The Role of R&D Technology in Asymmetric Research Joint Ventures

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Abstract

We characterize asymmetric equilibria in two-stage process innovation games and show that they are prevalent in the different models of R&D technology considered in the literature. Indeed, cooperation in R&D may be accompanied by high concentration in the product market. We show that while such an increase may be profitable, it may be socially inefficient.

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1 Introduction

Because the economics of R&D offer a congregation point for most forms of market imperfections, be they large fixed costs, uncertainty, or externalities, the design of optimal policies becomes a delicate exercise. In particular, the conflict between marginal cost pricing and the free-rider problem associated with R&D spillovers calls for a policy that can restore the firms' incentives to engage in R&D while simultaneously maintaining a competitive environment.

Solutions to the externality problems associated with R&D, such as Research Joint Ventures (RJV), have been pioneered by the seminal works of d'Aspremont and Jacquemin (1988) and Kamien, Muller, and Zang (1992).[1] In this literature, firms invest in R&D in a first stage and engage in Cournot or Bertrand competition in a second stage. Under RJVs, firms maximize *joint* profits, thereby alleviating free-riding by internalizing externalities.[2] As a consequence, RJVs dominate independent R&D and lead to higher social surplus, so long as knowledge spillovers are sufficiently large. Salant and Shaffer (1998), however, show that RJVs also dominate independent R&D when spillovers are *small,* provided the previous models' arguably artificial imposition of symmetry –where each firm engages in the same amount of R&D and produces the same amount of output– is dropped. Salant and Shaffer's result may surprise, since the socially desirable outcome is accompanied by potentially severe concentration in the product market. Is this a robust result or does it hinge on the R&D technology?

Two distinct R&D technologies, that is, mappings from R&D investment levels into cost reductions, are prevalent in the theoretical literature. Amir (2000) shows that R&D technologies can be characterized by the nature of R&D spillovers: either spillovers are in R&D inputs, that is, in R&D investments (as in In Kamien, Muller, and Zang), or they are in R&D output (as in d'Aspremont and Jacquemin), that is, in knowledge created.

[1] The large subsequent literature includes Suzumura (1992), De Bondt, Slaets, and Cassiman (1992), Vonortas (1994), Martin (1996), Leahy and Neary (1997), Salant and Shaffer (1998, 1999), Amir and Wooders (1998, 2003), Long and Soubeyran (1999, 2001), Amir (2000), Cabral (2000), and Martin (2002). For excellent surveys, see De Bondt (1997) and Martin and Scott (1998).

[2] D'Aspremont and Jacquemin consider the scenario where the firms participating in an RJV do *not* actively share R&D output. Kamien, Muller, and Zang (1992), on the other hand, introduced the concept of RJV *cartels*, where firms not only maximize joint profits, but also share all research output. These RJV cartels have the additional benefit of solving the duplication problem.

Salant and Shaffer assume that spillovers are in output (as did d'Aspremont and Jacquemin). Moreover, the R&D production function takes a specific functional form: a square-root function. Indeed, Salant and Shaffer (p. 207) do recognize that "the welfare changes associated with increased concentration remain to be determined in more general formulations."

In this paper, we revisit Salant and Shaffer's result for both types of spillovers and for a wider class of concave R&D technologies, extending beyond the square-root function. We do so by constructing economies for which higher concentration in the product market does allow for a more efficient allocation of R&D resources between firms and greater joint profits, yet implies a loss of social welfare. In other words, profit maximizing concentration in R&D and social welfare maximizing concentration in R&D do not generally coincide.

Moreover, we show that for output-side spillovers, asymmetry is more likely to occur when spillovers are small and the cost reduction function is not too concave. For input-side spillovers, on the other hand, asymmetry is more likely to be optimal when spillovers are large. This implies that the two models generate different implications for various R&D scenarios, a result in the spirit of Amir (2000), who shows that the two types of R&D technology lead to different policy recommendations regarding subsidies and R&D cooperation. In particular, he shows that joint labs are desirable for R&D cooperatives when spillovers are in input, while they are to be avoided when spillovers are in output. Our results are thus consistent with a growing literature analyzing the effect of different R&D technologies on R&D incentives.

