Welfare Improving Contracts in Cournot Markets *

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Abstract

The question this paper addresses is whether a government can regulate a Cournot oligopolist market to give higher level of welfare without changing either the strategic variable (output quantity) or the way prices are determined (by an auctioneer). The problem is set as a two-stage game played by profit-maximizing firms and a welfare-maximizing government. Firms are symmetric in capacity and technology but asymmetric in ownership. The government owns one firm and uses it strategically. The main policy implication of the model is that by owning and controlling one single firm, a government can regulate an entire industry and achieve welfare improvements. This is possible as the decision-making asymmetry among privately and publicly owned firms allows the government to change the context in which the quantity competition takes place. In addition, this paper shows that the social objectives of the government are not incompatible with profit maximization targets. The government improves the *total* welfare of the economy if and only if it maximizes profits in its own firm. We shall see that, in equilibrium, the publicly-owned firm maximizes profit either by producing the Stackelberg leader output or the competitive output.

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1 Introduction

It has long been recognized that oligopoly markets yield an inefficient equilibrium output (in the Paretian sense). The socially optimal level of output fails to be delivered due to the power firms possess to alter the market price.

In a Cournot oligopoly market firms compete through their output quantity choice. The equilibrium is given by the strategic interaction of firms that simultaneously and individually make their production decisions. In noncooperative game parlance, output quantity is the strategic variable used by firms.

Once the quantity output choice is made and production is brought to the market, the market clearing price is determined by an auctioneer who equates total supply and demand.

The question this paper addresses is whether a government can regulate Cournot oligopolist markets to give higher level of welfare without changing either the strategic variable (output quantity) or the way prices are determined (by an auctioneer).

Kreps and Scheinkman (1983), shows that *"the solution of oligopoly games depends on both the strategic variable that firms are assumed to employ and the context (game form) where this variables are employed"*(p.327). I take advantage of this finding and present a model where the government reduces firms' market power, not by directly controlling prices or production quantity, but by changing the context in which the Cournot (quantity) competition takes place.

The problem is set as follows. Assume a Cournot oligopolist market with one $(i = 0)$ publicly-owned firm and N $(i = 1, ...n)$ privately-owned firms that produce a homogeneous good. The $N + 1$ firms are symmetric and have an exogenously given upper bound on capacity output, k_i, such that $k_0 = ... = k_n = k$ and this constitutes a common knowledge among all firms.[1]

The N firms are profit maximizing and choose their output accordingly. On the other hand, the government pursues social objectives and uses its capacity strategically to reduce the deadweight loss caused by the market power firms possess.

Consider the following two-stage game played by profit maximizing firms and a welfare maximizing government. In the first-stage, the government announces the rules for the usage of its capacity k_0. In the second-stage, conditional on observing the government's announcement, firms individually and simultaneously make their output choices. This structure is to be specified in a contract set by the government.

This is a non-standard game in which N firms that pursue Cournot-like conjectures play a sequential game. I assume that the $N + 1$ firms are symmetric in capacity and technology. There is, nevertheless, an asymmetry regarding the ownership. This asymmetry is translated in terms of objectives and, as we shall see, provides the leadership role to the government.[2]

[1] Henceforth I will refer to privately owned firms simply as "firms" and to the publicly-owned firm as "government".

[2] Fershtman (1990), has already noticed that the ownership status may affect the market structure

The social objective of the government, which contrasts with firms' profit maximizing behaviour, gives the government the "first mover advantage" as its output choices are set in the first-stage of the game. Firms behave as followers and act conditionally on the government's decision. The game-form proposed in this paper produces an equilibrium very much *à la Stackelberg*.[3]

Depending on the level of the exogenously determined capacity parameter k_0 there are different solutions to this sequential game.

If the government's capacity is sufficiently large to bring the market-price to zero, the government can suggest an output level and threaten to flood the market unless firms produce the output suggested. Firms are left with the choice of either to produce zero or an output consistent with the government's objective. However, once this decision is taken the market price constitutes the mechanism that equates total supply and demand. If instead the government's capacity is smaller than this level, the threat of flooding the market is obviously not credible. Nevertheless, as we shall see, even in this case some welfare improvement can be achieved.

We are left to discuss how the government conveys its objective in terms of output level.[4] I assume that the government uses a *reference price* to put across its output target. This price is revealed to firms in the same contract that sets the rules for the usage of the government's capacity. However, if we want a kind of regulation that does not affect the strategic variable nor how prices are determined, the *reference price* cannot be a *ceiling price* imposed by the government.

I deal with this problem by assuming instead that, in the first stage, the government announces a *minimum price* and commits itself to buy an unlimited amount of output if the market clearing price is strictly smaller than the minimum price.

If the government's capacity is large enough to potentially drive the market price to zero, any strictly positive *minimum price* will obviously be strictly greater than the market-clearing price. Thus, firms can take the *minimum price* as their marginal revenue function and maximize profit by producing an output such that marginal revenue is equal to marginal cost. The *minimum price* happens to be a *ceiling price* although the government does not impose it.

If the government's capacity is smaller than this level, firms collectively ignore

in which firms operate.

[3]Dowrick (1986) shows that if firms have downward-sloping reaction functions and no predetermined asymmetry, each firm has an incentive to "act strongly" and take its preferred role. In a quantity space, when firms' reaction functions slope downwards the preferred role is to be a leader. If all firms take the preferred role they inflict in mutual damage. The author argues that firms may seek for a collusive solution to minimize mutual damage. For this reason the Stackelberg solution is not satisfactory when there is no predetermined asymmetry among firms.

In my model it is assumed that there is a predetermined asymmetry. The government pursues a social objective and uses its capacity strategically to achieve it. This means that it will always "act strongly" to take the leadership role. On the other hand, I assume that privately owned firms have downward sloping reaction functions. Under this assumption an equilibrium à *la Stakelberg* makes sense: if the government has the "first mover advantage" other firms will behave as followers.

[4]Recall that this paper assumes that the government does not impose either price or quantity.

the *minimum price* (and the threat is ineffective) provided they make a net positive profit producing *à la Stackelberg.*

The welfare improving contract proposed in this paper is based on a *threat* (to produce up to capacity) and an incentive (the *minimum price).* The credibility of both threat and incentive depends on the government's capacity and on technology conditions (the shape of the marginal cost curve).

