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Rent-seeking Contests under Symmetric and Asymmetric Information

*Cédric Wasser

* Humboldt University of Berlin

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Speaker: Prof. Dr. Urs Schweizer. · Department of Economics · University of Bonn · D-53113 Bonn,
Phone: +49(0228)739220 · Fax: +49(0228)739221

Rent-seeking Contests under Symmetric and Asymmetric Information

Cédric Wasser*

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Abstract

We consider a variant of the Tullock rent-seeking contest. Under symmetric information we determine equilibrium strategies and prove their uniqueness. Then, we assume contestants to be privately informed about their costs of effort. We prove existence of a pure-strategy equilibrium and provide a sufficient condition for uniqueness. Comparing different informational settings we find that if players are uncertain about the costs of all players, aggregate effort is lower than under both private and complete information. Yet, under additional assumptions, rent dissipation is still smaller in the latter settings. Numerical examples illustrate that there is no general ranking between private and complete information. The results depend on the distribution costs are drawn from and on the exact specification of the contest success function.

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*Humboldt University of Berlin, Institut für Wirtschaftstheorie I, Spandauer Str. 1, D–10178 Berlin, Germany; email: `cedric.wasser@wiwi.hu-berlin.de`. For valuable discussions and comments I am grateful to Philipp Denter, Jörg Franke, Thomas Giebe, Ulrich Kamecke, Yvan Lengwiler, Johannes Münster, Georg Nöldeke, Roland Strausz, Elmar Wolfstetter, as well as participants at the Young Researchers Workshop on Contests and Tournaments in Magdeburg and the 4th End-Of-Year Conference of Swiss Economists Abroad in Basel. Financial support from the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

1 Introduction

Many economic situations can be described as contests among players who invest costly effort to increase their probability of winning a prize. Examples are rent-seeking, lobbying, R&D races, election or advertising campaigns, litigation, and, of course, also military conflict as well as sports.[1] In all these situations, contestants might often be unsure about the abilities of their rivals for exerting effort or might not know their rivals' values for the prize. In addition, players might even be uncertain about their own ability or value. In this paper, we study how uncertainty and asymmetry of information affect the outcome of a contest compared to the complete information case.

Contests have been modeled in a variety of ways. A distinction may be made between perfectly and imperfectly discriminating contests, depending on whether the player who invested the highest effort wins with certainty or not. The (first-price) all-pay auction is a prominent example of the former and has been thoroughly studied both with symmetrically and asymmetrically informed contestants.[2] One of the most popular imperfectly discriminating contests is the rent-seeking contest by Tullock (1980). In the simplest version of that model, the winning probability of player i amounts to $x_i \big/ \sum_j x_j$ where x_i denotes i's effort. This is also known as the lottery contest.[3] A vast literature has developed extending Tullock's model in numerous directions. Yet, in contrast to the all-pay auction, there are only very few studies that depart from the basic assumption of players being all completely informed about every aspect of the game. Clearly, the case of asymmetric information in rent-seeking contests deserves greater attention.

Some progress has been made in the analysis of Tullock contests under asymmetric information for the case where there are only two players who both privately know their type (i.e., their valuation for the prize or their cost per unit of effort). Hurley and Shogren (1998a) numerically study the equilibrium of the lottery contest assuming types are drawn from two different discrete distributions. A more tractable distributional assumption allows Malueg and Yates (2004) to obtain a closed form solution for equilibrium efforts in the Tullock contest when there are only two possible types

[1] See Konrad (2009) for a recent survey on contest theory and its application to those examples.

[2] See, e.g., Baye, Kovenock, and de Vries (1996) for the symmetric and Krishna and Morgan (1997) for the asymmetric information case.

[3] The winning probability is equivalent to that in a lottery where each player i buys an amount x_i of lottery tickets and puts them into a box from which the winner is drawn.

for both players. A closed form for a different binary distribution is found by Münster (2009) who considers a repeated lottery contest. Without such rather specific distributional assumptions, however, equilibrium strategies can typically not be expressed in closed form. For a more general binary distribution, Katsenos (2009) explores a lottery contest that is preceded by a signaling stage. In a first step towards less restrictive distributional assumptions, Fey (2008) proves the existence of a symmetric pure-strategy equilibrium for a lottery contest with types drawn from a continuous uniform distribution.[4]

In this paper, we analyze a contest among $n \geq 2$ players where player i's winning probability is given by the contest success function $(x_i + \sigma) \Big/ \left(\sum_j x_j + n\sigma \right)$ with $\sigma \geq 0$. This is a variant of the lottery contest that has been proposed by Amegashie (2006). He argues that introducing the parameter σ allows for increasing the noise in the contest success function in a tractable way. Alternatively, σ can be thought of as a commonly known amount of effort that each player did already invest at an earlier stage (e.g., in order to enter the contest). Myerson and Wärneryd (2006) suggest a similar extension in order to remedy the problem of the Tullock contest success function being not strictly a member of the class axiomatized by Skaperdas (1996).[5]

We introduce uncertainty by assuming each player's constant marginal cost of effort to be drawn from a continuous probability distribution. Varying the amount of information contestants have regarding cost realizations, we obtain three different informational settings. On the one hand, we consider two flavors of symmetric information: either all players are completely informed about all marginal costs, or all players are unaware of the realization of all marginal costs (including their own). On the other hand, we focus on the case of asymmetric information where each player privately knows his marginal cost.

[4]There is also a small literature on one-sided asymmetric information, including Hurley and Shogren (1998b) who consider a lottery contest where one player's valuation for the prize is commonly known whereas the other player's is private information. In a similar setting, Denter and Sisak (2009) explore the uninformed player's incentives to acquire information. Assuming common values, Wärneryd (2003) studies a more general version of the Tullock contest under one-sided asymmetric information. This analysis is extended to multi-player contests in Wärneryd (2009).

[5]If $\sigma = 0$, winning probabilities are not defined if all players choose zero effort. It is usually assumed that in this case all players are equally likely to win the contest. The contest success function therefore exhibits a discontinuity: if no player invests any effort, player i can increase his probability of winning from $\frac{1}{n}$ to 1 by choosing an arbitrarily small but positive level of effort. An implication of this feature is that under complete information there are always at least two players that choose a strictly positive effort in equilibrium. Assuming $\sigma > 0$ removes the discontinuity and opens up the possibility of equilibria where only one player is active or where all players choose zero effort.

Analyzing the contest under symmetric information, we complement the discussion in Amegashie (2006) by determining equilibrium strategies in the general case and formally proving their uniqueness. For the uniqueness proof we adopt the approach of Cornes and Hartley (2005) and extend it to the case where $\sigma > 0$. Moreover, we find a way of formulating the equilibrium strategies that turns out to be very useful for comparing different informational settings to each other. Under asymmetric information we prove the existence of an equilibrium in monotone pure strategies, provided that $\sigma > 0$. In addition, we present a sufficient condition for the equilibrium to be unique. In contrast to Fey (2008) who develops his own existence proof for the uniform two-player case, we apply general results for Bayesian games derived by Athey (2001) as well as Mason and Valentinyi (2007).

Combining the equilibrium strategies determined under symmetric information with results characterizing equilibrium strategies under asymmetric information we find the following. If players are uncertain about the costs of all players, i.e., if they engage in a no information contest, ex ante expected aggregate effort is lower than under both private and complete information. Yet, under additional assumptions, rent dissipation is still smaller in the latter settings. In addition, our characterization of the private information equilibrium allows for a generalization of some of the numerical findings by Fey (2008) and Hurley and Shogren (1998a).

We complement the analytical results in this paper with additional insights obtained from approximating equilibrium efforts under asymmetric information numerically. A short discussion of the numerical methods we use can be found in Appendix B. In particular, our numerical examples illustrate the fact that there is no general ranking between the private and complete information contest in terms of expected efforts. The results depend on the distribution of costs, the number of players, and the parameter σ. In contrast, in the all-pay auction the two informational settings can be ranked clearly: Morath and Münster (2008) show that expected efforts are generally higher under private information than under complete information.

In the literature, in addition to differences in costs, also models where players differ with respect to their valuation for the prize are considered. In Section 6 we discuss to what extent our results also hold in the case where valuations instead of costs are randomly drawn. Whereas, using a simple transformation of variables, findings for a given information structure readily extend, this is in general not true for comparative results involving the no information contest.

Contrary to our results for the Tullock contest, for the all-pay auction with uncertain costs there is no general ranking in terms of expected efforts between no information and the other two settings. However, for the two-player all-pay auction with uncertainty regarding valuations Morath and Münster (2009) find expected efforts to be higher under no information than under private information. Hence, in the all-pay auction with value uncertainty a contest organizer who directly benefits from players' efforts would ex ante prefer no information over the other two informational settings. In contrast, in the Tullock contest with cost uncertainty we analyze in this paper the no information contest is the worst option for the contest organizer.

The paper is organized as follows. Section 2 describes the basic assumptions of the model. In Section 3 we analyze the contest under symmetric information. Section 4 is devoted to the asymmetric information case. In Section 5 we compare expected efforts and rent dissipation in the different informational settings. A variant of the model where values rather than costs are randomly drawn is considered in Section 6. Section 7 concludes. Some of the proofs are relegated to Appendix A, whereas Appendix B contains notes on the numerical methods we apply.

