Economic Analysis (2nd Edition)

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Abstract

A framework for economic analysis based on matrix algebra. In this edition, the mathematical model introduced in earlier works has been extended.

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PS:

In conventional economic analysis, the physical aspects of a given set of processes are captured by economic variables such as cost and returns to scale, which are in turn used to construct sets of equations to model the structure of the market where the processes are executed. While this approach is sufficient for applied research, it is not ideal for theoretical purposes because technological and physical constraints are treated as endogenous.

Utility theory and other techniques to model agent behavior in markets are only relevant where agents can execute more than one action, so they constitute an application of theory, not a core principle. Accordingly, a foundational theoretical model should not include any decision-making construct, but must contain the principles required to construct one.

This paper presents a generalized model that explicitly describes the physical nature of processes and uses that description to derive economic variables and relationships. The result of this approach is a systematic, compact framework that can be adapted to analyze a variety of micro and macro phenomena.

The introduction presents the definitions the model is constructed from, and chapters 1-3 present the structural model. Chapters 4-7 present the dynamics that define the operation of the model.

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Introduction

Section 1: Observation

Let structure be defined as a set of elements and its organization.
Let transformation be defined as the alteration of a structure.

Let observation be defined as the identification of a structure.
Let qualification be defined as the identification of a structure's attributes.
Let data be defined as observations and qualifications.

Section 2: Analysis

Let composition be defined as the derivation of a structure's attributes based on a given set of data.
Let decomposition be defined as the derivation of an element's attributes based on a given set of data.

Let a principle be defined as a concept that defines the state of a structure.
Let a protocol be defined as an implementation of a principle in a particular context.

Let induction be defined as the derivation of a principle based on a given set of data.
Let deduction be defined as the derivation of a protocol based on a given set of data.

Processes

Section 1: Composition

For a given transformation,
let a reactant be defined as an element that is added, removed, or reorganized;
let an accelerant be defined as an element that accelerates the transformation or allows it to occur but is not a reactant;
let a decelerant be defined as an element that decelerates the transformation or prevents it from occurring but is not a reactant;
let a regulator be defined as an element that acts as an accelerant or decelerant;
let an input be defined as an element that acts as a reactant or regulator;

let a product be defined as a structure formed by a transformation for a given purpose;
let a by-product be defined as a structure formed by a transformation for no given purpose;
let an output be defined as a structure that is a product or by-product;

let a resource be defined as an object that is an input or output.

Section 2: Organization

Let a track be defined as a set of α chronologically sequential transformations related by a common set of inputs. From this definition, a given track can converge with other tracks as well as diverge.

Let a process be defined as a set of β tracks related by a common set of outputs. Where $\beta > 1$, tracks can occur simultaneously or sequentially.

For analytical purposes, a process can be divided into subsets of transformations according to some set of criteria. Let these subsets be called stages.

Let a process descriptor be of the form

$$[I]/[O]^t$$

where [I] is an n x n matrix of inputs, [O] is an N x N matrix of outputs, and t is the time required to execute the process.

Let each element in [I] and [O] be of the form

$$(\gamma_Q^q)_S^T$$

where γ is the identifies of the resource, q is the number of units of the resource involved in one execution of the process, Q is the number of units of the resource involved in ten executions of the process, T is the set of chronological constraints the resource must satisfy, and S is the set of spatial constraints the resource must satisfy.

Identifiers can be technical or common names.

Economic Processes

Section 1: Composition

Let a good be defined as a phenomenon defined by a given set of specifications.
Let a type be defined as a set of goods defined by a given set of specifications.

Let production be defined as the process of transforming a set of inputs into a given set of outputs.
Let consumption be defined as the process of transforming a set of outputs into a given set of inputs.

Let search be defined as the process of locating a set of goods that satisfy a given set of criteria.
Let acquisition be defined as the process of obtaining a set of goods located in a search.
Let evaluation be defined as the process of determining the performance of a set of goods.

Let notification be defined as the process of presenting a set of information about a set of goods.
Let distribution be defined as the process of transferring a set of goods to a set of consumers.
Let development be defined as the process of creating a good that satisfies a given set of criteria.