The main policy implication of the model is that by owning and controlling one single firm, a government can regulate an entire industry and achieve welfare improvements. This is possible as the decision-making asymmetry among privately and publicly owned firms allows the government to change the context in which quantity competition takes place.

In addition, this paper shows that the social objectives of the government are not incompatible with profit maximization targets. The government improves the *total* welfare of the economy if and only if it behaves as a profit maximizing in its own firm. We shall see that, in equilibrium, the publicly-owned firm maximizes profit either by producing the Stackelberg leader output or the competitive output.

The paper is organized as follows. The next section presents the underlying economy and describes the main features of the (welfare improving) contract. Section 3 outlines the model and presents the solution to the two-stage game. Section 4 analyses the impact of the regulatory contracts on the total welfare of the economy. Section 5 is an example. Section 6 concludes and presents directions for further research.

2 The Underlying Economy

Assume a closed economy where the government is endowed with the capacity of producing up to k_0 units of a good to which a regulation is to be introduced. The government aims at designing welfare improving contracts and announces a minimum price to convey its output target. These contracts are to be enforced by law and exhibit the following characteristics:

1. *An Announcement* of a minimum price prior to firms' output decision;

2. *A Commitment* to buy an unlimited amount of output whenever the market-clearing price is strictly smaller than the announced minimum price;

3. *A Threat* of producing and selling out up to k_0 units if firms collectively ignore the minimum price.

Throughout the paper I refer to the set of measures that characterize the welfare improving contract as *"ACT"*, making a clear reference to the government's actions (to announce, to commit, to threaten).[5]

This is a stylized market in which firms engage themselves in a Cournot-like competition. There is no uncertainty in this economy and information is evenly distributed among firms and government. The game is made of two stages and lasts one sole period.

3 Outline of the Model with "*ACT*" Contract

There is the government' firm ($i = 0$) and N ($i = 1, ..., n$) private firms. These $N + 1$ firms produce a perfectly substitutable good for which the market demand function is $Q^d(P) = a - P$. Firms have an exogenously given upper bound on capacity output[6], $k_i \in R_{++}$, such that $k_0 = ... = k_n = k$[7]. Each firm i has identical twice differentiable cost function $c(y_i) = c(y)\ \forall i = 0, ..., n$, such that $c'(y) > 0$ and $c''(y) \geq 0$ at all $y > 0$ and $c(y) = 0$ if $y_i = 0$. Denote $C'(y) = \sum_{i=0}^{n} c'(y_i)$ and assume that the fixed cost is zero to all firm $i = 0, ..., n$.

The aggregate supply is $Q^S = \sum_{i=1}^{n} y_i + y_0$, where y_i is firm i's output choice and $0 \leq y_0 \leq k_0$ is the government's output level.

The game runs as follows. The government announces an *"ACT"* contract that includes a minimum price $P_{\min} \in R_+$. Upon observing the minimum price, $P_{\min}$, and the government's capacity level, k_0, firms simultaneously and individually make their output decision $y_{i\backslash 0} \in R_+$.[8] Firms have Cournot-like conjectures and choose the output level that equates marginal revenue and marginal cost. The government observes firms' output choice and puts up to k_0 units on the market if each firm i fails to produce $y_{i\backslash 0}$ such that $P_{\min} = C'(y)$.[9] The market clears.

Firms' payoffs are given by their profit function. The market-clearing price is $P(y_{i\backslash 0}, y_{-i\backslash 0}, y_0) = \max[0, a - \sum_{i=1}^{n} y_i - y_0]$ where $y_{-i\backslash 0}$ is the other firms output

[5] Although this contracts are set unilaterally firms have the right not to accept it. However, once firms accept the contract (i.e. they produce according to government's objective) they have rights guaranteed by law. I hope, in this case, the term "contract" is an acceptable abuse of terminology.

[6] Assume that the cost of installing capacity is already sunk when the game starts and therefore irrelevant.

[7] As a shorthand, with abuse of notation, I set $k_0 = 0$ when the government, although endowed with some capacity, cannot intervene in the market and carry on the *Threat*. Later in this paper I make this point clearer.

[8] $y_{i\backslash 0}$ stands for the output of a privately-owned firm i.

[9] If production is made simultaneously the government would just learn if firms produced according to the minimum price or not when the market clears. It means that the government would not be able to carry on the threat of intervening in the market if firms collectively reject the minimum price. An extra assumption has to be made to allow the government to observe firms' output choice. Assume that each firm i (in the beginning of the second-stage) is forced to reveal to the government their quantity output choice. Firms are free to choose any level of output. However, there will be a punishment if firms do not tell the truth.

choice. I assume $P(y_{i\backslash 0}, y_{-i\backslash 0}, y_0)$ is strictly positive on some bounded interval $[0, Y)$, on which it is twice differentiable, strictly decreasing and concave. For $y_0, y_{i\backslash 0} \geq Y$, $P(y_{i\backslash 0}, y_{-i\backslash 0}, y_0) = 0$. Let us define $\widehat{P}(P(\cdot,\cdot,\cdot), P_{\min}) = \max\{P(\cdot,\cdot,\cdot), P_{\min}\}$. Firm i's profit function is $\pi_{i\backslash 0} = \widehat{P}(\cdot,\cdot)y_{i\backslash 0} - c(y)$ that is strictly concave in $y_{i\backslash 0}$ on the range $\left(0, Y - y_{-i\backslash 0} - y_0\right]$ if $\widehat{P}(\cdot,\cdot) = P(\cdot,\cdot,\cdot)$ and concave in y_i on the range $[0, \infty)$ if $\widehat{P}(\cdot,\cdot) = P_{\min}$.

Assume a benevolent government that aims at maximizing the total welfare of the economy. Let us represent the government's payoff by the total value of the aggregate Marshallian surplus at the aggregate level of consumption X expressed as $S(X) = S_0 + \int_0^X \{P[s] - [c'(s) + P_{\min}]\}ds$, where S_0 is a constant of integration. It is easy to verify that this function is maximized when the aggregate level of consumption X^* is equal the competitive output (see Appendix).