2 The Model

There are $n \geq 2$ risk neutral players who compete in a contest for a single prize of value 1. Each player i invests a level of effort $x_i \geq 0$. Efforts are chosen simultaneously. Depending on the efforts of all players, the probability of player i winning the prize is given by the contest success function

$$p_i(\mathbf{x}) := \begin{cases} \dfrac{x_i + \sigma}{\sum_{j=1}^{n} x_j + n\sigma} & \text{if } \sum_{j=1}^{n} x_j + n\sigma > 0, \\ \dfrac{1}{n} & \text{otherwise} \end{cases} \tag{1}$$

where $\mathbf{x} := (x_1, x_2, \ldots, x_n)$ and $\sigma \geq 0$. Providing effort is costly. There are no fixed costs and each player i has constant marginal cost $c_i > 0$. Player i's payoff from taking part in the contest is therefore

$$u_i(\mathbf{x}, c_i) := p_i(\mathbf{x}) - c_i x_i.$$

Note that, instead of interpreting $p_i(\mathbf{x})$ as the probability of winning, we could also think of it as the share of the prize player i obtains, assuming the prize is divisible.

Let us now introduce uncertainty into our model by assuming that, for each player i, the parameter c_i is the realization of a random variable C_i which is continuously distributed according to F_i with density f_i and support $[\underline{c}_i, \overline{c}_i]$ where $0 < \underline{c}_i < \overline{c}_i$. This is commonly known to all players.

Consider the following timing. There is a point in time, T_1, where each player i privately learns the realization of his cost c_i. At some later point in time, T_2, all players are informed about the realizations of all cost parameters $\mathbf{c} := (c_1, c_2, \ldots, c_n)$. The time after T_2, between T_1 and T_2, and before T_1 is usually referred to as *ex post*, *interim*, and *ex ante*. Depending on the time at which we assume the contest to take place, we have the following three different types of contests.

Suppose the contest takes place ex post. As all players are informed about $\mathbf{c}$, we have a game of complete information which we will refer to as the *complete information contest*. Given $\mathbf{c}$, a Nash equilibrium of this game specifies an equilibrium effort level $x_i^*(\mathbf{c})$ for each player i such that

$$x_i^*(\mathbf{c}) \in \arg\max_{x_i} u_i(x_i, \mathbf{x}_{-i}^*(\mathbf{c}), c_i) \quad \forall i, \tag{2}$$

where $\mathbf{x}_{-i}^*(\mathbf{c}) := \left(x_1^*(\mathbf{c}), \ldots, x_{i-1}^*(\mathbf{c}), x_{i+1}^*(\mathbf{c}), \ldots, x_n^*(\mathbf{c})\right)$.

If the contest takes place at the interim stage, each player is only informed about his own costs. Let $\xi_i(c_i)$ denote the level of effort that player i chooses if his privately known cost parameter is c_i. A pure-strategy Bayesian Nash equilibrium for this *private information contest* specifies an equilibrium strategy $\xi_i : [\underline{c}_i, \overline{c}_i] \to \mathbb{R}_+$ for each player i such that

$$\xi_i(c_i) \in \arg\max_{x_i} E[u_i(x_i, \boldsymbol{\xi}_{-i}(\mathbf{C}_{-i}), c_i)] \quad \forall i, c_i \in [\underline{c}_i, \overline{c}_i], \tag{3}$$

where $\boldsymbol{\xi}_{-i}(\mathbf{c}_{-i}) := (\xi_1(c_1), \ldots, \xi_{i-1}(c_{i-1}), \xi_{i+1}(c_{i+1}), \ldots \xi_n(c_n))$.

Finally, suppose the contest takes place ex ante. In this case, players have no information concerning cost parameters $\mathbf{c}$ other than the distribution functions they are drawn from. We will call this variant the *no information contest*. In a Nash equilibrium each player i invests X_i such that

$$X_i \in \arg\max_{x_i} E\left[u_i(x_i, \mathbf{X}_{-i}, C_i)\right] \quad \forall i,$$

where $\mathbf{X}_{-i} := (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n)$.

Note that $E[u_i(x_i, \mathbf{X}_{-i}, C_i)] = u_i(x_i, \mathbf{X}_{-i}, E[C_i])$ which implies $X_i = x_i^*(E[\mathbf{C}])$. Thus, the no information contest is equivalent to the complete information contest where each player i's costs are commonly known to amount to $E[C_i]$.

In both the complete information contest and the no information contest all contestants hold exactly the same information, i.e., the contest takes place under symmetric information. By contrast, players in a private information contest all hold different information regarding cost parameters; they play a game under asymmetric information. In the following, we will, in turn, take up the task of analyzing equilibria first for the symmetric and then for the asymmetric information case.

3 Symmetric Information

Consider the complete information contest: all players know the realization of $\mathbf{c}$ at the time of their effort decision. For the following it is useful to perform a change of variables by setting $y_i := x_i + \sigma$ for all i. Define $Y := \sum_{i=1}^{n} y_i$ and $Y_{-i} := Y - y_i$. A contest where player i chooses effort $x_i \in [0, \infty)$ obtaining utility $u_i(\mathbf{x}, c_i)$ is equivalent to a contest where player i chooses $y_i \in [\sigma, \infty)$ obtaining utility

$$v_i(y_i, Y_{-i}, c_i) := \begin{cases} \dfrac{y_i}{Y_{-i} + y_i} - c_i\,(y_i - \sigma) & \text{if } Y_{-i} + y_i > 0, \\ \dfrac{1}{n} - c_i\,(y_i - \sigma) & \text{otherwise.} \end{cases}$$

Observe that if $Y_{-i} > 0$, $v_i(y_i, Y_{-i}, c_i)$ is strictly concave in y_i. Hence, the following first order condition describes the global maximum of $v_i(y_i, Y_{-i}, c_i)$ with respect to y_i:

$$\frac{Y_{-i}}{\left(Y_{-i} + y_i\right)^2} - c_i \leq 0, \quad \text{with equality if } y_i > \sigma. \tag{4}$$

Accordingly, player i's best response to $Y_{-i} > 0$ is[6]

$$y_i(Y_{-i}) := \max\left\{\sqrt{\frac{Y_{-i}}{c_i}} - Y_{-i}, \sigma\right\}.$$

In a pure-strategy Nash equilibrium $y_1^*, \ldots, y_n^*$ we must have $y_i^* = y_i(Y_{-i}^*)$ for all i.

[6]As $y_i \in [\sigma, \infty)$ for all i, $Y_{-i} > 0$ is always true if $\sigma > 0$. If $\sigma = 0$, the best response to $Y_{-i} = 0$ does not exist: for any $\tilde{y}_i > 0$, player i can reduce his expenses and still win the contest with probability 1 by choosing a $y_i \in (0, \tilde{y}_i)$ instead. With $\sigma = 0$, there can therefore be no Nash equilibria where less than two players choose strictly positive efforts. Consequently, there results no loss in generality from assuming $Y_{-i} > 0$ in the following.

We will prove existence and uniqueness of a pure-strategy Nash equilibrium using the *share function* approach proposed by Cornes and Hartley (2005). Also employing the terminology of those authors, we define the replacement function $r_i(Y)$ as being contestant i's best response to $Y_{-i} = Y - r_i(Y)$. Making use of (4) we have

$$r_i(Y) = \max\left\{Y - c_i Y^2, \sigma\right\}.$$

From this we obtain player i's share function $s_i(Y) := \frac{r_i(Y)}{Y}$ as

$$s_i(Y) = \max\{\alpha_i(Y), \beta(Y)\}$$

where

$$\alpha_i(Y) := 1 - c_i Y \quad \text{and} \quad \beta(Y) := \frac{\sigma}{Y}.$$

Let Y^* denote a value of Y that corresponds to a pure-strategy Nash equilibrium. Such an equilibrium requires that the sum of all share functions equals unity. Hence, Y^* is defined as a solution to

$$S(Y^*) := \sum_{i=1}^{n} s_i(Y^*) = 1$$

where $S(Y)$ is referred to as the aggregate share function.

Proposition 1. *The complete information contest has a unique Nash equilibrium in pure strategies. Suppose $c_1 \leq c_2 \leq \cdots \leq c_n$ and define $Y(0) := n\sigma$ as well as*

$$Y(m) := \frac{(m-1) + \sqrt{(m-1)^2 + 4(n-m)\sigma \sum_{i=1}^{m} c_i}}{2\sum_{i=1}^{m} c_i} \quad \text{for } m \in \{1, 2, \ldots, n\}. \tag{5}$$

Moreover, let

$$m^* := \arg \max_{m \in \{0,1,\ldots,n\}} Y(m).$$

In the unique pure-strategy equilibrium, each player i chooses effort

$$x_i^*(\mathbf{c}) = \begin{cases} Y(m^*)(1 - c_i Y(m^*)) - \sigma & \text{for } i \leq m^*, \\ 0 & \text{for } i > m^*. \end{cases} \tag{6}$$

Proof. First, suppose $\sigma = 0$. As $Y \to 0$, $s_i(Y) \to 1$ such that $S(Y) > 1$ for sufficiently small Y. For sufficiently large Y, we have $S(Y) = 0$. Furthermore, $S(Y)$ is continuous for all $Y > 0$ and strictly decreasing if $S(Y) > 0$. Hence, there is a unique $Y^* \in (0,\infty)$ that solves $S(Y^*) = 1$. Now, consider the case $\sigma > 0$. Since all individual y_i are restricted to the interval $[\sigma,\infty)$, we have $Y \in [n\sigma,\infty)$. For all $Y \in [n\sigma,\infty)$, $s_i(Y)$ and therefore also $S(Y)$ are continuous and strictly decreasing. Moreover, $S(n\sigma) \geq n\beta(n\sigma) = 1$ while $S(Y) = n\beta(Y) < 1$ for sufficiently large values of Y. Consequently, there exists a unique $Y^* \in [n\sigma,\infty)$ that satisfies $S(Y^*) = 1$. This establishes the first part of the proposition.