Section 2: Organization

Let supply be defined as the sequence of processes that results in the production of a given set of goods.
Let demand be defined as the sequence of processes that results in the consumption of a given set of goods.

Let a vertex be defined as a type where production or consumption occurs.
Let a vector be defined as a type that connects vertices.
Let an agents be defined as a type that can act as a vertex or vector.

Let a lattice be defined as a given set of vertices, vectors, and agents.

Economic Networks

Section 1: Composition

Let an economic network be defined as a set of lattices.

Let A_t be the set of agents encompassed by an economic network at time t, such that

$$A_t = [A_1 \ldots A_m] \qquad (1.1)$$

Let B_t be the set of agents in A_t that are producers at time t, such that

$$B_t = [B_1 \ldots B_i] \qquad (1.2)$$

where $i \leq m$.

Let C_t be the set of agents in A_t that are consumers at time t, such that

$$C_t = [C_1 \ldots C_r] \qquad (1.3)$$

where $r \leq m$. From these definitions, it is possible for a given agent to belong to subsets B_t and C_t at the same point in time.

Let N_t be an i x I matrix of the set of goods produced by the agents in B_t, such that

$$N_t = [N_1 \ldots N_I] \qquad (1.4)$$

for I types of goods.

Let n_t be an r x R matrix of the set of goods consumed by the agents in C_t, such that

$$n_t = [n_1 \ldots n_R] \qquad (1.5)$$

for R types of goods.

Let b_t be a 1 x I matrix of the set of goods produced by the agents in B_t, such that

$$b_t = [B_t][N_t] \qquad (1.6)$$

where B_t is a 1 x i matrix.

Let c_t be a 1 x R matrix of the set of goods consumed by the agents in C_t, such that

$$c_t = [C_t][n_t] \qquad (1.7)$$

where C_t is a 1 x r matrix.

From equation 1.6, the number of units of goods produced is given by

$$\sum_{g=1}^{i} \sum_{h=1}^{I} N_{gh} = \sum_{h=1}^{I} b_{1h} \qquad (1.8)$$

and the number of units of goods of the Eth type produced is given by

$$\sum_{g=1}^{i} N_{gE} = b_{1E} \qquad (1.9)$$

From equation 1.7, the number of units of goods consumed is given by

$$\sum_{j=1}^{r} \sum_{k=1}^{R} n_{jk} = \sum_{k=1}^{R} c_{1k} \qquad (1.10)$$

and the number of units of goods of the Eth type consumed is given by

$$\sum_{j=1}^{r} n_{jE} = c_{1E} \tag{1.11}$$

Section 2: Organization

Given the structure of an economic network, a set of ratios can be constructed:

i_t / m_t (r - 2.1)

r_t / m_t (r - 2.2)

i_t / r_t (r - 2.3)

I_t / R_t (r - 2.4)

$$\sum_{h=1}^{i} b_{1h} / i \quad \text{(r - 2.5)}$$

$$b_{1E} / i \quad \text{(r - 2.6)}$$

$$\sum_{k=1}^{R} c_{1k} / r \quad \text{(r - 2.7)}$$

$$c_{1E} / r \quad \text{(r - 2.8)}$$

$$b_{1E} / \sum_{h=1}^{I} b_{1h} \quad \text{(r - 2.9)}$$

$$c_{1E} / \sum_{k=1}^{R} c_{1k} \quad \text{(r - 2.10)}$$

$$b_{1E} / c_{1E} \quad \text{(r - 2.11)}$$

$$\sum_{h=1}^{I} b_{1h} / \sum_{k=1}^{R} c_{1k} \quad \text{(r - 2.12)}$$

First-Order Dynamics

Section 1: Accounting

A: Processes

A process value descriptor can be constructed by isolating the number of units of each resource involved in a process and the per unit value of each resource, where value is expressed in terms of a given resource.

Accordingly,

$$C = (\delta_Q^q)_S^T (\varepsilon_Q^q)_S^T \qquad (1.1)$$

where C is the cost of executing a process, δ is a 1x R matrix of inputs, and ε is an R x 1 matrix of per unit input prices.