3.1 The Equilibrium of the Game

In a standard Cournot-Nash model players (firms) move simultaneously and individually and the equilibrium of the game is given by the set of strategies that constitute a *Nash Equilibrium*.

As stated before, in this paper firms that pursue Cournot-like conjectures play a sequential game. Firms are called upon to play after perfectly observing $P_{\min}$ and k_0. The strategic choice that each firm $i\backslash 0$ has to make is either to take the minimum price ($P_{\min}$) as given and to produce accordingly, or simultaneously and individually to choose $y_{i\backslash 0} > 0$ given the government's threat. In the latter case firms behave *à la Stackelberg* with the government assuming the role of "leader". Recall that the government's threat is to put on the market up to k_0 units of output. It means that, when the competition is *à la Stackelberg*, $y_0 = k_0$.

The game is solved by backward induction to find the set of pure strategies that constitute *Subgame Perfect Nash Equilibrium*.

Let us define

$$R_{i\backslash 0}(y_{-i\backslash 0} + k_0; P_{\min}) = \underset{\substack{(i)\ 0 \leq y_{i\backslash 0} < Y - y_{-i\backslash 0} - k_0 \\ \text{or} \\ (ii)\ y_i \geq 0}}{\arg\max}\ \widehat{P}(\cdot,\cdot)y_{i\backslash 0} - c(y_{i\backslash 0})$$

That is $R_{i\backslash 0}(y_{-i\backslash 0}, k_0; P_{\min})$ is the best response function to a Stackelberg-like competition if $\widehat{P}(\cdot,\cdot) = P(y_{i\backslash 0}, y_{-i\backslash 0}, k_0)$ or the best response to a minimum price if $\widehat{P}(\cdot,\cdot) = P_{\min}$.

It is the solution with respect to $y_{i\backslash 0}$ of

$$\begin{aligned} (i)\ P'(y_{i\backslash 0},\cdot,\cdot)y_{i\backslash 0} + P(y_{i\backslash 0}, y_{-i\backslash 0}, k_0) &= c'(y) \text{ if } \widehat{P}(\cdot,\cdot) = P(y_{i\backslash 0}, y_{-i\backslash 0}, k_0) \\ &\quad \textit{or} \\ (ii)\ P_{\min} &= C'(y) \text{ if } \widehat{P}(\cdot,\cdot) = P_{\min} \end{aligned}$$

The next Lemma shows some facts of Cournot competition when there is an "*ACT*" contract.

Lemma 1 *(a) if $k_0 = 0$ and $P_{\min} < P(\cdot,\cdot,0)$ for every $c(y)$, $R_i(y_{-i})$ is nonincreasing in y_{-i}, and $R_i(y_{-i})$ is continuously differentiable and strictly decreasing over the range it is strictly positive.*

(b) if $0 < k_0 < Q^d(0)$ and $P_{\min} < P(\cdot,\cdot,k_0)$ for every $c(y)$, $R_{i\backslash 0}(y_{-i\backslash 0},k_0)$ is nonincreasing in $y_{-i\backslash 0}$ and k_0, and $R_{i\backslash 0}(y_{-i\backslash 0},k_0)$ is continuously differentiable and strictly decreasing over the range it is strictly positive.

(c) If $k_0 \geq Q^d(0)$, for every $c(y)$, $R_i(P_{\min})$ is monotone increasing in $P_{\min}$ and $R_i(P_{\min})$ is continuously differentiable over the range it is positive.

(d) $R^{'}(y_{-i\backslash 0},k_0) \geq -1$ with strictly inequality for $y_{i\backslash 0}$ such that $R_{i\backslash 0}(y_{-i\backslash 0},k_0) > 0$ and $0 \leq k_0 < Q^d(0)$ and $R^{'}(P_{\min}) = 0$ if $R_i(P_{\min}) > 0$ and $k_0 \geq Q^d(0)$.

(e) If $y_{i\backslash 0} > R_{-i\backslash 0}(y_{i\backslash 0},k_0)$ then $R_{-i\backslash 0}(R_{i\backslash 0}(y_{-i\backslash 0},k_0)) < y_{i\backslash 0}$.

Proof.

(a) For any y_{-i} and $k_0 = 0$ we have

$$P\left[R_i(y_{-i}) + y_{-i}\right] + R_i(y_{-i})P'\left[R_i(y_{-i}) + y_{-i}\right] = c'\left[R_i(y_{-i})\right]$$

If we increase y_{-i} and leave $R_i(y_{-i})$ constant this decreases the first term that is positive. The second term decreases and becomes more negative. The concavity of $y_iP(y_i + y_{-i}) - c(y_i)$ in y_i implies that to restore equality we must decrease $R_i(y_{-i})$. Differentiability of $R_i(y_{-i})$ follows from the smoothness of $P(\cdot,\cdot,\cdot)$ and $c(y_i)$.

(b) for any $y_{-i\backslash 0}$ and $0 < k_0 < Q^d(0)$, we have

$$\begin{aligned} & P\left[R_{i\backslash 0}(y_{-i\backslash 0},k_0) + y_{-i\backslash 0} + k_0\right] + R_{i\backslash 0}(y_{-i\backslash 0},k_0)P'\left[R_{i\backslash 0}(y_{-i\backslash 0},k_0) + y_{-i\backslash 0} + k_0\right] \\ = \; & c'\left[R_{i\backslash 0}(y_{-i\backslash 0},k_0)\right] \end{aligned}$$

Increase $y_{-i\backslash 0}$ while leaving $R_{i\backslash 0}(y_{-i\backslash 0},k_0)$. This decreases the first term that is positive and decreases the second. Thus the concavity of $y_{i\backslash 0}P(y_{i\backslash 0} + y_{-i\backslash 0}) - c(y_{i\backslash 0})$ in $y_{i\backslash 0}$ and as k_0 is given it implies that to restore the equality we must to decrease $R_{i\backslash 0}(y_{-i\backslash 0},-)$. As $P\left[R_{i\backslash 0}(y_{-i\backslash 0},k_0) + R_{-i\backslash 0}(y_{i\backslash 0},k_0) + k_0\right] = 0$ if $k_0 \geq Q^d(0)$ for any $y_{-i\backslash 0} \geq 0$, $R_i(y_{-i\backslash 0},k_0) = 0$.

