NETWORKS AND MARKET LAWS

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Abstract

The effects on the market price of a change in the number of buyers, in the number of sellers, in the prices sellers set and in the buyers' reservation prices are established in a simple network model of a market.

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1. Introduction

The standard course of introductory microeconomics still hinges on Marshall's (1966 [1920], pp. 276-291) modern version of the supply and demand model. The fact that the direction of price changes can be easily predicted in this model (Figs. 24-29 in Marshall (1966 [1920], pp. 385-386)) probably explains its success as a teaching model. For the standard case of increasing supply function and decreasing demand function, the so-called "market laws" hold that if the only change is an increase (decrease) in demand (supply) then the market price increases and if the only change is a decrease (increase) in supply (demand) then the market price decreases.

As a teaching model, the supply and demand model suffers nonetheless from some significant shortcomings. One of them is the lack of a convincing dynamic explanation of the underlying market mechanism. Specifically, it is difficult to reconcile the fact that the market price changes with the textbook conventional interpretation of the model according to which no agent in the market sets or alters the market price. In connection with this difficulty, there is the unrealistic premise that the market price is unique. When adopting this assumption, Marshall (1966 [1920], p. 284) himself was concerned with the extent to which it was "in accordance with the actual facts of life".

Network models of buyers and sellers provide an alternative to the supply and demand model that can cope with the above difficulties; see, for instance, Kranton and Minehart (2000, 2001) and Corominas-Bosch (2004). Though such models are more complex than the supply and demand model, they are conceptually and descriptively more satisfactory. Since there are grounds to believe that this type of model could replace the supply and demand model as the reference market model for teaching purposes, it is worth developing simple network models illustrating how this sort of model can compete with the suply and demand model at the teaching level.

The aim of this note is to suggest one such model with the purpose of stating results analogous to the "market laws" in the supply and demand model. The suggested model adopts an extremely simple network perspective: the commodity is homogeneous, buyers buy just one unit of the commodity, each seller sets his selling price and each buyer has a reservation price, which determines the buyer's network of potential sellers. The novel aspect of the model is the definition of "the" market price: it is associated with each buyer the average of prices in his network (understood as an expected price) and next the market price is defined as the average of the buyers' average price, so the market price is an aggregation of subjective prices rather an objective entity.

2. Model

Let a market for a certain commodity consists of a set S of $s \geq 1$ sellers and a set B of $b \geq s$ buyers. Buyer i's reservation price is $r_i > 0$, whereas the price set by seller j is $p_j > 0$. Each buyer is assumed to be willing to buy just one unit of the commodity. For $i \in B$, define $S_i := \{j \in S\colon r_i \leq p_j\}$ to be the set of sellers from which buyer i could buy the unit he is willing to buy. It is assumed that no reservation price is smaller than the minimum price. Consequently, for all $i \in B$, $S_i \neq \emptyset$. Since each buyer i is supposed to be equally likely to buy from any seller in S_i, define $\pi_i := \sum_{j \in S_i} \frac{p_j}{|S_i|}$ to be the price i expects to pay in order to obtain one unit of the commodity, where $|G|$ denotes the number of members of a finite set G. This presumes that each buyer i chooses one seller $j \in S_i$ (with every such seller having the same probability of being chosen) and buys the unit of the commodity from the chosen seller. Finally, by attributing the same weight to each subjective price π_i, let $P := \sum_{i \in B} \frac{\pi_i}{b}$ be the average price representing the expected price that a buyer chosen at random is expected to pay to obtain one unit of the commodity. This P will represent the <u>market price</u>. If some shock changes the price from P to P', define $\Delta P := P' - P$. The results presented in Section 3 (see Section 4 for the proofs) determine the effects on P of changes in: (i) the number of buyers; (ii) the number of sellers; (iii) a buyer's reservation price; and (iv) a seller's price.