2 The Main Result

The R&D technology specified by d'Aspremont and Jacquemin as well as Salant and Shaffer corresponds to a square-root cost-reduction function with output-side spillovers:

$$r(x_1, x_2) = \sqrt{x_1} + \beta\sqrt{x_2}, \tag{1}$$

where x_1 and x_2 represent the firms' investment levels and $\beta \in [0, 1]$ is the spillover parameter. We justify expression (1) as follows: simply let the cost of an *amount* y_1 of R&D undertaken by firm 1 be $C(y_1)$, with C strictly convex and $C(0) = 0$ and define $f(\cdot) \equiv C^{-1}(\cdot)$. If $x_1 \equiv C(y_1)$ denotes the amount of *dollars* invested in R&D by firm 1, then the cost reduction of firm 1 can be written as $r_1 = f(x_1) + \beta f(x_2)$, where f is strictly concave since C is strictly convex. Thus, if $C(y_1) = y_1^2 \equiv x_1$, as in d'Aspremont and Jacquemin's original paper, we obtain $r_1 = \sqrt{x_1} + \beta\sqrt{x_2}$. Here, however, our specification will be a more general:

$$r(x_1, x_2) = f(x_1) + \beta f(x_2). \tag{2}$$

Our setup is as follows: a given total dollar amount x is to be allocated between an industry's two firms,[3] with a share δ going to the R&D lab of firm 1 and the share $1-\delta$ going to the second firm's lab. (Without any loss in generality, we shall confine our attention to $\delta \geq 1/2$.) The marginal costs of production of firms 1 and 2 are then given by

$$\begin{cases} c_1(\delta) = A - f(\delta x) - \beta f((1-\delta)x) \\ c_2(\delta) = A - f((1-\delta)x) - \beta f(\delta x) \end{cases} \tag{3}$$

where A is the constant marginal cost in the absence of any R&D and the cost reduction function f is twice differentiable, increasing, and concave. Assume furthermore that $c_i(\delta) \geq 0$, for all $\delta \geq 1/2$. Finally, let the inverse demand function be given by $P = a - (Q_1 + Q_2)$, where P is the market price and Q_i is the quantity produced by firm i.

Then at the Cournot equilibrium, $P = (a + c_1 + c_2)/3$, $Q_i = (a - 2c_i + c_j)/3$, firm i's profit $\pi_i = (a - 2c_i + c_j)^2/9$, $i \neq j$, and consumer surplus CS=$(2a - c_1 - c_2)^2/18$. Define social welfare W as the sum of consumer surplus and joint profit. It is easy to verify that consumer surplus, profits, and welfare reach critical points at $\delta = 1/2$.

We are interested in cases where W is globally maximized at the symmetric allocation, but joint profit is not.[4] It can be shown that concavity of W at $\delta = 1/2$, i.e., $\frac{\partial^2 W}{\partial \delta^2} < 0$, is equivalent to

$$4.5\left(\frac{1-\beta}{1+\beta}\right)^2 < \frac{-f''(\frac{x}{2})(f(\frac{x}{2}) + \frac{a-A}{1+\beta})}{(f'(\frac{x}{2}))^2}. \tag{4}$$

[3] We are thus not restricting our attention to some optimal total R&D investment.

[4] It may be instructive to note that if joint profit is not maximized at the symmetric allocation, then no single *asymmetric* allocation can simultaneously maximize both joint profits and total surplus. Since $\frac{\partial W}{\partial \delta} = \frac{\partial \pi}{\partial \delta} + \frac{\partial CS}{\partial \delta}$, $\frac{\partial \pi}{\partial \delta} = 0$ for some δ^* implies $\frac{\partial W}{\partial \delta} = \frac{\partial CS}{\partial \delta}$. Since consumer surplus is strictly concave in δ and reaches a *global maximum* at $\delta = 1/2$, then for all $\delta > 1/2$, $\frac{\partial CS}{\partial \delta} < 0$. Therefore $\frac{\partial W}{\partial \delta} \neq 0$ at δ^*.