(c) For any $y_{-i\backslash 0}$ and $k_0 \geq Q^d(0)$, if $P_{\min} > 0$ we have $\widehat{P}(P(\cdot,\cdot,k_0),P_{\min}) = \max\{P(\cdot,\cdot,k_0),P_{\min}\} = P_{\min}$

(d) Let $R_{i\backslash 0}(y_{-i\backslash 0},k_0) = -\frac{\sum\limits_{i\neq 0}\left[\pi^{''}_{i,-i}(R_{i\backslash 0}(y_{-i\backslash 0},k_0),y_{-i\backslash 0})\right]}{\pi^{''}_{i,i}\left[R_{i\backslash 0}(y_{-i\backslash 0},k_0),y_{-i\backslash 0}\right]}$ be the slope of the best response function where $\pi^{''}_{i,-i}$ is the cross partial derivative.

A sufficient condition for reaction functions to intersect only once is that the derivatives of the reaction functions to be greater or equal than -1 over the relevant range.

The concavity of the profit function implies that $\pi^{''}_{i,i}\left(R_{i\backslash 0}(y_{-i\backslash 0},k_0),y_{-i\backslash 0}\right)=-1-c^{''}(y)<0$ and $\pi^{''}_{i,-i}\left(R_{i\backslash 0}(y_{-i\backslash 0},k_0),y_{-i\backslash 0}\right)=-1<0$. Thus, $\left|\pi^{''}_{i,i}\left(R_{i\backslash 0}(y_{-i\backslash 0},k_0),y_{-i\backslash 0}\right)\right|>\left|\sum_{i\neq -i\neq 0}\left[\pi^{''}_{i,-i}(R_{i\backslash 0}(y_{-i\backslash 0},k_0),y_{-i\backslash 0})\right]\right|$.

(e) Follows steps similar to (d) ■

The equilibrium that arises depends on the government's capacity level k_0. For every cost function $c(y)$ and $k_0=0$[10] there is a unique Cournot-Nash Equilibrium with each firm bringing to the market some quantity y_c^* and the Cournot-price is P_c^*. In this case the *Threat* of intervening in the market is not credible and ignored by firms. If $k_0=0$ each firm i ignores the *Announcement, Commitment,Threat* and produce the Cournot-like output.

If $k_0>0$ the *Threat* is credible. However, firms know that if $0<k_0<Q^d(0)$ they can still make net positive profit if they produce *à la Stackelberg* (rather than producing the Cournot output). In this case, for each level of $0<k_0<Q^d(0)$ there is a unique *à la Stackelberg* equilibrium with firms bringing forward some quantity y_k^*, so that the market-clearing price is P_k^*. So, if $0<k_0<Q^d(0)$ each firm i ignores the *Announcement and Commitment* but accepts the *Threat* and produces y_k^*.

If $k_0\geq Q^d(0)$ there is an unique equilibrium in which firms take P_{min} as given. Firms accept the *Announcement and Commitment* and produce according to the minimum price $P_{\min}$.[11]

The next proposition characterizes the *Nash equilibrium* obtained in the second-stage of the game.

Proposition 2 *Let y_c^* be the Cournot equilibrium output and y_k^* be the follower Stackelberg's output. In any (symmetric) pure strategy Nash Equilibrium of a sequential game where the government is endowed with a capacity level k_0 and chooses $P_{\min}\in R_+$, the unique equilibrium output choice to each firm i, y^*, is given*

(a) $y^*=y_c^*$, $\forall i=0,...n$, if $k_0=0$;

(b) $y^*=y_k^*,\forall i=1,...n$, and $y^*=k_0$ for $i=0$, if $0<k_0<Q^d(0)$;

(c) $y^*\in R_+$, $\forall i=1,...n$ s.t. $P^*_{\min}=C'(y)$ and $k_0\geq Q^d(0)$.

Proof. For (a), the Cournot output, see Lemma 1(a). For (b), the *à la Stackelberg*, see Lemma 1(b) . See Lemma 1(c) for the price taking output (c). For the uniqueness see Lemma 1(d). ■

In (a) the government has no means to *"ACT"* and change the context in which firms interact. Firms and government individually and simultaneously take their quantity output decision and produce $y^*=y_c^*$, the Cournot output.

On the other hand, if $0<k_0<Q^d(0)$, when firms are called upon to play they engage themselves in a Stackelberg-like competition. As the government is bound to

[10] See footnote 7.

[11] If the assumption of zero fixed cost is relaxed, firms can make losses even if $0<k_0<Q^d(0)$. If this is the case, firms are left with the option to produce according to the minimum price.

the "ACT" contract the outcome, $y^* = y_k^*$, $\forall i = 1, ...n$, if $0 < k_0 < Q^d(0)$, makes sense as equilibrium.

Lemma 1 and Proposition 2 show that firms comply to the minimum price if and only if $k_0 \geq Q^d(0)$.[12] However, the next Lemma shows that an "ACT" contract may be welfare improving despite the fact that firms ignore $P_{\min}$.

Lemma 3 *For any* $0 < k_0 < Q^d(0)$

(a) $Q_k^{s*} > Q_c^{s*}$*, where* Q_k^{s*} *and* Q_c^{s*} *are the aggregate supply when firms bring to the market the Stackelberg-like output and the Cournot output respectively and the governments sells out* k_0*.*

For any $k_0 \geq Q^d(0)$

(b) Let y^0 *be the perfectly competitive output level and* $P_{\min}^* = C'(y^0)$*, where* $C'(y^0) = \sum_{i=n} c'(y_i^0)$*. For every* $c'(y) > 0$ *and* $c''(y) > 0$*,* $P_{\min}^* = C'(y^0)$ *implies* $y^* = y^0$*.*

(c) For every $c'(y) > 0$ *and* $c''(y) = 0$ *such that* $c'(y) = \gamma$ *where* $\gamma > 0$*;* $P_{\min}^* = C'(y^0)$ *implies that* $y^* = y_c^*$*.*

Proof.