For each individual share function there is a $\hat{Y}_i \in [n\sigma,\infty)$ such that $s_i(Y) = \alpha_i(Y)$ for $Y < \hat{Y}_i$ and $s_i(Y) = \beta(Y)$ for $Y \geq \hat{Y}_i$. Recall that contestant i's effort is $x_i = s_i(Y)Y - \sigma$. Accordingly, player i chooses a strictly positive effort if $Y < \hat{Y}_i$. In this case, we refer to i as an active player. For $Y \geq \hat{Y}_i$, player i chooses zero effort. Assuming $c_1 \leq c_2 \leq \cdots \leq c_n$ implies $\hat{Y}_1 \geq \hat{Y}_2 \geq \cdots \geq \hat{Y}_n$. Hence, if $Y \in \left(\hat{Y}_m, \hat{Y}_{m+1}\right)$ for some $m \in \{1,2,\ldots,n-1\}$, then all $i \leq m$ are active while all $i > m$ choose zero effort. Similarly, if $Y < \hat{Y}_n$, all players are active, and if $Y \geq \hat{Y}_1$, all players choose zero effort.

Suppose the unique Nash equilibrium of the game is such that exactly m^* players are active with $m^* \in \{0,1,\ldots,n\}$. The corresponding Y^* solves

$$S(Y^*) = \sum_{i=1}^{m^*} \alpha_i(Y^*) + (n - m^*)\beta(Y^*) = 1.$$

The solution to this equation is $Y^* = Y(m^*)$ where the function $Y(\cdot)$ is defined in (5). Now, consider an $\tilde{m} \neq m^*$. $Y(\tilde{m})$ is the solution to

$$\sum_{i=1}^{\tilde{m}} \alpha_i(Y(\tilde{m})) + (n - \tilde{m})\beta(Y(\tilde{m})) = 1.$$

Because $\tilde{m}$ does not correspond to an equilibrium, we have either $\alpha_i(Y(\tilde{m})) < \beta(Y(\tilde{m}))$ for some $i \leq \tilde{m}$, or $\alpha_i(Y(\tilde{m})) > \beta(Y(\tilde{m}))$ for some $i > \tilde{m}$. This implies $S(Y(\tilde{m})) > 1$ and, since $S(Y)$ is strictly decreasing, $Y(\tilde{m}) < Y(m^*)$. Consequently, $m^* = \arg\max_m Y(m)$. Given m^*, we obtain (6) from $x_i^*(\mathbf{c}) = s_i(Y(m^*))Y(m^*) - \sigma$. □

Amegashie (2006) derives equilibrium efforts for the case where all players are active, i.e., choose strictly positive efforts. In addition, he provides a condition under which all players are inactive (exert zero effort) and mentions the possibility of equilibria where only some of the players are active. The number of active players, of course, depends on the realization of costs $\mathbf{c}$. Proposition 1 shows that the number of active players is uniquely determined as the number m that maximizes the function

$Y(m)$. In equilibrium, the m^* players with the lowest costs are active, exerting efforts according to (6).

In the standard lottery contest (where $\sigma = 0$), we have $Y(0) = Y(1) = 0$ whereas $Y(m) > 0$ for $m > 1$, implying that there are always at least two players active in equilibrium. In this case, the equilibrium described in Proposition 1 exactly corresponds to the known equilibrium of the standard lottery contest with asymmetric contestants.[7]

For the case that all contestants have the same marginal cost c, it can be shown that $Y(0) < (>)\, Y(n)$ implies $Y(m) < (>)\, Y(n)$ for all $m < n$. Hence, in a symmetric contest either $m^* = n$ or $m^* = 0$.

Corollary 1. *Suppose $c_i = c$ for all i. Then, for each i, $x_i^*(\mathbf{c}) = x^*(c)$ with*

$$x^*(c) := \max\left\{\frac{n-1}{n^2 c} - \sigma, 0\right\}.$$

As we have noted before, the no information contest is equivalent to the complete information contest where marginal costs of each player i amount to $E[C_i]$. Therefore, Proposition 1 applies to the no information contest as well.

Corollary 2. *The no information contest has a unique Nash equilibrium in pure strategies. Suppose $E[C_1] \leq E[C_2] \leq \cdots \leq E[C_n]$. Then, the equilibrium effort of player i is $X_i = x_i^*(E[\mathbf{C}])$ with $x_i^*(.)$ satisfying* (6).

Having derived the equilibrium for both the complete information contest as well as the no information contest, let us now turn to the private information contest.

4 Asymmetric Information

Suppose contestants engage in the private information contest, simultaneously deciding on their effort at the interim stage. If each player $j \neq i$ employs a strategy $\xi_j(c_j)$, the expected payoff for player i who has privately known cost c_i and exerts effort x_i is

$$E[u_i(x_i, \xi_{-i}(\mathbf{C}_{-i}), c_i)] = E[p_i(x_i, \xi_{-i}(\mathbf{C}_{-i}))] - c_i x_i.$$

Note that $E[u_i(x_i, \xi_{-i}(\mathbf{C}_{-i}), c_i)] \leq 1 - c_i x_i$ and that by choosing $x_i = 0$ player i can guarantee himself a nonnegative payoff. Therefore, effort levels $x_i > \frac{1}{c_i}$ are clearly

[7] See, for example, Corchón (2007).

dominated for type c_i of player i. Accordingly, we can restrict each player i's effort choice to the interval $[0, \frac{1}{\underline{c}_i}]$. In the following, we will apply general results from the literature on Bayesian games to our contest in order to study existence and uniqueness of a Bayesian Nash equilibrium.

Proposition 2. *Suppose $\sigma > 0$. For the private information contest, there exists a Bayesian Nash equilibrium in nonincreasing pure strategies. If $\sigma > \frac{n-1}{n^2} \max_{i,c} f_i(c)$, the Bayesian Nash equilibrium of the private information contest is unique.*

Proof. In order to apply results by Athey (2001) as well as Mason and Valentinyi (2007) to the private information contest, it is useful to perform the change of variables $t_i := -c_i$. With that and assuming $\sigma > 0$, contestant i's payoff is equivalent to

$$w_i(\mathbf{x}, t_i) := \frac{x_i + \sigma}{\sum_{j \neq i} x_j + x_i + n\sigma} + t_i x_i$$

where $x_i \in [0, \frac{1}{\underline{c}_i}]$ and $t_i \in [-\overline{c}_i, -\underline{c}_i]$. Each t_i is drawn from the distribution $\tilde{F}_i(t_i) := 1 - F_i(-t_i)$. Given a strategy $\gamma_j : [-\overline{c}_j, -\underline{c}_j] \to [0, \frac{1}{\underline{c}_j}]$ for each $j \neq i$, player i's interim expected payoff amounts to

$$W_i(x_i, t_i) := \int \dots \int_{\mathcal{T}_i} \frac{x_i + \sigma}{\sum_{j \neq i} \gamma_j(t_j) + x_i + n\sigma} \prod_{j \neq i} d\tilde{F}_j(t_j) + t_i x_i$$

where $\mathcal{T}_i := [-\overline{c}_1, -\underline{c}_1] \times \dots \times [-\overline{c}_{i-1}, -\underline{c}_{i-1}] \times [-\overline{c}_{i+1}, -\underline{c}_{i+1}] \times \dots \times [-\overline{c}_n, -\underline{c}_n]$.

From $\frac{\partial^2 W_i(x_i, t_i)}{\partial x_i \partial t_i} = 1$ for all i follows that the *Single crossing condition for games of incomplete information* in Athey (2001) is satisfied. Note that our model is consistent with Athey's assumption A1. Moreover, each player i's actions are restricted to the interval $[0, \frac{1}{\underline{c}_i}]$ and $w_i(\mathbf{x}, t_i)$ is continuous in $\mathbf{x}$ for all i as long as $\sigma > 0$. Hence, existence of an equilibrium in nondecreasing strategies $\gamma_i(t_i)$ follows from Corollary 2.1 in Athey (2001). Of course, this corresponds to nonincreasing strategies in the original game where types are described by c_i.

