Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

Working Paper nº 19/05

Unit and Fractional Roots in the Presence of Abrupt Changes with an Application to the Brazilian Inflation Rate

Luis Alberiko Gil-Alana

Facultad de Ciencias Económicas y Empresariales
Universidad de Navarra

Unit and fractional roots in the presence of abrupt changes with an application to the Brazilian inflation rate
Luis A. Gil-Alana
Working Paper No.19/05
November 2005
JEL Codes: C15; C22.

ABSTRACT

In this article we analyse the monthly structure of the Brazilian inflation rate by means of using fractionally integrated techniques. This series is characterized by strong government interventions to bring inflation to a low level. We use a testing procedure due to Robinson (1994) which permits us to model the underlying dynamics of the series in terms of an I(d) statistical model, with the government interventions being specified in terms of dummy variables. The results show that the series can be well described in terms of an I(0.75) process with some of the interventions having little impact on the series.

Luis A. Gil-Alana
Universidad de Navarra
Departamento de Economía
31080 Pamplona
SPAIN
alana@unav.es

1. Introduction

Modelling the nonstationarity in macroeconomic data is a matter that still remains controversial. Deterministic models based on linear (or quadratic) functions of time were shown to be inappropriate in many cases and, stochastic models based on first (or second) differences of the data were proposed, especially after the seminal paper by Nelson and Plosser (1982). Nevertheless, unit roots and linear time trends, each constitute extremely specialized models for nonstationarity, but each has the advantage of conceptual and computational simplicity, and they are naturally thought of as rival models, because a unit root without or with a drift implies a constant or linear trend function, the distinction being then in the disturbance terms. Following that work, a battery of test statistics were developed for testing unit roots (e.g., Said and Dickey, 1984; Phillips and Perron, 1988; Kwiatkowski et. al., 1992; etc.). Robinson (1994) also proposed tests of unit roots but, unlike the previous ones, which are embedded in autoregressive (AR) alternatives, they are nested in a fractional model of form:

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, ...,$$

where u_t is I(0), (properly defined in Section 2), and where the unit root null corresponds to d = 1.

On the other hand, Perron (1989, 1993) found that the 1929 crash shock and the 1973 oil price shock were a cause of nonrejection of the unit-root hypothesis, and that when these were taken into account, deterministic models were preferable. This question has been pursued by several authors (e.g., Christiano, 1992, Zivot and Andrews, 1992, etc.) arguing that the date of the break should be treated as unknown. This problem is somewhat related to the analysis of Franses and Haldrup (1994) who showed how unit root tests have liberal size distortions when a series with a unit root is contaminated by additive outliers. In another recent paper, Vogelsang (1999) proposes several procedures for testing unit roots in the presence of outliers.

In this article we try to connect both of these issues, testing unit and fractional roots in the presence of abrupt changes in the data. Fractional integration in the context of structural breaks is a topic that has been scarcely investigated. Beran (1994) proposes a class of M-estimators for long memory models, which is robust to the presence of outliers. He suggests that occasional outliers can force the estimate of the fractional differencing parameter to be close to 0 although there might be strong long memory in the data. Diebold and Inoue (2001) provide both theoretical and Monte Carlo evidence that structural breaks-based models and long memory processes are easily confused. Similarly, Granger and Hyung (1999) also developed a theory relating both types of models and Gil-Alana (2001a) shows that the order of integration of some series may be reduced by the inclusion of dummy variables for structural breaks in the regression model.

The analysis in this article is directly motivated by the time series properties of the Brazilian inflation rate. This series is characterized by a period of hyperinflation, starting by the end of the 1980s, and followed by government interventions to bring inflation to a low level for a short period of time. We use the same dataset as in Cati et al. (1999), i.e., the Brazilian monthly inflation rate (1974:1 – 1993:6). In that article, they show that the presence of outliers leads to a bias in the standard unit root tests in favour of stationarity where in fact the series is integrated of order one. In this article, we show, however, that the series is nonstationary but mean-reverting, with an order of integration smaller than one. The outline of this paper is as follows: Section 2 briefly describes a version of the tests of Robinson (1994) for testing I(d) statistical models, which allows us to include dummy variables to incorporate the government interventions. Section 3 applies different versions of

Robinson's (1994) tests to the Brazilian inflation rate while Section 4 contains some concluding comments.

2. Testing of I(d) models with the tests of Robinson (1994)

For the purpose of the present article, we define an I(0) process $\{u_t, t = 0, \pm 1, \ldots\}$ as a covariance stationary process with spectral density function that is positive and finite at the zero frequency. In this context, we say that x_t is I(d) if:

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{1}$$

$$x_t = 0, \qquad t \leq 0, \tag{2}$$

where the polynomial in (1) can be expressed in terms of its Binomial expansion such that for all real d,

$$(1 - L)^d = \sum_{j=0}^{\infty} \frac{\Gamma(d+1)(-1)^j}{\Gamma(d-j+1)\Gamma(j+1)} = 1 - dL + \frac{d(d-1)}{2} L^2 - \ldots,$$

where $\Gamma(x)$ means the gamma function. Clearly, if d = 0 in (1), $x_t = u_t$, and a 'weakly autocorrelated' x_t is allowed for. However, if d > 0, x_t is said to be long memory, so-called because of the strong association between observations widely separated in time.