(a) If $k_0 = 0$ (no "ACT "contract) firms produce the Cournot output and the aggregate supply is $Q_c^{s*} = \frac{N}{N+1}\left[a - c'(y_c^*)\right]$. If $0 < k_0 < Q^d(0)$, firms produce the Stackelberg-like output and the aggregate supply is $Q_k^{s*} = \frac{N}{N+1}\left[a - c'(y_k^*)\right] + \frac{k_0}{N+1}$. As $y_c^* > y_k^*$ (see **Lemma 1**) and given the convexity of the cost functions, $Q_k^{s*} > Q_c^{s*}$ and $P_k^* < P_c^*$ for every $0 < k_0 < Q^d(0)$.

(b)$k_0 \geq Q^d(0)$ implies that $P(\cdot, \cdot, k_0) = 0$. Thus the profit function that a profit maximizing firm faces is $\pi = P_{\min} y - c(y)$, that is strictly concave in y_i if $c'(y) > 0$ and $c''(y) > 0$. Therefore $y^* > 0$ maximizes the above profit function if and only if $\pi_y' = 0$. Then we verify that $\pi_y' = 0$ is unique at all $y > 0$ and implies $P_{\min} = c'(y)$.

Thus, if $P_{\min} = c'(y^0)$ each firm i maximizes profit producing $y^* = y^0$.

(c) Let $k_0 \geq Q^d(0)$ and $c'(y) = \gamma$ thus the profit function $\pi = P_{\min} y - \gamma\ y$ is linear monotone increasing in y. Therefore any $y^* > 0$ maximizes the above profit function if $\pi_y' = 0$. Then we verify that $\pi_y' = 0$ and implies $P_{\min} = \gamma$ at all $y > 0$. Thus if $P_{\min} = c'(y^0) = \gamma$ each firm i produces $y^* = y_c^*$ and the market clearing price is $P_c^* > \gamma$. ■

If the government is endowed with any capacity level $0 < k_0 < Q^d(0)$ and for every $c(y)$, an equilibrium with "ACT" contract seems to Pareto dominate[13] any other equilibrium in which $k_0 = 0$ (no"ACT" contract) . However, if $k_0 \geq Q^d(0)$ the

[12] If the government sets $P_{\min} > P(\cdot, \cdot, \cdot)$, to take advantage of the minimum price, firms have to produce y_i such that $P_{\min} = C'(y)$. If $0 < k_0 < Q^d(0)$ firms always deviate from $P_{\min}$ producing y_k^* and making net positive profit.

[13] For $0 < k_0 < Q^d(0)$, an "ACT" implies a higher level of output and lower prices. At the same time privately-owned firms still make net positive profit. Although it is good news to consumers, the effect on the total surplus of the economy has to be analyzed.

effect of an "*ACT*" contract is twofold and depends on the firms' cost function. If firms have strictly convex cost functions the government can induce firms to produce the competitive output by setting $P_{\min} = C'(y^0)$. In this case firms produce $y^* = y^0$ and the market-clearing price is $P(\cdot,\cdot,\cdot) = P_{\min} = C'(y^0)$. Under the "*ACT*" contract the government does not have to buy any amount of output as it is just committed to do so if the market clearing price is strictly smaller than the minimum price $P_{\min}$. In this case, the "message" the minimum price carries is unequivocal and $y^* = y^0, \forall i = 0, ..., n$, can be supported as the unique equilibrium.

On the other hand , when firms have non convex cost functions and the government announces $P_{\min} = C'(y^0) = \gamma$ the message the minimum price carries is misleading as any output $y > 0$ satisfies the condition $P_{\min} = \gamma$. As consequence, the government cannot carry on the *Threat* of producing at full capacity if firms make net positive profit. Also, the government has to be consistent to the *Commitment* if firms decide to overproduce and the market clearing price is strictly smaller than the minimum price.[14]

An "*ACT*" contract does not deliver any welfare improvement if the message coveyed by the "*ACT*" is misleading. This is much the same as the government's capacity was zero, what would make the *Threat* not credible.

The *Subgame Perfect Nash Equilibrium* to the whole game is given next.

Lemma 4 *In any pure strategy Subgame Perfect Nash Equilibrium in which a government endowed with the capacity level k_0 and announces $P_{\min} \in R_+$ as part of an "ACT" contract*

$$P^*_{\min} = \begin{cases} C'(y^0) \text{ if } k_0 \geq Q^d(0) \text{ ; } c'(y) > 0 \text{ and } c''(y) > 0 \text{ at all } y > 0 \\ P_{\min} \in R_+ \text{ such that } P_{\min} < P^*_k \text{ if } 0 < k_0 < Q^d(0) \\ 0 \text{ otherwise} \end{cases}$$

where y^0 is the competitive output and $C'(y^0) = \sum_{i=0}^{n} c'(y^0_i)$

Proof. The proof is straightforward given Proposition 2 and Lemma 3. ■

Proposition 5 *The n-tuple of strategies*

$$\begin{cases} [P^*_{\min} = 0;\ P(\cdot,\cdot,\cdot) = P^*_c;\ y^*_i = y^*_c\ ,\forall i = 1, ...n] \text{ if } k_0 = 0 \text{ or } c'(y) = \gamma,\ \gamma>0 \\ [P^*_{\min} < P^*_k;\ P(\cdot,\cdot,\cdot) = P^*_k;\ \ y^* = y^*_k\ ,\forall i = 1, ...n;\ y^* = k_0, \forall i = 0] \text{ if } 0 < k_0 < Q^d(0) \\ [P_{\min} = C'(y^0) = P(\cdot,\cdot,\cdot);\ y^* = y^0, \forall i = 0, ...n] \text{ if } k_0 \geq Q^d(0); c'(y) > 0 \text{ and } c''(y) > 0 \end{cases}$$

constitute the unique pure strategy Subgame Perfect Nash Equilibrium

Proof. See Proposition 2 and Lemma 4. ■

Lemma 4 states the conditions for existence of "*ACT*" contracts. The government has to be able to change the context where competition takes place. When it is not possible certainly another kind of intervention has to be prescribed.

[14] In equilibrium firms never overproduce and the government never buys unlimited amounts of output by the minimum price. For this reason, time inconsistence is not an issue in this model.

