A Theory of Capital Structure with Strategic Defaults and Priority Violations*

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Abstract

We reformulate the classic CSV model of financial contracting from Townsend (1979) and Gale & Hellwig (1985) to tackle criticisms raised against it voiced by Hart (1995), such as lack of optimal behavior at the repayment stage and an inability to allow for outside equity. As a result, we obtain a theory of capital structure that accommodates empirical regularities such as bankruptcies, strategic defaults of debt obligations, and violations of absolute priority rules as parts of the equilibrium description.

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1 Introduction

Financial contracts typically do not specify repayments to investors as a detailed function of all payoff relevant variables. For example, debt contracts normally do not specify

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repayments as a detailed function of the financial state of the firm, but rather puts some easily describable liability on the firm's cash flow through a fixed repayment obligation. One focal approach in the literature that attempts to model this feature of financial contracts is the Costly State Verification (CSV) approach. The core of this approach is that, upon the date of repayment, inside investors have superior information to the outside investors about the profitability of the firm, and therefore may try to divert cash from outside investors by underreporting the true cash-flow. Of course, this may in turn create an ex-ante governance problem in that external investors may be reluctant to finance the firm. The weapon outside investors can use to mitigate the cash diversion problem is to partially or fully verify the true profitability of the firm, by e.g. demanding an audit, declaring bankruptcy, or even discharge management and take control of the operations of the firm. Such a leveling of information can only take place at a certain cost of verification. Celebrated papers by Townsend (1979) and Gale & Hellwig (1985) derive debt contracts as optimal under such circumstances, i.e., contracts which promises a fixed repayment, and where the creditor verifies whenever the offered repayment falls below the promised repayment.

In spite of its elegance, the classroom CSV model suffers from some shortcomings. First, as pointed out by Hart (1995) and others, the debt contract derived under CSV relies on a commitment on the part of the lender to verify whenever the debt is not repaid in full, even if accepting a concession would be better for the lender, since verification is costly. As such, the equilibrium supporting the "optimal contract" may involve non-Nash strategies to be played by the creditor in default states, and – perhaps equally importantly – implies that the model cannot accommodate strategic defaults of debt obligations by the borrower. Second, as also pointed out by Hart (1995), while in practice debt typically coexists with equity as a financial claim on the firm, the standard CSV model is unable to explain the use of outside equity, and hence unable to account for capital structures with both debt and outside equity on the balance sheet.[1]

The motivation of the present paper is to recast the CSV model in response to these

[1] Indeed, as noted by Townsend (1979), "the [CSV] model as it stands may contribute to our understanding of closely held firms, but cannot explain the coexistence of publicly held shares and debt."

criticisms. For debt contracts, we require sequential rationality and allow for stochastic verification at the repayment stage. In other words, verification occurs only if it is an optimal strategy given the repayment offer by the borrower, and the verification strategy may be stochastic. This allows us to solve for an equilibrium where the manager offers the lender a debt repayment that depends on the true cash-flow of the firm, and the lender monitors with a probability that is increasing in the magnitude of the default. This lenience on part of the lender implies that there can be strategic defaults of debt repayments in equilibrium, in that the borrower defaults on his debt obligation even though he has sufficient cash on hand to avoid a default.

We also introduce outside equity in the CSV setting. While debt involves a fixed payment being promised to the outside investor, equity is issued with a promise to the investor of a fixed fraction of firm's cash flow. This fractional cash flow right is in turn supported by an unconditional right for the investor to intervene and verify. We solve for an equilibrium where the payout proposed to the investor by the entrepreneur is increasing in the true cash flow, and the investor intervenes with a probability that is decreasing in the size of the proposed payout.

Combining debt and equity in the model (where debt is the senior claimant) allows us to consider the choice of the optimal capital structure. We show that while the firm will never be funded by 100% equity (it will always want to issue at least some risky debt), it may be optimal for the firm to raise all its external funds in the form of debt. Less surprisingly, it will issue a mix of outside equity and debt when the intervention cost of equity is sufficiently low compared to that of debt. Moreover, we show that equilibria with a mixed capital structure involves a division of labor: creditors monitor the manager in bad states and outside equity monitors the manager in high states.

A financing mix can be optimal even if it implies that violations of absolute priority rules can occur in equilibrium, in that outside equity receives a positive repayment even if creditors are not repaid in full. It may be noted here that the literature generating AP-violations (e.g., Bebchuk, 2002) deals with AP-violations vis a vis the inside owner-entrepreneur. In our setting, there are AP-violations in the sense that both inside and outside equity receive positive payments even though debt is not paid in full.

The main message we obtain from the paper is that there exists a convenient way to escape the commitment assumption associated with the standard CSV model. In the equilibria we derive, investors make an optimal verification choice conditional on their (reasonable) beliefs about the true cash flow, and the entrepreneur makes repayment offers that depend on the true cash flow of the firm. With this starting point, we obtain plausible results on the possibility and properties of a mixed capital structure, encompassing such features as strategic defaults and priority violations.

We should emphasize at this point that our primary purpose with the paper is to understand the choice of capital structure, and the implications of a mixture. To do this we work with a limited contract space, consisting of standard debt and equity contracts. Our approach should here be contrasted with the mechanism design approach of several other papers in the CSV literature, such as Krasa & Villamil (2000) and Krasa et al. (2003), that derive optimal contracts under modified circumstances to those considered by Townsend (1979). We see pros and cons with both our more institutional approach and the more normative approach, and cannot see that there are compelling arguments to discard any of these approaches given the current state of knowledge.[2] However, since debt and equity contracts in real life can be of a somewhat richer variety than captured by our model, we have taken care to check our results for robustness against alternative formulations. All our attempts in this direction indicate that the basic results of the paper hold through with such modified contracts.

The CSV literature has taken alternative routes to solve the dilemma posed by the lack of sequential rationality of the basic CSV contracts. For example, Gale & Hellwig (1989) impose sequential rationality in a signaling game where the cash flow is fully revealed through the repayment offer from the inside investor to the outside investor. However, in Gale & Hellwig (1989) contracting plays no explicit role. In contrast, we allow for (debt or equity) contracts to be written on payoffs in the verification state, which enables us to endogenously determine the equilibrium contracts and repayment behavior.[3]

[2]The mechanism design approach has the advantage of endogenously determining contractual forms. However, such an approach also typically violate some important institutional rules; for example, there are legal limitations to which contracts can count as debt (to give a tax break), and legal rules that are designed to protect minority investors. Such rules are hard to model very precisely in a mathematical fashion but still clearly calls for a limitation of the available contracts.

[3]Reinganum & Wilde (1986) consider a tax-evasion game where a tax payer submits an income statement to the IRS, and the IRS makes a sequentially rational, random audit. The main difference to our

Others who consider outside equity and debt financing under incomplete contracting includes Fluck (1998), Fan & Sundaresan (2000), Myers (2000), and Anderson & Nyborg (2001), who operate in a symmetric-unverifiable information setup à la Grossman & Hart (1986) and Hart & Moore (1989). However, these papers focus on dynamic issues of repayment and do not derive an optimal mix of debt and outside equity.[4] There is a literature on strategic defaults and AP-violations that will be discussed in Section 4.

The rest of the paper is organized as follows. In Section 2 the basic model is presented. In Section 3 pure debt and pure equity financing is considered. In Section 4 we examine a mix of debt and equity, and Section 5 concludes.

