Evolutionary models' comparative analysis

Methodology proposition based on selected neo-schumpeterian models of industrial dynamics

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Abstract

A methodology of comparative analysis of evolutionary models is proposed. The main aim of this proposition is to identify to what extend different models can be called 'evolutionary ones'. Each model is analysed by searching for answers to following questions:

- Is the model dynamical one?
- Is it focused on far-from-equilibrium analysis?
- What are a unit of evolution and a unit of selection?
- Is diversity and heterogeneity of economic agents and their behaviour observed?
- Is search for innovation based on a concept of hereditary information (knowledge)?
- What kinds of innovation does the model describe?
- Does selection process lead to diversified rate of growth and spontaneity of development?
- How economic agents set prices?
- What kind of products are described by the model?
- Are decision making procedures and investment procedures present in the model?

Outline of selected schumpeterian models is accompanied by identification of crucial evolutionary characteristics of each model and a short indication of phenomena explained by that model.

Key-words: Evolutionary economics, neo-schumpeterian models, simulation

Since publication of a seminal work by Richard Nelson and Sidney Winter in 1982 evolutionary models proliferated enormously. After two decades of experience it is possible to distinguish two main streams of development within an evolutionary modelling of economic processes. The first one relates to the work of Schumpeter and the second one is based on the concept of cellular automata and relates to a general framework of artificial life and Agent-based Computational Economics (ACE). Intention of this short paper is to propose a comparative analysis of selected models much more from technical point of view then to make a review of evolutionary, neo-schumpeterian models (excellent reviews and surveys of evolutionary models can be found in Dosi *et al.* (1988), Saviotti, Metcalfe (1991), Nelson

(1995), Silverberg, Verspagen (1995)). Matter presented in this paper ought to be considered as the first attempt to build a 'comparative method (methodology)' of proliferating evolutionary models.

The paper will begin with short review of the models accompanied by identification of crucial evolutionary characteristics of these models. All selected models are **dynamical** ones so the first important feature to call them evolutionary ones is fulfilled by all selected models. Similarly all models are focused on **far-from-equilibrium analysis** but it is interesting to see to what extend this feature is really used in the process of simulation analysis of the models. Other crucial features of evolutionary process are not present not in all models. The question to what extend other 'evolutionary features' are present in the models will be the main aim of this paper. The features which seem to be crucial to call a model an evolutionary one are: **diversity and heterogeneity of economic agents (firms) and their behaviour**, **search for innovation based on a concept of hereditary information (knowledge)**, and **selection process** which leads to diversified rate of growth and spontaneity of development. Interesting question in relation to economic evolutionary models is presence of **decision making procedures**. In many models that procedure is not present in many others it has more or less complicated form.

Although some models are rather new ones, most of them were developed in 1990s, important question which ought to be stated is: what kind of phenomena (stylised facts) the models explain or shed new light on them? The answer seems to be rather difficult but at least some preliminary answer will be presented in the paper.

The easiest and the least controversial way of presentation would be to present the models in chronological order but it would be nothing more then cataloguing the models. Looking in the past history of Schumpterian tradition it seems possible to distinguish a few related but in some way independent streams of modelling efforts. The first is very closely associated with the work of Nelson and Winter (1982). To that tradition works of Winter (1984), Jonard, Yildizoglu (1998, 1999), Winter, Kaniovski, Dosi (1997, 2000), and Yildizoglu (2001) can be included. The other streams get inspirations from the work of Nelson and Winter but has essential distinguishing features. The second stream of models can be called '*Silverberg-Verspagen models'*. One distinguished feature of SV models is that technological progress is embedded in vintage capital. In the model presented in Silverberg (1985) and Silverberg *et al.* (1988) firms are self-financing using their cash and liquid interest bearing reserves. Idea that firms rely on rather simple rules of thumb or routines rather than explicit optimisation procedures is applied in models developed by Silverberg, Lehnert and Verspagen (Silverberg and Lehnert, 1993; Silverberg, Verspagen, 1994, 1994a, 1995, 1995a). These models can be seen as continuation of the work initiated by Gerald Silverberg in 1980s. The main difference between the Silverberg and Verspagen (1995) model and the ones presented in Silverberg (1985) and Silverberg and Lehnert (1993) is the way in which innovation is endogenized.

The third stream of models can be called '*Dosi et al. models'*, e.g., Chiaromonte and Dosi (1993), Dosi *et al.* (1993, 1994, 1995). Dosi *et al*. approach is highly bottom-up simulation. The aim of the authors seems to be to start from basic mechanisms of industrial development without making any assumption about possible modelled properties of the system and to obtain the well-known features (*stylised facts*) from the co-working of these basic mechanisms of development. Similar assumption was made by Kwasnicki in his model of industrial dynamics (Kwasnicka, Kwasnicki, 1992, 1996, Kwasnicki, 1994/1996, 2000). In a sense the work of Kwasnicki can be considered as separate modelling effort.

There are also numerous models that can be identified as having 'Schumpeterian flavour'. The model presented by Andersen (1997) is based on Pasinetti's scheme of the structural economic dynamics of a labour economy with inclusion of an evolutionary, micro-economic foundation. A proposition of Bruckner, Ebeling and Scharnhorst (1989), Bruckner, Ebeling, Jimenez Montano and Scharnhorst (1993) apply general n-dimensional birth-death transition model to describe technological development. Models proposed by Stan Metcalfe (1993, 1994) and Paul Windrum and Chris Birchenhall, (1998) are also shortly described. The main of this paper is to propose a methodology of comparative analysis of evolutionary models. Because of a natural limitation on the length of a paper, a large number of existing model (e.g., Englmann (1994), Iwai (1984, 1984a), Nelson and Wolff (1997), Saviotti and Mani (1993), and Verspagen (1991, 1993)) are not included in the analysis but proposed methodology can be applied also to these models.

Nelson and Winter model

It seems natural that when we think about evolutionary modelling rooted in Schumpeterian tradition we ought to start from the work of Nelson and Winter.[1] Nelson and Winter (NW) models were worked out in 1970s and 1980s and summarised in their book, sometime called 'bible of evolutionary economists' (Nelson, Winter, 1982). Nelson and Winter models suit frequently as a basis or a kind of pattern for inventing another evolutionary models. In NW model and in almost all models of Schumpeterian tradition a firm is a basic unit of evolution and also is a unit of selection.[2] Contrary to orthodox economics, concept of representative agent is not present in evolutionary models. Usually the economy is disaggregated into diverse individual firms influencing each other by nonlinear dynamic interactions describing search for innovation, competition (selection) and investment. In most simulation models agents use boundedly rational behavioural procedures. Learning and searching for innovation are modelled by allowing for mutation and imitation rules operating on the firms' operational parameters. Mutations are usually local within the routine space. Nelson and Winter apply a population perspective and they postulate that it is possible to specify the space in which innovative search takes place.

The assumption of macroeconomic properties flowing from microeconomic behaviour of economic agents (i.e. firms) is basic reason for necessity of using simulation to investigate these models. The first model that will be discussed is the one presented in Nelson and Winter (1982, ch. 9). This model can be seen also as the first evolutionary growth model.