For the uniqueness result we apply findings by Mason and Valentinyi (2007). First, we will show that their assumptions U1-U3 and D1-D2 hold in our model. For $\sigma > 0$ and $x_i \in [0, \frac{1}{\underline{c}_i}]$, $w_i(\mathbf{x}, t_i)$ is differentiable everywhere. Therefore, we can check U1 and U2 by looking at derivatives of $w_i(\mathbf{x}, t_i)$ as follows.[8] From $\frac{\partial^2 w_i(\mathbf{x}, t_i)}{\partial x_i \partial t_i} = 1$ for all i follows that U1 is satisfied with $\delta = 1$. U2 asks for $\left|\frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i}\right|$ to be bounded for all i. Note that

[8]This can be shown by appealing to the mean value theorem.

the fraction on the RHS of

$$\frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} = \frac{\sum_{j\neq i} x_j + (n-1)\sigma}{\left(\sum_{j\neq i} x_j + x_i + n\sigma\right)^2} + t_i$$

is clearly positive while it is maximized at $x_i = 0$ and $\sum_{j\neq i} x_j = 0$. Hence,

$$t_i < \frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} \leq \frac{n-1}{n^2\sigma} + t_i \tag{7}$$

such that U2 is satisfied with $\omega = \max_i \omega_i$ where $\omega_i := \max\left\{\overline{c}_i, \frac{n-1}{n^2\sigma} - \underline{c}_i\right\}$. For U3 we have to find a $\kappa \in (0, \infty)$ such that, for all $x_i \geq x_i'$, $\mathbf{x}_{-i}$, $\mathbf{x}'_{-i}$, t_i, i,

$$\left|\left(w_i(x_i, \mathbf{x}_{-i}, t_i) - w_i(x_i', \mathbf{x}_{-i}, t_i)\right) - \left(w_i(x_i, \mathbf{x}'_{-i}, t_i) - w_i(x_i', \mathbf{x}'_{-i}, t_i)\right)\right| \leq \kappa \left(x_i - x_i'\right).$$

A sufficient condition for this is that

$$\max_{\mathbf{x}} \frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} - \min_{\mathbf{x}} \frac{\partial w_i(\mathbf{x}, t_i)}{\partial x_i} \leq \kappa \qquad \forall i.$$

Using (7) we find that U3 holds with $\kappa = \frac{n-1}{n^2\sigma}$. As we have assumed that types are independently distributed, D1 is satisfied with $\iota = 0$ and D2 holds with $\nu = \max_{i,c} f_i(c)$. Finally, Theorem 4 in Mason and Valentinyi (2007) states that if $\delta > \iota\omega + \nu\kappa$, i.e., if $1 > \frac{n-1}{n^2\sigma} \max_{i,c} f_i(c)$, there is a unique equilibrium which is in nondecreasing pure strategies. □

For proving the existence result in Proposition 2 we have applied Athey (2001). Assuming each player's action space to be finite, Athey shows, using a fixed point theorem, that a pure-strategy equilibrium exists if a specific single crossing condition is satisfied. Moreover, under the condition that a player's payoff is everywhere continuous in the actions of all players, she proves that there is a sequence of such equilibria for finite-action games that converges to an equilibrium for the game where players choose from a continuum of actions. For the private information contest the single crossing condition generally holds, whereas continuity of payoffs is ensured by $\sigma > 0$.

Mason and Valentinyi (2007) show that if the effect of a player's own type on the expected payoff difference between two of his actions dominates the effect of his opponents' actions, the best response correspondence is a contraction which implies the existence of a unique equilibrium. In the private information contest, increasing the noise in determining the winner reduces the effect of his opponents' efforts on a player's payoff, leaving the effect of that player's costs unchanged. Hence, a sufficient condition for the equilibrium of the private information contest to be unique is that σ is large enough.

According to Proposition 2, a pure-strategy Bayesian Nash equilibrium exists for any $\sigma > 0$. Hence, the existence result also holds when, by choosing an arbitrarily small value for σ, looking at a contest that is arbitrarily similar to the standard lottery contest. Moreover, Fey (2008) proves the existence of such an equilibrium in a standard lottery contest between two players assuming costs are drawn from the same uniform distribution. When we derive properties of equilibrium strategies below, we will also include the case $\sigma = 0$ in our discussion.

As we have stated before, in a pure-strategy Bayesian Nash equilibrium of the private information contest with $\sigma \geq 0$, equilibrium strategies $\xi_1, \ldots, \xi_n$ solve

$$\xi_i(c_i) = \arg\max_{x_i \geq 0} E[p_i(x_i, \boldsymbol{\xi}_{-i}(\mathbf{C}_{-i}))] - c_i x_i \qquad \forall i \text{ and } c_i \in [\underline{c}_i, \overline{c}_i]. \tag{8}$$

Note that if $\sigma = 0$, we have a special case because of the discontinuity in $p_i(\mathbf{x})$. Yet the following result will allow us to simplify the exposition. Assuming $\sigma = 0$, suppose for every player i there is a proper interval $D_i \subseteq [\underline{c}_i, \overline{c}_i]$ such that $\xi_i(c_i) = 0$ for all $c_i \in D_i$. In this case, with some strictly positive probability, all of player i's competitors choose zero effort. Player i could therefore deviate from the equilibrium and increase, for this event, his contest success from $p_i = \frac{1}{n}$ to $p_i = 1$ by choosing an arbitrarily small but strictly positive effort for all $c_i \in D_i$. In a pure-strategy equilibrium for $\sigma = 0$ we must therefore have $\xi_i(c_i) > 0$ for all $c_i \in [\underline{c}_i, \overline{c}_i]$ for at least one player i. Returning to the general case where $\sigma \geq 0$, we can hence rewrite (8) as

$$\xi_i(c_i) = \arg\max_{x_i \geq 0} U_i(x_i, c_i) \qquad \forall i \text{ and } c_i \in [\underline{c}_i, \overline{c}_i]$$

where[9]

$$U_i(x_i, c_i) := E\left[\frac{x_i + \sigma}{\sum_{j \neq i} \xi_j(C_j) + x_i + n\sigma}\right] - c_i x_i.$$

[9]As we have shown, for $\sigma = 0$ there must be at least one player k that exerts strictly positive effort for all types. Expected payoffs of all $i \neq k$ are hence given by $U_i(x_i, c_i)$. In the case that with strictly positive probability all $i \neq k$ choose zero effort, using $U_k(0, c_k)$ for player k's expected payoff when choosing $x_k = 0$ is not correct. Yet, as we have argued above, k would not maximize his expected payoff by choosing $x_k = 0$.

Since $U_i(x_i, c_i)$ is strictly concave in x_i, the first order condition $\frac{\partial U_i(x_i,c_i)}{\partial x_i} \leq 0$, with equality if $x_i > 0$, defines the best response x_i for type c_i of player i. As in equilibrium player i chooses $x_i = \xi_i(c_i)$, we obtain, for each i, the equilibrium condition

$$E\left[\frac{\sum_{j\neq i}\xi_j(C_j)+(n-1)\sigma}{\left(\sum_{j\neq i}\xi_j(C_j)+\xi_i(c_i)+n\sigma\right)^2}\right] \leq c_i, \quad \text{with equality for } c_i \text{ where } \xi_i(c_i) > 0. \tag{9}$$

In general, there is no closed form solution to this system of equations. We can, however, still infer some properties of equilibrium efforts from condition (9), as we will do in the following lemma.

Lemma 1. *In the private information contest, player i's equilibrium strategy $\xi_i(c)$ has the following properties. There exists a $\tilde{c}_i \in [\underline{c}_i, \overline{c}_i]$ such that $\xi_i(c) = 0$ for $c > \tilde{c}_i$ while $\xi_i(c)$ is positive and strictly decreasing for $c < \tilde{c}_i$. If $\sigma > 0$, $\tilde{c}_i \leq \max\left\{\frac{n-1}{n^2\sigma}, \underline{c}_i\right\}$. If $\sigma = 0$, $\tilde{c}_i = \overline{c}_i$ for at least one $i \in \{1, 2, \ldots n\}$. Moreover,*

$$\xi_i(c) \leq \frac{1}{4c} - \sigma \quad \text{for } c < \tilde{c}_i.$$

The sum of ex ante expected equilibrium efforts satisfies

$$\sum_{i=1}^{n} E[\xi_i(C_i)] \geq \frac{n-1}{\sum_{i=1}^{n} E[C_i]} - n\sigma.$$

Proof. See Appendix A.1. □

One way to simplify the model is to assume that all costs are drawn from the same distribution, i.e., $F_i = F$ for all i. We exclusively focus in this case on a symmetric equilibrium where all players choose their effort according to the same equilibrium strategy $\xi(c)$.[10] For such a symmetric equilibrium condition (9) simplifies to a single

[10]In Appendix B of Kadan (2002) a variant of Theorem 1 by Athey (2001) is proved, stating that if types are all drawn from the same distribution, a symmetric pure-strategy equilibrium exists for finite-action games. As Theorem 2 by Athey (2001) continues to hold, the existence of a symmetric equilibrium for games with a continuum of actions follows. In turn, our Proposition 2 could be modified so as to yield existence of a symmetric equilibrium. Moreover, note that if the equilibrium is unique, it has to be symmetric. Fey (2008) proves the existence of a symmetric equilibrium for the standard two-player lottery contest where costs are drawn from the same uniform distribution.

equation:

$$E\left[\frac{\sum_{i=1}^{n-1}\xi(C_i)+(n-1)\sigma}{\left(\sum_{i=1}^{n-1}\xi(C_i)+\xi(c)+n\sigma\right)^2}\right]\leq c,\quad \text{with equality for } c \text{ where } \xi(c)>0.^{11} \tag{10}$$

Note that if $\sigma = 0$, $\xi(c) > 0$ for all c. This follows from the same argument we used above to show that $\xi_i(c_i) > 0$ for all c_i for at least one i.

