Competitive markets and "as if" methodology

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Abstract

Two results showing the limitations of the "as if" methodology are proved under relatively mild assumptions. In an interpretation of the results, a competitive market cannot simulate the outcome of a market M in which the single price assumption does not hold. In a second interpretation, the market *M* resulting from the aggregation of a finite number of competitive markets is not competitive. In both cases there is no ground to sustain the fiction that *M* operates as if it were competitive.

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1. Introduction

The term "'as if' methodology" (AIM) has come to define the methodological position expressed in Friedman (1953). For a critical review of Friedman's position, see Caldwell (1980; 1982, Ch. 8); for a defence, see Boland (1979; 1982, Ch. 9) and http://www.sfu.ca/~boland/summation.PDF.

In essence, the basic principle of AIM is that everything should be subordinated to the goal of obtaining a successful prediction; for a recent justification of AIM, see Nordberg and Røgeberg (2003). Friedman (1953) himself admits restrictions to the application of AIM: while considering the problem of determining the effect on retail prices of cigarettes of an increase in the federal cigarette tax, he ventures to predict that "broadly correct results will be obtained by treating cigarette firms as if they were producing an identical product and were in perfect competition", whereas he contends this presumption to be a false guide under the existence of price controls.

Motivated by this example, the aim of this note is to make apparent stronger limitations of AIM when attempting the "simulation" or "replication" of the outcomes of a non-competitive market through a competitive market. In fact, as in Friedman's example, textbooks at an elementary and even intermediate levels typically apply the competitive market model to the analysis of real markets that are far from being competitive. In addition, the outcome of a competitive market (market equilibrium) is also typically justified, even in more advanced textbooks, by resorting to mechanisms inconsistent with the competitive market structure.

For instance, Mas-Colell, Whinston and Green (1995 p. 315) justify equilibrium in a competitive market using arguments that violate the price-taking assumption. Similarly, Kreps (1990, p. 264) suggests four conditions describing how a market operates in order to define a competitive market. Implicit in those conditions is the presumption that each seller is able to set the price at which he is willing to exchange. This presumption is in principle at odds with the fact that a single price regulates all the exchanges in a competitive market. It is nonetheless claimed that the conditions ensure the existence of a single price in the market. In both cases, AIM seems to be implicitly in use.

Those considerations, and the nature itself of AIM, point to the problem of ascertaining the extent to which links between competitive and non-competitive markets can be consistently established. This note deals with what appears to be a favourable case for the application of AIM: a real market for a certain commodity is assumed to be accurately represented by a non-competitive market M which differs from a competitive market essentially only in that the single price assumption fails. This appears to be the situation Friedman considers in his example, as he invokes a competitive market to explain what happens to retail *prices* in an actual market. The two results in this note show that, under relatively mild conditions, it is not possible for a competitive market m to consistently replicate the outcomes of M because there is no way to translate the information these outcomes express in terms that are meaningful in m. Section 2 defines the framework in which the results are obtained; these are stated and proved in Section 3; and briefly discussed in Section 4.

2. Framework

Let Ω be a non-empty set of states and $N = \{1, ... , n\}$, with $n \geq 2$, a finite set whose members designate producers-sellers of a given commodity C in a certain non-competitive market M (true representation of a real market). For each $i \in N$, there are functions $p_i : \Omega \rightarrow \Re^+$ and $q_i : \Omega \rightarrow \Re^+$, with $\Re^+$ being the set of non-negative real numbers. The interpretation is that, in state $\omega \in \Omega$, $i \in N$ produces $q_i(\omega)$ units of C and sets price $p_i(\omega)$ for each one of those $q_i(\omega)$ units of C. In trying to conceptualize M as close as possible to a competitive market, it is presumed that producers believe M to be competitive but, having incomplete information, do not know with full accuracy which is the market price. Thus, in state ω, producer i resorts to available information to compute an estimate of what he believes to be the market price; once he estimates price $p_i(\omega)$, he merely ascertains what is the amount of production $q_i(\omega) = s_i(p_i(\omega))$ that his (standard) supply function s_i indicates that must be produced at price $p_i(\omega)$.