Robinson (1994) proposed a Lagrange Multiplier (LM) test of the null hypothesis:

$$H_o: \; d = d_o \tag{3}$$

for any real value d_o, in a model given by:

$$y_t = \beta' z_t + x_t, \qquad t = 1, 2, \ldots. \tag{4}$$

and (1), where y_t is the time series we observe; β is a (kx1) vector of unknown parameters; and z_t is a (kx1) vector of deterministic regressors that may include, for example, an intercept ($z_t \equiv 1$); a linear time trend ($z_t = (1, t)'$) or dummy variables). Specifically, the test statistic is given by:

$$\hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2}, \qquad (5)$$

where T is the sample size and

$$\hat{a} = \frac{-2\pi}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)\, g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T}\sum_{j=1}^{T-1} g(\lambda_j;\hat{\tau})^{-1}\, I(\lambda_j);$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\lambda_j)^2 - \sum_{j=1}^{T-1}\psi(\lambda_j)\hat{\varepsilon}(\lambda_j)' \times \left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\hat{\varepsilon}(\lambda_j)'\right)^{-1} \times \sum_{j=1}^{T-1}\hat{\varepsilon}(\lambda_j)\psi(\lambda_j)\right),$$

$$\psi(\lambda_j) = \log\left|2\sin\frac{\lambda_j}{2}\right|; \qquad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial\tau}\log g(\lambda_j;\hat{\tau}); \qquad \lambda_j = \frac{2\pi\, j}{T}.$$

$I(\lambda_j)$ is the periodogram of $\hat{u}_t$, where

$$\hat{u}_t = (1-L)^{d_o}\, y_t - \hat{\beta}'\, w_t; \qquad w_t = (1-L)^{d_o}\, z_t; \qquad \hat{\beta} = \left(\sum_{t=1}^{T} w_t\, w_t'\right)^{-1} \sum_{t=1}^{T} w_t\,(1-L)^{d_o}\, y_t.$$

The function g above is a known function coming from the spectral density function of u_t,

$$f(\lambda;\sigma^2;\tau) = \frac{\sigma^2}{2\pi} g(\lambda;\tau), \qquad -\pi < \lambda \le \pi,$$

evaluated at $\hat{\tau}$ = arg min $\sigma^2(\tau)$. Note that the functional form of the test statistic will be affected by the specification we adopt for the I(0) disturbances u_t in (1). Thus, for example, if u_t is white noise, the test statistic greatly simplifies since $g \equiv 1$ and $\hat{A}$ below (5) becomes:

$$\frac{2}{T}\sum_{j=1}^{T-1}\psi(\lambda_j)^2,$$

which can be asymptotically approximated by $\pi^2/6$. However, the I(0) disturbances can also be weakly autocorrelated. If u_t is an AR(p) process of form: $\phi_p(L)u_t = \varepsilon_t$, with white noise ε_t,

$$g(\lambda;\tau) = \left|\phi_p(e^{i\lambda})\right|^{-2},$$

so that the AR coefficients are function of τ, and $\varepsilon(\lambda)$ is now a (px1) vector with l^{th} element given by:

$$2\left\{\cos l\lambda \;-\; \sum_{r=1}^{p}\tau_r \cos(l-r)\lambda\right\} g(\lambda;\tau).$$

In this article, we also make use of other less conventional forms of I(0) processes. In particular, we also employ the Bloomfield (1973) exponential spectral model. This is a non-parametric approach of modelling the disturbances, where u_t is exclusively specified in terms of its spectral density function, which is given by:

$$f(\lambda;\sigma^2;\tau) \;=\; \frac{\sigma^2}{2\pi}\exp\left\{\sum_{r=1}^{m}\tau_r \cos\lambda r\right\}. \qquad (6)$$

Like the stationary AR(p) case, this model has exponentially decaying autocorrelations and thus, using this specification, we do not need to rely on so many parameters as in the ARMA processes, which always results tedious in terms of estimation, testing and model specification.

Based on H_o (3), Robinson (1994) established that under certain regularity conditions:

$$\hat{r} \;\;\rightarrow_d\;\; N(0,1) \qquad as \quad T \;\rightarrow\; \infty. \qquad (7)$$

The conditions on u_t in (7) are far more general than Gaussianity, with a moment condition only of order 2 required. Thus, an approximate one-sided 100α%-level test of (3) against alternatives H_1: $d > d_o$ is given by the rule: "Reject H_o (3) if $\hat{r} > z_\alpha$", where the probability that a standard normal variate exceeds z_α is α. Conversely, an approximate one-sided 100α%-level test of (3) against alternatives: H_1: $d < d_o$ is given by the rule: "Reject H_o (3) if $\hat{r} < -z_\alpha$". As these rules indicate, we are in a classical large sample testing situation for reasons described by Robinson (1994), who also showed that the above test is efficient in the Pitman sense, that against local alternatives H_1: $d = d_o + \delta T^{-1/2}$ for $\delta \neq 0$, it has an asymptotic normal distribution with variance 1 and mean which cannot, (when u_t is Gaussian), be exceeded in absolute value by that of any rival regular statistic. This version of the tests of Robinson (1994) was used in empirical applications in Gil-Alana and Robinson (1997) and Gil-Alana (2000) and, other versions of his tests, based on seasonal (quarterly and monthly) and cyclical models can be respectively found in Gil-Alana and Robinson (2001) and Gil-Alana (1999, 2001b). In the following section, the tests of Robinson (1994) are applied to the Brazilian inflation rate.

3. The Brazilian inflation rate

The series used in this paper is the monthly Brazilian inflation rate for the time period 1974:1 to 1993:6, and we use the same dataset as in Cati et al. (1999). Figures 1 and 2 show plots of the original series and its first differences respectively. Looking at the original data, we observe that the series is characterized in the 80's by several sudden drops that are important in magnitude. These are the outcome of the various shock plans instituted by the government in an attempt to stop the process of high and increasing inflation. They correspond to the time periods 86:3, 87:7, 89:2, 90:3 and 91:2. As in Cati et al. (1999), we stop the sample in 93:6 in order to avoid incorporating the Real Plan which is still in effect. The plot of the first differences clearly shows the importance of the outliers corresponding to these shock plans.