4 The *"ACT"* Contract and The Economic Welfare

Lemma 3 (section 3) shows that an *"ACT"* contract may Pareto dominate any other equilibrium in which $k_0 = 0$. This follows the fact that the total output with *"ACT"* contracts is higher than the Cournot output for any $k_0 > 0$. Higher level of output and lower prices is good news to consumers. However, recall that the government target is to maximize the *total* surplus of the economy. Thus, what has to be investigated is whether or not the contract, as set in the previous section, fulfills its role. In this section I analyze the impact of an *"ACT"* contract in terms of total welfare.

The deadweight loss caused by the firms' market power can be measured using the change in the Marshallian aggregate surplus (see Appendix and section 3).

The expression belows shows the welfare loss the *Cournot competition* brings along.[15]

$$\int_{Ny_c^*}^{Ny^0} [P(s) - c'(s)]ds > 0$$

Let us introduce now the *"ACT"* contract and analyze the case in which $k_0 > 0$ and firms have strictly convex cost functions.

If $k_0 > Q^d(0)$, it has been showed that, in equilibrium, firms produce the competitive output. In this case, the deadweight loss is eliminated as firms behave as price takers and produce the competitive equilibrium. On the other hand, if the shape of firms' marginal cost prevents the government to enforce an *"ACT"* contract, we are back to the case where the industry produce the *Cournot output*. When $k_0 > Q^d(0)$, the total surplus of the economy is maximized and the government maximizes profit by producing the competitive output.

The interesting case to look at is the one in which the government assumes the leadership role and the equilibrium output is *à la Stackelberg*. In this particular case, we know that total welfare of the economy is not maximized because the government is unable to force firms to produce the competitive output. However, one should expect welfare improvements as the industry moves from the Cournot equilibrium output to *à la Stackelberg* equilibrium output.

Let us use the change in the Marshallian aggregate surplus to measure the deadweight loss of the *à la Stackelberg* output. Recall that, according to **Proposition 2**, when $0 < k_0 < Q^d(0)$ firms produce the symmetric *à la Stackelberg* output $y^* = y_k^*$, and the government produces $y^* = k_0$. Using the change in the Marshallian aggregate

[15] When competitive output is produced, the Marshillian aggregate surplus is maximized as $P(s) - c'(s) = 0$. To prove that just note that $S''(X) > 0$ for all x_i. It means that the concavity of $S(X)$ implies that the function is maximized if $S'(X) = 0$. Then, the next step is to prove that $S'(X) = P(s) - c'(s)$. (see Appendix and Mas Collel et al (1985, chap. 10)).

surplus to measure the deadweight loss of the *à la Stackelberg* output, we have,

$$\int_{Ny_k^*+k_0}^{(N+1)y^0} [P(s) - c'(s)]ds - c'(k_0)$$

The first term of the Marshallian aggregate surplus is greater than zero as the profit maximization conditions in Stackelberg competition implies a price to be greater than the marginal cost. However, the whole expression can be greater, smaller or equal to zero according to the government's capacity level k_0.

Assume that the government has a capacity level $0 \leq k_0 < Q^d(0)$ such that $Ny_k^* + k_0 \geq (N+1)y^0$. In this case, the change in the Marshallian aggregate surplus works out as

$$P[Ny_k + k_0] - P[(N+1)y^0] - c'(y^0)\Big]_{Ny_k^*+k_0}^{Ny^0} < 0$$

If $0 \leq k_0 < Q^d(0)$ and the aggregate supply is greater than the competitive output, the government makes losses producing up to capacity with a welfare reducing effect.

This problem can be avoided however by assuming that the government also has profit maximizing objectives. Thus, in the first stage of the game, instead of threatening to produce up to capacity, the government announces and produces the "standard" Stackelberg leader's output. In the second stage, firms behave in the same way as discussed in the previous section.

The government's strategy, when its capacity level is $0 < k_0 < Q^d(0)$ and it behaves as a "standard" Stackelberg leader, is to choose y_0 to maximize the profit function $\pi_0 = P(y_i)y_0 - c(y)$. The government has the "first mover advantage" and chooses y_0 such that

$$y_0 = \arg\max_{0 \leq y_0 < Y - y_{-i\backslash 0}} P(R_{i\backslash 0}(y_0))y_0 - c(y)$$

As in the usual case, $R_{i\backslash 0}$ is a decreasing function, and so the government can decrease other players output by increasing its own (see Lemma 1). Thus, it implies that the government's Stackelberg output is higher than in Cournot equilibrium and the payoff of the other firms is lower.

Corollary 6 and 7 characterize the *Subgame Perfect Nash Equilibrium* for the second-stage and for the whole game, respectively, when total welfare is to be taken into account.

Corollary 6 (Proposition 2) *In any pure strategy Subgame Perfect Nash Equilibrium of a sequential game where the government uses its capacity strategically to give total welfare improvements the equilibrium output choice, y^*, is*

(a) $y^* = y_c^*$, $\forall i = 0, ...n$, if $k_0 = 0$;

(b) $y^* = y_k^*, \forall i = 1, ...n$, and $y^* = y_K^*$ for $i = 0$, if $0 < k_0 < Q^d(0)$;

(c) $y^* \in R_+$,$\forall i = 1, ...n$, s.t. $P_{\min}^* = c'(y)$ and $k_0 \geq Q^d(0)$.

where y_K^* is the leader's Stackelberg output.

Corollary 7 (Proposition 5) *The n-tuple of strategies*

$$\begin{cases} [P^*_{\min} = 0;\ P(\cdot,\cdot,\cdot) = P^*_c;\ y^*_i = y^*_c\ ,\forall i = 0,...n]\ \text{if}\ k_0 = 0\ \text{or}\ c'(y) = \gamma,\ \gamma{>}0 \\ [P^*_{\min} < P^*_k;\ P(\cdot,\cdot,\cdot) = P^*_k;\ y^* = y^*_k\ ,\forall i = 1,...n;\ y^* = y^*_K, \forall i = 0]\ \text{if}\ 0 < k_0 < Q^d(0) \\ [P_{\min}\ = C'(y^0) = P(\cdot,\cdot,\cdot);\ y^* = y^0, \forall i = 0,...n]\ \text{if}\ k_0 \geq Q^d(0); c'(y) > 0\ \text{and}\ c''(y) > 0 \end{cases}$$

constitute a pure strategy Subgame Perfect Nash Equilibrium in a game in which the government aims to give total welfare improvements.