2 The basic setup

The entrepreneur operates in a competitive market for financing, and has a choice between debt financing and equity financing. There are two stages, the investment stage and the payoff stage. Let the cash flow x in the payoff stage be a stochastic variable with density function $f(x)$, $x \in [x_L, x_H]$, where $0 < x_L < x_H$. The expected cash flow $\int_{x_L}^{x_H} xf(x)dx$ is denoted by Ex, and the required investment amount is given by I. The entrepreneur has zero funds. The NPV of the project (gross of verification costs) is therefore $Ex - I$. The riskless interest rate is zero, and all agents are assumed to be risk neutral. Contracts can only specify payouts to the investors in the verification state. A *(pure) debt contract* specifies the payout to the creditor as $\min[D, x]$, where D is the contractible variable. A *(pure) equity contract* specifies the payout to the outside equity holder as βx, where

setting is that the "contract" between a tax-payer and the IRS (proportional taxation with a penalty for misreporting) is exogenously imposed by a third party (the "policy makers") rather than being determined by competitive forces. Povel and Raith (2002) consider a related setting involving a loss in future private benefits to manager from being acquitted. As with Gale & Hellwig (1989) and Reinganum & Wilde (1986), their setting is different because the verification state payoffs are not contracted upon. Persons (1997) imposes sequential rationality and stochastic monitoring in a CSV setting but restricts attention to the two-state case. Krasa & Villamil (2000) derive optimal contracts in a setting under sequential rationality in a setting with limited commitment by the investor. They focus on equilibria without renegotiation (by fixing beliefs such that offers are either accepted with probability one or zero), in contrast to our approach. The issue of equilibrium selection is discussed further in a later footnote.

[4] Boyd & Smith (1999) show that the optimal contract in a CSV type of setting can involve a mix of debt and equity. However, the payoff to outside equity in their paper is only supported by the observable part of the firm's cash flow, and hence their paper does not explain the use of equity financing to projects that generate unobservable cash-flows.

$\beta \in (0, 1]$ is the contractible variable. In Section 4, we consider the case where the entrepreneur may finance the project through a mix of debt and equity.

The realized cash flow is observed freely by the entrepreneur-manager, but can be observed by the outside investors only at a positive cost, denoted by c_D for debt, and c_E for equity. One interpretation is that c_D is a bankruptcy cost, and that c_E is the cost for outside equity holders of taking control of the firm.[5] Less dramatically, c_D and c_E could reflect the creditors' and the outside equity holders' respective cost for performing a thorough audit. For several reasons, it is difficult to put any tight restrictions on the relative magnitude of c_D and c_E, one being that debt and equity holders may have different information about the operations of the firm.[6] At this point, we therefore merely assume that $0 < c_D, c_E < x_L$, i.e., that there is liquidity in the firm ex post to cover the verification cost.[7] Later, we will discuss the relative magnitude of c_D and c_E in more detail. Regardless of financing, the entrepreneur is the residual claimant.

3 Polar cases: debt or equity

For clarity of exposition, we first consider pure debt and pure equity financing in this section, and consider the possibility of a mixture between debt and equity in Section 4.

3.1 Pure debt financing

Debt is issued with a face value $D \in \Re_{++}$ along with a right on the part of the lender to verify (intervene) if D is not paid in full. We assume that the creditor will be reimbursed for the costs of collecting the contracted payment D, with D representing the maximum amount that the creditor can collect net of verification costs. Thus, the creditor obtains

[5] We are implicitly assuming that the manager does not lose private benefits from the shareholders taking control, and that the outside option of the manager (other career options) are independent of whether the shareholders take control or not. These assumptions simplify the analysis, but do not change the qualitative insights. A related change of assumptions would take into account managerial moral hazard, by modeling managerial effort or risk taking as a function of the financial structure.

[6] Another reason for c_E being different from c_D is that since the control rights for debt and equity differ, creditors and equity holders may have different incentives to invest in a cheap monitoring technology ex-ante. This argument is further discussed in Section 4.

[7] The liquidity restriction $c_D, c_E < x_L$ could be made endogenous by requring the entrepeneur to borrow more than I, in order to keep a liquidity reserve for bad states.

a net payoff $\min[D + c_D, x] - c_D = \min[D, x - c_D]$ after verification. This feature is consistent with the bankruptcy law in most countries. At any extent our results would be exactly the same if the creditor pays the verification cost.

In the current setup, it is hard to imagine debt contracts with a different structure than that described above. One possibility would be the use of convertible debt, as in Cornelli & Yosha (2003). Since we assume that there are no events to condition a conversion upon other than the repayment proposal of the manager, convertibles must either be (weakly) dominated by straight debt or by equity in the current setting, and can therefore be abstracted from. An extension of the model where the investors learn about x before the repayment offer by the manager would make convertibles a more interesting security to issue for the manager.

First the parties agree upon a debt obligation D (taken as given at this point). Then the true cash flow is realized and observed privately by the entrepreneur. The entrepreneur makes a (deterministic) repayment offer $\tilde{D} : [x_L, x_H] \to [0, x_H]$, where $\tilde{D} \leq x$ due to the zero initial funds of the entrepreneur. Notice that the entrepreneur making a repayment offer $\tilde{D} < D$ is equivalent to proposing for the creditor to make a concession $D - \tilde{D}$ on the debt claim. Given an offer $\tilde{D} < D$ by the entrepreneur, the creditor either accepts or rejects the concession proposal. If the creditor accepts, he receives $\tilde{D}$, and the manager gets the residual $x - \tilde{D}$. If the creditor rejects, he verifies and receives a payoff according to the written contract.[8] A strategy for the creditor is an accept probability $Q(\tilde{D})$, where $Q(.)$ is a mapping from the set of possible repayments $[0, x_H]$ to a probability on $[0, 1]$. For $\tilde{D} \geq D$, the contract dictates that $Q(\tilde{D}) = 1$, since the creditor may only reject offers less than D. For $\tilde{D} < D$, then $Q(\tilde{D})$ is the probability that the creditor accepts the concession on the debt claim proposed by the manager.

We rule out pre-commitment in the verification strategy $Q(.)$ by considering equilibria that involves optimal play by the investor Nash play at all paths in the game. This

[8] Our approach here is similar to that in Anderson & Sundaresan (1996) and Mella-Barral & Perraudin (1997). Potentially, there is a third action open to the creditor, namely to put a counter-offer on the table. If the costs of making counter-offers are large relative to the cost for the manager to make counter-counter-offers, the solution of a Rubinstein (1982) type of bargaining game between the manager and the creditor would give the creditor less than accepting the offer $\tilde{D}$, and hence this third action would not be relevant. Fan & Sundaresan (2000) consider a setting which allows for varying relative bargaining strength of the inside equity holders and the creditors.

assumption is plausible for example for bank or venture capital type of debt, where the relation between the borrower and the lender is of close character, and where concessions made are not necessarily observed by the market, and hence induces no loss of reputation for the creditor. Moreover, we focus on equilibria with stochastic monitoring by the creditor for $\tilde{D} < D$.[9] Consequently, for $\tilde{D}(x)$ to be part of an equilibrium, the creditor must be indifferent between accepting and rejecting the offer, and the only candidate equilibrium debtor strategy is,

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{1}$$

Since the function $\tilde{D}(x)$ is strictly increasing for $x \in [x_L, D + c_D]$, an offer implicitly defines a reported cash flow, $\tilde{x}$.

The question is now whether there exists a function $Q(.)$ such that the manager has incentives to play the strategy in (1). It turns out that there exists a unique solution to this problem, which moreover can be given a closed-form characterization. Quite remarkably, the solution does not depend on the cash flow distribution $f(x)$. The intuition is that the investor's accept/reject strategy serves to control the manager's reporting behavior, and the manager *knows* the realization of x, so the prior distribution of x is irrelevant.