(Figures 1 and 2 about here)

Denoting the inflation series y_t, we initially employ throughout the model (1), (2) and (4) with $z_t = (1, t)'$, $t \geq 1$, $z_t = (0, 0)'$ otherwise, so

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots, \tag{8}$$

$$(1 - L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{9}$$

treating separately the cases $\beta_0 = \beta_1 = 0$ a priori, β_0 unknown and $\beta_1 = 0$ a priori and (β_0, β_1) unknown, i.e., we consider the cases of no regressors, an intercept, and an intercept and a linear time trend. We model the I(0) u_t to be both, white noise and to have weak parametric autocorrelation, in the latter case assuming AR(1), MA(1) and Bloomfield(1) disturbances.

The test statistic reported in Table 1 (and also in Tables 2 and 3) is the one-sided one given by (5), so that significantly positive values of this are consistent with alternatives with higher orders of integration, whereas significantly negative ones are consistent with smaller values of d. We apply the tests for the time periods: 74:1 – 86:3, 86:4 – 93:6, and for the whole sample 74:1 – 93:6. Starting with the first subsample, we observe that practically all the non-rejection values of d oscillate between 0.25 and 0.75. We also observe a non-rejection value if d = 1 for the case of no regressors and white noise disturbances. In general, the results seem quite robust to the different specifications of z_t, however, they substantially vary depending on how we specify the I(0) disturbances. Thus, if u_t is white noise, the non-rejected values are 0.75 and 1. If u_t is AR(1), we observe several other non-rejections. However, in this case, we also observe a lack of monotonic decrease in the value of the test statistic with respect to d, for small values of d. Such monotonicity is a characteristic of any reasonable statistic, given correct specification and adequate sample size, because, for example, we would which that if d = 1 is rejected against d > 1, an even more significant result in this direction should be expected when d = 0.75 or d = 0.50 are tested. However, in the event of misspecification, monotonicity is not necessarily to be expected: frequently misspecification inflates both numerator and denominator of $\hat{r}$, to varying degrees, and thus affects $\hat{r}$ in a complicated way. Modelling u_t in terms of a MA(1) process or with the Bloomfield(1) exponential model, monotonicity is always obtained and the non-rejection values of d are 0.50 (in case of MA disturbances) and 0.50 and 0.25 with the Bloomfield model.

(Table 1 about here)

The results for the second subsample are a bit more ambiguous, given the higher proportion of non-rejection values, which could be largely due to the smaller sample size. These values widely range between 0 and 1.25, again observing higher values of d if u_t is white noise rather than autocorrelated. We observe that the non-fractional cases (d = 0 and d = 1) cannot be rejected in some cases. Thus, for example, the unit root null cannot be rejected if u_t is white noise or AR(1). On the contrary, d = 0 results non-rejected if u_t is also AR(1) or if follows the Bloomfield (1) exponential model.

Finally, we also present the results for the whole sample period. They are less ambiguous than in the previous cases. Thus, if u_t is white noise, d = 1 appears as the only non-rejection value for the three specifications in z_t. This may contradicts the results in Cati et al. (1999), where they found strong evidence against unit roots using tests of Dickey and Fuller (1979), Phillips and Perron (1988) and Stock (1990). However, as mentioned in Section 1, these tests are based on AR models and do not consider fractionally integrated alternatives. We also observe in this table that imposing weakly autocorrelated disturbances, the unit root null hypothesis is rejected in favour of less nonstationary (or even stationary) alternatives, and the non-rejection values of d are now 0.25 and 0.50.

The results presented in Table 1 may be affected by the presence of outliers due to the government interventions during the 80's and early 90's. Thus, in Table 2, we recalculate the tests of Robinson (1994), but this time including in z_t, five dummy variables (D_{it}) to incorporate these outliers. Thus, instead of (9), we have:

$$y_t \;=\; \alpha \;+\; \beta t \;+\; \sum_{i=1}^{5} \gamma_i D_{it} \;+\; x_t, \qquad t \;=\; 1, 2, \ldots, \tag{10}$$

where D_{it} = 1 I(t = T_i), and T_i corresponding to the time periods of each of the government interventions. Other types of dummy (step and slope) variables were also considered but the coefficients were insignificant in practically all cases. Note that the estimates are least squares and are based on the differenced series so that they have short memory under the null. Furthermore, the inclusion of impulse dummies also permits us to incorporate lag effects throughout the autocorrelated disturbances. Again, we perform $\hat{r}$ given by (5), testing H_o (3) in model (9) and (10), for values d_o = 0, (0.25), 2, and white noise and autocorrelated disturbances. The results are given in Table 2 and the non-rejection values of d correspond now to d = 1 and 1.25 in case of white noise u_t; d = 0.50 and 0.75 for AR(1) and MA(1) disturbances; and d = 0.25 and 0.50 when using the Bloomfield exponential spectral model. Higher orders for the AR, MA and Bloomfield models were also considered and the results were very similar to those reported across the tables, implying that higher autocorrelated orders were unnecessary when describing the short run dynamics of the series. We also observe in this table that the significance of the coefficients in the regression model (10) substantially vary depending on the degree of integration. Thus, D_1 appears significant if d ≥ 0.75; D_2 if d ≥ 0.50; D_3 if d ≥ 0.25 and D_4 if d = 0, 1.75 and 2. D_5 always results insignificant across the different values of d. In view of this, we report in Table 3, the results for the same statistic as in Table 2, but taking into account only those regressors that were significant in Table 2.

(Tables 2 and 3 about here)

We see that the non-rejection values practically coincide with those given in Table 2. The only exception corresponds to the case of Bloomfield disturbances and d = 0.50. This model cannot be rejected in Table 2 but is rejected in Table 3 when including only the significant coefficients.