The state of the evolutionary process of an industry at any moment t is described by the capital stock and the behavioural rules of each firm. The state in the next moment (t+1) is determined by the state in previous moment. In this growth model firms use production techniques which are characterised by fixed labour and capital coefficients. Firms manufacture homogeneous products, so the model describes only process innovation. It is assumed that firms produce using a Leontief production function, therefore substitution between labour and capital is not explicitly present in the model. Invention occurs as a result of firms' search activities. Firms search for new combinations of labour and capital

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[1] Many authors have programmed the Nelson-Winter model. Let us mention only three implementations available through the Internet. One was done within the DRUIDIC (Dynamic Reconstruction of Unfolding Industrial Diversity by Interactive Computing) project. The NW models programmed in Maple V can be found in Esben Sloth Andersen homepage (http://www.business.auc.dk/evolution/esa/). Murat Yildizoglu programmed the NW models in Java (http://cournot.u-strasbg.fr/yildi/NelWin.htm). The NW model programmed by Bart Verspagen (http://www.tm.tue.nl/ecis/bart/) accompanies a new textbook written by Verspagen (2001).

[2] Distinguishing a unit of evolution and a unit of selection seems to be important. In biological evolution an organism is a unit of evolution as well as a unit of selection. In 'artificial' evolution (as e.g, knowledge evolution or industrial evolution) we observe a separation of these two aspects of evolutionary processes. The unit of evolution is an individual (knowledge evolution) or a firm (industrial evolution) and the unit of selection is an idea, a theory or a product, a commodity, that is, a result of efforts of the unit of evolution (Kwasnicki, 1994/1996).

coefficient. Changes of these both coefficients are not correlated therefore a phenomenon that resembles substitution between labour and capital may be observed in the simulated process. Search activities are determined by satisfying behaviour, in a sense that a new technique is adopted only if the expected rate of return is higher than the firm's present rate of return. The search process may take two different forms: local search (mutation) or imitation. In the first case, firms search for new techniques, yet not present in the industrial practice. The term *local* search indicates that each undiscovered technique has a probability of being discovered which linearly declines with a suitably defined technological distance from the current technology. Imitation allows particular firm to find techniques currently employed by other firms but not yet used in its own production process. The probability of given technique imitation is proportional to its share in output. It is assumed that if a firm is engaged in search it can use only one type of the search. Selection of actually used type of search is a random event with a fixed probability for each type. An additional source of novelty in the economy is entry by new firms, which also search for innovation. Potential entrant enters the industry if it discovers a production technique which promises a rate of return over 16% but it has still 0.25 probability that it actually enters the market. A value for initial capital stock of entering firm is drawn randomly.

The rate of return on techniques is the main selection force in the NW model. A firm's investment in capital is equal to its profit diminished by a fixed fraction, which depends on paid dividends and capital depreciation. A firm's capital stock shrinks if profit of that firm is negative. Therefore we have second selection force which imposes withdrawing firms from the market if they do not pace of technological progress of its competitors.

To calibrate the above sketched model for the case of the Solow data on total factor productivity for the United States in the first half of the twentieth century it was assumed that firms produce homogenous product named GNP. Using that model, Nelson and Winter address the question whether these time series of the calibrated model correspond in a broad qualitative sense to the ones actually observed by Solow.

The most developed and documented NW model which deals with the evolution of the production techniques and other behavioural rules of an industry producing a homogeneous product is frequently named as "Schumpeterian competition" (Nelson, Winter, 1982, ch. 12; Winter, 1984). A firm is a unit of evolution as well as a unit of selection in that model. As in the formerly sketched model, a number of firms produce single homogenous product. Techniques used by different firms differ in output per unit of capital, i.e. in capital productivity A. All other technique factors, as e.g. return to scale and input coefficient are assumed to be equal for all firms. Technical change (increase of the productivity of capital) takes the form of process innovations and process imitations. Each firm chooses a technique with the highest productivity out of the three possible techniques (i.e. currently used and found through innovative and imitative processes). Probability that firms innovate or imitate depends on R&D funds determined in proportion to the level of physical capital (respectively r_{in}, r_{im}). Profit per unit of capital is calculated by including R&D costs as ordinary cost elements. The maximum investment of a firm depends on current profit plus loans from the banks (calculated in proportion to the profit). The firm's desired investment is determined by the unit costs, a mark-up factor influenced by the market share of the firm, and the rate of depreciation. The investment process has no time-lags. By multiplying the capital stock with the new level of productivity, we have the production capacity of the firms of the industry in next period. Products price is not firm specific but is equal to all firms and flows from the downward-sloping demand function to balance supply and demand. Investment decision of each firm is based on investment function, which depends on the firm's market share, price elasticity of the demand function, firm's unit profit and bank policy. Firms can take credit but there is no procedure of repaying it.

Winter (1984) presents an interesting elaboration of search activity and entry. Firms are partitioned into two types: primarily innovative or imitative. It allows Winter to apply a notion of technological *regime* depending on whether the source of technical progress is external to the firm (e.g., from public scientific knowledge bases) or from firms' own accumulated technological capabilities. These two regimes are named as the *entrepreneurial* and the *routinized*. Specific parameters exogenously impose the type of investigated regime.

Because of stochastic factors related to the process of innovation-imitation search for innovation and nonlinearities of the production-investment equations it is not possible to find analytical solutions of NW models. It is also not possible to find stochastic characteristics of this process, as e.g., average and standard deviation of firms' production. The only way to investigate these models is to use computer simulation techniques of random numbers generation and get estimated values of general stochastic characteristics, or observe peculiarities of any single realisation of the industrial process.

In original model R&D policy of each firm is rigid (in a sense that the percentage of capital spend for innovation and imitation do not change in a course of time). In (Winter, 1984, and in similar way in Verspagen, 2001) R&D policies of each firm adapt accordingly to the long-run performance of the firm – if the performance is above the average performance of the industry the policies are not changed but if it falls below the average performance it is modified (with assumed probability) in such a way that there is tendency toward average R&D industry spending for innovation and imitation.

A firm grows (or shrinks, in terms of its market share and long-run performance index) accordingly to its profit (or loss) gained in each year (instant of time). A firm is withdrawn from the market if its capital fall below assumed minimal capital or if its long-run performance index falls below the assumed value. Firms can imitate and innovate. Improving productivity of capital is the main aim of innovative process.

Phenomena explained: differences between entrepreneurial and routinized regimes. Skewed distribution of firms' size. Diversity of market shares of all incumbents at any moment of time as well variability of the market shares of each firm in a course of time.

Murat Yildizoglu (2001) does interesting extension of the NW model where investment in physical capital and investment in R&D procedures are essentially modified. Firms invest a fraction of their gross profit on R&D (although there is minimal investment imposed, just to 'keep live' R&D potential). There are two types of firms, so called NWFirms (which invest fixed proportion of their profit in R&D) and GenFirms, which learn (using individual genetic algorithm procedure) and adjust their R&D strategies to the conditions of the industry. R&D investment of GenFirms does not pay back immediately and each R&D strategy must be used for many periods before proving its efficiency. The average gross profit rate in assumed learning period gives the fitness of selected strategy. Because of financing R&D activity from current profit, procedure of investment in physical capital is modified in Yildizoglu proposition. Namely, capital investment results directly from the arbitrage of firms between R&D investment and capital expansion – i.e., investment into new capital is equal to the remaining part of profit after financing R&D activity. Yildizoglu has made numerous simulation runs and two general findings can be drawn from the obtained results. First, learning should not be ignored in models of industry dynamics, in other case performance of industries and social welfare would be severely underestimated. Second, learning gives competitive advantage and in general it can be said that learning firms dominate industry.