For the purposes at hand, this framework can be simplified to a structure $(N, D, (s_i)_{i\in N})$, where $D \subseteq \Re^+$ is a domain of possible prices and, for $i \in N$, $s_i : D \rightarrow \Re^+$ is a standard individual supply function. The structure $(N, D, (s_i)_{i\in N})$ is supposed to provide true data from the working of M. Specifically, at a given state of the world, M generates two observable n-tuples, a vector price $(p_1, \ldots , p_n)$ and a supply vector $(s_1(p_1), \ldots , s_n(p_n))$. AIM is then invoked to associate with M a competitive market m in such a way that a correspondence can be established between the outcomes of M and those of m. Since M and m can be viewed as generating information in different languages, the correspondence defines the translation rules between languages. Without such a correspondence, it is not possible to determine whether treating M as if it were m makes m yield "broadly correct" predictions in M: if the increase of a tax in M raises some prices but lowers some others, it is necessary some rule to clarify whether this means an increase or a reduction in “the” price in order to compare the result with what occurs in m, where there is a unique price.

The first problem is then to define a merging function $P : D^n \rightarrow \Re^+$ that constructs a price level $P(p_1, \ldots , p_n)$ for M from the corresponding vector price $(p_1, \ldots , p_n)$ in M. If m is assumed to simulate or replicate M (or, simply, be the “as if” counterpart of M) then this P would actually embody the way to associate price vector $(p_1, \ldots , p_n)$ from M with price $p = P(p_1, \ldots , p_n)$ in m. But once prices from M and m have been linked, a second problem emerges: everything determined by prices (quantities supplied) should also be linked in a way consistent with the price link. In particular, it is necessary to define a second merging function $S : \text{rang}(P) \rightarrow \Re^+$ such that $S(P(p_1, \ldots , p_n)) = \sum_{i\in N} s_i(p_i)$, where $\text{rang}(P) = \{x \in \Re^+\text{: for some } \xi \in D^n, P(\xi) = x\}$. Just as P defines a single price for M (the one supposed to be replicated by m), S defines the (hypothetical) market supply function that associates with price level $P(p_1, \ldots , p_n)$ the total amount of production generated under price vector $(p_1, \ldots , p_n)$, that is, the amount $\sum_{i\in N} s_i(p_i)$.

Accordingly, the links between M and m are at least two. On the one hand, when M generates the price vector $(p_1, \ldots , p_n)$, m must generate the price $P(p_1, \ldots , p_n)$. On the other, when M generates the output vector $(s_1(p_1), \ldots , s_n(p_n))$, m must generate the same total output $\sum_{i\in N} s_i(p_i)$ and make it equal to the corresponding total output $S(P(p_1, \ldots , p_n))$.

In this interpretation, it is possible to treat M as if it were m provided there are such functions $P : D^n \rightarrow \Re^+$ and $S : \text{rang}(P) \rightarrow \Re^+$. It is through P and S that the “as if” fiction operates, acting as a procedure to translate M into m or, more precisely, to make m capable of simulating or replicating outcomes of M. Hence, the way M forms prices and determines the total supply could be simulated by a competitive market, so that, under P and S, it could be argued that M operates as if there were a single price.

In this respect, it could be claimed that there exists a competitive “rationalization” of the non-competitiveness of M: the departure from m consisting in the existence of several prices for the commodity can be accommodated through AIM by defining P and S. The existence of such functions would make m pass a “robustness” test: the “perturbed” market M obtained from the competitive market m by dropping the single price assumption remains, in a way, competitive. This robustness interpretation would be related to results like the one showing that a competitive market can be thought of as the limit of certain oligopolistic markets; see Mas-Colell, Whinston and Green (1995, pp. 411-412).

3. Results

There are plausible requirements to be imposed on P, S and the functions s_i. Proposition 3.1 suggests four such conditions and shows that no P and S can be found that satisfy all of them, together with the primary "as if" condition $S(P(\xi)) = \sum_{i\in N} s_i(\xi_i)$ for all $\xi \in D^n$.

Condition (i) is a unanimity property: for all $p \in D$, $P(p, \ldots, p) = p$. This is reasonable under the presumption that P selects a representative price for every price vector, for in the case in which all producers set the same price p, p itself appears to be the best candidate to represent the price vector $(p, \ldots, p)$.

Condition (ii) means that it is not always worth while to produce: for every producer there is some price inducing the producer to leave the market (he does not produce).

Condition (iii) is in part symmetric with respect to (ii) and expresses the existence of competitors: there is some price inducing at least two producers to be in the market.

Finally, condition (iv) holds that if, for some price vector $\xi \in D^n$, only one producer is in the market then this producer's price should count as "the" market price. The justification is that when there is only one producer serving the market, the prices the rest of potential producers would have liked to receive is irrelevant information to define a representative price for the commodity. For $i \in N$, $x \in D$ and ξ of D^n, define (x^i, ξ_{-i}) to be the member of $\zeta \in D^n$ such that $\zeta_i = x$ and, for all $j \in N\backslash\{i\}$, $\zeta_j = \xi_j$.