To illustrate the preceding definitions, consider the market with $B = \{1, 2, 3, 4\}$ and $S = \{1, 2, 3\}$ such that, for $i \in B$, $r_i = i$ and, for $j \in S$, $p_j = j$. In this case, $S_1 = \{1\}$, $S_2 = \{1, 2\}$, $S_3 = S_4 = S$, $\pi_1 = 1$, $\pi_2 = 3/2$, $\pi_3 = \pi_4 = 2$ and $P := (\pi_1 + \pi_2 + \pi_3 + \pi_4) / 4 = 1.625$.

3. Results

Choose any ordering $(p_1, p_2, \ldots, p_s)$ of the set of prices $\{p_j\colon j \in S\}$ such that $p_1 \leq p_2 \leq \ldots \leq p_s$. Consider the sequence $(a_1, a_2, \ldots, a_s)$, where, for $t \in \{1, \ldots, s\}$, $a_t := (a_1 + a_2 + \ldots + a_s) / t$. With a_t being the first member in the sequence $(a_1, a_2, \ldots, a_s)$ such that $a_t > P$, define $r^+ := p_t$. The value r^+ corresponds to the price p_j which is closest from above to the average price P (in the example of Section 2, $r^+ = 2$.). The corresponding seller j could be considered the "average" seller when prices close to P from above are regarded as better approximations to P than prices close to P from below.

Proposition 1. <u>Buyers entering and leaving the market</u>. (a) If $k \notin B$ is a new buyer entering the market with reservation price r_k then $\Delta P > 0$ if, and only if, $r_k \geq r^+$, where r^+

is computed before k enters. (b) If $k \in B$ is a buyer leaving the market then $\Delta P < 0$ if, and only if, $r_k \geq r^+$, where r^+ is computed before k leaves.

By Proposition 1(a), when an additional buyer enters the market, the market price P does not diminish if, and only if, the buyer's reservation price is smaller than the reservation price generating the subjective price that is closest to P from above. Therefore, the incorporation of a new buyer will cause an increase in the market price if, and only if, his reservation price is sufficiently high. This conforms with the corresponding result in the supply and demand model: a buyer entering the market will not affect the market price unless the maximum price he is willing to pay is sufficiently high (in particular, above the equilibrium market price). Similarly, by Proposition 1(b), for a buyer leaving the market to cause a reduction in the market price his reservation price has to be sufficiently high (observe that causing a reduction in P by leaving the market is equivalent to causing an increase in P by entering the market).

Proposition 2. Sellers entering and leaving the market. (a) If $k \notin S$ is a new seller entering the market with price p_k and $C := \{i \in B\text{: } r_i \geq p_k\}$ then $\Delta P < 0$ if, and only if, $p_k < \dfrac{\sum_{i \in C} \frac{\pi_i}{|S_i|+1}}{\sum_{i \in C} \frac{1}{|S_i|+1}}$. (b) If, for all $i \in B$, S_i has at least two members, $k \in S$ is a seller leaving the market and $C := \{i \in B\text{: } r_i \geq p_k\}$ then $\Delta P > 0$ if, and only if, $p_k < \dfrac{\sum_{i \in C} \frac{\pi_i}{|S_i|-1}}{\sum_{i \in C} \frac{1}{|S_i|-1}}$.

Proposition 2(a) states that, for the addition of a new seller to lower the market price, the seller's price should be sufficiently low. Proposition 2(b) expresses a symmetric result: a seller leaving the market will not cause an increase in the market price unless the seller's price was sufficiently small. Those results also conform to the supply and demand logic: if a new seller starts supplying for a sufficiently high price (above the equilibrium price), his inclusion does not produce any effect on the equilibrium price.

Proposition 3. Sellers modifying their reservation prices. (a) $\Delta P \geq 0$ if the reservation price of a buyer increases. (b) $\Delta P \leq 0$ if the reservation price of a buyer decreases.

Proposition 3 is interesting in presenting clear-cut results: if a buyer i is willing to pay more then his subjective price π_i will never diminish (since his set S_i either remains the same, if the new reservation price does not allow i to buy from another seller, or is enlarged, if i can buy from another seller) and therefore the market price will accordingly not diminish. Inversely if a buyer is willing to pay less.