Denote the manager's utility as a function of the "report" $\tilde{x}$ [with an implied offer $\min(D, \tilde{x} - c_D)$] and the true state x by $U(\tilde{x}; x)$, for simplicity just written $U(\tilde{x})$. For the manager's incentive-compatibility constraint to hold, it must be the case that $U(\tilde{x})$ is maximized for $\tilde{x} = x$. The question is whether there exists a verification function $Q(.)$ such that truth-telling is indeed the optimal strategy.

The manager has no interest in offering the lender a payment that exceeds D, and the lender's right to demand verification is contingent on offers less than D. Consider

[9] A note on equilibrium selection: There can exist sequentially rational equilibria with deterministic monitoring but these equilibria have the unreasonable feature that an offer slightly less than D are rejected by the creditor, which would not be optimal play by the creditor given that he assigns sufficiently high probability to the cash flow being in the intermediate range (a similar argument can be made against equilibria where the entrepreneur plays a mixed strategy). Our approach to the contracting problem is in this respect similar to that in Gale & Hellwig (1989), and can be seen in contrast to e.g., Krasa & Villamil (2000, 2003), which focus on such pooling equilibria.

therefore values of $\tilde{x}$ on the interval $[x_L, D + c_D]$, and let $d := x - \tilde{x}$ be the magnitude of cash flow misreporting. First consider the case $x \in [x_L, D + c_D]$. We then have that,

$$\begin{aligned} U(\widehat{x}) &= Q(\tilde{x})[c_D + d] + [1 - Q(\tilde{x})]0 \\ &= Q(\tilde{x})[c_D + d] \end{aligned} \tag{2}$$

In words, since the manager gets nothing if the creditor rejects the concession pledge, the expected utility of the manager after making a report $\tilde{x}$ just equals the concession proposal $(c_D + d)$ multiplied by the probability of the creditor accepting the proposal. We now maximize the manager's utility with respect to $\tilde{x}$, where it is assumed that $Q(\tilde{x})$ is differentiable.[10]

$$\frac{dU(\tilde{x})}{d\tilde{x}} = \frac{dQ(\tilde{x})}{d\tilde{x}}[c_D + d] - Q(\tilde{x}) = 0 \tag{3}$$

For truthful announcement to be optimal, this function must be maximized for $d = 0$, and hence,

$$Q(\tilde{x}) - \frac{dQ(\tilde{x})}{d\tilde{x}} c_D = 0 \tag{4}$$

Solving this differential equation yields,

$$Q(\tilde{x}) = K e^{\frac{\tilde{x}}{c_D}} \tag{5}$$

where K is an integration constant. Using the corner condition $Q(D + c_D) = 1$,[11] we can determine this constant to obtain,

$$Q(\tilde{x}) = \begin{cases} e^{-\frac{D + c_D - \tilde{x}}{c_D}}, \ \tilde{x} \in [x_L, D + c_D] \\ 1, \text{ for } \tilde{x} \in [D + c_D, x_H] \end{cases} \tag{6}$$

Note that $Q(.)$ in (6) induces truth-telling for $x < D + c_D$.[12] Hence we then have the

[10] The equilibrium $Q(.)$ function must be continuous. Were it not for some x, the manager would be made better off by setting the announced x slightly higher than the true x (to thereby pay out only slightly more but have discontinuos jump in accept probability).

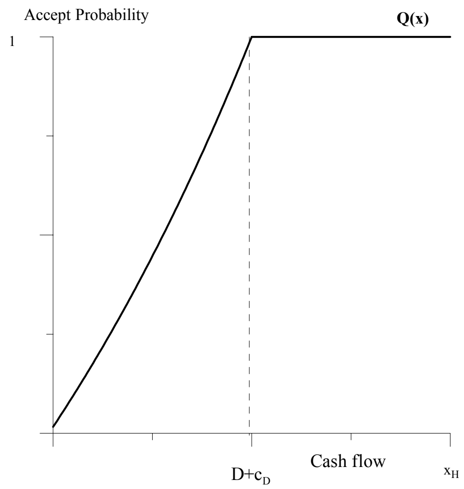
[11] This condition follows from the continuity requirement mentioned in the previous footnote.

[12] To see that the second order condition for maximum is satisfied, differentiate $U(\widehat{x})$ twice with respect to $\hat{x}$, which yields $\frac{Q(.)}{c_D^2}(d - c_D)$ which is clearly negative for $d = 0$.

following result.

Proposition 1 *(Debt) In equilibrium, the manager offers* $\tilde{D} = D$ *if* $x \geq D + c_D$. *If* $x < D + c_D$, *the manager defaults by offering* $x - c_D$, *and the lender accepts with probability* $Q(x) = e^{-\frac{D+c_D-x}{c_D}}$.

We can illustrate the proposition in a figure.



The true cash flow of the firm is on the horizontal axis, and the accept probability of the creditor on the vertical axis. The function $Q(x)$ is the equilibrium accept probability, given that the offer from the lender is represented by the function $\tilde{D}(x)$ in (1). The accept probability $Q(.)$ is inversely related to the extent of the default $D - x$, which is intuitive because understating the true cash flow must be costly to induce truth-telling. It implies that the lender will be less lenient with firms with large defaults. If we think of the lender accepting the entrepreneur's offer as the firm successfully restructuring its debt out of court and the lender rejecting the offer as the firm going to formal bankruptcy (under e.g., Ch. 11), then the proposition implies that firms are more likely to enter formal bankruptcy the larger their default. The intuition for convexity of $Q(.)$ is that it is more tempting for the manager to underreport the cash flow when x is relatively high, so that the steepness of $Q(.)$ must be higher for higher reports.

Notice that the borrower, expecting the lender to be lenient with defaults (with positive probability), for $x \in [D, D + c_D]$ has an incentive to offer a lower repayment than D even though he has sufficient cash to avoid default. In other words, we get strategic defaults in equilibrium for $x \in [D, D + c_D]$.[13]

We should emphasize that the perhaps the most plausible interpretation of the mixed strategy played by the creditor is that the entrepreneur faces a market of possible financiers, and where each financier may play a pure strategy on when to verify (e.g., to verify for any default larger than z, where z is some positive constant), so that the mixed strategy reflects the average behavior played by potential creditors, not the strategy played by each possible creditor. Under this interpretation, the offer function $\tilde{D}(x)$ is a best response to the average or expected play by creditors, not necessarily the best response to the particular creditor played (this is a standard interpretation of mixed strategy equilibria in the game-theoretic literature, see e.g., Rubinstein, 1991). The same interpretation is applicable to the equilibrium we derive under pure equity and under mixed financing.

We have assumed that verification state payoffs can only depend on x. Alternatively, we could enrich the contractual space by allowing verification state payoffs to depend both on x (resources available) as before, and the report $\tilde{x}$ (this assumes that reports are contractible). Specifically, the contract could specify a punishment for the manager if caught lying ($\tilde{x} \neq x$), an idea explored by Mookherjee & Png (1989) and Persons (1997). In Appendix C, we consider such contracts and show that they would yield qualitatively the same results as the current contracts.

3.2 Pure equity financing

We model outside equity as a linear contract that gives the investor a fractional right, $\beta \in (0, 1]$, to the firm's cash flow. Linearity is consistent with laws protecting minority shareholders, in that a smaller ownership share should give proportionally the same cash flow rights (interpreted broadly as dividends, liquidation proceeds, or a takeover premium) as a larger ownership share.[14]

[13] That strategic defaults are empirically important is shown by Esty and Megginson (2003), which in an empirical analysis of international project financing argue that lending syndicates are structured to deter strategic defaults rather than to improve monitoring incentives of lenders.