The welfare analysis of the *"ACT"* contract shows that if $0 < k_0 < Q^d(0)$, the government improves *total* welfare of the economy if and only if it pursues profit maximizing objectives in its own firm.

In the introduction of this paper I claimed that government's social objectives are not incompatible with profit maximizing targets. The results displayed by the model with *"ACT"* contracts shows, nevertheless, that to follow a profit maximizing *rationale* is a necessary condition to improve the *total* welafre of the economy.

5 An Example

Let there be one ($i = 1$) privately-owned firm and one publicly-owned firm ($i = 2$). The cost function to each firm $i = 1, 2$ is $c(y_i) = y_i^2 = y^2$. The demand function is $Q^d = 12 - P$ and the aggregate supply $Q^S = y_1 + y_2$. From the demand function and the aggregate supply we have the market-clearing price which is $P(y_1 + y_2) = [12 - (y_1 + y_2)]$. The profit maximization problem to each firm is given by $\underset{y_i}{Max}\ \pi_i = Py_i - c(y_i)$.

The first order conditions for the profit maximization problem is;

$$\begin{cases} P'(y_1 + y_2)y_i + P^*(y_1 + y_2) = 2y_i & \text{if } P(\cdot) > P_{\min} \\ P_{\min} = 2y_i & \text{if } P(\cdot) \leq P_{\min} \end{cases} \tag{FOC}$$

Let us analyze four different scenarios.

Case 0: The Competitive Output.
The competitive output produced for each firm i is

$$\begin{aligned} y_i^0 &= \frac{12 - 2y}{2} \\ y_i^0 &= 3,\ \forall i = 1, 2 \end{aligned}$$

The aggregate supply is $Q_0^* = 6$ and the market-clearing price is $P_0^* = 6$

Case 1: Standard Cournot Model (no *"ACT"* contracts)
The best response functions to each firm $i = 1, 2$ if $P(\cdot) > P_{\min}$ are

$$b_1(y_2) = \frac{12-y_2-2y_1}{2}$$
$$b_2(y_1) = \frac{12-y_1-2y_2}{2}$$

The Nash Equilibrium is given by the intersection of the two best response functions. In equilibrium, given firms' symmetry

$$y_{1c}^* = y_{2c}^* = y_c^* = \frac{12 - 2y_c^*}{3}$$

$$y_c^* = 2.4$$

The aggregate supply is $Q_c^S = 4.8$, the market-clearing price $P_c^* = 7.2$ and the profit to each firm i is $\pi_c^* = 11.52$.

Welfare Analysis

Deadweight loss of the Cournot output is measured by the change in the Marshallian aggregate surplus.

$$\int_{Q_c^*}^{Q_0^*} [P(s) - c'(s)]\, ds = \int_{4.8}^{6} (14.4 - 9.6) ds = 5.76 > 0$$

Case 2: Equilibrium "à la Stackelberg" $(0 < k_0 < 12)$

Assume $k_0 = 6$ and the government produces up to capacity.

In the first period, the government announces its output choice $y_1 = 6$. In the second period, firm 2 observes the government's output choice and chooses y_2 such that

$$y_{2k}^* = \frac{12-6}{4}$$
$$y_{2k}^* = 1.5$$

The aggregate supply in this case is $Q_k^* = 7.5$ and the market-clearing price is $P_k^* = 4.5$. Firm 2 profit is $\pi_2^* = 4.5$ and firm 1 (the government) profit is $\pi_1^* = -9$.

Welfare Analysis

Deadweight loss of the *à la Stackelberg* output is measured by the change in the Marshallian aggregate surplus.

$$\int_{Q_0^*}^{Q_k^*} [P(s) - c'(s)]\, ds - c'(k_0) = \int_{6}^{7.5} (9 - 3) dx - 12 = -3 < 0$$

Case 3: The Standard Stackelberg Equilibrium $(0 < k_0 < 12)$

The timing of the game is as follows. In the first period the government announces its output choice y_1. In the second-period, upon observing the government's choice, firm 2 makes its output decision.

Solving backwards we have firm 2's best response to the government's output choice as

$$b_2(y_1) = \frac{12 - y_1}{4} \tag{1}$$

In the second-period, the government chooses y_1 to maximize its profit function given that $b_2(y_1)$.

$$\begin{aligned} \pi_1 &= P(y_1 + b_2(y_1))y_1 - c(y) \qquad (2) \\ \pi_1 &= (12 - y_1 - b_2(y_1))y_1 - c(y) \end{aligned}$$

Substituting eq.(1) into eq.(2) we have

$$\pi_1 = (12 - y_1 - \left(\frac{a - y_1}{4}\right))y_1 - c(y)$$

From the first order condition we have

$$\begin{aligned} \pi_1' &= 0 \\ y_1^* &= 2.6 \end{aligned}$$

Substituting y_1^* into eq.1 we find that

$$y_2^* = 2.35$$

The aggregate supply $Q_K^* = 4.95$ is greater than the Cournot aggregate supply, $Q_K^S = 4.8$ and the market price is $P_K^* = 7.05$. Firm 1's (the government) profit is $\pi_1^* = 11.57$ and firm 2 profit is $\pi_2^* = 11.05$.

Welfare Analysis

Deadweight loss of the Stackelberg output measured by the change in the Marshallian aggregate surplus is

$$\int_{Q_K^*}^{Q_0^*} [P(s) - c'(s)]\, ds = \int_{4.95}^{6} (14.10 - 4.70 - 5.2)ds = 4.41 > 0$$

Case 3: Price Taker Equilibrium $(k_0 \geq 12)$

Assume $k_0 = 12$. The best-response to each firm i is

$$\begin{aligned} b_1(y_2, k_0) &= \tfrac{10 - y_2 - 10 - 2y_1}{2} \\ b_2(y_1, k_0) &= \tfrac{10 - y_1 - 10 - 2y_2}{2} \end{aligned}$$

	Q^*	P^*	π_1^*	π_2^*	Deadweight loss
Competitive Equilibrium	6	6	9	9	0
Cournot Equilibrium	4.8	7.2	11.52	11.52	5.76
à la Stackelberg Equilibrium, $k_0 = 6$	7.5	4.5	-9	4.5	-3
"Standard" Stackelberg equilibrium	4.95	7.05	11.57	11.05	4.41
Price taker equilibrium, $k_0 \geq 12$, $P_{min} = 6$	6	6	9	9	0

Table 1: Welfare Comparison

The intercept of the two best response function is in the intersection of the two axes where each firm i produces $y = 0$.