Proposition 3.1. Let $N = \{1, \ldots, n\}$, with $n \geq 2$, and $D \subseteq \Re^+$ be non-empty sets. For all $i \in N$, suppose there are functions $s_i : D \to \Re^+$, $P : D^n \to \Re^+$ and $S : \text{rang}(P) \to \Re^+$. If

(i) for all $p \in D$, $P(p, \ldots, p) = p$

(ii) for every $i \in N$ there is $p \in D$ such that $s_i(p) = 0$

(iii) there are $p \in D$, $i \in N$ and $j \in N\backslash\{i\}$ such that $s_i(p) \neq 0$ and $s_j(p) \neq 0$ and

(iv) for all $i \in N$ and $\xi \in D^n$, if $s_j(\xi_j) = 0$ for all $j \in N\backslash\{i\}$ and $s_i(\xi_i) \neq 0$ then $P(\xi) = \xi_i$

then it is not the case that

(v) for all $\xi \in D^n$, $S(P(\xi)) = \sum_{i\in N} s_i(\xi_i)$.

Proof. Assume (i)-(v). By (iii), let $p \in D$, $i \in N$ and $j \in N\backslash\{i\}$ satisfy $s_i(p) \neq 0 \neq s_j(p)$. By (ii), there is $\zeta \in D^n$ such that, for all $k \in N$, $s_k(\zeta_k) = 0$. By (v), $S(P(p^i, \zeta_{-i})) = \sum_{k\in N\backslash\{i\}} s_k(\zeta_k) + s_i(p)$. By (iv), $P(p^i, \zeta_{-i}) = p$. Hence, by (ii), (iii) and (iv), $S(p) = S(P(p^i, \zeta_{-i})) =$

$\sum_{k\in N\setminus\{i\}} s_k(\zeta_k) + s_i(p) = s_i(p) \neq 0$. By (i), $S(p) = S(P(p, \ldots, p))$. As $S(p) = s_i(p)$, by (v), $S(P(p, \ldots, p)) = \sum_{k\in N} s_k(p)$. Thus, $s_i(p) = \sum_{k\in N\setminus\{i\}} s_k(p) + s_i(p)$, so $\sum_{k\in N\setminus\{i, j\}} s_k(p) + s_j(p) = 0$. Since, for all $k \in N$ and $q \in D$, $s_k(q) \geq 0$, it follows that $s_j(p) = 0$, contradicting $s_j(p) \neq 0$.■

Proposition 3.2 states another impossibility result when the following three conditions are added to "$S(P(\xi)) = \sum_{i\in N} s_i(\xi_i)$ for all $\xi \in D^n$". Condition (i) is, as in Proposition 3.1, the unanimity principle. Condition (ii) requires from P to restrict its values to the set D of prices that producers can set. The converse, $D \subseteq \text{rang}(P)$, follows from the unanimity principle. By condition (ii), every price that could count as a price set by some producer can also count as a market price and vice versa. Finally, condition (iii) asserts that there is some price in D encouraging some producer to produce and, moreover, that either (a) there is a second producer producing for some price or (b) that the total supply function does not coincide with the individual supply of some producer, so that no producer can always be identified with the total supply of the market. Condition (iii) again expresses the existence in some case of a minimal amount of competition in the market.

Proposition 3.2. Let $N = \{1, \ldots, n\}$, with $n \geq 2$, and $D \subseteq \Re^+$ be non-empty sets. For all $i \in N$, suppose there are functions $s_i : D \to \Re^+$, $P : D^n \to \Re^+$ and $S : \text{rang}(P) \to \Re^+$. If

(i) for all $p \in D$, $P(p, \ldots, p) = p$
(ii) $\text{rang}(P) = D$ and
(iii) there are $q \in N$ and $\xi \in D^n$ such that $s_q(\xi_q) \neq 0$ and
  (a) either there are $k \in N\setminus\{q\}$ and $\zeta \in D^n$ with $s_k(\zeta_k) \neq 0$
  (b) or there is no $k \in N$ such that $s_k = S$

then it is not the case that

(iv) for all $\xi \in D^n$, $S(P(\xi)) = \sum_{i\in N} s_i(\xi_i)$.