Proposition 4. Buyers modifying their prices. If seller k's price changes from p_k to p'_k then define $\Delta p_k := p'_k - p_k$ and $\nabla p_k := p_k - p'_k$. (a) Suppose that, for all $i \in B$, S_i has at least two members. If seller $k \in S$ increases his price from p_k to p'_k, $C := \{i \in B: p'_k > r_i \geq p_k\}$ and $D := \{i \in B: r_i \geq p'_k\}$ then $\Delta P > 0$ if, and only if, $\Delta p_k > \dfrac{\sum_{i \in C} \frac{p_k - \pi_i}{|S_i| - 1}}{\sum_{i \in D} \frac{1}{|S_i|}}$. (b) If seller $k \in S$ reduces his price from p_k to p'_k, $C := \{i \in B: p'_k \leq r_i < p_k\}$ and $D := \{i \in B: r_i > p_k\}$ then $\Delta P < 0$ if, and only if, $\nabla p_k < \dfrac{\sum_{i \in C} \frac{p'_k - \pi_i}{|S_i| + 1}}{\sum_{i \in D} \frac{1}{|S_i|}}$.

The subjective definition of market price adopted in this paper leads to the apparently paradoxical results of Proposition 4: an increase (reduction) in a seller's price will not raise (lower) the market price unless the increase (reduction) is sufficiently high (small). The reason is that a change in a seller's price causes two opposite effects on P. Consider, for instance, a reduction in seller k's price p_k. On the one hand, this reduction induces lower subjective prices π_i on those buyers that could previously buy from k (namely, the set of buyers that belong to D) because they can now buy from k at a lower price. But, on the other hand, there are those buyers (represented by the set C) that could not buy from k before the reduction in p_k but could buy from k after the reduction. For these buyers, it could be that the new price p'_k, despite being smaller than the initial p_k, is higher than their previous average subjective price, for which reason the new subjective price could raise. The net result on P will depend on the relative strength of these two effects. This relative strength in his turn will depend on the magnitude of the change in the seller's price. Proposition 4 identifies the magnitude of the changes that guarantee one result or another.

As an example to illustrate the two effects on P of a reduction in a seller's price, let $S = \{1, 2, 3\}$, with $p_1 = 1$, $p_2 = 2$ and $p_3 = 4$. Suppose that two buyers, 1 and 2, are such that $r_1 = 4$ and $r_2 = 3$. If p_3 is reduced from 4 to 3, then buyer 1's expected price π_1 is reduced from 7/3 to 2, whereas buyer 2's is raised from 3/2 to 2.

4. Proofs

Proof of Proposition 1. (a) Let P be the average price before k enters the market and P' the price afterwards. It is clear that $\Delta P > 0$ if, and only if, $\pi_k > P$. Since $\pi_k := \frac{\sum_{j\in S_k} p_j}{|S_k|}$, if $r_k \geq r^+$, by definition of r^+, $\frac{\sum_{j\in S_k} p_j}{|S_k|} > P$, so $\pi_k > P$; and if $r_k < r^+$, by definition of r^+, $\frac{\sum_{j\in S_k} p_j}{|S_k|} \leq P$ and, hence, $\pi_k \leq P$. (b) Let P be the average price before k enters the market and P' the price afterwards. Since $\Delta P < 0$ if, and only if, $\pi_k > P$, the result follows from the fact that $r_k \geq r^+$ implies $\pi_k > P$ and the fact that $r_k < r^+$ implies $\pi_k \leq P$.□