[14] The presence of executive options, which presumably are exercised when the firm is doing well, would generate concavity in the outside investors' payout. This issue is left for future research.

As with D in the case of debt financing, β will be determined by the funding requirement and the outside investor's participation constraint, but can be viewed as exogenous at this point. The cash flow right associated with equity is supported by an *unconditional* right for the outside shareholder to intervene. The combination fractional cash flow right and unconditional right to intervene is consistent with equity as observed in practice, and is the same type of approach as e.g., Myers (2000) and Andersen & Nyborg (2001). If equity is allowed to have a conditional control right, it would make equity more debt-like, but would yield qualitatively the same type of results. We furthermore assume that the verification cost under outside equity is borne by the investor. This assumption implies for example that a shareholder cannot be reimbursed for costs of engaging in a proxy contest. Our results do not depend on this formulation.[15]

A strategy by the entrepreneur is an offer-function $\tilde{E}(x)$, where $\tilde{E} : [x_L, x_H] \to [0, x_H]$, and $\tilde{E} \leq x$. A strategy for the equity holder is an accept function $P(\tilde{E})$, where $P(.)$ is a mapping from the set of possible repayments $[0, x_H]$ to a probability on $[0, 1]$. As with debt, we consider sequentially rational equilibria of the game between the manager and the equity holder. This means that the shareholders cannot precommit to a monitoring strategy, see e.g., Admati & Pfleiderer (1994) for a similar type of assumption. Given the cash flow right β, and the assumption that intervention costs are covered by the investor, the investor receives $\beta x - c_E$ in net payoff if he decides to intervene, where c_E is the intervention cost.

Analogous to the case of pure debt financing, we consider signaling equilibria where the entrepreneur plays a pure strategy. Thus, for a given β and $\tilde{E}$ the equity holder must be indifferent on whether to verify, or

$$\tilde{E}(x) = \beta x - c_E \tag{7}$$

Since the function $\tilde{E}(x)$ in (7) is strictly increasing, an offer implicitly defines a reported

[15] Letting the firm absorb the verification cost, as in the case of debt financing, implies that the shareholder is offered $\tilde{E}(x) = \beta(x - c_E)$ in equilibrium, rather than $\tilde{E}(x) = \beta x - c_E$. The equilibrium accept probability, given β, is independent of who bears the intervention cost c_E. However, the required ownership fraction β in the alternative formulation will be less, since the investor receives $\beta(x - c_E)$ in equilibrium rather than $\beta x - c_E$. This gives a higher accept probability $P(.)$ and hence lower expected verification costs, but no other qualitative change in the results.

cash flow, $\tilde{x}$. The question again is whether there exists a function $P(\tilde{x})$ such that truthful reporting is indeed obtained in equilibrium. Imposing the corner condition $P(x_H) = 1$, this problem conveniently turns out to have a unique solution, which can be given a closed-form characterization.

Proposition 2 *(Outside equity) In equilibrium, the manager offers the investor* $\beta x - c_E$*, and the investor accepts the manager's offer with probability* $P(x) = e^{-\beta \frac{x_H - x}{c_E}}$, $x \in [x_L, x_H]$.

Proof. See Appendix A. ■

The probability of the outside equity holder intervening is decreasing in the size of the payment that the entrepreneur offers. This is intuitive, the higher the earnings and the higher the dividend payout the less is the chance that shareholders will find it necessary to intervene. Note also that there is a positive probability of intervention for all x, in contrast to what the case is with debt financing.

As can readily be seen, for a given $\tilde{x}$, the shareholder's accept probability $P(\tilde{x})$ is decreasing in his ownership stake β. Intuitively, higher outside ownership increases the potential for the insider to divert cash away from the outsider by under-reporting the true cash flow, which in turn forces the outsider to intervene with a greater probability in order to induce truth-telling. The straightforward implication is that a higher outside ownership implies more active owners, in terms of intervening more frequently.

We may notice that β cannot be arbitrarily small for equity financing to work, because there must be sufficient incentives for the equity holder to intervene after being offered a (low) payment.[16] As we shall see later, this property of equity implies that small projects (a low I) will be 100% debt financed.

We now turn to the case where the firm may be both debt and equity financed.

[16] More specifically, if $\beta < \frac{c_E}{x_L}$ then the equity holder will not have incentives to monitor when x_L is (truthfully) reported. But then the manager will always report x_L and an equilibrium cannot exist. Hence equity financing implies that $\beta \geq \frac{c_E}{x_L} >> 0$. If a liquidity reserve can be provided ex-ante, by e.g., the outside investors providing more than I, then the minimum β can be decreased, but must still be bounded away from zero.

4 Capital structure

We now consider the possibility of the entrepreneur issuing both debt and equity to finance the project. We take the creditor to be the senior claimant and the outside equity holder to be the junior claimant, meaning that the entrepreneur settles his accounts with the creditor before proposing a payout to the outside equity holder. The objective of the manager is to pick the financial structure that minimizes expected verification costs, subject to the constraint that the outside investors are willing to participate.

First, the manager funds the amount I with a fraction α in the form of debt and $(1-\alpha)$ in the form of equity, where $\alpha \in [0,1]$, and D and β are agreed upon.[17] The cash flow is then realized and observed only by the manager. Upon observing the true cash flow, the manager offers a debt repayment $\tilde{D}$ to the creditor, which the creditor accepts with probability $q(\tilde{D})$. If the creditor rejects the offer, he incurs the cost c_D and gets the net payout $\min[D, x-c_D]$, while the equity holder gets $\max[0, \beta(x-D-c_D)]$.[18] The entrepreneur's payoff is the residual. If the creditor accepts the manager's offer $\tilde{D}$, the manager proceeds to the shareholder with a repayment offer $\tilde{E}$, which the shareholder accepts with probability $p(\tilde{E})$. By conditioning p only on $\tilde{E}$, we are implicitly assuming that the equity holder does not observe $\tilde{D}$, only whether the creditor chose to verify or not. The case where $\tilde{D}$ is observable to the equity holder, so that p is a function of both $\tilde{E}$ and $\tilde{D}$, has qualitatively similar properties, but is algebraically more complex, and is considered in Appendix D. If the shareholder accepts the offer $\tilde{E}$, the manager retains $x-\tilde{D}-\tilde{E}$. If the shareholder rejects the offer, and verifies, the shareholder receives $\beta(x-\tilde{D})-c_E$, and the manager gets the residual. We assume that the creditor by accepting waives any future rights to the cash flow. This is consistent with bankruptcy law as practiced in e.g., the U.S. where repudiation is limited to situations under which the creditor can show that he was coerced to accept the firm's offer (see Berglöf, Roland,

[17] The two contracts β and D are assumed to be agreed upon in a manner that excludes opportunistic behavior by a subset of the three agents at the contracting stage. Stylistically, we can think of the manager solving for the optimal β and D (that satisfies the participation constraints), and then offering and signing the two contracts simultaneously.

[18] Reasonably, we assume that contracts between the creditor and the equity holder involving the creditor (equity holder) subcontracting the intervention action to the equity holder (creditor) are prohibitively costly to enforce.

and von Thadden (2000) for a discussion).

If a (sequentially rational) equilibrium with a mixed capital structure exists, it must have a similar structure to the equilibria of pure debt and pure equity, in that the manager offers repayments that (implicitly) reveals the true cash flow, and where the creditor and the shareholder play a mixed strategy in certain states. We first derive the accept probability functions $q(.)$ and $p(.)$ of debt and equity, respectively, taking the capital structure α as given on the interior of (0,1). Then we derive results on the optimal capital structure.