Therefore, if

$$4.5\left(\frac{1-\beta}{1+\beta}\right)^2 < \frac{-f''(\frac{x}{2})f(\frac{x}{2})}{(f'(\frac{x}{2}))^2}, \tag{5}$$

then inequality (4) holds for all $a > A$. Note that inequality (5)'s right-hand side can be interpreted as a measure of curvature of the cost-reduction technology. Joint profit, on the other hand, reaches a local *minimum* at $\delta = 1/2$ if $\frac{\partial^2(\pi_1+\pi_2)}{\partial\delta^2} > 0$, i.e. if,

$$\frac{-f''(\frac{x}{2})(f(\frac{x}{2})+\frac{a-A}{1+\beta})}{(f'(\frac{x}{2}))^2} < 9\left(\frac{1-\beta}{1+\beta}\right)^2. \tag{6}$$

It follows that if

$$\frac{-f''(\frac{x}{2})f(\frac{x}{2})}{(f'(\frac{x}{2}))^2} < 9\left(\frac{1-\beta}{1+\beta}\right)^2, \tag{7}$$

then there exists *an* $a > A$ such that joint profit is *not* maximized at the symmetric allocation of R&D resources.

In summary, if we can find a cost-reduction technology satisfying inequality (5), then total welfare has a local maximum at $\delta = 1/2$ for all $a > A$, and there exists an $a > A$ such that joint profits are *not* maximized at the symmetric allocation. (Of course, in the examples we analyze below, we check that the local maximum is also a global one.)

In the case of the literature's commonly assumed (or implied) square-root cost-reduction function, for example, conditions (5) and (7) simplify to

$$4.5\left(\frac{1-\beta}{1+\beta}\right)^2 < 1 < 9\left(\frac{1-\beta}{1+\beta}\right)^2$$

i.e.,

$$\frac{3-\sqrt{2}}{3+\sqrt{2}} = 0.359 < \beta < \frac{1}{2}.$$

For each economy, we can easily construct examples where firms' preferences for a lop-sided investment into R&D and higher concentration in the product market is detrimental to social welfare. Figure 1 shows joint profit, consumer surplus (dashed line), and social welfare (thick line) as functions of allocation parameter δ, when $\beta = 0.4$ and $a = A + 3$.[5]

[5]For better visualization, the functions are scaled to share the same value at $\delta = 1/2$.

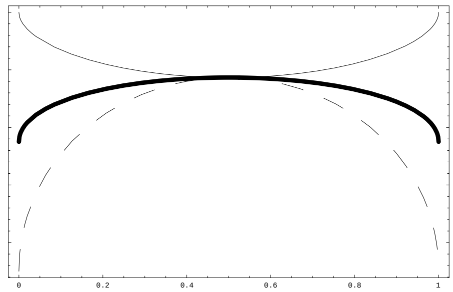

Figure 1: Joint Profit and Welfare as a function of δ

We have, so far, explored the R&D technology originally assumed by d'Aspremont and Jacquemin and further investigated by Salant and Shaffer, namely cost-reduction with output-side spillovers. In contrast, when spillovers are in R&D inputs, as assumed by Kamien, Muller, and Zang, for example, the constant marginal cost functions can be written as

$$\begin{cases} c_1 = A - f(\delta x + \beta(1-\delta)x), \\ c_2 = A - f((1-\delta)x + \beta\delta x), \end{cases} \tag{8}$$

where f is also increasing and concave.

Here, too, we can construct examples where a symmetric allocation maximizes total welfare but fails to maximize joint profits. We find that concavity of total surplus at the symmetric allocation is satisfied if

$$4.5 < \frac{-f''(\alpha)(f(\alpha) + a - A)}{(f'(\alpha))^2}, \tag{9}$$

where $\alpha = \frac{x(1+\beta)}{2}$, whereas profit is convex at the symmetric allocation if

$$\frac{-f''(\alpha)(f(\alpha) + a - A)}{(f'(\alpha))^2} < 9. \tag{10}$$

Therefore, if

$$4.5 < \frac{-f''(\alpha)f(\alpha)}{(f'(\alpha))^2}, \tag{11}$$

there exists an $a > A$ such that both inequalities (10) and (11) are satisfied.