Similarly,

$$R = (\zeta_Q^q)_S^T (\eta_Q^q)_S^T \qquad (1.2)$$

where R is the revenue generated by a process, ζ is a 1 x I matrix of outputs, and η is an I x 1 matrix of per unit output prices.

Isolating the q or Q terms from 1.1 gives

$$C = q_\delta p_\varepsilon \qquad (1.3)$$

where q denotes quantity and p denotes price.

Isolating the q or Q terms from 1.2 gives

$$R = q_\zeta p_\eta \qquad (1.4)$$

where q denotes quantity and p denotes price.

B: Economic Processes

For the set of economic processes executed by a set of agents,

$$\begin{aligned} R_t &= q_\zeta p_\eta \\ &= R_{production} \\ &= b_t Y_t \end{aligned} \qquad (1.5)$$

where Y_t is an I x 1 matrix of prices, and

$$\begin{aligned} C_t &= q_\delta p_\varepsilon \\ &= C_{consumption} \\ &= c_t Z_t \end{aligned} \qquad (1.6)$$

where Z_t is an R x 1 matrix of prices.

Where π_t is profit at time t,

$$\begin{aligned} \pi_t &= R_t - C_t \\ &= b_t Y_t - c_t Z_t \end{aligned} \qquad (1.11)$$

Accordingly,

$$\Pi_t = \sum_{t=1}^{T} \pi_t \qquad (1.12)$$

for $t = 1...T$ periods.

Second-Order Dynamics

Section 1: Scale

Let scale be defined as the relationship between the price and quantity of a good, such that

$$P_{\theta t} = x_{\theta t} Q_{1t} + x_{1t} \tag{1.1}$$

where x_{1t} is a Q_{1t} independent component of $P_{\theta t}$.

Let increasing returns to scale (IRS) be defined as a scale where $x_{\theta t} < 0$. Let constant returns to scale (CRS) be defined as a scale where $x_{\theta t} = 0$. Let decreasing returns to scale (DRS) be defined as a scale where $x_{\theta t} > 0$.

Section 2: Elasticity

Let elasticity be defined as the relationship between the quantity and price of a good, such that

$$Q_{\iota t} = x_{\iota t} P_{\theta t} + x_{2t} \tag{2.1}$$

where x_{2t} is a $P_{\theta t}$ independent component of $Q_{\iota t}$.

Let inelastic elasticity (IE) be defined as an elasticity where $x_{\iota t} < -1$. Let neutral elasticity (NE) be defined as an elasticity where $x_{\iota t} = -1$. Let elastic elasticity (EE) be defined as an elasticity where $x_{\iota t} > -1$.

Section 3: Symmetry

Let symmetry be defined as the relationship between the profits of separate economic processes, such that

$$\pi_{\kappa t} = x_{\kappa t} \pi_{\iota t} + x_{3t} \tag{3.1}$$

where x_{3t} is a $\pi_{\iota t}$ independent component of $\pi_{\kappa t}$.

Let increasing symmetry (IS) be defined as a symmetry where $x_{\kappa t} > 0$. Let neutral symmetry (NS) be defined as a symmetry where $x_{\kappa t} = 0$. Let decreasing symmetry (DS) be defined as a symmetry where $x_{\kappa t} < 0$.

Section 4: Conclusion

Equation 3.1 can be expressed as

$$\begin{aligned} \pi_{\kappa t} &= x_{\kappa t}(q_{\zeta\iota t} p_{\eta\theta t} - q_{\delta\iota t} p_{\varepsilon\theta t}) + \bar{q}_{\zeta\iota t} \bar{p}_{\eta\theta t} - \bar{q}_{\delta\iota t} \bar{p}_{\varepsilon\theta t} \\ &= x_{\kappa t}(b_{\iota t} Y_{\theta t} - c_{\iota t} Z_{\theta t}) + \bar{b}_{\iota t} \bar{Y}_{\theta t} - \bar{c}_{\iota t} \bar{Z}_{\theta t} \end{aligned} \tag{4.1}$$

Let $\Pi_{\kappa t}$ be defined as

$$\Pi_{\kappa t} = \sum_{t=1}^{T} \pi_{\kappa t} \tag{4.2}$$

for t = 1...T periods.