Table 4 summarises the selected models according to the results in Table 3. That is, we write the estimated models based on (9) and (10), in which the null hypothesis H_o (3) was not rejected and all the coefficients were significantly different from zero. We see that all except model 7 are nonstationary (d ≥ 0.5), and the significant dummies appear to be D_1,

D_2 and D_3 in models 1, 2, 3, and 4; the time trend and D_2 and D_3 in models 5 and 6; while the time trend and D_3 are the only significant regressors in model 7.

(Table 4 about here)

A more difficult task is to determine which is the correct model specification across he different models presented in that table. We display in the last column of Table 4 several diagnostic tests carried out on the residuals. In particular, we perform tests of no serial correlation, functional form, normality and homoscedasticity using Microfit. For serial correlation, we use a Lagrange Multiplier test of residuals serial correlation (Godfrey, 1978a,b); for the functional form, the Ramsey's (1969) RESET test using the square of the fitted values; for normality, a test based on skewness and kurtosis of residuas, (Bera and Jarque, 1981) while for homoscedasticity, a test based on the regresion of squared residuals on squared fitted values.

We observe that if d = 1.25 or 1 (Models 1 and 2), the models fail in relation to the homocedasticity property.(the p-values are respectively 0.007 and 0.002). Models 4, 5 and 6 (corresponding to d = 0.75 and 0.50) fail in relation to the functional form, (p-values: 0.0001, 0.0003 and 0.001) while Model 7 (d = 0.25 and Bloomfield disturbances) cannot be evaluated because of its non-parametric autocorrelated structure. Thus, we see that Model 3 is the only one which passes all the diagnostics on the residuals, and it corresponds to:

$$y_t \;=\; \gamma_1 D_{1t} \;+\; \gamma_2 D_{2t} \;+\; \gamma_3 D_{3t} \;+\; x_t, \qquad\qquad t \;=\; 1, 2, ..., \qquad (11)$$

$$(1 \;-\; L)^{0.75}\, x_t \;=\; u_t; \qquad u_t \;=\; \phi u_{t-1} \;+\; \varepsilon_t, \qquad\qquad t \;=\; 1, 2, ..., \qquad (12)$$

giving the estimates: γ_1 = -7.469; γ_2 = -9.040; γ_3 = -16.457 and ϕ = -0.472. (The LM tests give values in this model of 4.74; 5.21, 6.36 and 1.14 respectively for no serial correlation, functional form, normality and homoscedasticity, which are all adequate at the 10% significance level). Thus, the impact of the government interventions appears especially relevant for the data in the cases of the first three plans, (i.e., 86:3, 87:7 and 89:2), while the fourth and fifth interventions, even being large in magnitude (in particular the fourth) have little impact on the series.

(Figures 3 and 4 about here)

To evaluate the responses of inflation to the plan shocks, we need to derive the impulse response functions. We do this next. Let $(1 + 0.472L)(1 - L)^{0.75} = a(L)$, and calling $d_1(L) = -7.469a(L)$; $d_2(L) = -9.040a(L)$; $d_3(L) = -16.457a(L)$, the model in (11) and (12) becomes:

$$a(L)\, y_t \;=\; d_1(L)\, D_{1t} \;+\; d_2(L)\, D_{2t} \;+\; d_3(L)\, D_{3t} \;+\; \varepsilon_t, \qquad t \;=\; 1, 2, ...,$$

and using a power expansion of a(L), d_1(L), d_2(L) and d_3(L) in terms of its lags, with $D_{1j} = D_{2j} = D_{3j} = 0$ for $j \leq 0$, we obtain:

$$y_t \;=\; \sum_{j=1}^{T-1} a_j\, y_{t-j} \;+\; \sum_{j=1}^{t-1} d_{1j}\, D_{1t-j} \;+\; \sum_{j=1}^{t-1} d_{2j}\, D_{2t-j} \;+\; \sum_{j=1}^{t-1} d_{3j}\, D_{3t-j} \;+\; \varepsilon_t, \qquad t \;=\; 1,\; 2,\; ...., \qquad (13)$$

where a_j are the coefficients of the impulse response functions, and d_{1j}, d_{2j} and d_{3j} are the impacts of the shock plans on the inflation. Figure 3 summarizes these values for the impulse responses and Figure 4 for the impacts of the plans. We observe through the a_j's that the effect of a shock on inflation tends to die away in the long run, though it takes a very long period to disappear completely. In fact, we see that even 50 periods after the initial shock, 20% of its effect still remains on the series. The impact of the shocks is higher for the third plan (89:2) than for the others (86:3 and 87:7) and they also take a long time to disappear completely. Impulse responses, however, should be interpreted with great care because of the underlying assumptions. An impulse response assumes that the system is initially shocked once by some amount, but that the system is absent of any other perturbation from then on. This is a scenario that we rarely see in stochastic dynamic economies. We should rather see the economy in a system that is continuously perturbed with random fluctuations coming either from news, policy decisions or changing environments.

An argument that can be employed against this type of procedures is that Robinson's (1994) method is not robust in the presence of outliers. In that respect, other methods like Beran's (1994) M-estimator might be more appropriate. However, it should also be noted that the tests of Robinson (1994) have the advantage of permitting us to include dummy variables for the outliers, and given that the model is estimated under the null, which is supposed to be I(0), we can test the significance of the dummies via t-tests. Additionally, the significancy of such coefficients can be used to examine the impacts of government interventions, something that we are unable to do in case of using the method of Beran (1994) in spite of its robustness.