Proof of Proposition 2. (a) If P is the average price before k enters the market and P' the price afterwards, $P' < P \Leftrightarrow \frac{1}{b}\left(\sum_{i\in B\setminus C}\pi_i + \sum_{i\in C}\frac{p_k + \sum_{j\in S_i} p_j}{|S_i|+1}\right) < \frac{1}{b}\left(\sum_{i\in B}\pi_i\right) \Leftrightarrow \sum_{i\in C}\frac{\sum_{j\in S_i} p_j}{|S_i|+1} + p_k\sum_{i\in C}\frac{1}{|S_i|+1} < \sum_{i\in C}\pi_i \Leftrightarrow \sum_{i\in C}\frac{|S_i|}{|S_i|+1}\frac{\sum_{j\in S_i} p_j}{|S_i|} + p_k\sum_{i\in C}\frac{1}{|S_i|+1} < \sum_{i\in C}\pi_i \Leftrightarrow \sum_{i\in C}\frac{|S_i|}{|S_i|+1}\pi_i + p_k\sum_{i\in C}\frac{1}{|S_i|+1} < \sum_{i\in C}\pi_i \Leftrightarrow p_k\sum_{i\in C}\frac{1}{|S_i|+1} < \sum_{i\in C}\frac{\pi_i}{|S_i|+1}$. (b) If P is the average price before k leaves the market and P' the price afterwards, $P' > P \Leftrightarrow \frac{1}{b}\left(\sum_{i\in B\setminus C}\pi_i + \sum_{i\in C}\frac{-p_k + \sum_{j\in S_i} p_j}{|S_i|-1}\right) > \frac{1}{b}\left(\sum_{i\in B}\pi_i\right) \Leftrightarrow \sum_{i\in C}\frac{|S_i|}{|S_i|-1}\frac{\sum_{j\in S_i} p_j}{|S_i|} - p_k\sum_{i\in C}\frac{1}{|S_i|-1} > \sum_{i\in C}\pi_i \Leftrightarrow \sum_{i\in C}\frac{|S_i|}{|S_i|-1}\pi_i - \sum_{i\in C}\pi_i > p_k\sum_{i\in C}\frac{1}{|S_i|-1} \Leftrightarrow p_k\sum_{i\in C}\frac{1}{|S_i|-1} < \sum_{i\in C}\frac{\pi_i}{|S_i|-1}$.□

Proof of Proposition 3. (a) Let P be the average price before k's reservation price increases, P' the price afterwards and similarly for π_k, π'_k, S_k and S'_k. Clearly, $\Delta P > 0 \Leftrightarrow \pi'_k > \pi_k$. With $|S'_k - S_k| = n$, since $S_k \subseteq S'_k$ and $\pi'_k := \sum_{j\in S'_k}\frac{p_j}{|S'_k|} = \sum_{j\in S_k}\frac{p_j}{|S_k|+n} + \sum_{j\in S'_k\setminus S_k}\frac{p_j}{|S_k|+n} = \frac{|S_k|}{|S_k|+n}\sum_{j\in S_k}\frac{p_j}{|S_k|} + \sum_{j\in S'_k\setminus S_k}\frac{p_j}{|S_k|+n} = \frac{|S_k|\pi_k}{|S_k|+n} + \frac{\sum_{j\in S'_k\setminus S_k} p_j}{|S_k|+n}$, it follows that $\pi'_k > \pi_k \Leftrightarrow \frac{\sum_{j\in S'_k\setminus S_k} p_j}{|S'_k - S_k|} > \pi_k$. That is, the average of the prices p_j such that $r_k < p_j \leq r'_k$ (representing the new sellers from which k can buy after the increase in his reservation price) is greater than his initial average price π_k. But $\pi_k \leq r_k$ and, for each p_j with $j \in S'_k$

$- S_k$, $p_j > r_k$. Consequently, any increase in r_k such that $S'_k \neq S_k$ yields $\frac{\sum_{j\in S'_k \setminus S_k} p_j}{|S'_k - S_k|} > \pi_k$ and, accordingly, $\Delta P > 0$. Conversely, $\Delta P > 0$ requires an increase in r_k such that $S'_k \neq S_k$, which guarantees $\frac{\sum_{j\in S'_k \setminus S_k} p_j}{|S'_k - S_k|} > \pi_k$. Finally, it is clear that an increase in r_k such that $S'_k = S_k$ is equivalent to $\Delta P = 0$. (b) The proof is analogous to the one in case (a).□