Note that if the cost-reduction function f takes the square-root form, inequality (10) cannot be satisfied, since

$$\frac{-f''(\alpha)f(\alpha)}{(f'(\alpha))^2} = 1 < 4.5.$$

On the other hand, for the class of cost-reduction functions $f(y) = y^z$, $0 < z \leq 1$, the measure of concavity is

$$\frac{-f''(\alpha)f(\alpha)}{(f'(\alpha))^2} = \frac{1-z}{z}.$$

Inequalities (7) and (9) are therefore satisfied if

$$\frac{1}{10} < z < \frac{2}{11}.$$

An example is illustrated in Figure 2, where joint profit, consumer surplus (dashed line), and social welfare (thick line) are plotted as functions of allocation parameter δ; the parameters are $z = 0.15$, $\beta = 0.3$ and $a = A + 1$. Joint

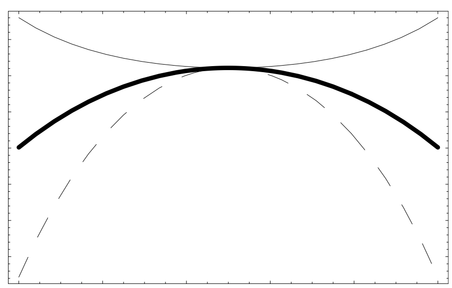

Figure 2: Joint Profit and Welfare as a function of δ

profit is maximized at an asymmetric allocation, but welfare is maximized at the symmetric one.

3 Comparative Statics

As suggested by inequalities (7) and (11), the results depend on the curvature of the cost-reduction function. For purposes of illustration, we will confine our attention to the class of power functions $f(\alpha) = \alpha^z$, $0 \leq z \leq 1$; then $f'(\alpha) = z\alpha^{z-1}$, $f''(\alpha) = z(z-1)\alpha^{z-2}$, $f'(\alpha)^2 = z^2\alpha^{2z-2}$, and

$$\frac{-f''(\alpha)f(\alpha)}{(f'(\alpha))^2} = \frac{1-z}{z}.$$

The less the cost-reduction function is concave, the smaller is the first-stage efficiency loss associated with an asymmetric allocation of R&D resources. For instance, in the case of full spillovers in output, the gain from a *symmetric* allocation of a \$2 R&D investment would be

$$\left(1^{.5} + 1^{.5}\right) - \left(2^{.5} + 0^{.5}\right) = 0.59$$

if we assume a square-root cost-reduction function, but would be a lower

$$\left(1^{.8} + 1^{.8}\right) - \left(2^{.8} + 0^{.8}\right) = 0.26$$

if we assumed a less concave $f(x) = x^{.8}$.

Intuitively, then, when the cost-reduction function is sufficiently concave and spillovers are sufficiently large, firms do best by allocating their R&D resources symmetrically; but as spillovers become smaller, the focus of Salant and Shaffer's analysis, or when the cost-reduction function becomes less concave, a symmetric allocation may no longer be profit-maximizing. Eventually, with further decreases in concavity and spillovers, a symmetric allocation may not even be *welfare* maximizing. This insight is shown in Figure 3. On the other hand, when spillovers are in R&D input, we observe the opposite effect: as shown in Figure 4, the optimality of a *symmetric* allocation of R&D resources becomes increasingly likely as spillovers are smaller.

4 Ceteris Paribus?

The objective of the RJV literature has been to evaluate the combined benefits from product competition and from internalization of R&D spillovers.

One may thus wonder what share of the benefits to RJVs can be credited to research consolidation and what share of the benefits can be credited to the restructuring of the output sector.