Modelling of production sphere in extension of the NW model by N. Jonard, and M. Yildizoglu (1998, 1999) is very close to the Nelson and Winter (1982) model but technology space is modified, namely all possible technologies are ordered in sequence and identified with natural numbers. Firms devote resources to the systematic search for new production possibilities and the random outcome of this search belongs to a technological trajectory with an intrinsic productivity level having a shape of logistic equation. In contrary to the NW model, profit gain by each firm can accumulate and can capitalise with assumed interest rate. Each firm devotes a fraction of its cash-flow to R&D activities plus minimal R&D investment necessary to 'keep alive' R&D activities. R&D investment has permanent priority and if there is no possible to finance it from current and cumulated profit a fraction of capital stock is sold to cover this financing. Therefore the capital can be reduced not only by its natural depreciation but also as the result of R&D activity.

Firms can innovate, namely, accordingly to the Poisson process a firm can 'jump' a random number of technologies along assumed trajectory. Average number of steps in the Poisson process depends on R&D funds allocated by each firm to search through innovation. Firms can also imitate, and the probability of imitation depends also on R&D funds. Tendency to imitate neighbouring technologies (i.e. those less distanced) in present in the model.

As the authors write, they strongly depart form Nelson and Winter (1982) investment policy, making it more realistic. The firm can choose between investing into physical capital and investing in financial market (what depends on expected returns on investment). Each firm estimates relationship between physical capital investment and profit (via simple regression) and takes this relationship into account in the investment decision process (by taking into consideration expected profit in future). The firm expands its capacity as long as the return on the capital investment exceeds the return on financial market.

Beside interaction of firms via a standard way through the market price authors also add interaction through so called firms' space, i.e. socio-economic network specifying trading partners, participants to co-operative agreement, proximity in corporate culture or geographical neighbourhood. It is assumed that the interaction structure takes the shape of two-dimensional closed lattice, firms being described by pair of coordinates corresponding to horizontal and vertical location. Imitative draw entails the acquisition of the highest productivity technology in the neighbourhood of given firm. Because of localised learning and limited diffusion, firms are heterogeneous in terms of technologies and productivities. Selection process and nature of innovation are very similar in nature to the original NW model.

Phenomena explained: the way in which localised learning and externalities influence industry dynamics; firms and technologies are highly diversified.

One of the most recent efforts of developing Nelson and Winter proposition of evolutionary modelling is work done by S. Witner, Y. Kaniovsky and G. Dosi (Winter, Kaniovski, Dosi, 1997 and 2000). The authors present a model which is a "baseline not merely in the sense of a standard for comparison, but also as a starting point for future work" (Winter, Kaniovski, Dosi, 1997). In the most general term the model encompasses a stochastic system driven by the persistent random arrival of new firms and a systematic selection process linking investments to realised profitability. In (Winter, Kaniovski, Dosi, 2000) three versions of the model are presented. In the first version competitiveness of any firm is ultimately determined by its capital per unit of output a_i, i.e. inverse of original NW model concept of the productivity of capital A_i. In that version learning (innovation is observed only in entering firms (only newcomers learn to improve the productivity of capital)). The improvement is modelled by a random process with exponential tendency of productivity of capital improvements (i.e. the ratio of productivity improvement has stochastic tendency to decrease in the course of the productivity improvement, a kind of a law of diminishing return).

As in the original model decreasing continuous demand function H impose uniform price for all firms, i.e., $p = H(Q_t)$, where Q_t is total productive capacity of the industry.

Total gross investment per unit of capital for i-th firm is equal to $\lambda H(Q_t) / va_i$ where,

λ – the share of the gross profit which does not leak out as the interest payment and shareholder's dividends (i.e. this parameter is a measure for the propensity to invest);
ν– price per unit of physical capital.

Taking into account the depreciation rate of the capital stock d, at the end of period t capital of firm i is equal to:

$$Q_t^i a_i (1 - d + \frac{\lambda}{\nu a_i} H(Q_t)$$

If an incumbent firm's capital drops below the assumed fraction of minimal capital for entering firm then the firm is withdrawn from the industry (i.e., die, exit).
Therefore we can say that in the first version of the model learning is observed only in entrants and no innovation are made by incumbents. Innovation in this verison is realted only to improvement of productivity of capital. At any period a number of new firm enter the market; for each entrant, level of capital per unit of output is randomly determined on a basis of the highest productivity of capital attainable to newcomers in the industry at time t.

In the second version learning (innovation) concerns only labour productivity. Productivity of capital (capital/output ratio for all firms) is kept at the constant level. Therefore the competitiveness of a firm is determined by variable cost per unit of output. As in the first version incumbents do not innovate, and the only source of innovation are entrants (newcomers) who learn how to improve the productivity of labour (via random process in similar mode as in the first version). Improvement of productivity of labour is visible in decreasing variable cost per unit of labour, m_i. Investment rule in the second version implies that in the end of period t capital of firm i is equal to

$$Q_t^i C (1 - d + \frac{\lambda}{\nu C} \max(H(Q_t) - m_i, 0)),$$

where C is capital per unit of output; $H(Q_t) - m_i$ is naturally the gross profit per unit of output at t for firm i; λ is the share of the gross profit re-invested.

In the third version newcomers are allowed to innovate with respect to both capital and labour efficiencies (the third version is analysed only through simulation study).

The models presented are in fact the NW model simplified to allow for formal representation enabling analytical enquire of its properties. In both version asymptotic or long-run behaviour properties of the industry are theoretically analysed – three theorems, one lemma for the first version and two theorems and one lemma for the second version. Simulation study of both models is also presented. The proposed approach could be considered as a progress in evolutionary modelling of economic processes – at least from the point of theoretical tractability of an evolutionary model. Great majority of evolutionary models is studied only via computer simulation and only a few of them, those rather least complex, are studied analytically to search for their more general properties. The model proposed by Winter, Kaniovski and Dosi describes real phenomena at a satisfactory level and thanks to its proper formal representation it is possible to formulate its long-run properties e.g. in a form of theorems.
Phenomena explained: Skewed firms size distribution. Life time distribution of firms. Persistence of market turbulence (so called turbulence index is a measure of market shares fluctuation of all firms present in the market and entering the market), persistent fluctuations of aggregate characteristics of the industry development (price, production capacity, total production, etc.)

Silverberg-Verspagen models
One distinguished feature of Silverberg and Verspagen (SV) models is that technological progress is embedded in vintage capital. In the model presented in Silverberg (1985) and

(Silverberg et al., 1988) firms are self-financing using their cash and liquid interest bearing reserves. A firm is a unit of evolution but selection acts on products. There are no explicit investment procedures but investment plan of each firm is based on its financial strength. Capital growth depends on the amount of capital capacity scrapped and on a capacity utilisation (there are no constraints on investment by firm's profit or bank rules). Each firm's investment ability governs the realisation of the investment plan (Is it realised partly or in a whole in the best available technology?). Concurrently to the investment process the oldest vintage is continuously scrapped. Textbooks notions of "demand' and "supply" are not present in the model. Instead of it firms behaviour is placed in more realistic spaces of orders, order backlog, delivery delay, rate of capacity utilization, shipment, etc. The current level of production is constrained by firm's maximum capacity and the production of each firm depends on prime unit labour cost (i.e., an average over all capital vintages).