4. Conclusions

The monthly structure of the Brazilian inflation rate (74:1 – 93:6) has been investigated in this article by means of fractionally integrated techniques. This series is characterized by a period of hyperinflation, starting by the end of the 80's and followed by government interventions to bring inflation to a low level for a short period of time. We have made use of a testing procedure due to Robinson (1994) that allows us to consider I(d) statistical models and at the same time to incorporate dummy variables for the government interventions, with no effect on the standard limit distribution of the tests. These tests are also the most efficient ones when directed against the appropriate alternatives. When the dummy variables are not included in the regression model, the results of Robinson's (1994) tests indicate that the series may be I(1) if the underlying disturbances are white noise. However, if they are autocorrelated, the order of integration seems to be smaller, oscillating between 0.25 and 0.50. This contradicts the results in Cati et al. (1994), which found that the series is I(1) but, in that paper, they do not consider fractionally integrated alternatives. Incorporating the dummies, the results also substantially vary depending on how we specify the disturbances. Thus, if they are white noise, the orders of integration are 1 and 1.25. If u_t is AR(1) or MA(1), d appears to be 0.50 and 0.75, while using the Bloomfield (1973) exponential model, the series seems to be stationary with d = 0.25. In order to choose which might be the best model specification across the potential models, we look at several diagnostic tests carried out on the residuals of the selected models, the results indicating that the series may be well described in terms of an I(0.75) process with three dummy variables for the first three interventions (86:3, 87:7 and 89:2). Thus, the fourth and fifth interventions (even being large in magnitude) have little impact in the underlying structure of the series.

It would be worthwhile proceeding to get point estimates of d, perhaps especially in the Bloomfield case, where the results indicate that the series may be stationary as opposed to the nonstationary results obtained for the remaining cases. However, not only would this be computationally more expensive, but it is then in any case confidence intervals rather than point estimates which should be stressed, while available rules of inference require preliminary integer differencing to achieve stationarity and invertibility. In addition, most of these methods (e.g., Dahlhaus, 1989; Sowell, 1992) estimate the fractional differencing parameter including at most an intercept and/or a linear time trend but do not permit the inclusion of dummy variables for the outliers. The approach used in this article generates simply computed diagnostics for departures from any real d. It is thus not at all surprising that, when fractional hypotheses are entertained, some evidence supporting them appears, because this might happen even when the unit-root model is highly suitable. However, even though our practise of computing test statistics for a wide range of null hypotheses lead to ambiguous conclusions, often the bulk of these hypotheses are rejected, suggesting that the optimal local power properties of the tests may be supported by reasonable performance against non-local alternatives. Furthermore, the diagnostic tests carried out on the residuals of the selected models give further support for fractional models when describing the Brazilian inflation rate.

Several other lines of research are under way which should prove relevant to the analysis of these and other macroeconomic data. Extensions of the tests of Robinson (1994) to include structural breaks at unknown periods of time are being developed. Also, the estimation of fractional models in the context of Bloomfield (1973) exponential spectral disturbances is being examined. Work is also proceeding on multivariate extensions of Robinson's (1994) tests, and this would lead to an alternative approach to the study of cointegration. How these approaches may affect to the conclusions obtained in this article still remains to be investigated.

References

Bera, A.K. and C.M. Jarque, 1981, An efficient large-sample test for normality of observations and regression residuals, Australian National University, Working Papers in Econometrics 40, Camberra, Australia.

Beran, J., 1994, On the class of M-estimators for long memory Gaussian models, Biometrika 81, 755-766.

Bloomfield, P.J., 1973, An exponential model for the spectrum of a scalar time series, Biometrika 60, 217-226.

Cati, R.C., M.G.P. Garcia and P. Perron, 1999, Unit roots in the presence of abrupt governmental interventions with an application to Brazilian data, Journal of Applied Econometrics 14, 27-56.

Christiano, I.J., 1992, Searching for a break in GNP, Journal of Business and Economic Statistics 10, 237-250.

Dahlhaus, R., 1989, Efficient parameter estimation of self-similar processes, Annals of Statistics 17, 1749-1766.

Dickey, D.A. and W.A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427-431.

Diebold, F.X. and A. Inoue, 2001, Long memory and regime switching, Journal of Econometrics 105, 131-159.

Franses, P.H. and N. Haldrup,1994, The effect of additive outliers on tests for unit roots and cointegration, Journal of Business and Economic Statistics 12, 471-478.

Gil-Alana, L.A., 1999, Fractional integration with monthly data, Economic Modelling 16, 613-629.

Gil-Alana, L.A., 2000, Mean reversion in the real exchange rates, Economics Letters 69, 285-288.

Gil-Alana, L.A., 2001a, A fractionally integrated model with a mean shift for the US and the UK real oil prices, Economic Modelling 18, 643-658.

Gil-Alana, L.A., 2001b, Testing stochastic cycles in macroeconomic time series, Journal of Time Series Analysis 22, 411-430.

Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, Journal of Econometrics 80, 241-268.

Gil-Alana, L.A. and P.M. Robinson, 2001, Testing of seasonal fractional integration in the UK and Japanese consumption and income, Journal of Applied Econometrics 16, 95-114.

Godfrey, L.G., 1978a, Testing against general autoregressive and moving average error models when the regressors include lagged dependent variables, Econometrica 46, 1293-1301.

Godfrey, L.G., 1978b, Testing for higher order serial correlation in regressor equations when the regressors include lagged dependent variables, Econometrica 46, 1303-1310.

Granger, C.W.J. and Hyung, 1999, Occasional structural breaks and long memory, Discussion Paper 99-14, University of California, San Diego.

Kwiatkowski, D., P.C.B. Phillips, P. Schmidt and Y. Shin, 1992, Testing the null hypothesis of stationarity against the alternative of a unit root, Journal of Econometrics 54, 159-178.

Nelson, C.R. and C.I. Plosser, 1982, Trends and random walks in macroeconomic time series, Journal of Monetary Economics 10, 139-162.