With input-side spillovers, it is clearly wasteful to have more than a single plant and a single lab. To see why, we look at the optimal allocation of an investment of x *dollars* in R&D and the optimal allocation of an arbitrary quantity of output Q between the two firms so as to minimize the cost of producing Q. If firm 1 invests δx and firm 2 invests $(1-\delta)x$ *dollars* in R&D ($\delta \in [0,1]$), then the marginal costs of firms 1 and 2 are given by equations (8). Without loss of generality, let $\delta \geq 1/2$; this implies that $c_1(\delta) \leq c_2(\delta)$ since

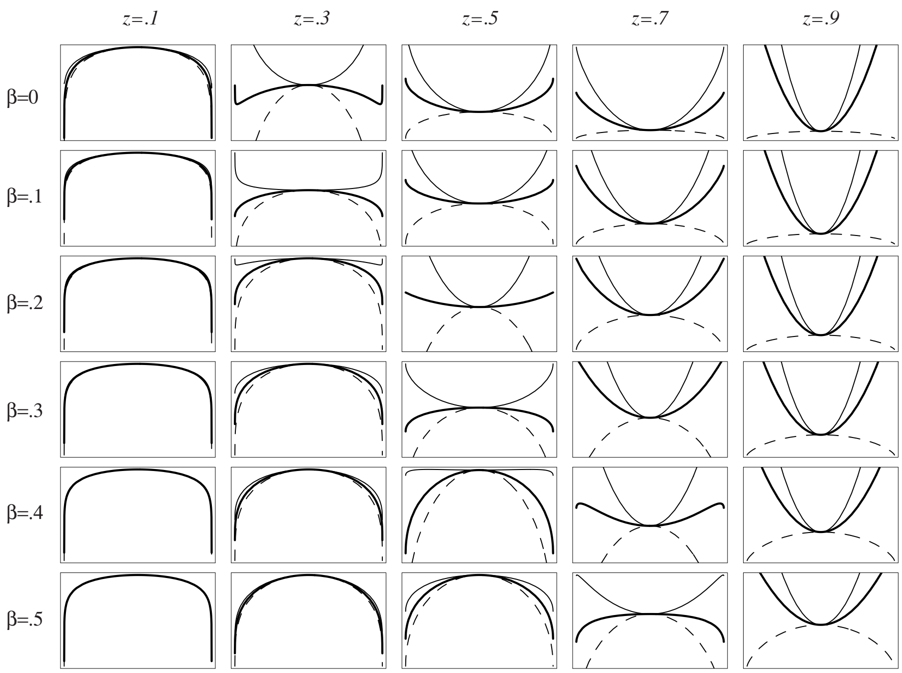

Figure 3: Output-side Spillovers, $a = A + 3$

$\beta \leq 1$. Therefore, the total cost of producing an arbitrary quantity Q of output is minimized at $\delta = 1$:

$$
\begin{aligned}
& \min_{\frac{1}{2} \leq \delta \leq 1} c_1(\delta) \cdot \left(\frac{Q}{2} + k(\delta) \right) + c_2(\delta) \cdot \left(\frac{Q}{2} - k(\delta) \right) \\
& \text{where} \quad 0 \leq k(\delta) \leq Q/2 \quad \text{since} \quad \delta \geq 1/2 \\
= \; & \min_{\frac{1}{2} \leq \delta \leq 1} c_1(\delta) \cdot Q \quad \text{since} \quad c_1(\delta) \leq c_2(\delta) \\
= \; & \min_{\frac{1}{2} \leq \delta \leq 1} (A - f(\delta x + \beta(1 - \delta)x)) \cdot Q \\
= \; & (A - f(x)) \cdot Q
\end{aligned}
$$

With output-side spillovers, on the other hand, the concavity of f implies an interior solution in general. While it still makes sense to only have a single plant, both labs would get positive, though generally uneven funding, as long as spillovers are sufficiently large: the reason lies in the tradeoff between duplication of R&D investment levels and diminishing returns to R&D expenditures. If the spillovers are small, however, then the social cost