However, if there exists a $P_{\min} > 0$ the best response to each firm i is to produce y_i such that $P_{\min} = C'(y^*)$.

Let $P_{\min} = 6$, the best response to each firm is

$$\begin{aligned} b_1(P_{\min}) &= \frac{6}{2} = 3 \\ b_2(P_{\min}) &= \frac{6}{2} = 3 \end{aligned}$$

In this case, the total output is equal to the competitive one (see **case 0**). Both firms maximize profit and the total welfare of the economy is also maximized.

Table 1 summarizes the results of the example.

The social objectives of the government are not incompatible with profit maximization targets. If the government behaves as a "standard" Stackelberg leader it gives welfare improvements and at the same time maximizes profit in its own firm. On the other hand, if the government behaves *à la Stackelberg* it may make losses by producing up to capacity depending on the capacity level k_0.

6 Conclusion

I presented a model in which a government, endowed with the capacity level k_0, uses this capacity strategically to change the market structure and provide a higher level of welfare.

I considered a Cournot oligopolist market in which firms are symmetric in technology and capacity and produce a homogeneous good. The asymmetry in ownership and in decision-making objectives gives the government the leadership role in this market. I showed that, in equilibrium, privately-owned firms that originally pursued Cournot-like conjecture behave either as Stackelberg-followers or price-takers.

According to the capacity level k_0, condition on technology and to the government's objectives there are three possible equilibria.

(i) *à la Stackelberg Equilibrium*: If the government aims at maximizing the **consumers surplus**, for every cost function $c(y)$ there is a unique *à la Stackelberg* equilibrium for each $0 < k_0 < Q^d(0)$. Each privately-owned firm produce $y^* = y_k^*$ and the government produces $y^* = k_0$

(ii) *"Standard" Stackelberg Equilibrium:* If the government aims at maximizing the **total surplus of the economy**, for every cost function $c(y)$ there is a unique *"Standard" Stackelberg* equilibrium for each $0 < k_0 < Q^d(0)$. Each privately-owned firm produce $y^* = y_k^*$ and the government produces the Stakelberg leader's output $y^* = y_K^*$.

(iii) *Price-taker Equilibrium:* If firms have strictly convex cost functions and $k_0 \geq Q^d(0)$ there is an unique equilibrium in which firms produce the competitive output.

In the *à la Stackelberg equilibrium* the government produces up to capacity. The aggregate supply is greater and price is lower, if compared with the Cournot equilibrium output. However, if the aggregate supply is greater than the competitive equilibrium output the government make losses by producing up to capacity. For a certain level of k_0, the deadweight loss implied by the *à la Stackelberg* equilibrium can be smaller than (in absolute terms) the deadweight loss implied by the Cournot equilibrium output.

In the *"Standard" Stakelberg Equilibrium* the government combines social objectives with profit maximizing targets. There is a total welfare improvement (compared with the Cournot equilibrium) and the government maximizes profits in its own firm. Finally, in the *Price-taker Equilibrium,* the total welfare of the economy is maximized as firms and government produce the competitive equilibrium output.

The results of this paper are not directly transferable to the reality. Nevertheless, the structure of behaviour in this sequential game, in which the government assumes the role of leader, can be an alternative approach to the regulation problem posed in oligopoly markets.

There are several variations of this model that may be worth investigating. We could allow the government and firms to choose capacity in the first-stage and then engage themselves in a Bertrand price competition, as suggested by Kreps and Scheinkman (1983). In addition, the model can be extended to a multi period game to analyze if the properties of the equilibria are maintained.

A Appendix

Consider an economy with $j = 1, ...J$ consumers and $i = 0, ..., N$ firms that produce a good l. Let $(y_0, ...y_n)$ be the amount of output produced by each firm i. Let consumer j's preference over consumption bundles $(x_1, ..., x_j)$ be described by the quasi-linear utility function

$$U(x_j) = m_j + \phi(x_j)$$

where m_j is the numeraire good.

Mas Collel et al. (1995, Chapter 10 section 10.E) describes the *Marshalian Aggregate Surplus* as

$$S(x_1, ..., x_j, y_0, ...y_n) = \sum_{j=1}^{J} \phi(x_j) - \sum_{n=1}^{N} c(y_i)$$

A change in the consumption and production level of good l leads to an increase in welfare if and only if it increases the value of the *Marshalian Aggregate Surplus.*

Let us define the *Marshalian Aggregate Surplus*

$$S(x_1, ..., x_j, y_0, ...y_n) = \sum_{j=1}^{J} \phi[x_j] - \left[\sum_{n=0}^{N} c[y_i] - \sum_{n=1}^{N} P_{\min} y_i\right]$$

where the differential $(dx_1, ..., dx_j, dy_1, ...dy_n)$ is:

$dS(dx_1, ..., dx_j, dy_1, ...dy_n) = \phi'[x_j] \sum_{j=1}^{J} dx_i - [c'[y_i] - P_{\min}] \sum_{n=1}^{N} dy_i$

From Mas Collel et al. (1995) we have that $\phi'[x_j] = P[x_j]$ for all j, and $c'[y_i] = C'(y)$ for all i. So we get

$$dS = P[X] \sum_{j=1}^{J} dx_i - [C'[Y] + P_{\min}] \sum_{n=1}^{N} dy_i$$

where $X = \sum_{j=1}^{J} x_i$ and $Y = \sum_{n=1}^{N} y_i$.

For market feasibility condition we know that $X = Y + k_0$.and so $\sum_{j=1}^{J} dx_i = \sum_{n=1}^{N} dy_i + dk_{0.}$ As $dk = 0$ we have

$$dS = [P[X] - [C'[Y] + P_{\min}]] \, dX \tag{3}$$

Denoting (3) in terms of integral

$$S(X) = S_0 + \int_{0}^{X} [P[s] - [C'[s] + P_{\min}]] \, ds$$

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