Studying a numerical approximation to the symmetric equilibrium strategy $\xi(c)$ for the case where $n = 2$, $\sigma = 0$, and costs are drawn from the uniform distribution on $[0.01, 1.01]$, Fey (2008) finds that, for each c, $\xi(c)$ is smaller than the equilibrium effort in the complete information contest where both players are commonly known to have cost c. From Lemma 1 with $n = 2$, $\xi(c) \leq \max\left\{\frac{1}{4c}-\sigma, 0\right\}$ where, according to Corollary 1, the RHS is exactly the equilibrium effort of the symmetric complete information contest. Hence, we have shown that Fey's finding generally holds for any distribution F and also extends to contests with $\sigma > 0$.[12]

Although the equilibrium condition is simplified when assuming $F_i = F$ for all i and focusing on a symmetric equilibrium, there is in general no closed form solution for $\xi(c)$. Given a specific assumption concerning F, however, numerical methods can be applied to (10) so as to compute an approximation to the symmetric equilibrium strategy $\xi(c)$. Appendix B contains some notes on the methods we employed to find the numerical results presented in this paper.

For the case where costs are uniformly distributed on $[0.5, 1.5]$, numerical approximations to the equilibrium strategy are shown in Figure 1. The solid line represents $\xi(c)$ when $n = 2$ and $\sigma = 0$, the dotted and dashed lines display the effect of increasing n and σ, respectively. Increasing the number of players in general reduces the influence a single player's effort has on winning probabilities, as for any symmetric strategy the sum of efforts of player i's opponents increases. Yet the sum of opponents' efforts is not simply scaled upwards, its distribution changes as well: as n increases, the variance of average efforts decreases. When more players take part in the contest, the symmetric equilibrium strategy generally requires players with high costs to reduce their efforts. As n is increased from 2 to 3, players with very low costs

[11] $\sum_{i=1}^{n-1}\xi(C_i)$ is chosen arbitrarily and could be replaced by any sum over $n-1$ distinct $i \in \{1, 2, \ldots, n\}$.

[12] However, as our own numerical results suggest, $\xi(c)$ does not have this property if $n > 2$.

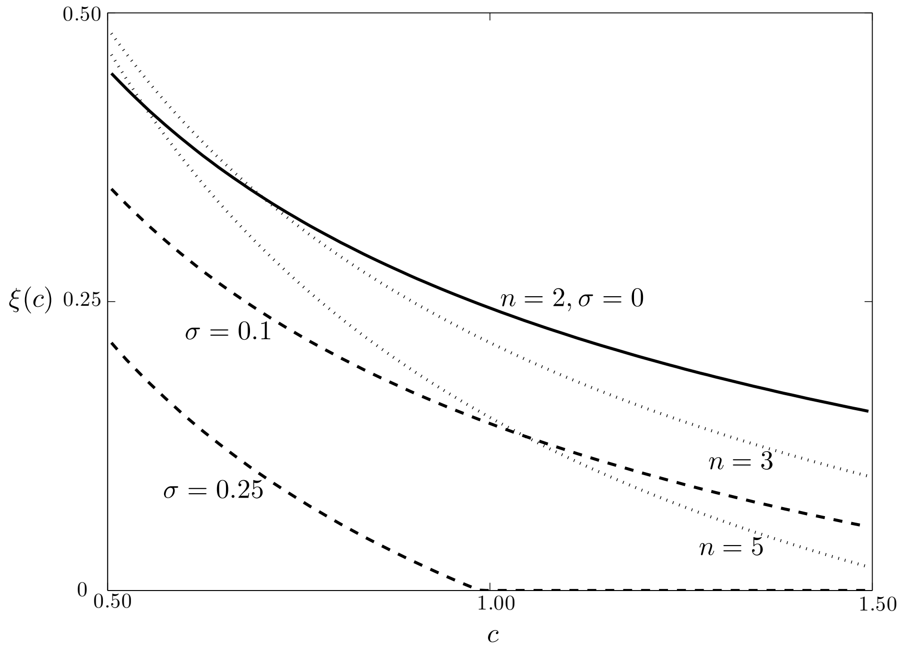

Figure 1: Symmetric equilibrium strategy $\xi(c)$ for uniformly distributed costs.

exert more effort. If n increases further, all types reduce their efforts, yet players with low costs do so by less than players with high costs. A higher σ increases the noise in determining the winner, making players' efforts less effective in changing winning probabilities. Consequently, equilibrium efforts for all types decrease in response to an increase in σ.

5 Expected Efforts and Rent Dissipation

Having analyzed equilibrium efforts for all three contests, we are now ready to study the impact of uncertainty and asymmetry of information on the behavior of contestants. Comparing the ex ante expected sum of efforts in the complete information contest to the sum of efforts in the no information contest we find that the former is at least as high as the latter.

Proposition 3. *Under symmetric information, uncertainty concerning* $\mathbf{c}$ *reduces the ex ante expected sum of efforts:*

$$\sum_{i=1}^{n} X_i \leq \sum_{i=1}^{n} E[x_i^*(\mathbf{C})].$$

Proof. See Appendix A.2. □

According to Proposition 3, if players are generally uncertain about the costs of all players, they exert less effort than when all cost parameters are commonly known. As we show below, under the assumption that expected costs are the same for all contestants, effort under uncertainty regarding all costs is also lower than when each player is privately informed about his own cost parameter.

Proposition 4. *Suppose* $E[C_1] = E[C_2] = \cdots = E[C_n]$*. Then, the ex ante expected sum of efforts is lower in the no information contest than in the private information contest, i.e.,*

$$\sum_{i=1}^{n} X_i \leq \sum_{i=1}^{n} E[\xi_i(C_i)]. \tag{11}$$

Proof. By Corollaries 1 and 2, if $E[C_1] = E[C_2] = \cdots = E[C_n]$, the sum of efforts in the no information contest is

$$\sum_{i=1}^{n} X_i = \max\left\{\frac{n-1}{\sum_{i=1}^{n} E[C_i]} - n\sigma, 0\right\}.$$

From Lemma 1 immediately follows (11). □

If all F_i have the same mean, the no information contest yields the smallest sum of efforts. An organizer of a contest who is interested in maximizing ex ante expected efforts thus prefers to let the contest take place at the interim or ex post stage.

Comparing expected efforts in the complete information contest and the private information contest to each other is more difficult. Malueg and Yates (2004) consider the standard Tullock contest between two players. A player's type is either high or low, each with (unconditional) probability $\frac{1}{2}$. Malueg and Yates (2004) find a player's interim expected effort in the complete information contest to exactly match his effort in the private information contest. However, this result is not robust. Analyzing a corresponding variant of our model where costs are independently drawn from $\{c_L, c_H\}$

		B					
		0.5	1	2	4	8	
	0.5	-0.88	0.90	1.51	0.97	0.36	
	1	-1.27	-0.61	0.27	0.58	0.36	
A	2	-0.52	-0.54	-0.30	0.05	0.19	$\times 10^{-3}$
	4	-0.12	-0.17	-0.18	-0.11	0.01	
	8	-0.02	-0.03	-0.05	-0.05	-0.03	

Table 1: $E[x_i^*(\mathbf{C})] - E[\xi_i(C_i)]$ if $n = 2$, $\sigma = 0$, and F is the beta distribution on $[0.5, 1.5]$ with parameters A and B.

with $c_H > c_L$ and q denoting the probability for c_L, we find that interim expected efforts are higher in the complete information contest than in the private information contest if $q \in (\frac{1}{2}, 1)$ whereas the opposite is true if $q \in (0, \frac{1}{2})$.

In order to numerically study equilibrium efforts for continuous distributions of types, we now introduce a specific distributional assumption. Let all costs be drawn from the same (generalized) beta distribution on $[0.5, 1.5]$ with parameters $A, B > 0$. The corresponding probability density function amounts to

$$f(c) = \frac{(c - 0.5)^{A-1}(1.5 - c)^{B-1}}{\int_0^1 z^{A-1}(1-z)^{B-1}\, dz} \qquad \text{if } c \in [0.5, 1.5]$$

and $f(c) = 0$ otherwise. Changing the free parameters A, B of the beta distribution allows for obtaining a variety of differently shaped densities. Assuming $n = 2$ and $\sigma = 0$, Table 1 reports the difference between the ex ante expected effort in the complete information contest and that in the private information contest for different combinations of A and B. Note that $A = B = 1$ corresponds to the uniform distribution. Moving from the binary uniform distribution considered by Malueg and Yates (2004) to a continuous uniform distribution, we hence observe that the complete information contest leads to lower ex ante expected efforts than the private information contest. In fact, according to Table 1 private information efforts are higher for all parameter choices where $A \geq B$. If B is sufficiently larger than A, however, complete information efforts exceed private information efforts. For $A = B$ the density f is symmetric about the mean; if $A > (<) B$, f is negatively (positively) skewed. Hence, our numerical results are well in accord with the intuition provided by the binary

		$n=2$	$n=3$	$n=5$	$n=7$	$n=10$
	complete info.	0.2616	0.2297	0.1706	0.1368	0.1068
$\sigma=0$	private info.	0.2622	0.2383	0.1786	0.1419	0.1100
	no info.	0.2500	0.2222	0.1600	0.1224	0.0900
	complete info.	0.1616	0.1352	0.0876	0.0584	0.0311
$\sigma=0.1$	private info.	0.1622	0.1383	0.0870	0.0565	0.0291
	no info.	0.1500	0.1222	0.0600	0.0224	0
	complete info.	0.0440	0.0373	0.0095	0	0
$\sigma=0.25$	private info.	0.0440	0.0369	0.0092	0	0
	no info.	0	0	0	0	0

Table 2: Ex ante expected effort by player i for uniform F.

case. If the distribution places more weight on high costs, the private information contest yields higher expected efforts than the complete information contest.