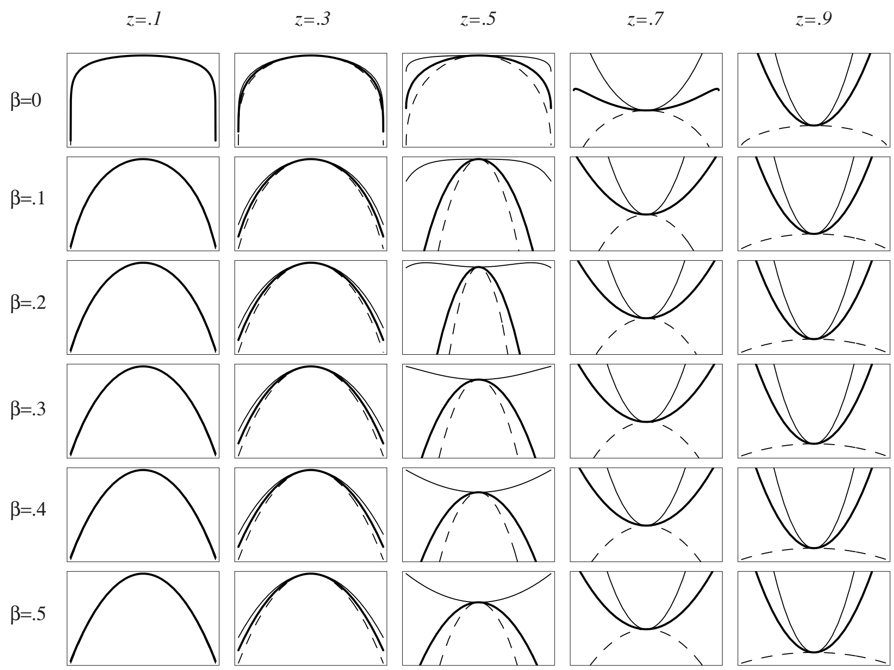

Figure 4: Input-side Spillovers, $a = A + 30$

of producing any level of output is minimized by having a single lab in the R&D sector and a single plant at the output stage. (We prove this claim in the appendix.)

As shown in Figure 5, the socially optimal allocation δ^* of investments among labs is decreasing in the degree of spillovers β. Therefore, the smaller β, the more R&D investments will be allocated to a preferred lab over all others. At one extreme, for β sufficiently low, all resources go to a single lab; at the other extreme, with full spillovers ($\beta = 1$), all labs are treated equally.[6]

5 Conclusion

This paper is a contribution to the strand of literature dealing with the comparison of different R&D technologies and knowledge spillovers. We have

[6] Amir and Wooders (1999) explore the role of the spillovers rate on intra industry heterogeneity.

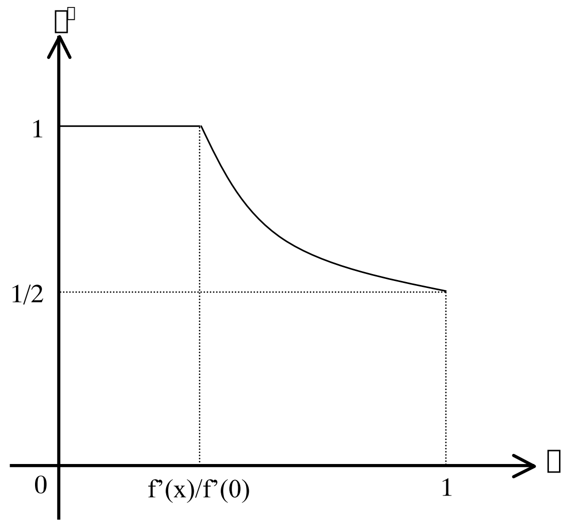

Figure 5: Optimal Allocation of R&D Investment

shown how the literature's two main models of R&D technology lead to different conclusions with respect to the optimal allocation of R&D and the resulting concentration in the product market. Moreover, profit and welfare maximixing allocation of R&D do not generally coincide.

At this point, it is useful to remark that we have been comparing and contrasting two polar technologies, even though they may not necessarily be mutually exclusive modeling choices. A new literature is emerging in which both types of R&D technologies are treated simultaneously. Martin (2002), for example, develops a model with stochastic innovation that includes both input *and* output side spillovers and investigates the effect of input side spillovers and the degree of appropriability of R&D output on R&D incentives.[7]

In conclusion, it is critical that policy makers carefully assess the nature of R&D spillovers in an industry before adopting any policy on research joint ventures or R&D subsidies.

[7]He shows that a successful innovator's payoff increases with greater appropriability. However, in industries where input side spillovers are low, firm value is maximized when appropriability is low.