Perron, P., 1989, The great crash, the oil price shock and the unit root hypothesis, Econometrica 57, 1361-1401.

Perron, P., 1993, Trends, unit roots and structural change in macroeconomic time seires, Unpublished manuscript, University of Montreal, Montreal.

Phillips, P.C.B. and P. Perron, 1988, Testing for a unit root in time series regression, Biometrika 75, 335-346.

Ramsey, J.B., 1969, Test for specification errors in classical linear least squares regression analysis, Journal of the Royal Statistical Society, Series B, 350-371.

Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420-1437.

Said, S.E. and D.A. Dickey, 1984, Testing for unit roots in autoregressive moving average models of unknown order, Biometrika 71, 599-607.

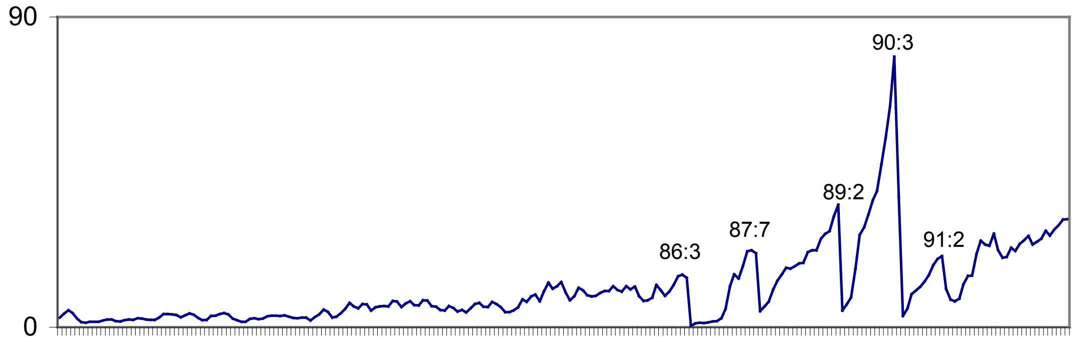
Sowell, F., 1992, Maximum likelihood estimation of stationary univariate fractionally integrated time series models, Journal of Econometrics 53, 165-188.

Stock, J.H., 1990, A class of tests for integration and cointegration, mimeo, Kennedy School of Government, Harvard University.

Vogelsang, T.J., 1999, Two simple procedures for testing a unit roots when there are additive outliers, Journal of Time Series Analysis 20, 237-252.

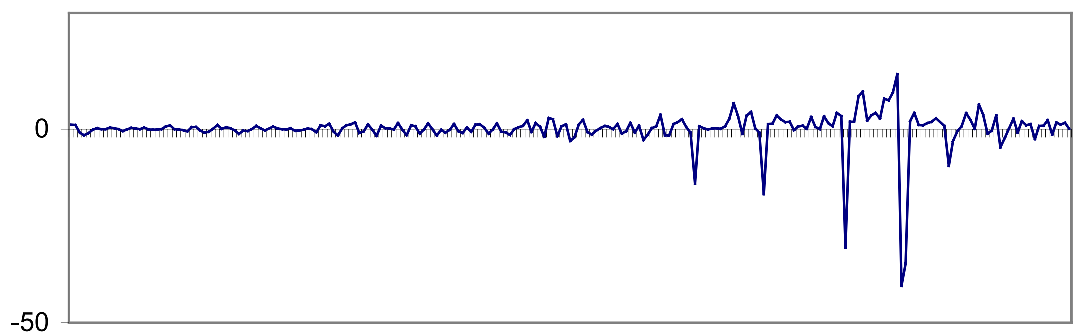
Zivot, E. and D.W.K. Andrews, 1992, Further evidence of the great crash, the oil price shock and the unit root hypothesis, Journal of Business and Economics Statistics 10, 251-270.