Market share equation, which form fundamental mathematical description of competitive process, is formally identical to the equation first introduced into mathematical biology by R.A. Fisher in 1930 and in last decades is used in a variety of context by Eigen, Schuster, Ebeling, Feistel, and others. The equation differs from most biological applications „in that the competitiveness parameters rather being constant or simple functions of other variables, themselves change over time in complex ways in response to the strategies pursued by firms and feedbacks from the rest of the system". Market share equation has the following form:

$$\frac{df_i}{dt} = a(E_i - \overline{E})f_i$$

where:

f_i – market share of firm i; E_i – competitiveness of products manufactured by firm i, $\overline{E}$ is average competitiveness $\overline{E} = \sum_i E_i f_i$.

The competitiveness is a linear combination of logarithms of price of firm's i products (p_i) and delivery delay (dd_i).

$$E_i = -\ln p_i - b \cdot dd_i$$

Silverberg proposes specific pricing policy, which describes a compromise between strict cost-plus pricing (markup rule), and competitive advantage of a firm (the price increases if the competitiveness is higher then average competitiveness and is reduced otherwise). This represent a compromise between short and long term profitability targets. Each firm adjust its products' price through dynamical markup based on the equation:

$$\frac{dp_i}{dt} = c(p_{ci} - p_i) + d(E_i - \overline{E})$$

where p_{ci} *and* p_i are log of mark up price (on unit prime cost) and log of market price, respectively.

Experience acquired by individual firm during its development can "leak" out and became available to the rest of the industry. Logistic equation describes learning dynamics and through that way internal skill level of each firm evolves.

From some point of view the model describes the process of diffusion of new technology in the case in which a best practice technology is apparent to all agents. Standard methods of investment policy guarantee diffusion of technical progress within the industry. Innovation process exists in a form of learning dynamics embodied through so called "internal skill level" of each firm.

Silverberg model is a set of differential equations with discredited representation of vintage capital in the computer implementation. It is highly nonlinear model, nonlinearity is present in almost all differential equations. The replication equation of Fisher mode is itself nonlinear but also its parameters (i.e. competitiveness) are function of price and delivery delay, which changes in turn are governed by a set of difference equations. The only way to investigate properties of this model is making numerical simulation on the computer.

Phenomena explained by the model relates to diffusion of new technology (best practice technology is apparent to all agents) and "optimal" time to adopt new technology (two technological trajectories representing at any time the maximum productivities attainable in best practice vintages of respective technologies). Both technologies are changing at some rate but the second technology is always absolutely superior in productivity. Superior technology is available at some moment of time.

Similar idea that firms rely on rather simple rules of thumb or routines rather than explicit optimization procedures is applied in models developed by Silverberg, Lehnert and Verspagen (Silverberg, Lehnert, 1993, 1993a; Silverberg, Verspagen, 1994, 1994a, 1994b, 1995, 1995a).

These models can be seen as continuation of the work initiated by Silverberg in 1980s. Although a unit of evolution in this model is a firm but unit of selection is more related to a technology than to a product. In this series of models firms undertake behavioural *imitation* with increasing probability the more unsatisfactory their performance is. Contrary to the former model worked by Silverberg, in the later models stochastic elements are present, namely those related to innovation emergence. The main difference between the Silverberg and Verspagen (1995) model and the ones presented in Silverberg (1985) and Silverberg and Lehnert (1993) is the way in which innovation is endogenized. It is assumed that in each discrete period, firms devote resources (R&D) to the systematic search for new production possibilities (i.e., new types of capital).

Firms must determine how much to spend on R&D in relation to either their profits or their sales. Technical change comes about as a result of the profit-seeking activities of each firm. Therefore, as in almost all evolutionary models growth is endogenized. Such important feature of modern industrial development as increasing returns, spillover and other phenomena known from the economics of innovation are also included in some of these models. Decision problem is considered in the context of *bounded rationality* – firms (decision-makers) have only vague ideas about final consequences of their actions.

All a time a fixed number of firms operate in the economy and the models are constructed around three basic blocks. The first block consists of equations for the rate of capital accumulation, the diffusion of new technologies in the total capital stock of the firms, and the real wage rate. The wage rate depends positively on the unemployment rate (in a fashion of real Philips curve). Each firm has variable number of different types of capital that it utilises to produce a homogenous product. New capital arises from the accumulation of profit (there is possibility to finance a growth of one type capital by the profit gained form 'activity' of the other type of capital). Each type of capital (vintage) is characterised by fixed technical coefficients, c and a (i.e. capital coefficient and labour productivity, respectively). Capital coefficient is assumed to be fixed thorough the economy and time, and labour productivity is assumed to change under the influence of technical progress. Profit rate of given capital is equal to $(1-w/a)/c$.

Share of the labour force employed on each capital stock (called employment share) changes dynamically in such a way that labour employed for each capital stock is proportional to that capital and inverse to the labour productivity. Therefore selection mechanism generates that technologies with above-average labour productivity (i.e., more profitable) engage more employment (their employment shares increase) and concurrently below-average technologies (backward) tend to vanish.

The equations describe how economy evolves with a given set of technologies. Selection takes place either at levels of firms and technologies. The second block describes how new technologies and firms are introduced into the economy. The last block describes the way of influence of the evolving economy and firm learning on the firms' innovative behaviour. Collective learning phenomena are present in this block.

Each firm has a variable number of different types of capital goods utilised in production. Profit is the only source of capital accumulation. Innovation rate depends on R&D funds, which consist of firm-specific portions of profit and sales. Profits gained from different vintages of capital may be redistributed in such a way that more profitable types of capital accumulate even faster and less profitable even slower, than would otherwise be the case.

Basic equations of firms' dynamics describe the share of the labour force employed on each capital stock. Production is assumed to be always equal to production capacity. It is assumed that the ratio between R&D expenditures and R&D labour input is equal to a fraction of the economy-wide labour productivity.

The wage rate is determined by the differential equation following the idea of Phillips curve. Assumed Phillips curve ensures that real wages tend to track labour productivity in the long run. The employment share equation describes how more profitable technologies (in terms of their labour productivity) tend to increase their employment share, concurrently backward technologies tend to vanish. The wage rate equation and the employment share equation form a selection mechanism in the described economy. New technologies are continuously introduced, that implies that all technologies, after an initial phase of market penetration, will be eventually superseded from the production system. New type of capital (vintage) is created each time an innovation occurs. Because of fixed labour productivity of each vintage and increasing of real wages over time it happens that at some stage of development every technology generate negative profits. It is assumed that these losses are financed by an equivalent decrease of the capital stock. It means that losses imply capital scrapping to cover the losses.

Entry of a new firm occurs only as a result of competition and an exit of an incumbent firm. Exit occurs whenever a firm's employment share falls below assumed threshold value. Therefore exit of incumbent firms is completely endogenous and entry only occurs in case of exit, so that the total number of firms is constant.