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Appendix

Proof of Claim in Section 4:
If firm 1 invests δx and firm 2 invests $(1-\delta)x$ *dollars* in R&D ($\delta \in [0,1]$), then the marginal costs of firm 1 and 2 are given by

$$\begin{cases} c_1(\delta) = A - f(\delta x) - \beta f((1-\delta)x) \\ c_2(\delta) = A - f((1-\delta)x) - \beta f(\delta x) \end{cases} \tag{12}$$

Without loss of generality let $\delta \geq 1/2$; this implies that $c_1(\delta) \leq c_2(\delta)$. Therefore, the total minimum cost of producing an arbitrary quantity of output Q, $C(Q)$, is obtained by

$$\begin{aligned} & \min_{\frac{1}{2}\leq\delta\leq 1} c_1(\delta)\cdot\left[\frac{Q}{2}+k(\delta)\right]+c_2(\delta)\cdot\left[\frac{Q}{2}-k(\delta)\right] \\ = & \min_{\frac{1}{2}\leq\delta\leq 1} c_1(\delta)\cdot Q \quad \text{since} \quad c_1(\delta)\leq c_2(\delta) \\ = & \min_{\frac{1}{2}\leq\delta\leq 1} [A-f(\delta x)-\beta f((1-\delta)x)]\cdot Q. \end{aligned}$$

To minimize $c_1(\delta)$, we maximize cost-reduction $R \equiv f(\delta x)+\beta f((1-\delta)x)$. Note that the second order condition is satisfied:

$$\frac{d^2R}{d\delta^2} = x^2\left(f''(\delta x)+\beta f''((1-\delta)x)\right) < 0 \quad \text{since } f'' < 0.$$

As for the first-order condition, we have

$$\frac{dR}{d\delta} = x\left(f'(\delta x)-\beta f'((1-\delta)x)\right).$$

If $\frac{dR}{d\delta} > 0$ for any δ, then $\delta = 1$ maximizes R. This happens if $f'(\delta x) > \beta f'((1-\delta)x)$ for any δ. A sufficient condition for this is

$$\min_{\frac{1}{2}\leq\delta\leq 1} f'(\delta x) > \max_{\frac{1}{2}\leq\delta\leq 1} \beta f'((1-\delta)x).$$

Since f' is decreasing,

$$\min_{\frac{1}{2}\leq\delta\leq 1} f'(\delta x) = f'(x).$$

Similarly,

$$\max_{\frac{1}{2}\leq\delta\leq 1} \beta f'((1-\delta)x) = \beta f'(0).$$

Hence if $\beta \leq \frac{f'(x)}{f'(0)}$, then $\delta = 1$ maximizes R. Thus, for sufficiently small spillovers, it is cheaper to have one firm invest in R&D in the first stage and a single firm produce the total output in the second stage. Similarly, if $f'(\delta x) < \beta f'((1-\delta)x)$ for any δ, then $\delta = \frac{1}{2}$ maximizes R. A sufficient condition for this is

$$\begin{aligned} \max_{\frac{1}{2}\leq\delta\leq 1} f'(\delta x) &< \min_{\frac{1}{2}\leq\delta\leq 1} \beta f'((1-\delta)x) \\ \text{or } f'\left(\frac{x}{2}\right) &< \beta f'\left(\frac{x}{2}\right). \end{aligned}$$

But this is not possible, since $\beta \leq 1$. Hence for $\beta > \frac{f'(x)}{f'(0)}$, an interior solution δ^* exists and satisfies the first order condition $f'(\delta^* x) = \beta f'((1-\delta^*)x)$ or $\beta = \frac{f'(\delta^* x)}{f'((1-\delta^*)x)}$. Note that $\frac{f'(\delta x)}{f'((1-\delta)x)}$ is decreasing in δ since $f'' < 0$. It follows that δ^* is decreasing in β. If $\beta \leq \frac{f'(x)}{f'(0)}$, then $\delta^* = 1$. If $\beta > \frac{f'[x]}{f'[0]}$, then $\delta^* < 1$ and reaches a minimum $\delta^* = \frac{1}{2}$ when $\beta = 1$. This implies that for large spillovers, it is cheaper to have two firms invest in R&D in the first stage and one firm produce the total output in the second stage. ■