For the creditor's indifference condition to hold, it must as under pure debt financing be the case that,

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{8}$$

Given this strategy, consider the equity subgame. Consider first the case in which the manager does not default on his debt obligation (by offering $\tilde{D} = D$), in which case the creditor has no choice but to accept the offer. After D is paid out to the creditor, the manager proceeds to the shareholder with an offer $\tilde{E}$, where $\tilde{E} \in [0, x - D]$. For the equity holder's indifference condition to hold,

$$\tilde{E}(x) = \beta(x - D) - c_E \tag{9}$$

Again, this offer implicitly contains a report $\tilde{x}$. For truthful reporting to occur in this subgame, it must be the case that $p(\tilde{x}) = P(\tilde{x}), \forall \tilde{x}$, i.e., the solution to the equity subgame is identical to the equilibrium of the pure equity financing case, considered in the previous section. This observation is proved in Appendix A.

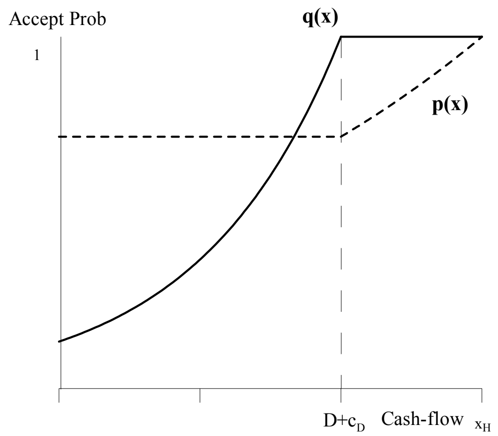
Consider now the case where the manager asks for a debt concession by offering $\tilde{D} < D$. Conditional on the creditor accepting the offer $\tilde{D} < D$, there remains c_D $(= x - \tilde{D})$ of the cash-flow, and the equity holder is offered $\beta c_D - c_E$, which he accepts with probability $p(D + c_D) := \bar{p}$.

If the equity holder was expected to never verify after a debt concession (i.e., $\bar{p} = 1$) then $q(.) = Q(.)$, i.e., the creditor would follow the same monitoring strategy as under pure debt financing (treating D as fixed). However, since the shareholder will have incentives to monitor after a concession (i.e., $\bar{p} < 1$), the creditor is more lenient under mixed financing than under pure debt financing. Hence the main new feature of the accept functions under a mixture is that, holding D constant, the creditor will be more lenient, because he takes into account that the shareholder will also monitor.

Formally, the equilibrium has the following structure: if $x \geq D + c_D$, the entrepreneur offers D to the creditor and $\beta(x - D) - c_E$ to the shareholder, which the shareholder accepts with probability $p(x) = P(x) = e^{-\beta \frac{x_H - x}{c_E}}$. If on the other hand $x < D + c_D$, the entrepreneur offers $x - c_D$ to the creditor, which the creditor accepts with probability $q(x) = \mathbf{e}^{\psi(x - D - c_D)}$, where $\psi := \frac{(1-\beta)+\bar{p}\beta}{(1-\beta)c_D + \bar{p}c_E}$. The proofs of these statements are in Appendix A. Informally, we have

Proposition 3 *(Capital mix) If the firm's external finance is a mixture of debt and equity, the creditor serves to discipline the manager in low states, and the equity holder disciplines the manager in high states. Strategic defaults and priority violations occur in equilibrium.*

The proposition can be illustrated in the following figure.



The true cash flow of the firm is on the horizontal axis, and the accept probabilities on the vertical axis. The function $q(x)$ is the equilibrium accept probability by the creditor, given that the offer from the lender is represented by the function $\tilde{D}(x)$. The function $p(x)$ is the equilibrium accept probability by the shareholder (recall that he is given an offer only if the creditor has accepted), given that the offer from the lender is represented by (12). For $x < D + c_D$, there is a positive probability of the creditor monitoring, while the probability of the shareholder monitoring (conditional on the creditor not monitoring) is constant (since the repayments are the same). For $x \geq D + c_D$, there is a zero probability of the creditor verifying, and a positive (and decreasing) probability of the shareholder verifying. Hence, there is a division of labor in equilibrium: the creditor disciplines the entrepreneur by monitoring in the bad states, and the shareholder disciplines the entrepreneur in good states.

Strategic defaults occur in equilibrium for $x \in [D, D + c_D]$, by which the manager defaults even though the firm has sufficient cash on hand to pay out the full debt value D. In the region $x \in [x_L, D + c_D]$ the lender will accept payments less than D without demanding a verification (with probability $q(x)$), and at the same time the repayment to the shareholder is strictly positive. This leniency on the part of the lender constitutes an absolute priority violation (AP-violation), since it implies that the equity holder receives a positive payoff (with probability $q(.)$) even though the lender is not paid the full value of his debt contract.

These results are of some practical interest, as strategic defaults and violations of priority rules are common explanations for why risk premia on corporate debt significantly exceed those implied by Merton (1974). Strategic defaults occur in the present setting because it is costly for the creditor to collect his payment as specified by the contract. The presence of this cost puts a sufficient wedge between the creditor's proper payment under the contract and what the insider is actually willing to offer, thus leading to strategic defaults for $x \in [D, D + c_D]$. As shown by Bergman & Callen (1991) and Mella-Barral & Perraudin (1997) a similar type of effect can occur in symmetric information models, where there is some costs for outside investors to invoke bankruptcy. It may be pointed out, however, that in the present setting, there are AP-violations in the sense that both inside and outside equity receive positive payments even though debt is not paid in full, while the literature on AP-violations (including the papers referred to above) focuses on inside equity. The empirical literature on AP-violations (e.g., Franks & Torous, 1989) obtains measures of the sum of AP-violations of internal and external junior claimants, and hence is easier comparable to the present model.

In a recent contribution, Bebchuck (2002) studies the effects of AP-violations on the ex-ante risk shifting incentives of borrowers, finding that debt that permits AP-violations induces stronger risk shifting incentives than debt that does not. The effect identified by Bebchuck can be generated in the present setting as well. While AP-violations in his setup are imposed exogenously by giving the borrower a fixed fraction of the firm's assets in any default state, the AP-violations in the present setting arise endogenously, due to the frictions created by the verification costs. By showing that debt with AP-violations may induce stronger risk shifting incentives than debt without AP-violations, Bebchuck (2002) identifies an important ex-ante cost of allowing for AP-violations. It may be noted though that this insight is generated by comparing a *riskless* project to that of a risky (less valuable) project. Although using a riskless project as benchmark provides for a clean experiment, the effects on ex-ante risk shifting incentives from AP-violations become more ambigous once the benchmark project is assumed risky as well. In such a case, whether AP-violations will generate greater or less risk shifting incentives will depend on factors such as the amount of debt that the firm issues and the underlying returns generating distribution.

Relatedly, Longhofer (1997) derives the optimal contract under the traditional CSV-assumption that the creditor is able to pre-commit to verify. He finds that AP-violations unambigously increase the cost of debt by increasing the verification region. However, while in Longhofer (1997) AP-violations are imposed exogenously, in the present setting they obtain endogeneously. And comparing the present contract to that of the traditional debt contract of the CSV setting under which the creditor commits to verify whenever debt is not paid in full, and hence do not allow for AP-violations, reveals that AP-violations will allow lower funding costs in many cases. In other words, debt without a commitment to verify, and hence debt that allows AP-violations, will in many cases be less costly than debt under which the creditor commits to verifiy, even under CSV.