It can be said that the model describers closed economy with innovating firms, generating technical change through specific learning mechanisms based on two genetic operators, namely mutation and imitation. It is also assumed that the more profitable a firm is, the less likely it will change its strategy by imitating another firm. If a firm has decided to imitate, the probability of selection of another firm to imitate is proportional to its market share in output. If neither imitation nor mutation occur, the firm simply retains its strategy from the previous period.

At each period firms devote R&D resources to search for new production possibilities, represented by new types of capital. Nature of this search process is stochastic. Each time an innovation emerges, new type of assumed quantity of capital is generated with assumed increase of its labour productivity compare to the best practice labour productivity of that firm. Because of stochastic introducing more efficient vintages of capital by competitors, a firm can be withdrawn from the market because of its negative profit (it is assumed that the loses are financed by an equivalent decrease of the capital stock, i.e. capital is scrapped).

Emergence of innovation for each firm is governed by a Poisson process with assumed formula of arrival rate (the arrival rate depends on so called innovation potential function and can be simple linear function of the potential or more sophisticated, non-linear, logistic function).The innovation potential of each firm is determined by this firm R&D funds and average R&D spending (averaged by market share weighting) – i.e., we observe two forms of spillover effect – economy wide and firm-specific. Additionally, innovation potential is augmented by a measure of its distance from the best practice frontier. R&D funds are modified randomly (with assumed distribution) what is called mutation, but also each firm can imitate R&D strategy of other incumbent firms.

Phenomena explained: the bell shape of diffusion function of successive technologies; phases of economic development (e.g. 'mercantilism' or 'industrial revolutions') can be associated with different R&D strategies; convergence of firms' R&D spending to the some pattern through spontaneous process; well identified three stages of industry development (strong concentration of the market, competitive stage with technical change slightly going up, and the evolutionary steady state characterised by rapid technical change and a competitive market); convergence of some indicators (e.g. GDP per capita related to the best country (frontier) – simulated and real variables are characterised by very similar skewed power spectrum density function.

Dosi et al. models

Complex phenomena observed in the SV models, generated, among others, by interrelationship of large number of competing firms, numerous vintages of capital of each firm, and nonlinearities and stochastic factors, cause that analytical tractability of these models is impossible. Therefore the only way to deal with the models is computer simulation. The same can be said about numerous models developed by Giovanni Dosi and his collaborators, e.g., Chiaromonte and Dosi (1993), Dosi *et al*. (1993), Dosi *et al.* (1994), Dosi *et al*. (1995).

An example of the family of these models is the model aimed to explain classical phenomena of skewness of firms' size distribution from an evolutionary point of view (Dosi *et al*., 1993, 1995). They assume that an 'industry' is composed of several 'sectors', each corresponding to particular technological and market regimes. Each 'sector' is composed of 'micro sectors' (i.e., groups of relatively homogenous products or technologies – learning, changes in market shares, entry and exit occur within microsectors). Therefore it can be said that unit of evolution is a microsector and unit of selection is a firm. Each firm is characterised by its age (*a*), size (*s*), and competitiveness (*e*). A firm's size and its competitiveness depend on learning. The dynamic of the markets to which the firm belongs influences also the firm's size. The authors consider two archetypes of industrial development named as 'Schumpeter Mark I' and 'Schumpeter Mark II', additionally they analyse also 'Intemediate Regime'. Differences between those archetypes (regimes) lay mainly in nature of innovation activity.

Competitiveness is positive real number, which reflects the technological and organisational capabilities of each firm. Under 'Schumpeter I' the only sources of innovation are newcomers. Incumbents never learn (i.e. their competitiveness is constant during their lifetime) and a competitiveness of entrants at time (*t*+1) is

$$e_i(t+1) = \bar{e}(t)(1-k+\frac{g}{\lambda_1})$$

$\bar{e} = \sum e_j f_j$ is the average competitiveness of incumbent, weighted with their market shares, g (here and in below equations) is random variable drawn from a Poisson distribution; k and λ_1 are parameters characterising entry barriers and levels of generically available opportunities for innovations).

In 'Schumpeter II' learning by incumbents is highly cumulative and a growth of competitiveness is governed by equations:

$$e_j(t+1) = e_j(t)(1+h_j(t))$$

$$h_j(t+1) = (1+\ln(1+\frac{e_j(t)}{\bar{e}(t)}))\frac{g_I}{\lambda_2}$$

Competitiveness of newcomers (entering firms) changes in similar fashion as in 'Schumpeter I' although with different values of parameters.

In the 'Intermediate Regime' learning of incumbents goes in similar way as entrants and their competitiveness changes accordingly to the equation:

$$e_j(t+1) = e_j(t)(1+\frac{g_I}{\lambda_4})$$

In any discrete period a random number of firms try to enter the market and concurrently firms that do not perform well enough (i.e., their competitiveness is much below the average competitiveness or their market share is very small) are withdraw from the market (i.e., thy exit the industry).

Selection process influences market shares of all firms and the market shares of each firm are governed by a the Fisher type replicator equation:

$$\Delta f_k(t,t+1) = A(\frac{e_k(t)}{\bar{e}(t)} - 1) f_k(t),$$

i.e. market share of a firm grows if its competitiveness is higher then average and falls if is below the average competitiveness in an microsector.

Market size of each microsector is not constant. It is assumed that there is limited lifetime of the microsectors and the size of the microsector changes in a mode of 'the bell shape function' with the maximum at the middle of the cycle. New microsectors are randomly born with assumed distribution of time-span between the birth of new microsectors.

In 'Schumpeter II' firms can be 'multi-product' entities, i.e. can exploit their knowledge across different microsectors via entry and acquisition of incumbent firms.

Phenomena explained: a bell shape of firms' market shares over time, skewed distribution of firm size, industry concentration variance according to different regimes, interfirm asymmetries and turbulence (defined as variability of market share changes of all firms operating on the market: $\sum | f_k(t+1) - f_k(t) |$)

In the Chiaromonte and Dosi (1993) model, a firm is characterised by a single labour coefficient. The pricing strategy is based upon firm-specific demand expectations. Two important features characterise each firm, namely its technological capability (what is made in the form of input coefficients), and economic strategies, which determines how much resources the firm invest in the search for new technologies and what is products' price.

The search process takes place in a two-dimensional space of 'technological paradigms' and labour coefficients. Firms either produce 'machines' (each of which is characterised by a set of coordinates in the two dimensional plane), or they produce consumption goods (to which they need machines as inputs). Therefore we can speak about two-sector economy. The innovation process differs between these two sectors. In the first sector (producing capital goods) the number of R&D workers determines the success of innovation. For given innovation, the new capital good's productivity is drawn randomly. In the consumption goods sector, firms possess a skill level for each available capital good type. Learning process contributes to increasing skill level. This learning process has both public and private features. Correctness of firms' prediction of their skill level is limited. Actual labour productivity is a function of the capital good's characteristics and the firm's skill level. Selection of capital good potentially employed by a firm from consumption sector is based on maximisation 'utility' function involving labour productivity, prices, and the order backlog. Competitiveness of a firm depends on products prices and unfulfilled demand in the previous period (the backlog of orders).