Third-Order Dynamics

Constraints can be imposed by protocols or the structure of an economic network, or constructed from assumptions. Given second-order dynamics, there are nine types of constraints:

$$x_{\kappa t} = \upsilon_t \quad (1.1)$$

$$b_{\iota t} = \phi_{bt} \quad (1.2)$$

$$Y_{\theta t} = \phi_{Yt} \quad (1.3)$$

$$b_{\iota t} = \chi_{ct} \quad (1.4)$$

$$Z_{\theta t} = \chi_{Zt} \quad (1.5)$$

$$\bar{b}_{\iota t} = \psi_{bt} \quad (1.6)$$

$$\bar{Y}_{\theta t} = \psi_{Yt} \quad (1.7)$$

$$\bar{c}_{\iota t} = \omega_{ct} \quad (1.8)$$

$$\bar{Z}_{\theta t} = \omega_{Zt} \quad (1.9)$$

These constraints can be integrated into the equation for $\Pi_{\kappa t}$ using Lagrange multipliers, producing the equation

$$\Pi_{\kappa t} = \sum_{t=1}^{T}[\pi_{\kappa t} - \lambda_{1t}(x_{\kappa t} - \upsilon_t) - \lambda_{2t}(b_{\iota t} - \phi_{bt}) - \lambda_{3t}(Y_{\theta t} - \phi_{Yt}) - \lambda_{4t}(c_{\iota t} - \chi_{ct}) - \lambda_{5t}(Z_{\theta t} - \chi_{Zt})$$
$$- \lambda_{6t}(\bar{b}_{\iota t} - \psi_{bt}) - \lambda_{7t}(\bar{Y}_{\theta t} - \psi_{Yt}) - \lambda_{8t}(\bar{c}_{\iota t} - \omega_{ct}) - \lambda_{9t}(\bar{Z}_{\theta t} - \omega_{Zt})]$$

Fourth-Order Dynamics

Section 1: Type 1 Statistics

Let M be mass.

$$M_{xt} = (\sum_{h=1}^{I} b_{1h}) + (\sum_{k=1}^{R} c_{1k}) \tag{1.1}$$

$$M_{yt} = b_t Y_t + c_t Z_t \tag{1.2}$$

Let v be velocity.

$$V_{xt} = [(\sum_{h=1}^{I} b_{1h}) + (\sum_{k=1}^{R} c_{1k})] / t$$
$$= M_{xt} / t \tag{1.3}$$

$$V_{yt} = (b_t Y_t + c_t Z_t) / t$$
$$= M_{yt} / t \tag{1.4}$$

Section 2: Type 2 Statistics

Let g be growth.

$$g_{xt} = (M_{xt} - M_{xt-1}) / t \tag{2.1}$$
$$g_{yt} = (M_{yt} - M_{yt-1}) / t \tag{2.2}$$

Let a be acceleration.

$$a_{xt} = (V_{xt} - V_{xt-1}) / t$$
$$= (M_{xt} - M_{xt-1}) / t^2 \tag{2.3}$$

$$a_{yt} = (V_{yt} - V_{yt-1}) / t$$
$$= (M_{yt} - M_{yt-1}) / t^2 \tag{2.4}$$

Section 3: Type 3 Statistics

Let P be momentum.

$$P_{xt} = (M_{xt})(V_{xt})$$
$$= (M_{xt})^2 / t \tag{3.1}$$

$$P_{yt} = (M_{yt})(V_{yt})$$
$$= (M_{yt})^2 / t \tag{3.2}$$

Let F be force.

$$F_{xt} = (g_{xt})(a_{xt})$$
$$= (M_{xt} - M_{xt-1})^2 / t^3 \tag{3.3}$$

$$F_{yt} = (g_{yt})(a_{yt})$$
$$= (M_{yt} - M_{yt-1})^2 / t^3 \tag{3.4}$$

Section 4: Type 4 Statistics

A: Lower Thresholds for Type 1 Statistics

For a given A_t, let $C_{t(min)}$ be the minimum set of consumers at time t and let $n_{t(min)}$ be the minimum set of goods they consume, where there are α types of goods.