If the optimal capital structure is mixed, we must have that $\beta c_D \geq c_E$. On the left hand side of this expression is the payoff for the shareholder if he verifies (given a low cash flow), and on the right hand side is his cost of entering the verification state. If the right hand side exceeds the left hand side it would not pay for the shareholder to monitor after the manager announces a low cash flow, in which case the manager would have incentives to misreport the true cash flow, and an equilibrium with a mixed capital structure cannot exist. From this observation there follows two necessary conditions for a mix to occur. First, the equity holder's stake β in the firm must be bounded away from zero (i.e., $\beta \geq \frac{c_E}{c_D}$). This is consistent with the idea from Admati et al. (1994) that to be effective monitors each shareholder must hold a sufficient stake in the firm to cover private monitoring costs. The second condition for mix to occur is that $c_E < c_D$, which follows from $\beta \in (0, 1]$. This condition is discussed further in Section 4.3.

4.1 Optimal capital structure

Let us now analyze the optimal capital structure, where we can obtain some insights although closed-form solutions are not feasible. For a given D and β, the expected verification cost is given by,

$$\begin{aligned} V(D, \beta) &= c_D \int_{x_L}^{D+c_D} [1 - q(x; .)] f(x) dx + c_E \int_{x_L}^{D+c_D} q(x; .)[1 - \bar{p}] f(x) dx \\ &\quad + c_E \int_{D+c_D}^{x_H} [1 - p(x; .)] f(x) dx \end{aligned} \tag{10}$$

The first two terms is the expected verification costs for low cash flows ($x \in [x_L, D + c_D]$), and the third term is the expected verification cost for high cash flows ($x \in [D + c_D, x_H]$). The objective of the entrepreneur is to pick the α that minimizes this expression, subject to the participation constraints of the investors. Notice that for $\alpha = 0$, i.e., pure equity financing, the first and the second term in (10) drop out, and $p(x; .) = P(x; .)$. For $\alpha = 1$, i.e., pure debt financing, the second and the third term of (10) drop, and $q(x; .) = Q(x; .)$. Trivially, when $c_D = c_E = 0$, any choice of capital structure will be optimal.

The first observation we can make about optimal capital structure follows from the necessary condition for mix $\beta c_D \geq c_E$.

Proposition 4 *High-NPV firms will be externally financed by debt only.*

Proof. For the outside equity holder to have incentives to monitor, he must have an ownership share that exceeds $\frac{c_E}{c_D}$. This implies that the (expected) verification cost is discontinuous in the point $I_E = 0$, where $I_E := (1 - \alpha)I$. On the other hand, the expected verification cost is continuous in the point $I_D = 0$. This implies that firms with high NPV (low I) will be 100% debt financed. ■

So far we have taken D and β as exogenous. To make further headway we need to include the outside investors' participation constraints, and endogenize D and β. For the creditor to be willing to participate, his expected payout equals his financing contribution, αI,[19]

$$\int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx = \alpha I \tag{11}$$

Notice that the creditor's expected utility is a function of D, but not β, since debt is the senior claimant. Likewise, for the shareholder's participation constraint to hold, we must have that

$$\int_{x_L}^{D+c_D} (\beta c_D - c_E) f(x) dx + \int_{D+c_D}^{x_H} [\beta(x - D) - c_E] f(x) dx = (1 - \alpha) I \tag{12}$$

Combining (11) and (12), we can obtain β as a function of D alone,

$$\beta(D) = \frac{I - \int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx + c_E}{\int_{x_L}^{D+c_D} c_D f(x) dx + \int_{D+c_D}^{x_H} (x - D) f(x) dx} \tag{13}$$

To find the optimal capital structure, it is more convenient to let D rather than α be the choice variable of the entrepreneur. The first order condition for minimum for the expected verification costs then becomes

$$\frac{dV}{dD} = \frac{\partial V}{\partial D} + \frac{\partial V}{\partial \beta} \frac{\partial \beta}{\partial D} = 0 \tag{14}$$

The first and the second partial derivative on the right hand side can be evaluated from (10), and the third can be evaluated from (13).[20]

Equipped with these expressions, we have the following.

[19] It can easily be verified that the participation constraints must be binding.

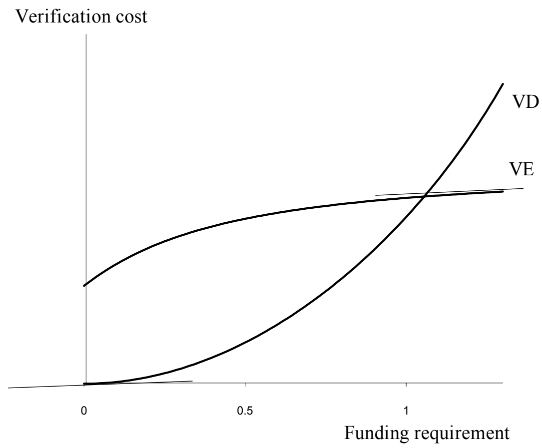
[20] Assuming that the value of α that minimizes verification costs, α^*, is on the interior of (0,1), the optimum condition $\frac{dV}{dD} = 0$ will hold for the optimal face value of debt, D^*, and hence the optimal capital structure α^* implicitly, since α is a function of D from equation (11). In other words, D^* uniquely determines α^*.

Proposition 5 *The firm will never be 100% equity financed.*

Proof. Letting D go to 0 in (14) gives a negative expression, as shown in Appendix B. ■

Note that this result does not hinge on debt being risk free, as shown in the proof.

We have established that the firm will be 100% debt financed for a sufficiently low funding requirement, and that the firm will never be 100% equity financed (even if the verification cost for equity should be much lower than the verification cost for debt). Some intuition for these results can be captured by a figure, which compares the expected cost of verification in the two polar cases $\alpha = 0$ and $\alpha = 1$ (i.e., pure equity financing and pure debt financing).



The figure shows the cost of capital (verification cost) for pure debt financing (VD) and pure equity financing (VE), as a function of the funding requirement I. For a low funding requirement, pure debt is the better financing due to the discontinuity of VE in the point $I = 0$, which arises because a $\beta >> 0$ is required for the equity holder to have incentives to monitor ex-post. That gives intuition for Proposition 4. For a higher funding requirement, VD exceeds VE, and one may think that pure equity dominates a mixed financing. However, having a mix of capital has a lower verification cost than pure equity, because it is on the margin cheaper to issue debt than to issue more equity. This can be captured by comparing the gradient of VD at a low level of debt with the gradient of VE with a high level of equity, drawn in the figure. That gives intuition for

Proposition 5. What drives the convexity of VD is that the face value of debt D, via the creditor's participation constraint, is convex in the amount of funding αI delivered by the creditor. The reason for this is that default probability of debt is increasing in D, and that this factor becomes more important as αI and thus D is increased.[21]

Our theory of capital structure is related to the trade-off theory of capital structure, where the optimal mix of debt and equity obtains at the point where the tax benefit of debt, on the margin, equals the expected bankruptcy cost. In our setting, the optimal mix of debt and (outside) equity obtains when the expected intervention cost associated with debt, on the margin, equals the expected intervention cost of equity. A distinction to the trade-off theory is that the latter does not make a distinction between inside and outside equity, since equity in this theory serves just as a buffer against bankruptcy. In the present theory, however, outside equity has an active role in disciplining the manager and there is therefore a clear distinction between inside and outside equity. In the choice between inside and outside equity as a financing mean, the entrepreneur would choose inside equity because the verification cost is lower. As such, our theory has predictions reminiscent of pecking-order: inside funds are preferred, then debt, and if the funding need is large, issue outside equity along with debt.