Similarly in (Dosi *at al.*, 1994) model a firm is characterised by a single labour coefficient but the search space is more similar to the one in Nelson and Winter model. Unit of evolution in this model is sector with homogenous products and a unit of selection is a firm. The probability of an innovation depends on R&D employment. Increase of productivity of labour force is possible through imitation and innovation. The probability of innovation (and imitation) depends on investment in search, which is measured by current and lagged number of searching workers (researchers). In case of success of innovation activity a productivity of successful firm is increased with assumed increment rate.

New techniques can be imitated but with search cost. Probability of imitation depends also on a number of workers engaged in imitative search. Learning is 'local' and knowledge is partly tacit, therefore a firm imitate less distanced techniques eager (with higher probability) then far distanced techniques (distance is measured by difference between productivity used by that firm and productivity of technique potentially to be imitated).

Out of these three techniques, i.e., current technique applied by the firm, techniques gained by imitation and innovation, a firm selects technique with maximum productivity.

Labour is the only input in both search and production. Number of workers engaged in search for imitation and for innovation depends on R&D funds allocated by a firm (which is a constant share of firm's previous-period turnover) and current wage level of a country where the firm operates.

Price is determined in two-stage process. First, the 'desired ' price is calculated on a basis of mark-up procedure, in relation to the wage level in the current and the previous period and the firm's productivity coefficient. In the second stage, the actual price variation by each firm on its own domestic market is computed (the price variation depends in logarithmic way on the actual price of the firm, the markup price calculated in the first stage, desired market share and market shares in two previous moments).

Prices and exchange rate determine the competitiveness of each firm in each market, i.e., competitiveness of firm *i* operating on the country *j* and particular market *k* is equal to:

$$E_{ij}^{k}(t) = \frac{1}{p_{ij}(t)} \rho_{j}(t)$$

where ρ_j is the exchange rate of country *j*, and p_{ij} is price of firm *i* in country *j*. Through that way technological competence (labour productivity), wages, pricing rules contribute to competitiveness formation.

A domestic demand of each country depends on total earned wages by workers in that country (i.e., is equal to sum all wages earned by all workers in all sectors of the country).
Demand is distributed among individual producers (both domestic and foreign) accordingly to their relative competitiveness (i.e., calculated as multiplication of market share of given firm on specific market by the overall demand for that product on that market)

It is assumed that firms produce as demand indicate but under the condition that the demand does not exceed calculated from year to year credit-constrained maximum growth of a firm. Market share dynamics of firm *i* in market *k* is calculated using replicator equation in a form:

$$\Delta f_{i}^{k}(t, t+1) = a_{11}\left(\frac{E_{i}^{k}(t)}{E^{k}(t)} - 1\right) f_{i}^{k}(t)$$

where E^k is the average competitiveness and a_{11} is parameter controlling 'selectiveness' of the market.

The market shares (the replication equation) are translated into actual production levels by considering the size of the aggregate market, which is endogenous to the model. The total size of the market is the minimum of aggregate demand and supply. Demand for labour to be employed in production by firm *i* is just the real output divided by labour productivity of the firm.

A firm exit the market (die) if its market share falls below assumed critical level. Dead firms are replaced by new entrants with initial productivity equal to average productivity in the sector in the country where birth occurs, plus a white noise.

Phenomena explained: Preliminary observations of aggregate dynamics and national accounts (as e.g. national income at constant prices, exports, imports, wage dynamic) are isomorphic to the ‘stylised facts’ of real open economies development.

Dosi *et al*. approach is highly bottom-up simulation. The aim of the authors seems to be to start from basic mechanisms of industrial development without making any assumption about possible modelled properties of the system and to obtain the well-known properties (stylised facts) from the co-working of these basic mechanisms of development. Similar assumption was made by Kwasnicki in his model of industrial dynamics (Kwasnicka, Kwasnicki, 1992, Kwasnicki, 1994/1996).

Kwasnicki’s model

This model describes the behaviour of a number of competing firms producing functionally equivalent, but not homogeneous, products. A unit of evolution is a firm and a unit of selection is a product. One of the distinguished features of the model is the coupling of technological development and economic processes. A firm is characterised by a set of routines applied by the firm. In order to improve its position in the industry and in the market, each a firm searches for new routines and new combinations of routines (innovation) to reduce the unit costs of production, increase the productivity of capital, and to improve the competitiveness of its products in the market. Each firm may simultaneously produce products with different prices and different values of the characteristics, i.e., the firm may be a multi-unit operation. Different units of the same firm manufacture products by employing different sets of routines.

Simulation of industry development is done in discrete time in four steps:

(1) Search for innovation (i.e., search for new sets of routines, which potentially may replace the old set currently employed by a firm).

(2) Firms’ decision making process (calculation and comparison of investment, production, net income, profit, and some other characteristics of development which may be attained by employing the old and the new sets of routines. Decisions of each firm on: (a) continuation of production by employing old routines or modernizing production, and (b) opening (or not) of new units).

(3) Entry of new firms.

(4) Selling process (market evaluation (selection) of the offered pool of products; calculation of firms’ characteristics: production sold, shares in global production and global sales, total profits, profit rates, research funds, etc).

Technological change is endogenized and the probability of finding an innovation (new set of routines) depends on the R&D funds allocated to in-house research (‘mutation’) and imitation. There are two types of routines: *active*, that is, routines employed by the firm in its everyday practice, and *latent*, that is, routines which are stored by the firm but not actually applied. Latent routines may be included in the active set of routines at a future time. The set of routines employed by a firm may evolve. There are four basic mechanisms for generating new sets of routines, namely: *mutation*, *recombination*, *transition* and *transposition.*

On the basis of its expectations of future market development and expected decisions of its competitors, each firm decides on the price of its products, investment and the quantity of production which it expects to sell on the market. Inclusion of the element of expectations in the decision making process makes it boundedly rational. Current investment capability and the possibility of credit taking are also considered by each firm. In establishing the product price and future level of production firms take into account short term elements (profit increasing) and long term elements (to be present on the market as long as possible).

The productivity of capital, variable costs of production and product characteristics are the functions of routines employed by a firm. Each routine has multiple, pleiotropic effects, that is, it may affect many characteristics of products, as well as productivity, and the variable costs of production. Similarly, the productivity of capital, unit costs of production and each characteristic of the product can be function of a number of routines (polygeneity).

Attractiveness (competitiveness) of the product on the market depends on the values of the product characteristics and its price. The competitiveness of products with characteristics z and price p is equal to

$$c_i(t) = c_i(t, p, z) = \frac{q(t, z)}{(p(t))^{\alpha}}, \qquad z = (z_1, z_2, \ldots, z_m),$$

where $q(t,z)$ is the technical competitiveness, z a vector of product characteristics, and α price elasticity. Products with better products characteristics and lower price are preferred by consumers. The selection equation of the competition process describes changes of the firms' shares in global output.

The selection equation describing competition among firms (products) in the market has the following form (f_i is the market share of products manufactured by firm i):

$$f_i(t) = f_i(t-1)\frac{c_i(t)}{c(t)},$$

where $c(t)$ is the average competitiveness of products offered for sale,

$$c(t) = \sum_i f_i(t-1)c_i(t).$$

This means that the share (f_i) of firm i is growing if its competitiveness is higher then the average competitiveness and is decreasing if product performance is below the average within the industry. The product competitiveness can be improved by offering technologically better products (what is done through introducing innovations) or by lowering the price.