From these definitions,

$$c_{t(min)} = [C_{t(min)}][n_{t(min)}] \tag{4.1}$$

At minimum mass,

$$b_{t(min)} = [0] \tag{4.2}$$

Accordingly,

$$M_{xt(\min)} = \sum_{k=1}^{\alpha} c_{1k} = c_{1k(\min)} \tag{4.3}$$

$$M_{yt(min)} = c_{1k(min)}Z_{t(min)} \tag{4.4}$$

$$V_{xt(min)} = M_{xt(min)} / t \tag{4.5}$$

$$V_{yt(min)} = M_{yt(min)} / t \tag{4.6}$$

B: Upper Thresholds for Type 1 Statistics

For a given A_t, let $B_{t(max)}$ be the maximum set of producers and let $N_{t(max)}$ be the a maximum set of goods they produce, where there are Ω types of goods.

From these definitions,

$$b_{t(max)} = [B_{t(max)}][N_{t(max)}] \tag{4.7}$$

At maximum velocity c_t is identical to $b_{t(max)}$, so

$$b_{t(max)} = c_{t(max)} \tag{4.8}$$

Accordingly,

$$M_{xt(\max)} = (\sum_{h=1}^{\Omega} b_{1h}) + (\sum_{k=1}^{\Omega} c_{1k}) = 2\sum_{h=1}^{\Omega} b_{1h} \tag{4.9}$$

$$M_{yt(min)} = b_{t(max)}Y_t + c_{t(max)}Z_t = 2b_{t(max)}Y_t \tag{4.10}$$

$$V_{xt(max)} = M_{xt(max)} / t \tag{4.11}$$

$$V_{yt(max)} = M_{yt(max)} / t \tag{4.12}$$

<u>Section 5: Type 5 Statistics</u>

A: Lower Thresholds for Type 2 Statistics

$$g_{xt(min)} = (M_{xt(min)} - M_{xt-1}) / t \tag{5.1}$$
$$g_{yt(min)} = (M_{yt(min)} - M_{yt-1}) / t \tag{5.2}$$

$$a_{xt(min)} = (M_{xt(min)} - M_{xt-1}) / t^2 \tag{5.3}$$
$$a_{yt(min)} = (M_{yt(min)} - M_{yt-1}) / t^2 \tag{5.4}$$

B: Upper Thresholds for Type 2 Statistics

$$g_{xt(max)} = (M_{xt(max)} - M_{xt-1}) / t \quad (5.5)$$
$$g_{yt(max)} = (M_{yt(max)} - M_{yt-1}) / t \quad (5.6)$$

$$a_{xt(max)} = (M_{xt(max)} - M_{xt-1}) / t^2 \quad (5.7)$$
$$a_{yt(max)} = (M_{yt(max)} - M_{yt-1}) / t^2 \quad (5.8)$$

<u>Section 6: Type 6 Statistics</u>

A: Lower Thresholds for Type 3 Statistics

$$P_{xt(min)} = (M_{xt(min)})(V_{xt(min)}) = [M_{xt(min)}]^2 / t \quad (6.1)$$

$$P_{yt(min)} = (M_{yt(min)})(V_{yt(min)}) = [M_{yt(min)}]^2 / t \quad (6.2)$$

$$F_{xt(min)} = (g_{xt(min)})(a_{xt(min)}) = [M_{xt(min)} - M_{xt-1}]^2 / t^3 \quad (6.3)$$

$$F_{yt(min)} = (g_{yt(min)})(a_{yt(min)}) = [M_{yt(min)} - M_{yt-1}]^2 / t^3 \quad (6.4)$$

B: Upper Thresholds for Type 3 Statistics

$$P_{xt(max)} = (M_{xt(max)})(V_{xt(max)}) = [M_{xt(max)}]^2 / t \quad (6.5)$$

$$P_{yt(max)} = (M_{yt(max)})(V_{yt(max)}) = [M_{yt(max)}]^2 / t \quad (6.6)$$

$$F_{xt(max)} = (g_{xt(max)})(a_{xt(max)}) = [M_{xt(max)} - M_{xt-1}]^2 / t^3 \quad (6.7)$$

$$F_{yt(max)} = (g_{yt(max)})(a_{yt(max)}) = [M_{yt(max)} - M_{yt-1}]^2 / t^3 \quad (6.8)$$