To make the equilibrium structure more concrete, let us now consider an example.

4.2 A typical example

Recall that the cash flow x follows the density function $f(x)$ with support $[x_L, x_H]$, the funding requirement equals I, and the cost of verification is c_D and c_E for debt and equity, respectively. Now let

Example 1 $f(x) = \dfrac{1}{x_H - x_L}$, $x_L = 1.2$, $x_H = 3.8$, $c_D = \frac{1}{2}$, $c_E = \frac{1}{5}$, $I = 1.4$.

Denoting the optimum values by a * topscript, we get that for these parameter values, $D^* = .80$, $\beta^* = .47$, $\alpha^* = .57$, and $V^* = .17$, where D^* is the optimum face value of debt, β^* is the optimum ownership share of the outside equity holder, α^* is the fraction of I

[21] Although the convexity of VD holds for all the examples we have considered, we do not have a general proof of it.

financed by debt, and V^* is the expected verification cost.[22] Hence we get a mixed capital structure, where 57% of the capital is raised through issuing debt. By defining the debt ratio of the firm as the (expected) value of debt, αI, divided by the value of the firm, $Ex - V$, i.e., $g := \frac{\alpha I}{Ex - V}$, we get that $g^* = .34$.

Interpreting c_D as a bankruptcy cost, $c_D = \frac{1}{2}$ gives a bankruptcy cost of 21% of the firm's market value $Ex - V^* = 2.33$. This magnitude is consistent with the empirical evidence on bankruptcy costs of $10-20\%$ of the firm's market value, as found by Andrade and Kaplan (1998), and the 25% found by Altman (1985).

To get an idea of how the optimal capital structure changes as a function of the exogenous variables, let us perform three comparative statics exercises; increasing the funding requirement, decreasing the verification cost for equity, and changing risk by changing the support of the distribution. The example is typical in that changing parameter and distribution assumptions, we were unable to generate examples that did not have identical (qualitative) comparative statics features.

By increasing the funding requirement to $I = 1.5$ in Example 1, and keeping the other parameters unchanged, we get $D^* = .76$, $\beta^* = .54$, $\alpha^* = .51$, and $g^* = .33$. Hence increasing the funding requirement leads to a lower debt ratio, which is as expected given Proposition 4. Decreasing the verification cost, by setting c_E equal to e.g., .15 in Example 1, we get $D^* = .75$, $\beta^* = .46$, $\alpha^* = .54$, and $g^* = .23$, hence also a decrease in the debt ratio.

We can also decrease risk in Example 1, by setting $x_L = 1.3$ and $x_H = 3.7$. In that case, we obtain $D^* = .94$, $\beta^* = .42$, $\alpha^* = .67$, and $g^* = .40$. Hence, when decreasing risk, we get that 67% of the capital is raised through issuing debt, in contrast to 57% before, and the firm's debt ratio increases from 34% to 40%. This result is consistent with empirical evidence of less risky firms having a higher debt ratio than more risky firms (see the survey by Harris & Raviv, 1991, and for more recent evidence, Fama and French, 2002). We can sum up these findings in a remark.

Remark 1 *The following gives a lower debt ratio,*

i)Increasing the funding requirement

[22] The numbers are generated in Maple V, and the worksheets are available from the authors.

ii)Decreasing c_E

iii)Decreasing risk

4.3 Monitoring costs

To obtain a mixed capital structure, we need that $c_E < c_D$. Although we are not aware of systematical empirical work comparing the intervention cost of debt and the intervention cost of equity, this condition is consistent with the following two lines of thought.

First, if intervention by creditors involves liquidation of the firm, while intervention by the equity holder involves continued operation of the firm, we can interpret the condition $c_E < c_D$ as saying that the *social cost* of intervention by the creditor (cost of taking control plus reduced value due to liquidation) is higher than the social cost of intervention by the equity holder (cost of taking control). Observe that the creditor is always reimbursed the c_D in the verification state, and hence we are free to interpret c_D as a social cost rather than the private cost incurred by the creditor.

The second justification for $c_E < c_D$ is as follows. In an extended model, we can imagine the verification costs as being determined by ex-ante investments by the claimants. The idea is that if the investors can make investments in monitoring technology before x is realized, then creditors will have less incentives than equity holders to invest, and we get $c_E < c_D$ as part of the equilibrium description. Let us illustrate this idea. Under a (pure) debt contract, the creditor's expected utility equals,

$$U_D = \int_{x_L}^{D+c_D} (x - c_D) f(x) dx + \int_{D+c_D}^{x_H} D f(x) dx \tag{15}$$

The marginal increase in utility from decreasing c_D equals,

$$-\frac{\partial U_D}{\partial c_D} = p := \int_{x_L}^{D+c_D} f(x) dx < 1 \tag{16}$$

Hence the marginal gain from decreasing verification cost equals the probability p of the entrepreneur making a debt concession pledge, i.e., whenever $x \in [x_L, D + c_D]$.

In contrast, under a (pure) equity contract, the shareholder's expected utility equals,

$$U_E = \int_{x_L}^{x_H} (\beta x - c_E) f(x) dx \tag{17}$$

The marginal increase in utility from decreasing c_E equals,

$$-\frac{\partial U_E}{\partial c_E} = 1 \tag{18}$$

Hence the equityholder gains from decreasing c_E independently of the realization of x.

A comparison of the derivatives (16) and (18) reveals that the marginal gain from improving monitoring technology is higher for an equity holder than for a creditor.

Suppose that the creditor and the investor have access to an identical technology for making the monitoring less costly. Such investments can take place after the contracts $\{D,\beta\}$ are signed, but before the entrepreneur makes his repayment offer. Then, in equilibrium, the equity holder will make a higher investment in monitoring technology than a creditor, thus leading to $c_E < c_D$ in equilibrium. Of course, the magnitude of $c_D - c_E$ will depend on the specific cost function associated with improvements in monitoring technology, but the main message is clear: there are good economic reasons to believe that $c_E < c_D$ in equilibrium that is entirely due to the different payoff structure for a debt and for an equity claim.[23]

5 Conclusion

We have constructed a theory of capital structure that accommodates strategic defaults and priority violations, phenomena that are well-known from the empirical and theoretical literature, but has not been integrated into a theory of capital structure before. In addition, the model produces implications that are consistent with several other stylized

[23] Under a mixed financing, the incentives for making investments in monitoring will be the same for the creditor as under pure debt financing, while the incentives for the equity holder will be reduced because the creditor may not accept the repayment offer from the entrepreneur (and there is consequently a positive probability that the investment in c_E will be wasted). However, the simple message from comparing (16) and (18) will still be present, interacting with other effects. We leave a further exploration of this issue for further work.

facts, such as bankruptcies occurring in equilibrium, a division of labor in disciplining the entrepreneur between different security holders, and the debt ratio of the firm increasing in its NPV. Also, we find that firms with low funding requirements will issue only debt, while firms with sufficently high funding requirements will issue a mix of debt and equity. In other words, firms may be all debt financed, but will never be all equity financed, even if the intervention cost of equity is much lower than that of debt.

The basis of the theory is the cash diversion problem. Under the simplest interpretation this says that the manager steals the money. Although this phenomenon may be important in undeveloped economies (Shleifer & Vishnu 1997 provide some examples), the cash diversion in the model may more reasonably be seen as a short form of a situation where the manager may divert the cash into unprofitable pet projects if given the opportunity, which creates a need for investors to discipline. In contrast to some earlier models of costly state verification, our added features are to require sequential rationality and to allow for equity contracts in addition to debt.