The model is rooted in the tradition established by Nelson and Winter. The main similarities to the NW model lay in the concept of routines and endogenized innovations. Important departures of this model from the NW model consist of a more realistic concept of innovation covering product and process innovations, diversity of price (there is no uniform price for all firms but firms individually decide about the price), inclusion in the decision making module the concept of agents expectations (on future market behaviour and decisions of other competitors).

Phenomena explained: Textbook properties of industry development (profit and concentration level of the industry; high industry concentration when economy of scale is present, higher profit for expanding market, etc.). Objectives of firms – 'maximizers vs. satisfiers'. Diversity of price and heterogeneity of products; innovation and temporal monopoly of the pioneer firm (Schumpeter's temporary monopoly), skewed distributions of business firms' size, path-dependence and cumulative causation, industry turbulence; industry development along different innovation regimes.

Other models

Firm is a unit of evolution as well as unit of selection in a model worked out by Metcalfe (1993, 1994). The model is based on the Downie (1958) concept of competing firms within evolutionary frame. Growth rate of each firm is proportional to its unit profit, i.e., $g_i = f_i(p - h_i)$, where p is price (given exogenously for the industry), h_i is unit cost level of firm i, and f_i is propensity to accumulate (i.e., the fraction of profit reinvested, divided by the common output:capital ratio). Downie assumed that the propensity to accumulate is common for all firms (in fact he considered a case of two competing firms). Downie identified two reasons for differences in propensity to accumulate, namely differential ability to borrow in the capital market and differential capital:output ratios between the firms. Metcalfe adds managerial differences in the willingness and ability to expand the firm; for example, the economic life of less efficient firms is prolonged if a more efficient firm has the greater capital:output ratio, or is less willing to growth.

Competitive process differs in Metcalfe approach in that it is not one dimensional (as in Downie) but two dimensional, namely associated with f_i and h_i. Such proposition of evolutionary (although it seems that self-organisational would be the better word) description of competitive process leads Metcalfe to numerous proposition of interesting properties of this process (e.g. those related to the famous Fisher principle).

Metcalfe considers more general case then that proposed by Downie (e.g., there is more then two firms). A market share (s_i) of profitable firms ($p>h_i$) is equal:

$$\frac{ds_i}{dt} = s_i(g_i - g_s),$$

where g_s is the aggregate growth rate of output:

$$g_s = \sum s_i g_i = \bar{f}_s(p - \bar{h}_s) - C_s(f,h),$$

$\bar{f}_s = \sum f_i s_i$ is the average propensity to accumulate and $\bar{h}_s = \sum h_i s_i$ is the average level of unit costs. $C_s(f,h) = \sum s_i(f_i - \bar{f}_s)(h_i - \bar{h}_s)$ is the critical covariance between unit costs and propensities to accumulate.

Because of mentioned two dimensionality of competitive process, it is no longer true that a firm with below average unit costs is necessary increasing its market share. By analysing dynamics of the whole process, rather then focusing on individual firms, the author calculates averaged behaviour, expressed, e.g., in terms of rates of change of the moments of the joint distribution of f and h.

There are no decision procedures, values of unit costs and propensities to accumulate are the model's parameters. Selection process takes the form of replicator equation (similar to that proposed by Fisher).

Phenomena explained: changes in the average behaviour in a group of firms are connected in a precise way to measures of the variety in behaviour across that group (analogy to the Fisher's principle). Deeper understanding of patterns of growth and accumulation to which competition is central phenomenon.

A two-population model that comprises a population of firms and a population of consumers, which adapt to and learn about preferences of each other is presented by Paul Windrum and Chris Birchenhall (Windrum, Birchenhall, 1998). The adaptive learning is mediated by the technological designs that are traded in the market. A unit of evolution is a firm and a unit of selection is a product design.

Selection process acts through adaptive learning of both populations. The main feature of the model is that it simulates interaction of a number of consumers (partitioned into a number of types) and producers (firms). In each period the following sequence is repeated:

- consumers allocate their purchases across the firms (out of firms' offer of sale which consist of maximum quantity of goods and their prices);
- population of consumers replicates;
- firms make adjustment of their offer.

Each consumer will attempt to buy the most attractive offer if this offer is better than the option named 'not buying' (if there is no stock available a consumer tries to buy the second best, etc). A consumer of type i is characterised by a quantity of money m_i and a utility function of the form:

$$u_i(x, p) = v_i(m_i - p) + w_i(x),$$

where x is the characteristic vector of a good and p is the price of the good; v is the indirect utility of money that can be obtained in other markets (this function has a constant form); $w(x)$ is the direct utility of consuming the good with characteristic vector x.

Distribution of consumers across the set of types is governed by a form of replicator equation in such a way that proportion (share) of type i consumers grows in proportion to the utility attained by that type (related to the maximum value of the utility at current time t).

After trading, the firms adjust their offer by modifying prices and quantities produced. Price markup and target market shares are fixed in the model so the only task is to adjust the firm's capacity and level of production. Adjustment of the capacity depends on a firm's profits (and losses). Real production is minimum of two values: a firm's capacity and a firm's target level of production. Investment of a firm depends on the wealth of the firm, which is defined as all collected profits (or losses) of that firm during its life.

Firms can innovate, i.e. can modify products characteristics x (each characteristics can mutate with given probability, and if it mutates, characteristics values are randomly changed accordingly to assumed normal distribution). The firm compares the mutated design with the old design and allows the mutated design to be put into production if it is assessed to be improvement. A kind of imitation of existing designs is present in the model, it takes a form of transfer of values of selected number of characteristics from randomly selected firm to that being 'under imitation procedure'. The transfer is accepted if it improves the utility of target consumer type. We can say that the innovation type present in the model is a product innovation embodied in its design.
Phenomena explained: emergence of the dominant design (as explained by the cycle theory of innovation) – but the emergence of a single design within a single niche market is a special case of the more general problem presented in the paper and there are possible another market outcomes. Evolving consumer preferences (ignored in the traditional product life cycle) significantly influence direction of technological innovation.

Andersen (1997) presents a model based on Pasinetti's scheme of the structural economic dynamics of a labour economy with inclusion of an evolutionary, micro-economic foundation. The model describes the evolution of an economic system with a varying number of sectors, each of which is producing a different consumption good. The essence of this model is the assumption that consumers have a hierarchy of goods, and they consume a higher-order good when they are fully provided with the lower-order goods. Labour and knowledge are basic production factors. Innovative process allows firms to increase their productivity with respect to individual goods. Therefore in the long-term perspective labour becomes available for the production of new consumption goods. The hierarchy of goods and the assumption about sequential fulfilment causes the emergence of "technological unemployment, which emerges if goods are not provided to a sufficient degree". Slow productivity development in the production of new goods leads to a slowdown in the overall rate of growth, and it can occur irrespectively of productivity growth in old sectors. To raise long-term growth the concept of "anticipatory R&D" is introduced.

A micro-based simulation model of national economy, which integrates micro activities, was, developed by Gunnar Eliasson (Eliasson, 1985, 1989). The project of micro-to-macro model was initiated in 1975 and was calibrated to describe the development of the Swedish economy. Firms and household are the basic units of the model. It is not a fully evolutionary model, but contains some evolutionary features and Schumpeterian innovative behaviour.

