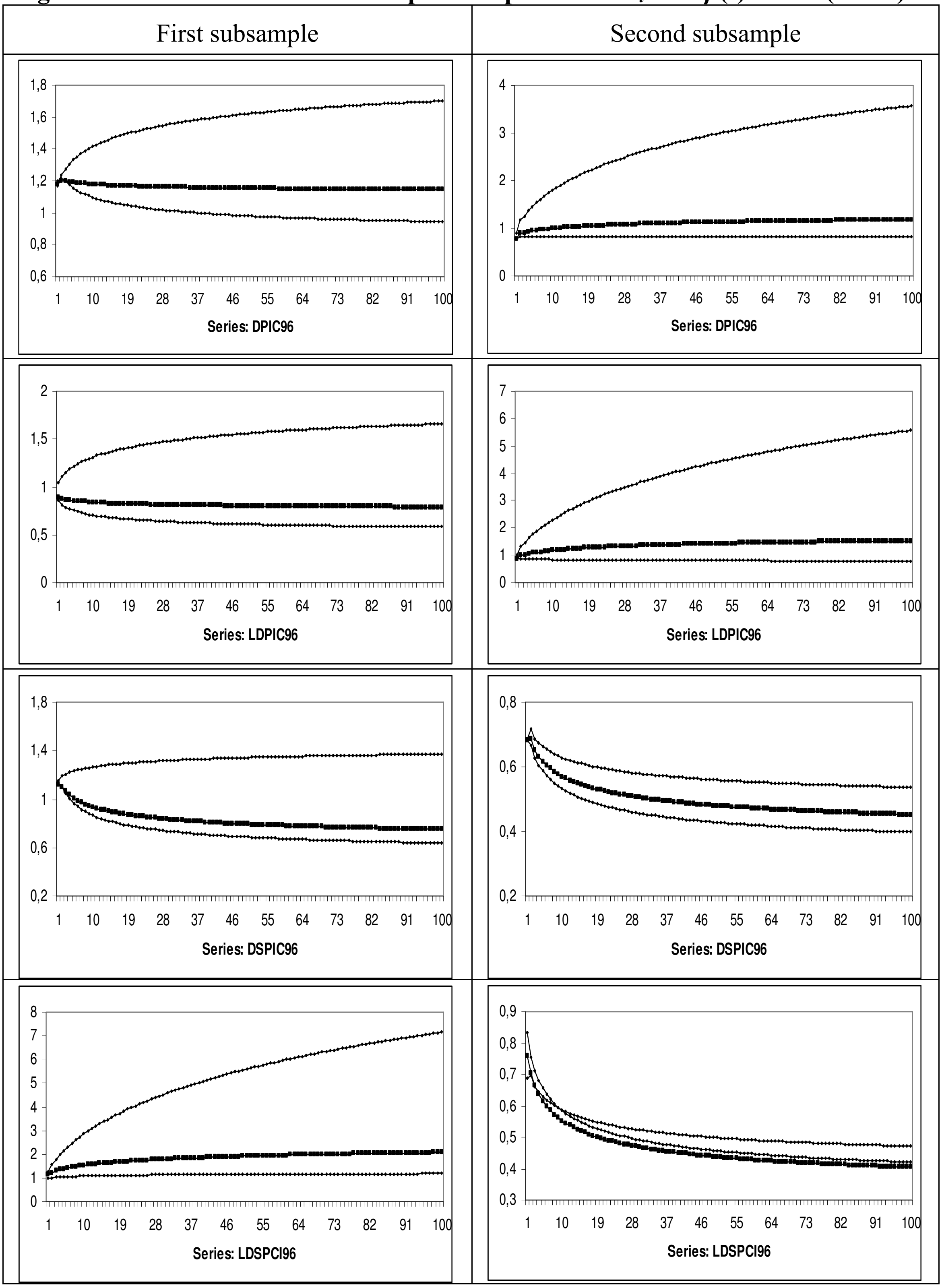



**Monthly**




The full sample periods are 1947:Q1-2008:Q2 and 1959:M1-2008:M7, respectively.

**Figure 6: Discounted cumulative impulse responses for $Y_t$ with $\beta(r)$ = 0.95 ($r$=5%)**




The thin lines refer to the 95% confidence band.