Redundancy and Reliability

Lian Cheng

Institute of Quantitative & Technical Economics, Chinese Academy of Social Sciences

11 July 2005

Abstract

Redundancy is a commonly employed strategy to enhance the reliability of systems involving human factors. However, this paper shows that a system with higher level of redundancy can be less reliable due to free-rider effects.

Keywords: redundancy; reliability; free-rider effect

JEL classification: D23; D82

1. Introduction

Redundancy is a commonly employed strategy to enhance the reliability of systems involving human factors. For example, in an organization we often see two or more staffs are assigned to a key operation, although anyone of them can complete the task independently. And whenever people show concern for the reliability of the operation, a natural reaction is to send more staffs there. However, as some authors argued (e.g., Sagan, 2004), higher level of redundancy may actually lead to less reliability. This article builds a formal model to support this view. The intuition behind is that the reliability of a redundant system depends on two elements: the number of the parallel subunits and the possibility that one of them fail to work. Unfortunately, in a redundant system involving human factors the measure to increase the former often also increases the later because of the free-rider effect. Whether increasing the level of redundancy enhances the reliability depends on which effect dominates.

2. The Model

Consider a team whose task is to guard a system against possible errors. Each team member monitors the system independently. An unfound error will cause a crash of the system. And if that happens, each team member will be punished. The team has n members and the object function of the ith team member is

$$\phi_0\phi_1\phi_2\cdots\phi_i\cdots\phi_{n-1}\phi_n B + c(\phi_i) \tag{1}$$

, where ϕ_0 is the possibility that there is an error in the system and ϕ_i is the possibility that the ith member does not find the error (individual fail possibility). B is the punishment for a system crash. $c(\phi_i)$ is the cost of the ith member associated with his effort devoted to system monitoring (individual monitoring cost) and

$$c'(\phi_i) < 0,\ c''(\phi_i) > 0,\ c(1) = 0,\ c(0) = +\infty$$

.

Using the first order condition for minimizing 1

$$\phi_0\phi_1\phi_2\cdots\phi_{i-1}\phi_{i+1}\cdots\phi_{n-1}\phi_n B + c'(\phi_i) = 0 \tag{2}$$

, we get a unique symmetric equilibrium where every team member chooses an optimal individual fail possibility ϕ^*:

$$\phi_0\phi^{*n-1}B + c'(\phi^*) = 0 \tag{3}$$

. Taking the derivative of ϕ^* with respect to n yields the relationship between the individual fail possibility and the number of team members

$$\frac{d\phi^*}{dn} = \frac{\ln \phi^*}{\frac{c''(\phi^*)}{c'(\phi^*)} - (n-1)\phi^{*-1}} \tag{4}$$

. Since $\phi^* \in (0,1)$, $\frac{d\phi^*}{dn} > 0$ and we have

Lemma 1 *The individual fail possibility increases with the number of team members.*

Lemma 1 implies a free-rider effect: as more people join the team, the risk of system crash on the current effort level deceases, so each team member reduces his effort and relies more on other members to detect the error.

Now we turn to the impact of increasing the number of team member on the possibility of a system crash $\frac{d(\phi_0 \phi^{*n})}{dn}$. Using equation 3, we can write the derivative as $\frac{d\left[\phi^* c'(\phi^*)/B\right]}{dn}$ and then $\frac{1}{B} \cdot \frac{d\left[\phi^* c'(\phi^*)\right]}{d\phi^*} \cdot \frac{d\phi^*}{dn}$. As $\frac{d\phi^*}{dn} > 0$, its sign depends only on that of $\frac{d\left[\phi^* c'(\phi^*)\right]}{d\phi^*}$, thus

$$sign\left[\frac{d\left(\phi_0 \phi^{*n}\right)}{dn}\right] = sign\left[1 + \frac{c''(\phi^*)}{c'(\phi^*)}\phi^*\right] \tag{5}$$

. The right side of the equation is negative if and only if $\frac{c''(\phi^*)}{c'(\phi^*)}\phi^* < -1$, or $\left|\frac{c''(\phi^*)}{c'(\phi^*)}\phi^*\right| > 1$. So we have

Proposition 2 *The possibility of a system crash decreases with the number of team members only when the elasticity of marginal individual monitoring cost with respect to individual fail possibility is larger than* 1*.*

The intuition for Proposition 2 is straightforward. The addition of new team members will unavoidably decrease the effort of every member, but if a little relaxation of effort can alleviate the individual monitoring cost enough, this negative effect will not be too large to keep the original level of system reliability.

3. Conclusion

In this paper we show that a system with higher level of redundancy can be less reliable due to free-rider effects. This result implies that increasing the level of redundancy may be not an efficient strategy to ensure reliability, even without the consideration of budget constraint.

4. References

Sagan, S. D., 2004. "The problem of redundancy problem: will more nuclear security forces produce more nuclear security?" Risk Analysis. 24, 935-946.