Under the assumption that all costs are uniformly distributed on $[0.5, 1.5]$, Table 2 presents a player's ex ante expected effort in each type of contest for various combinations of n and σ. For $\sigma = 0$ expected effort generally is highest for the private information contest whereas, as predicted by Propositions 3 and 4, expected effort in the no information contest is lowest. Note that while a single player's effort in all three contests is decreasing in n, the sum of expected efforts is increasing. For $\sigma > 0$ we observe that the complete rather than the private information contest induces the highest expected efforts if the number of players becomes sufficiently large.

A measure that is often studied in models of rent-seeking contests is the ratio between total expenses and the value of the prize. This ratio determines to what extent the rent the winner obtains is dissipated through contestants' investment of resources. In our model, players compete for a rent of value 1. Therefore, rent dissipation is simply defined as $R := \sum_{i=1}^{n} c_i x_i$. In the remainder of this section we try to shed some light on how uncertainty and asymmetry of information affect rent dissipation. For the case where all F_i have the same mean, σ is small enough, and all types choose strictly positive effort in equilibrium, we find that when general uncertainty prevails the prize is dissipated to a larger extent than when players are privately informed.

Proposition 5. *Suppose* $E[C_1] = E[C_2] = \cdots = E[C_n] < \frac{n-1}{n^2\sigma}$ *and* $\xi_i(c_i) > 0 \forall i, c_i \in [\underline{c}_i, \overline{c}_i]$. *Then, ex ante expected rent dissipation is weakly larger in the no information contest than in the private information contest.*

Proof. See Appendix A.3. □

In general, if we assume the costs of all players to be drawn from the same distribution F, all players expect ex ante to win the prize of value 1 with the same probability, regardless of the type of contest they engage in. Consequently, they prefer the contest with the lowest expected rent dissipation. Proposition 5 implies that in this case, if σ is small enough, from an ex ante perspective contestants prefer the private information contest over the no information contest.

If all costs are drawn from the same distribution F, Corollaries 1 and 2 imply for ex ante expected rent dissipation in the no information contest

$$E[R] = \max\left\{\frac{n-1}{n} - \sigma n E[C], 0\right\}.$$

If $\sigma > 0$, there is no dissipation at all when $n \to \infty$. Of course, the reason for this is that for large n the additional noise in determining the winner is increased to such an extent that the outcome is entirely independent of players' efforts. In contrast, if $\sigma = 0$, we obtain the classic result that the rent is fully dissipated when the number of players is very large. For the standard lottery contest under complete information Cornes and Hartley (2005) show that if m players are active in equilibrium, rent dissipation is bounded above by $\frac{m-1}{m}$ which is strictly smaller than $\frac{n-1}{n}$ for all $m < n$. Hence, if $\sigma = 0$, ex ante expected rent dissipation is larger in the no information contest than in the complete information contest, implying contestants ex ante prefer the latter.

Again assuming all costs to be uniformly distributed on $[0.5, 1.5]$, Table 3 reports ex ante expected rent dissipation in all three contests for different values for n and σ. If $\sigma = 0$, the no information contest induces the largest dissipation, followed by the private information contest and the complete information contest. For $\sigma = 0.25$ this ranking is reversed. In addition, rent dissipation is generally decreasing in n whereas it is increasing for $\sigma = 0$. With $\sigma = 0.1$, rent dissipation is lowest in the complete information contest and increasing in n if $n \leq 3$ whereas it is lowest in the no information contest and decreasing in n if the number of players is sufficiently

		$n=2$	$n=3$	$n=5$	$n=7$	$n=10$
	complete info.	0.4780	0.5919	0.6534	0.6833	0.7136
$\sigma=0$	private info.	0.4787	0.6249	0.7206	0.7484	0.7752
	no info.	0.5000	0.6667	0.8000	0.8571	0.9000
	complete info.	0.2780	0.3148	0.2951	0.2545	0.1797
$\sigma=0.1$	private info.	0.2787	0.3250	0.2944	0.2468	0.1679
	no info.	0.3000	0.3667	0.3000	0.1571	0
	complete info.	0.0571	0.0682	0.0259	0	0
$\sigma=0.25$	private info.	0.0570	0.0673	0.0248	0	0
	no info.	0	0	0	0	0

Table 3: Ex ante expected rent dissipation for uniform F.

large. As we observe for $n=5$, it is also possible that rent dissipation is lowest in the private information contest.

6 From Uncertain Costs to Uncertain Values

In the literature on contests among asymmetric players, those players are sometimes assumed to differ in their valuation for the prize rather than in their abilities or costs. Most importantly, in the related studies by Hurley and Shogren (1998a) as well as Malueg and Yates (2004) contestants are privately informed about their values. In the following, we examine to what extent the results obtained in preceding sections carry over to models with uncertain values.

Suppose $c_i=1$ for all i, but each player i values the prize v_i rather than 1. For each i, valuation v_i is a realization of the random variable V_i that is distributed according to the continuous distribution function $\tilde{F}_i$ on $[\underline{v}_i, \overline{v}_i]$ with $0<\underline{v}_i<\overline{v}_i$. Accordingly, player i's ex post payoff amounts to

$$\tilde{u}_i(\mathbf{x}, v_i) := p_i(\mathbf{x})v_i - x_i.$$

Let $\tilde{x}_i^*(\mathbf{v})$, $\tilde{\xi}_i(v_i)$, and $\tilde{X}_i$ denote player i's equilibrium strategies in the complete, private, and no information contest for this modified setup. The equilibrium strategies have to satisfy

$$\tilde{x}_i^*(\mathbf{v}) \in \arg\max_{x_i} \tilde{u}_i(x_i, \tilde{\mathbf{x}}_{-i}^*(\mathbf{v}), v_i) \quad \forall i, \tag{12}$$

$$\tilde{\xi}_i(v_i) \in \arg\max_{x_i} E[\tilde{u}_i(x_i, \tilde{\boldsymbol{\xi}}_{-i}(\mathbf{V}_{-i}), v_i)] \quad \forall i, v_i \in [\underline{v}_i, \overline{v}_i], \tag{13}$$

$$\tilde{X}_i \in \arg\max_{x_i} E[\tilde{u}_i(x_i, \tilde{\mathbf{X}}_{-i}, V_i)] \quad \forall i.$$

Let, for all i, V_i be a transformation of the random variable C_i such that

$$V_i = \frac{1}{C_i} \quad \text{and therefore} \quad \tilde{F}_i(v_i) = 1 - F_i(\tfrac{1}{v_i}). \tag{14}$$

With this transformation of variables the maximization problems in (12) and (13) coincide with those in (2) and (3) for the original model.[13] Consequently,

$$\tilde{x}_i^*(\mathbf{v}) = x_i^*(\tfrac{1}{v_1}, \ldots, \tfrac{1}{v_n}) \quad \text{and} \quad \tilde{\xi}_i(v_i) = \xi_i(\tfrac{1}{v_i}).$$

All our results for the original model concerning the complete and private information contest and their comparison to each other therefore directly extend to the case with uncertain values.

Now consider the no information contest. Similar to the original model, we have $\tilde{X}_i = \tilde{x}_i^*(E[\mathbf{V}])$. Hence, under transformation of variables (14), Jensen's inequality implies

$$\tilde{X}_i = x_i^*(\tfrac{1}{E[V_1]}, \ldots, \tfrac{1}{E[V_n]}) \geq x_i^*(E[\tfrac{1}{V_1}], \ldots, E[\tfrac{1}{V_n}]) = X_i.$$

If (14), efforts under cost uncertainty are smaller than when values are uncertain.[14] As a result, Propositions 3 and 4 stating that expected efforts in the no information contest is smaller than in the other two contests do not extend to the model with uncertain valuations. Yet all the cases where we found rent dissipation to be largest in the no information contest, Proposition 5 in particular, also apply to the modified model.

[13]Note that, for each player i, $\tilde{u}_i(x_i, \tilde{\mathbf{x}}_{-i}^*(\mathbf{v}), v_i) = v_i u_i(x_i, \tilde{\mathbf{x}}_{-i}^*(\mathbf{v}), \frac{1}{v_i})$ and $E[\tilde{u}_i(x_i, \tilde{\boldsymbol{\xi}}_{-i}(\mathbf{V}_{-i}), v_i)] = v_i E[u_i(x_i, \tilde{\boldsymbol{\xi}}_{-i}(\mathbf{V}_{-i}), \frac{1}{v_i})]$ where v_i is a positive constant.

[14]Suppose the prize is measured in dollars and effort in hours, such that $c_i = \frac{1}{v_i}$ is the price of one hour in dollars. As long as player i knows this price, his optimization problem is unchanged when expressing payoffs in terms of hours rather than dollars. However, if the price c_i is random, i's payoff measured in dollars follows a different distribution than if measured in hours. That is why optimal effort choice in the no information contest changes when moving from the original to the modified setup.