Proof. Assume (i)-(iv). By (iii), let $q \in N$ and $\xi \in D^n$ satisfy

$$s_q(\xi_q) \neq 0. \tag{1}$$

Choose $j \in N\setminus\{q\}$ and $\zeta \in D^n$ such that $\zeta_q = \xi_q$. With $z := \zeta_j$, let $f : D \to \text{rang}(P)$ be the function such that $f(x) = P(x^j, \zeta_{-j})$. Clearly, $P(\zeta) = f(z)$. By (i), $S(f(z)) = S(P(f(z), \ldots, f(z)))$. By (iv), $S(P(f(z), \ldots, f(z))) = \sum_{i\in N} s_i(f(z))$. By (iv), $S(f(z)) = S(P(\zeta)) = \sum_{i\in N\setminus\{j\}} s_i(\zeta_i) + s_j(z)$. Hence, $\sum_{i\in N} s_i(f(z)) = \sum_{i\in N\setminus\{j\}} s_i(\zeta_i) + s_j(z)$. Set $\alpha := \sum_{i\in N\setminus\{j\}} s_i(\zeta_i)$, so

$$s_j(z) = \sum_{i\in N\setminus\{j\}} s_i(f(z)) + s_j(f(z)) - \alpha. \qquad (2)$$

By (ii), $f(z) \in D$. When $f(z)$ replaces z, the preceding reasoning leads to conclude that

$$s_j(f(z)) = \sum_{i\in N\setminus\{j\}} s_i(f^2(z)) + s_j(f^2(z)) - \alpha \qquad (3)$$

where $f^2(z) = f(f(z))$. Inserting (3) into (2) yields $s_j(z) = \sum_{i\in N\setminus\{j\}} [s_i(f(z)) + s_i(f^2(z))] + s_j(f^2(z)) - 2\alpha$. By (ii), $f^2(z) \in D$ and the same reasoning yields $s_j(z) = \sum_{i\in N\setminus\{j\}} [s_i(f(z)) + s_i(f^2(z)) + s_i(f^3(z))] + s_j(f^3(z)) - 3\alpha$. Since, for all $r \geq 1$, $f^r(z) \in \text{rang}(P)$, it follows that, for all $r \geq 1$, $s_j(z) = g(z, r) - r\alpha$, where $g(z, r) := \sum_{1\leq t\leq r} \sum_{i\in N\setminus\{j\}} s_i(f^t(z)) + s_j(f^r(z))$. Given that $s_j(z)$ is a number independent of r, the limit of $g(z, r) - r\alpha$ as r goes to infinity is the constant value $s_j(z)$. If $\alpha = 0$ then $0 = \alpha = \sum_{i\in N\setminus\{j\}} s_i(\zeta_i)$; in particular, since the values of each s_i are non-negative, $s_k(\zeta_k) = s_k(\xi_k) = 0$, contradicting (1). If $\alpha \neq 0$ then, in view of (1), the sequence $\{r\alpha\}_{r\geq 1}$ approaches infinity, so the limit of $g(z, r) - r\alpha$ as r tends to infinity must be 0. Since both $j \in N\setminus\{k\}$ and $\zeta_j = z$ were arbitrary, for all $j \in N\setminus\{q\}$ and $x \in D$, $s_j(z) = 0$. This contradicts (a). If it is (b) that holds then, by the preceding result, (i) and (iv), for all $x \in D$, $S(x) = S(P(x, \ldots, x)) = \sum_{i\in N} s_i(x) = s_q(x)$, contradicting (b).■

4. Comments

Propositions 3.1 and 3.2 identify two situations in which the way a non-competitive market *M* operates cannot be replicated or simulated by a competitive market *m* and, therefore, one cannot have recourse to the fiction that it is possible to deal with markets like *M* as if it were competitive. In addition, the non-existence of *P* and *S* seems to remove the possibility of a non-competitive interpretation of the way a competitive market is supposed to work: without *P* and *S*, there does not appear to be room to contend that *M* operates as if there were a single price determining total supply.

The same formal results can be interpreted in a different context and for a different problem. Suppose the aim is to ascertain whether *n* competitive markets for the same commodity can, from a theoretical point of view, be treated as if they constituted a unique competitive market. In this case, members of the set *N* could designate geographically scattered competitive markets for the same commodity. For $i \in N$, p_i would be the equilibrium price in competitive market *i* and $s_i(p_i)$ would be the total supply in market *i* at market price p_i. The question is then whether those *n* markets can be merged to form one competitive market that reproduces the result of aggregating the outcomes of the *n* markets. This problem is formally identical to the one described in

Section 2: the construction of the summarizing market would require a single price $P(p_1, \ldots, p_n)$ and a competitive supply function S such that $S(P(p_1, \ldots, p_n)) = \sum_{i\in N} s_i(p_i)$. Propositions 3.1 and 3.2 establish conditions under which an all-embracing competitive market cannot represent independent competitive markets. Under the assumed conditions, these results make it difficult to sustain the fiction that the market resulting from the aggregation of competitive markets operates as if it were competitive.

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