Proof of Proposition 4. Let P be the market price before the change in p_k and P' the price after the change. (a) $P' > P \Leftrightarrow \frac{1}{b}\left(\sum_{i\in B\setminus(C\cup D)}\pi_i + \sum_{i\in C}\frac{\sum_{j\in S_i\setminus\{k\}} p_j}{|S_i|-1} + \sum_{i\in D}\frac{p'_k + \sum_{j\in S_i\setminus\{k\}} p_j}{|S_i|}\right) > \frac{1}{b}\left(\sum_{i\in B}\pi_i\right) \Leftrightarrow \sum_{i\in C}\frac{p_k - p_k + \sum_{j\in S_i\setminus\{k\}} p_j}{|S_i|-1} + \sum_{i\in D}\frac{p'_k - p_k}{|S_i|} + \sum_{i\in D}\frac{p_k + \sum_{j\in S_i\setminus\{k\}} p_j}{|S_i|} > \sum_{i\in C\cup D}\pi_i \Leftrightarrow \sum_{i\in C}\left(\frac{|S_i|}{|S_i|-1}\right)\left(\frac{-p_k + \sum_{j\in S_i} p_j}{|S_i|}\right) + \sum_{i\in D}\frac{\Delta p_k}{|S_i|} + \sum_{i\in D}\frac{\sum_{j\in S_i} p_j}{|S_i|} > \sum_{i\in C\cup D}\pi_i \Leftrightarrow \sum_{i\in C}\frac{|S_i|\pi_i}{|S_i|-1} - \sum_{i\in C}\frac{p_k}{|S_i|-1} + \sum_{i\in D}\frac{\Delta p_k}{|S_i|} + \sum_{i\in D}\pi_i > \sum_{i\in C\cup D}\pi_i \Leftrightarrow \sum_{i\in C}\frac{\pi_i}{|S_i|-1} + \sum_{i\in D}\frac{\Delta p_k}{|S_i|} > \sum_{i\in C}\frac{p_k}{|S_i|-1} \Leftrightarrow \Delta p_k \sum_{i\in D}\frac{1}{|S_i|} > \sum_{i\in C}\frac{p_k - \pi_i}{|S_i|-1}$. (b) $P' < P \Leftrightarrow \frac{1}{b}\left(\sum_{i\in B\setminus(C\cup D)}\pi_i + \sum_{i\in C}\frac{p'_k + \sum_{j\in S_i} p_j}{|S_i|+1} + \sum_{i\in D}\frac{p'_k + \sum_{j\in S_i\setminus\{k\}} p_j}{|S_i|}\right) < \frac{1}{b}\left(\sum_{i\in B}\pi_i\right) \Leftrightarrow \sum_{i\in C}\frac{p'_k}{|S_i|+1} + \sum_{i\in C}\left(\frac{|S_i|}{|S_i|+1}\right)\left(\frac{\sum_{j\in S_i} p_j}{|S_i|}\right) + \sum_{i\in D}\frac{p_k - p_k + p'_k + \sum_{j\in S_i\setminus\{k\}} p_j}{|S_i|} < \sum_{i\in C\cup D}\pi_i \Leftrightarrow \sum_{i\in C}\frac{p'_k}{|S_i|+1} + \sum_{i\in C}\frac{|S_i|\pi_i}{|S_i|+1} + \sum_{i\in D}\frac{\Delta p_k}{|S_i|} + \sum_{i\in D}\pi_i < \sum_{i\in C\cup D}\pi_i \Leftrightarrow \sum_{i\in C}\frac{p'_k}{|S_i|+1} - \sum_{i\in C}\frac{\pi_i}{|S_i|+1} + \sum_{i\in D}\frac{\Delta p_k}{|S_i|} < 0 \Leftrightarrow \sum_{i\in C}\frac{p'_k - \pi_i}{|S_i|+1} < \nabla p_k \sum_{i\in D}\frac{1}{|S_i|}$.□

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