Studying a numerical example of the standard lottery contest where values for two players are drawn from two different distributions with the same mean, Hurley and Shogren (1998a) find that a player's ex ante expected effort in the no information contest exceeds that in the private information contest. This is exactly the opposite of what Proposition 4 states. Making use of results derived in preceding sections, we establish the following.

Proposition 6. *Suppose $n = 2$, $E[V_1] = E[V_2]$, and $\underline{v}_i \geq 4\sigma$ for $i = 1,2$. Then, ex ante expected efforts in the no information contest are higher than in the private information contest:*

$$\tilde{X}_i \geq E[\tilde{\xi}_i(V_i)] \qquad \textit{for } i = 1,2.$$

Proof. Let $\tilde{\mu} := E[V_1] = E[V_2]$. From Lemma 1 follows, with $\underline{v}_i \geq 4\sigma$,

$$\tilde{\xi}_i(v_i) = \xi_i(\tfrac{1}{v_i}) \leq \frac{1}{4}v_i - \sigma$$

implying

$$E[\tilde{\xi}_i(V_i)] \leq \frac{1}{4}\tilde{\mu} - \sigma.$$

According to Corollaries 1 and 2,

$$\tilde{X}_i = x_i^*(\tfrac{1}{E[V_1]}, \tfrac{1}{E[V_2]}) = \frac{1}{4}\tilde{\mu} - \sigma. \qquad \square$$

Proposition 6 generalizes the numerical result by Hurley and Shogren (1998a) to any standard two-player lottery contest with values drawn from two distributions with equal means. Moreover, provided that the additional noise σ is not too large, the result continues to hold for $\sigma > 0$. Interestingly, Morath and Münster (2009) find the same ranking of expected efforts to generally hold for the two-player all-pay auction with uncertain values.[15]

7 Conclusion

In order to study the impact of uncertainty and asymmetry of information on the behavior in imperfectly discriminating contests, we compare three different informational settings to each other. The model we employ is the Tullock lottery contest,

[15]Note that, for the same reason as in the rent-seeking contest, their result does not extend to the all-pay auction with uncertain costs of effort.

augmented by an additional noise parameter σ. By considering more than two players and types that are drawn from general continuous probability distributions, we extend the analysis of rent-seeking contests under asymmetric information.

For both the no information and the complete information contest we determine unique equilibrium strategies. For any $\sigma > 0$ we prove that the private information contest has an equilibrium in monotone pure strategies. In addition, we find the equilibrium to be unique if σ is big enough. Apart from analytically deriving properties of the equilibrium strategies, we also identify numerical methods suitable for computing approximations to those strategies. The simple application of Athey (2001) we present for proving existence of a pure-strategy equilibrium in the private information contest can readily be extended to a more general class of contest success functions. Most importantly, this class includes the winning probabilities axiomatized by Skaperdas (1996) that take the form $g(x_i)\Big/\sum_j g(x_j)$ where $g(\cdot)$ is an increasing and strictly positive function. Analyzing the corresponding equilibrium strategies is an interesting task for future research.

In general, ex ante expected aggregate effort is lowest in the no information contest. Yet at the same time we find that rent dissipation in the no information contest is larger than in the other two contests if σ is small enough. In this case, if types are all drawn from the same distribution, both contestants and a contest organizer benefiting from players' efforts would prefer the private and the complete information contest over the no information contest. Hence, we would expect contestants to try to gather information before competing. Moreover, the organizer would have an incentive to encourage such behavior. Our analysis can therefore be seen as a first step for future work on acquisition and provision of information in imperfectly discriminating contests.

As our numerical examples illustrate, a general ranking of the complete and the private information contest in terms of expected efforts is not possible. Which of the two contests yields higher efforts depends on the distribution of types, the exact specification of the contest success function, and the number of players. Numerical results for $\sigma > 0$ and uniformly distributed costs suggest that if there are relatively few players (and σ is not too big), the no information contest induces the largest rent dissipation, followed by the private information contest and the complete information contest. If the number of players is sufficiently large, however, the ranking is reversed.

Our results concerning the equilibrium of the three types of contests also extend to an alternative formulation of the model where values rather than costs are randomly drawn. Comparing the no information contest to the other two contests tends to yield different conclusions, though. This is an issue that is not restricted to our specific contest format. It should be kept in mind when comparing results in the literature that involve no information contests.

Appendix A: Proofs

A.1 Proof of Lemma 1

Observe that the fraction on the LHS of (9) is strictly decreasing in $\xi_i(c_i)$. Hence, if $\xi_i(\hat{c}) > 0$ for some $\hat{c}$, then $\xi_i(c) > \xi_i(\hat{c})$ for all $c < \hat{c}$. Consequently, there must be a $\tilde{c}_i \in [\underline{c}_i, \overline{c}_i]$ such that $\xi_i(c) = 0$ for $c > \tilde{c}_i$ while $\xi_i(c)$ is positive and strictly decreasing for $c < \tilde{c}_i$.

Suppose $\sigma > 0$ and $c_i < \tilde{c}_i$. In this case (9) holds with equality. Note that the fraction on the LHS of (9) is maximized if $\sum_{j=1}^{n} \xi_j(c_j) = 0$, which implies $c_i \leq \frac{n-1}{n^2\sigma}$. Therefore, we must have $\tilde{c}_i \leq \frac{n-1}{n^2\sigma}$. Now, let $\sigma = 0$. As we have argued above, there must be at least one player choosing strictly positive effort for all types, i.e., $\tilde{c}_i = \overline{c}_i$ for at least one player i.

Assume $c_i < \tilde{c}_i$. Multiplying (9) on both sides with $\xi_i(c_i) + \sigma$ yields

$$E\left[\frac{\left(\sum_{j\neq i} \xi_j(C_j) + (n-1)\sigma\right)(\xi_i(c_i)+\sigma)}{\left(\sum_{j\neq i} \xi_j(C_j) + \xi_i(c_i) + n\sigma\right)^2}\right] = c_i(\xi_i(c_i)+\sigma).$$

Since

$$\frac{\left(\sum_{j\neq i} \xi_j(C_j) + (n-1)\sigma\right)(\xi_i(c_i)+\sigma)}{\left(\sum_{j\neq i} \xi_j(C_j) + \xi_i(c_i) + n\sigma\right)^2} = \frac{\xi_i(c_i)+\sigma}{\sum_{j\neq i} \xi_j(c_j) + \xi_i(c_i) + n\sigma}\left(1 - \frac{\xi_i(c_i)+\sigma}{\sum_{j\neq i} \xi_j(c_j) + \xi_i(c_i) + n\sigma}\right) \leq \frac{1}{4},$$

we obtain

$$\frac{1}{4} \geq c_i(\xi_i(c_i)+\sigma) \quad \text{or} \quad \xi_i(c_i) \leq \frac{1}{4c_i} - \sigma.$$

Replacing c_i by the random variable C_i, taking expectation on both sides of (9), and summing over all i, we obtain

$$E\left[\frac{(n-1)\sum_{i=1}^{n}\xi_i(C_i)+n(n-1)\sigma}{\left(\sum_{i=1}^{n}\xi_i(C_i)+n\sigma\right)^2}\right]\leq\sum_{i=1}^{n}E[C_i].$$

This can be rearranged to yield

$$E\left[\frac{1}{\sum_{i=1}^{n}\xi_i(C_i)+n\sigma}\right]\leq\frac{1}{n-1}\sum_{i=1}^{n}E[C_i].$$

Applying Jensen's inequality we find

$$\frac{1}{E\left[\sum_{i=1}^{n}\xi_i(C_i)\right]+n\sigma}\leq E\left[\frac{1}{\sum_{i=1}^{n}\xi_i(C_i)+n\sigma}\right]$$

and therefore

$$\sum_{i=1}^{n}E\left[\xi_i(C_i)\right]\geq\frac{n-1}{\sum_{i=1}^{n}E[C_i]}-n\sigma.$$

□

A.2 Proof of Proposition 3

Let $E[C_1]\leq E[C_2]\leq\cdots\leq E[C_n]$ and suppose expected costs are such that in the no information contest $m^*>0$ players choose a strictly positive effort. According to Corollary 2,

$$\begin{aligned}\sum_{i=1}^{n}X_i&=Y(m^*)-n\sigma\\&=\frac{(m^*-1)+\sqrt{(m^*-1)^2+4(n-m^*)\sigma\sum_{i=1}^{m^*}E[C_i]}}{2\sum_{i=1}^{m^*}E[C_i]}-n\sigma.\end{aligned}$$

Now, consider the complete information contest. From Proposition 1,

$$\begin{aligned}\sum_{i=1}^{n}x_i^*(\mathbf{c})&=\max_m Y(m)-n\sigma\geq Y(m^*)-n\sigma\\&\geq\frac{(m^*-1)+\sqrt{(m^*-1)^2+4(n-m^*)\sigma\sum_{i=1}^{m^*}z_i}}{2\sum_{i=1}^{m^*}z_i}-n\sigma\end{aligned}$$

where $z_1, z_2, \dots, z_n$ is a reordering of $c_1, c_2, \dots, c_n$ such that $z_1 \le z_2 \le \dots \le z_n$. Taking expectations, we have

$$\sum_{i=1}^{n} E[x_i^*(\mathbf{C})] \ge E\left[\frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} Z_i}}{2\sum_{i=1}^{m^*} Z_i}\right] - n\sigma.$$