FIGURE 1



Brazilian inflation rate, 1974:1 to 1993:6

FIGURE 2



First differences of the Brazilian inflation rate

FIGURE 3

Impulse response function of the Brazilian inflation rate

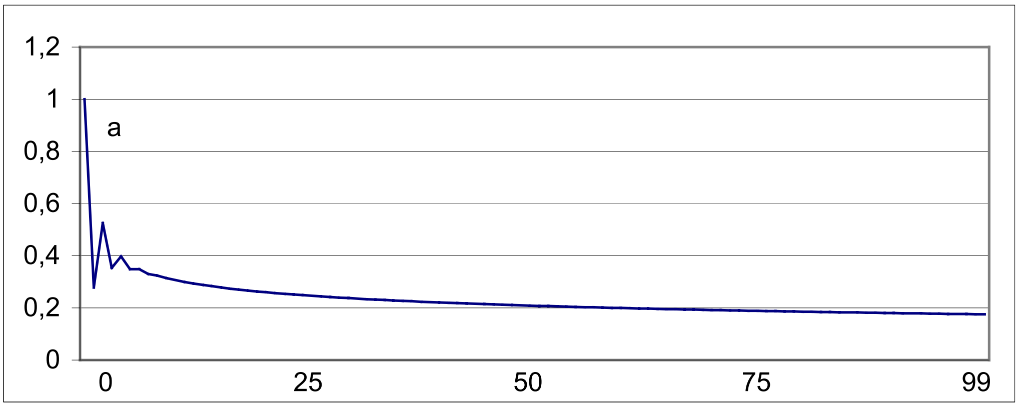


FIGURE 4

Impacts of government interventions on the Brazilian inflation rate

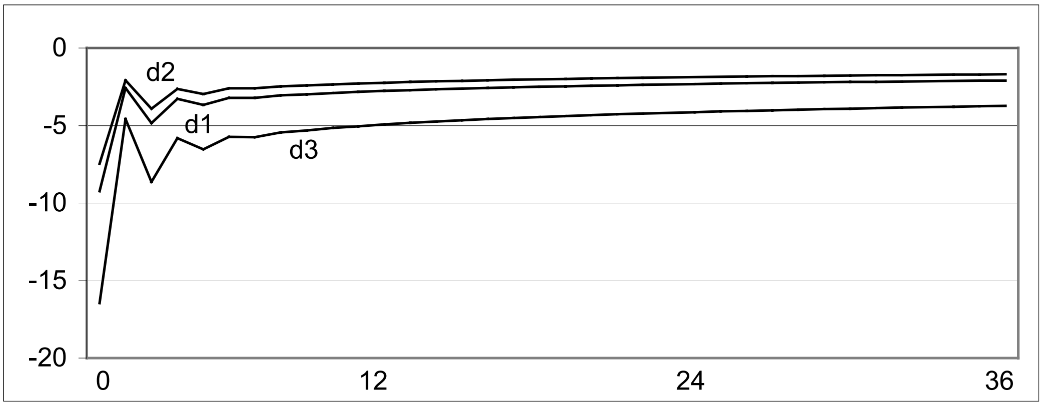


TABLE 1

Testing H_o (3) in (8) and (9) with $\hat{r}$ given by (5)

Sample	u_t	z_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
74.1 – 86.3	White noise	---	24.75	15.27	5.26	**0.78’**	**-1.49’**	-2.82	-3.62	-4.14	-4.51
		1	24.75	16.80	5.94	**0.42’**	-1.70	-2.86	-3.58	-4.08	-4.45
		(1, t)’	14.80	8.63	3.59	**0.23’**	-1.70	-2.85	-3.56	-4.03	-4.40
	AR (1)	---	-1.26’	-0.28’	**0.02’**	**-1.45’**	-2.51	-3.26	-3.73	-4.01	-4.20
		1	-1.26’	-1.21’	**-0.08’**	**-1.57’**	-2.61	-3.28	-3.71	-3.99	-4.18
		(1, t)’	-0.62’	**0.32’**	**-0.57’**	-1.70	-2.61	-3.26	-3.67	-3.89	-4.06
	MA (1)	---	18.95	7.52	**-0.49’**	-2.73	-3.51	-5.01	-6.39	-7.29	-7.93
		1	18.95	9.28	**-0.009’**	-3.12	-3.89	-5.07	-6.32	-7.18	-7.82
		(1, t)’	8.58	2.22	**-1.57’**	-3.25	-3.88	-5.05	-6.29	-7.09	-7.73
	Bloomfield (1)	---	10.22	3.65	**-0.54’**	-2.45	-3.35	-4.03	-4.42	-4.65	-4.79
		1	10.22	4.98	**-0.19’**	-2.77	-3.74	-4.24	-4.57	-4.74	-4.86
		(1, t)’	2.75	**0.34’**	**-1.61’**	-2.89	-3.73	-4.21	-4.50	-4.63	-4.70
Sample	u_t	z_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
86.4 – 93.6	White noise	---	9.55	5.90	3.60	1.67	**0.10’**	**-1.09’**	-2.00	-2.69	-3.21
		1	9.55	6.24	3.56	**1.64’**	**0.10’**	**-1.09’**	-2.00	-2.69	-3.21
		(1, t)’	9.53	6.44	3.83	1.72	**0.10’**	**-1.10’**	-2.00	-2.69	-3.21
	AR (1)	---	**0.90’**	**0.29’**	**-0.47’**	**-1.07’**	**-1.53’**	-1.90	-2.21	-2.47	-2.71
		1	**0.90’**	**-0.01’**	**-0.61’**	**-1.09’**	**-1.53’**	-1.90	-2.21	-2.47	-2.71
		(1, t)’	**1.37’**	**0.38’**	**-0.41’**	**-1.05’**	**-1.53’**	-1.90	-2.21	-2.47	-2.71
	MA (1)	---	5.88	2.36	**0.30’**	**-1.12’**	-2.19	-2.88	-3.71	-4.94	-5.88
		1	5.88	2.56	**0.29’**	**-1.14’**	-2.20	-2.88	-3.71	-4.94	-5.89
		(1, t)’	5.87	2.79	**0.47’**	**-1.09’**	-2.19	-2.89	-3.71	-4.94	-5.88
	Bloomfield (1)	---	**1.33’**	**0.63’**	**-1.54’**	-2.17	-2.65	-3.01	-3.30	-3.56	-3.74
		1	**1.33’**	**-0.33’**	**-1.53’**	-2.20	-2.65	-3.01	-3.30	-3.56	-3.74
		(1, t)’	**1.21’**	**-0.39’**	**-1.36’**	-2.13	-2.65	-3.02	-3.31	-3.56	-3.73
Sample	u_t	z_t / d_o	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
74.1 – 93.6	White noise	---	27.75	13.49	6.12	2.49	**0.04’**	-1.76	-3.12	-4.14	-4.92
		1	27.75	14.85	6.35	2.48	**0.03’**	-1.76	-3.12	-4.14	-4.91
		(1, t)’	16.13	10.15	5.20	2.44	**0.03’**	-1.76	-3.12	-4.14	-4.91
	AR (1)	---	1.66	**1.44’**	**-0.69’**	-2.01	-2.78	-3.34	-3.81	-4.22	-4.58
		1	1.66	**1.18’**	**-0.69’**	-2.03	-2.78	-3.34	-3.81	-4.22	-4.58
		(1, t)’	3.01	**0.96’**	**-0.86’**	-2.04	-2.78	-3.34	-3.81	-4.22	-4.58
	MA (1)	---	20.51	7.04	**0.96’**	-1.69	-3.23	-4.19	-5.41	-7.14	-8.45
		1	20.51	8.11	**1.13’**	-1.69	-3.24	-4.20	-5.41	-7.14	-8.45
		(1, t)’	10.54	4.56	**0.64’**	-1.72	-3.24	-4.19	-5.41	-7.14	-8.45
	Bloomfield (1)	---	9.97	1.75	**-1.64’**	-2.92	-3.68	-4.21	-4.63	-5.00	-5.25
		1	9.97	2.41	**-1.49’**	-2.93	-3.69	-4.22	-4.63	-5.00	-5.25
		(1, t)’	2.53	**-0.15’**	**-1.79’**	-2.96	-3.69	-4.21	-4.63	-4.99	-5.24

‘ and in bold : Non-rejection values of the null hypothesis H_o (3) in (9) and (10) at the 95% significance level.