Technical change is introduced at the firm level through new investment. The decisions of firms' managers are mathematically modelled by a search process for proper decisions based on a trail and error procedure. To be closer to reality the principle of 'maintain or improve profit' (MIP) is included in the submodel describing the behaviour of a firm. Long-term investment decisions and short-term production search are also included in the submodel of a firm's behaviour. Long-term economic development primarily depends on the capital market. Investment and growth of potential capacity at the micro level is driven by the difference between the perceived rate of return of the firm and the interest rate.

Another approach to describe innovation processes is proposed by Bruckner, Ebeling, Jimenez Montaño and Scharnhorst (1993). They start from observation of physicists that "relationship between micro- and macro-level descriptions become important and led to questions of fundamental relevance" and that "relatively independent of the nature of the subsystems mainly the manner of their coordination is important for the demonstration of the well-known macroscopic phenomena of spontaneous structure formation." The authors apply general n-dimensional birth-death transition model to describe technological development. It is assumed that firms contain different plants using different technologies. In general term, the system is described by a number of fields (which in a case of technological process are production units used by different firms applying specific technology i). Elementary process of self-reproduction, spontaneous generation, self amplification (i.e. non-linear self-reproduction), sponsoring, error reproduction, cooperative and non-cooperative exchange, spontaneous decline and self-inhibition is a base theoretical concept of the model. Development of the system is described by a Master Equation system defining probability distribution of technologies.

Summary

Last 20 years of evolutionary modelling in economics surely can be named as the period of great radiation. One can find analogy to periods of great species radiation in biological evolution. Two main streams of research, already mentioned, are the Schumpeterian tradition and Agent-based Computational Economics (ACE). Within each stream proliferation of models is enormous but still there is open question if out of these efforts a single paradigm will emerge or a few independent straits of research will coexist.

In Table 1 a summary of comparative analysis of selected Schumpeterian models is presented. There are numerous traits suitable to compare these models, among them are: units of evolution and selection, type of firms modelled, explicitness of presence of decision procedures and investment rules, how selection is modelled, what is a nature of innovation, how price is set, and what kind of products are considered.

In most models firm is a unit of evolution as well as a unit of selection, in some other models product is treated as unit of selection and firm as unit of evolution. Concerning type of modelled firms, there are two approaches: treat a firm as one unit operation (in such case introduced innovation transforms a whole firm) or treat a firm as multi-unit operation. In the second case there are also two methods of modelling: each innovation is associated with new vintage of capital or radical innovation is introduced in a new, relatively small, separate unit (a kind of daughter-unit; incremental innovations transforms 'mother-unit).

Decision procedures (rules) are distinguished feature of human activity and surely are present in all evolutionary models but in many models the presence of this procedures is implicit. In a model with explicit decision rules they relates mainly do R&D process (how much to spend, should R&D financing be rigid or flexible (adaptable, learning), how much spend for innovation (in-house research) and for imitation, etc.). In some models the decisions on level of investment, production and price are explicitly modelled. Sometime they have a rather sophisticated form in which decisions on specific values of investment, production and/or price are made in highly interactive and iterative way instead of rather simple mathematical formulas.

Crucial features of evolutionary models are selection mechanism and innovation nature of modelled process. There is no common way of modelling selection process. In roughly half analysed models selection is modelled using different forms of a replicator equation. In some models the selection goes via price mechanism (price is set as uniform for all firms, the price and different cost of production generate diversified profit which in turn governs investment abilities of firms and through this way generates variety of grow rates of modelled firms). Spectrum of modelled types of innovation is really enormous. Some of them can be identified with traditional process and product innovations, and are related to increase of efficiency of capital use (increase of productivity of capital), diminishing unit cost of production (e.g., labour productivity improvement) or improving products performance (e.g., design improvement). In some other models they have a form of internal skill improvement or simply improvement of ‘abstract’ competitiveness index.

The last features enclosed in Table 1 is price setting and a nature of modelled products. Price can be set exogenously (and keep constant), calculated from marked demand function and set as uniform price for all products of all firms, or price can be set by firms individually and can be firm specific. Price setting procedure is closely related with type of products modelled. If products are homogenous the price is usually uniform for all firms but if there is heterogeneity of products (and they differ e.g. in their technical characteristics) then price is firm specific (diversity of price accompanied with heterogeneity of products are observed, what makes that the models are closer to reality).

There is diversity of phenomena explained by different models. This characteristic is not presented in Table 1 but some of them were indicated in the paper in the end of description of each model.

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Table 1. Features of selected neo-schumpeterian models (a summary)

Model (in an order of years of publications)	**Unit of evolution/ unit of selection Firm type**	**Decision rules/ investment rules**	**Selection**	**Nature of innovation**	**Price**	**Product**
(Nelson, Winter, 1982) (Winter, 1984)	firm/firm one unit firm	investment function; R&D funds (rigid and/or flexible)	through price mechanism; investment constraints lead to capital grows or scrapping	productivity of capital improvement innovation/imitation	uniform for all firms (via marked demand function)	homogenous
(Silverberg, 1985) (Silverberg, Dosi, Orsenigo, 1988)	firm/product vintage capital firm	no explicit investment procedures	replicator equation	firms' internal skill level improvement	firm specific, heterogeneous (dynamical mark-up)	partly heterogeneous (products differ in price and delivery delay)
(Kwasnicka, Kwasnicki, 1992, Kwasnicki, 1994/1996).	firm/product multi-unit firm	investment, production and price evaluated in interactive way	replicator dynamics	products' technical improvement, variable cost reduction, capital productivity improvement	firm specific, heterogeneous	heterogeneous (technical characteristics and price)
(Silverberg, Lehnert, 1993, 1993a) (Silverberg, Verspagen, 1994, 1994a, 1994b, 1995, 1995a)	firm/firm vintage capital firm	R&D investment (learning procedure)	through efficient use of employment (employment share of more efficient firms increase increases)	labour productivity improvement	no explicit price of products	'hidden' homogenous product
(Metcalfe, 1993, 1994)	firm/firm one unit firm	no explicit decision rules	replicator dynamics	no explicit innovation process	constant, exogenous price	no product
(Dosi *et al*., 1993, 1995)	microsector/firm multi-unit firm	no explicit decision procedures	"quasi-replicator dynamics"	improvement of competitiveness index	no explicit price of products	'hidden' homogenous product
(Dosi *at al*., 1994)	sector/firm multi-unit firm	R&D investment and its structure (innovation vs.	replicator dynamics	improving labour productivity; innovation and	heterogeneous price	homogenous product (but with different price)

		imitation)		imitation		
(Windrum, Birchenhall, 1998)	firm/product design one unit firm	no explicit decision rules	Selection through adaptive learning of producers and consumers	design characteristic; improvement innovation and imitation	constant, exogenous price	heterogeneous product design
(Yildizoglu, 2001)	firm/firm one unit firm	R&D rigid and learning (using GA) Investment – simplified NW rule	through price mechanism; investment constraints lead to capital grows or scrapping	productivity of capital improving innovation/imitation	uniform for all firms (via marked demand function)	homogenous
(Winter, Kaniovski, Dosi, 1997, 2000).	firm/firm one unit firm	simple investment rule via current profit gained	through price mechanism; investment constraints lead to capital grows or scrapping	capital productivity or labour productivity improvement (entering firms only)	uniform for all firms	homogenous