Note that the term we take the expectation of on the RHS is decreasing and convex in $\sum_{i=1}^{m^*} Z_i$. Jensen's inequality thus implies

$$\sum_{i=1}^{n} E[x_i^*(\mathbf{C})] \ge \frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} E[Z_i]}}{2\sum_{i=1}^{m^*} E[Z_i]} - n\sigma.$$

Since the expected sum of the first m order statistics cannot be larger than the sum of the m smallest means, i.e., $\sum_{i=1}^{m} E[Z_i] \le \sum_{i=1}^{m} E[C_i]$ we finally obtain

$$\sum_{i=1}^{n} E[x_i^*(\mathbf{C})] \ge \frac{(m^*-1) + \sqrt{(m^*-1)^2 + 4(n-m^*)\sigma \sum_{i=1}^{m^*} E[C_i]}}{2\sum_{i=1}^{m^*} E[C_i]} - n\sigma \qquad \square$$

A.3 Proof of Proposition 5

Because $\xi_i(c_i) > 0$, (9) holds with equality for all c_i. Multiplying (9) on both sides with $\xi_i(c_i) + \sigma$ yields

$$E\left[\frac{\left(\sum_{j\neq i} \xi_j(C_j) + (n-1)\sigma\right)(\xi_i(c_i)+\sigma)}{\left(\sum_{j\neq i} \xi_j(C_j) + \xi_i(c_i) + n\sigma\right)^2}\right] = c_i(\xi_i(c_i)+\sigma).$$

Replacing c_i by the random variable C_i, taking expectations on both sides, and summing over all i, we obtain

$$E\left[\frac{\sum_{i=1}^{n}\sum_{j\neq i}(\xi_j(C_j)+\sigma)(\xi_i(C_i)+\sigma)}{\left(\sum_{i=1}^{n} \xi_i(C_i) + n\sigma\right)^2}\right] = \sum_{i=1}^{n} E[C_i(\xi_i(C_i))] + \sigma \sum_{i=1}^{n} E[C_i]. \quad (15)$$

The fraction on the LHS of (15) is bounded by $\frac{n-1}{n}$. To see this, note that

$$\begin{aligned}
&(n-1)\left(\sum_{i=1}^{n}\xi_i(C_i)+n\sigma\right)^2-n\sum_{i=1}^{n}\sum_{j\neq i}(\xi_j(C_j)+\sigma)(\xi_i(C_i)+\sigma)\\
&=(n-1)\left(\sum_{i=1}^{n}(\xi_i(C_i)+\sigma)^2+\sum_{i=1}^{n}\sum_{j\neq i}(\xi_i(C_i)+\sigma)(\xi_j(C_j)+\sigma)\right)\\
&\qquad\qquad\qquad\qquad-n\sum_{i=1}^{n}\sum_{j\neq i}(\xi_j(C_j)+\sigma)(\xi_i(C_i)+\sigma)\\
&=(n-1)\sum_{i=1}^{n}(\xi_i(C_i)+\sigma)^2-\sum_{i=1}^{n}\sum_{j\neq i}(\xi_i(C_i)+\sigma)(\xi_j(C_j)+\sigma)\\
&=\frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}\left\{(\xi_i(C_i)+\sigma)^2+(\xi_j(C_j)+\sigma)^2-2(\xi_i(C_i)+\sigma)(\xi_j(C_j)+\sigma)\right\}\\
&=\frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}\left(\xi_i(C_i)-\xi_j(C_j)\right)^2\geq 0.
\end{aligned}$$

Hence,

$$\frac{\sum_{i=1}^{n}\sum_{j\neq i}(\xi_j(C_j)+\sigma)(\xi_i(C_i)+\sigma)}{\left(\sum_{i=1}^{n}\xi_i(C_i)+n\sigma\right)^2}\leq\frac{n-1}{n}$$

and therefore (15) implies for expected rent dissipation in the private information contest

$$\sum_{i=1}^{n}E[C_i(\xi_i(C_i))]\leq\frac{n-1}{n}-\sigma\sum_{i=1}^{n}E[C_i].$$

Due to Corollaries 1 and 2 combined with $E[C_1]=E[C_2]=\cdots=E[C_n]<\frac{n-1}{n^2\sigma}$, expected rent dissipation in the no information contest amounts to

$$\sum_{i=1}^{n}E[C_iX_i]=\frac{n-1}{n}-\sigma\sum_{i=1}^{n}E[C_i].$$ □

Appendix B: Notes on Numerical Methods

For the private information contest, equilibrium strategies can in general not be obtained in closed form. In such a case, progress in studying contestants' behavior can be made by approximating equilibrium strategies numerically. In this appendix we provide a short discussion of the computational methods we applied to obtain the numerical results presented in Sections 4 and 5.

For all our numerical results we assume $F_i = F$ for all i, i.e., all costs are drawn from the same distribution. In this case, the symmetric equilibrium strategy $\xi(c)$ has to fulfill condition (10) which can be restated as

$$H(\xi,c)\begin{cases} =0 & \text{if} \quad \xi(c)>0 \\ \leq 0 & \text{if} \quad \xi(c)=0 \end{cases} \tag{16}$$

where

$$H(\xi,c):=\int_{\underline{c}}^{\overline{c}}\cdots\int_{\underline{c}}^{\overline{c}}\frac{\sum_{j=1}^{n-1}\xi(k_j)+(n-1)\sigma}{\left(\sum_{j=1}^{n-1}\xi(k_j)+\xi(c)+n\sigma\right)^2}f(k_1)dk_1\dots f(k_{n-1})dk_{n-1}-c.$$

We approximate $\xi(c)$ numerically by a discrete function on a grid of points in $[\underline{c},\overline{c}]$. Denoting the size of the grid by g, we consider the set of points

$$\widehat{\mathbf{c}}=\left\{c^1,c^2,\dots,c^g\right\} \qquad \text{where } c^i=\frac{2i-1}{2g}\left(\overline{c}-\underline{c}\right)+\underline{c}.$$

Our goal is now to find a set of function values $\widehat{\boldsymbol{\xi}}=\left\{\hat{\xi}^1,\hat{\xi}^2,\dots,\hat{\xi}^g\right\}$ corresponding to $\widehat{\mathbf{c}}$ that represents a good approximation of the continuous function $\xi(c)$. With a discrete version of $H(\xi,c)$, denoted by $\hat{H}(\widehat{\boldsymbol{\xi}},c^i)$, at hand, standard iterative algorithms can be applied to compute a $\widehat{\boldsymbol{\xi}}$ that fulfills a discrete approximation to condition (16).

How to compute $\hat{H}(\widehat{\boldsymbol{\xi}},c^i)$? Note that $H(\xi,c)$ consists of an $n-1$-dimensional integral. The simplest method for approximating this integral on $\widehat{\mathbf{c}}$, repeatedly summing the areas of rectangles, requires a number of function evaluations that grows exponentially in n. For $n>3$ and a reasonable grid size (e.g., $g=100$), the computation of $\hat{H}(\widehat{\boldsymbol{\xi}},c^i)$ becomes so slow that finding a good approximation to $\xi(c)$ using iterative algorithms is impossible (even in the simplest case where $\sigma=0$). A more efficient method to compute integrals in multiple dimensions is Monte Carlo integration. Applying this method, we evaluate the integrand at a uniformly distributed sequence of pseudorandom points in $\widehat{\mathbf{c}}^{n-1}$ and take the average. We can further improve our results by choosing points from a low-discrepancy sequence, such as the Sobol sequence, instead of pseudorandom points. This is sometimes called quasi-Monte Carlo integration and yields, for the same number of function evaluations, more accurate results.[16]

As we have argued in Section 4, for $\sigma = 0$, $\xi(c) > 0$ for all c. Accordingly, (16) simplifies to $H(\xi, c) = 0$ for all c. A numerical approximation to $\xi(c)$ is a $\widehat{\xi}$ that solves $\hat{H}(\widehat{\xi}, c^i) = 0$ for all $c^i \in \hat{\mathbf{c}}$. We numerically solve this system of g equations with g unknowns using the trust-region dogleg algorithm as implemented in the function `fsolve` that is provided with the Matlab Optimization Toolbox.

The case where $\sigma > 0$ is computationally more expensive. To make it suitable for the algorithm we apply, we restate the discrete version of condition (16) as

$$\widehat{\xi} = \arg\min_{\overline{\xi}} \sum_{i=1}^{g} D(\overline{\xi}, c^i)^2 \quad \text{s.t.} \quad \hat{\xi}^i \geq 0 \ \ \forall i \tag{17}$$

where

$$D(\widehat{\xi}, c^i) := \begin{cases} 0 & \text{if } \hat{\xi}^i = 0 \text{ and } \hat{H}(\widehat{\xi}, c^i) \leq 0, \\ \hat{H}(\widehat{\xi}, c^i) & \text{otherwise.} \end{cases}$$

The minimization problem with inequality constraints (17) can be solved numerically using the active-set algorithm that is implemented in the function `fmincon` of the Matlab Optimization Toolbox.

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