TABLE 2					
Testing H_o (3) in (9) and (10) with $\hat{r}$ given by (5)					
d	Significant regressors	White noise	AR(1)	MA(1)	Bloomfield (1)
0.00	1; t; D_4	15.59	2.30	9.76	2.83
0.25	T; D_3	10.53	1.86	5.44	**-0.75'**
0.50	T; D_2; D_3	6.00	**-0.93'**	**1.19'**	**-1.44'**
0.75	D_1; D_2; D_3;	3.43	**-1.29'**	**-1.10'**	-3.92
1.00	D_1; D_2; D_3;	**1.19'**	-2.41	-2.92	-4.51
1.25	D_1; D_2; D_3;	**-0.51'**	-3.22	-4.25	-4.98
1.50	D_1; D_2; D_3;	-1.82	-3.84	-5.17	-5.29
1.75	D_1; D_2; D_3; D_4	-2.83	-4.33	-5.81	-5.56
2.00	D_1; D_2; D_3; D_4	-3.63	-4.73	-6.25	-5.76

' and in bold: Non-rejection values of the null hypothesis H_o (3) in (9) and (10) at the 95% significant level.

TABLE 3					
Testing H_o (3) in (9) and (10) with $\hat{r}$ given by (5) including only significant regressors					
d	Regressors	White noise	AR(1)	MA(1)	Bloomfield (1)
0.00	1; t; D_4	15.33	2.66	9.18	3.24
0.25	T; D_3	10.74	1.86	5.54	**-0.63'**
0.50	T; D_2; D_3	6.40	**0.07'**	**1.56'**	-2.65
0.75	D_1; D_2; D_3;	3.42	**-1.35'**	**-1.15'**	-3.72
1.00	D_1; D_2; D_3;	**1.07'**	-2.49	-3.03	-4.37
1.25	D_1; D_2; D_3;	**-0.67'**	-3.28	-4.27	-4.81
1.50	D_1; D_2; D_3;	-2.01	-3.87	-5.12	-5.16
1.75	D_1; D_2; D_3; D_4	-2.89	-4.34	-5.76	-5.52
2.00	D_1; D_2; D_3; D_4	-3.66	-4.67	-6.31	-5.65

' and in bold: Non-rejection values of the null hypothesis H_o (3) at the 95% significant level.

TABLE 4		
Selected models for the Brazilian inflation rate according to Table 3		
Model		Diagnostics
1	$y_t = \underset{(2.80)}{-7.346\,D_{1t}} - \underset{(2.80)}{8.887\,D_{2t}} - \underset{(2.80)}{16.222\,D_{3t}} + x_t$ $(1 - L)^{1.25}\, x_t = \varepsilon_t.$	A; B; C
2	$y_t = \underset{(3.20)}{-7.395\,D_{1t}} - \underset{(3.20)}{9.040\,D_{2t}} - \underset{(3.20)}{16.250\,D_{3t}} + x_t$ $(1 - L)\, x_t = \varepsilon_t.$	A; B; C
3	$y_t = \underset{(3.78)}{-7.469\,D_{1t}} - \underset{(3.78)}{9.244\,D_{2t}} - \underset{(3.78)}{16.457\,D_{3t}} + x_t$ $(1 - L)^{0.75}\, x_t = u_t; \quad u_t = \underset{(0.057)}{-0.472\, u_{t-1}} + \varepsilon_t.$	A; B; C; D
4	$y_t = \underset{(3.78)}{-7.469\,D_{1t}} - \underset{(3.78)}{9.244\,D_{2t}} - \underset{(3.78)}{16.457\,D_{3t}} + x_t$ $(1 - L)^{0.75}\, x_t = u_t; \quad u_t = \varepsilon_t + \underset{(17.277)}{0.701\, \varepsilon_{t-1}}.$	A; C; D
5	$y_t = \underset{(0.027)}{0.110\,t} - \underset{(4.62)}{9.533\,D_{2t}} - \underset{(4.62)}{16.900\,D_{3t}} + x_t$ $(1 - L)^{0.50}\, x_t = u_t; \quad u_t = \underset{(0.052)}{0.590\, u_{t-1}} + \varepsilon_t.$	A; C; D
6	$y_t = \underset{(0.027)}{0.110\,t} - \underset{(4.62)}{9.533\,D_{2t}} - \underset{(4.62)}{16.900\,D_{3t}} + x_t$ $(1 - L)^{0.50}\, x_t = u_t; \quad u_t = \varepsilon_t + \underset{(19.872)}{0.727\, \varepsilon_{t-1}}.$	A; C; D
7	$y_t = \underset{(0.009)}{0.104\,t} - \underset{(5.93)}{17.004\,D_{3t}} + x_t$ $(1 - L)^{0.25}\, x_t = u_t; \quad u_t \approx \textit{Bloomfield}\,(1).$	--- --- --- ----

*: Non-rejections at the 99% significant level of A): No serial correlation; B): Functional form; C): Normality, and D): Homocedasticity. Standard errors in parenthesis.