For future work, we see a range of possible extensions of the present framework. First, it may be of interest to introduce dynamics in the model, to discuss such issues as dividend policy and delays in debt repayments. A second extension of our work would be to discuss commitment debt (where the creditors commit to verifying whenever the proposed repayment falls short of some treshold) vis-a-vis non-commitment debt (considered in the paper) and to allow for different seniority in debt claims. Dispersed investors in the securities market may have commitment through their free-rider status, while banks do not. A preliminary result from our analysis of this question indicates that non-commitment (bank) debt dominates commitment (security) debt for projects with a cash flow distribution which is skewed to the left, which is intuitively appealing, as the non-commitment debt would rely on verifying less often in low cash-flow states. Third, it would be of interest to extend the current setup to accommodate investments in monitoring technology. Equity holders may have stronger incentives than debt holders to invest in a monitoring technology, but on the other hand it is not obvious how the private costs of investment levels correspond to the social costs. If claimants may have private costs that differ from social costs when investing in monitoring technology this may have interesting implications for security design that lies outside the scope of the present paper.

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7 Appendix A

Here we prove Proposition 2, and then prove Proposition 3.

7.1 Proof of Proposition 2

For the manager to prefer announcing truthfully, it must be the case that,

$$U(\tilde{x}) = P(\tilde{x})[x - \beta\tilde{x} + c_E] + [1 - P(\tilde{x})](1 - \beta)x \qquad \text{(A1)}$$

is maximized for truthful reporting, i.e., $\tilde{x} = x$. Differentiating (A1) with respect to $\tilde{x}$ and setting $\tilde{x} = x$ we obtain the differential equation,

$$P(\tilde{x})\beta - \frac{dP(\tilde{x})}{d\tilde{x}} c_E = 0 \qquad \text{(A2)}$$

Solving this differential equation yields,

$$P(\tilde{x}) = K\mathbf{e}^{\dfrac{\beta\tilde{x}}{c_E}} \tag{A3}$$

By using the corner condition $P(x_H) = 1$, we obtain that the probability of the shareholder accepting the announcement $\tilde{x}$ (with an implied offer $\beta\tilde{x} - c_E$ to the investor) equals,

$$P(\tilde{x}) = \mathbf{e}^{-\dfrac{\beta(x_H - \tilde{x})}{c_E}}, \; x, \tilde{x} \in [x_L, x_H] \tag{A4}$$

The second order condition for a true announcement being optimal can be easily checked to hold: $\dfrac{d^2U(\hat{x})}{d\hat{x}^2} = \dfrac{\beta^2}{c_E}[p'd - p] = \dfrac{\beta^2}{c_E}[\dfrac{\beta}{c_E}pd - p] = p\dfrac{\beta^2}{c_E}[\dfrac{\beta}{c_E}d - 1] < 0$ for $d = 0$.

7.2 Proof of Proposition 3

For a truth-telling equilibrium to exist, as before it must be the case that,

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{A5}$$

There are two cases of interest, $\tilde{D} = D$ and $\tilde{D} < D$. When $\tilde{D} = D$, the true cash-flow is not fully revealed, and we enter the equity subgame, with the same solution as before, i.e.,

$$p(\tilde{x}) = \mathbf{e}^{-\dfrac{\beta(x_H - \tilde{x})}{c_E}} \tag{A6}$$

The reason for this is the following. Note that after D is repaid,

$$\begin{aligned} U(\tilde{x}) &= p(\tilde{x})[x - D - \beta(\tilde{x} - D) + c_E] + (1 - p(\tilde{x}))[(1 - \beta)(x - D)] \qquad \text{(A7)} \\ &= (1 - \beta)(x - D) + p(\tilde{x})[c_E - \beta(\tilde{x} - x)] \end{aligned}$$

Differentiating with respect to $\tilde{x}$ and substituting for $\tilde{x} = x$, one obtains the first order condition for truthful reporting,

$$\frac{dU(x)}{dx} = p(x)\beta - \frac{dp(x)}{dx}c_E = 0 \tag{A8}$$

Using the corner condition $p(x_H) = 1$ and solving the differential equation, we obtain the $p(\tilde{x})$ function from (A6). As for pure debt and pure equity financing, it can easily be seen that the second order conditions for maximum hold.

Now consider the case $\tilde{D} < D$, which must occur when $x < D + c_D$. There are then two cases, the creditor accepting the offer and the creditor rejecting the offer. If the creditor rejects the offer, both the manager and the shareholder receive zero. If the creditor accepts the offer, there remains c_D of the cash flow, and for the equity holder to be indifferent between accepting and not accepting, the manager must offer him $\beta c_D - c_E$ which, by continuity of $p(.)$ is accepted with probability $p(D + c_D)$. The utility of the manager in this case is

$$U(x) = q(x)[(1-\beta)c_D + \bar{p}c_E] \tag{A9}$$

Suppose now that the manager reports $\tilde{x} < x$, with the implied offer to the creditor of $\tilde{x} - c_D$, where $d = x - \tilde{x}$. If accepted, the manager is now left with $c_D + d$ and offers the shareholder an amount $\beta c_D - c_E$ (where d is sufficiently large to ensure $c_D + d \geq \beta c_D - c_E$ or $d \geq (1-\beta)c_D + c_E$), which the shareholder accepts with probability $\bar{p}$. The utility of the manager from such misreporting becomes,

$$\begin{aligned} U(\tilde{x}) &= q(\tilde{x})[(1-\beta)c_D + \bar{p}(c_E + d) + (1-\bar{p})(1-\beta)d] \\ &= q(\tilde{x})[(1-\beta)c_D + \bar{p}c_E + [(1-\beta) + \bar{p}\beta]d] \end{aligned} \tag{A10}$$

Differentiating with respect to $\tilde{x}$ and substituting for $\tilde{x} = x$ yields the first order condition,

$$\frac{dU(x)}{dx} = -\frac{dq(x)}{dx}[(1-\beta)c_D + \bar{p}c_E] + q(x)[(1-\beta) + \bar{p}\beta] = 0 \tag{A11}$$

Solving then for $\frac{dU(x)}{dx} = 0$, using the corner condition $q(D + c_D) = 1$, we obtain

$$q(x) = \mathbf{e}^{\psi(x-D-c_D)} \tag{A12}$$

where $\psi := \frac{(1-\beta)+\bar{p}\beta}{(1-\beta)c_D+\bar{p}c_E}$, as stated in the text.

8 Appendix B

As described in the main text, we can write the expected verification as purely a function of D, by combining equation (10) and equation (13). Differentiating V with respect to D in equation (10) we then get,

$$\begin{aligned} \frac{dV}{dD} = \; & c_D[1 - q(D + c_D)]f(D + c_D) + \int_{x_L}^{D+c_D} \frac{\partial[1 - q(x)]}{\partial D} f(x)dx \\ & -c_E[1 - q(D + c_D)]f(D + c_D) + \int_{D+c}^{x_H} \frac{\partial[1 - p(x)]}{\partial D} f(x)dx \\ & +c_E q(D + c_D)[1 - \bar{p}]f(D + c_D) \\ & + \int_{x_L}^{D+c_D} \frac{\partial[q(x)(1 - \bar{p}]}{\partial D} f(x)dx \end{aligned} \tag{B1}$$

Noting that $1 - q(D + c_D) = 0$ and $\frac{\partial[1-p(x)]}{\partial D} = \frac{1}{c_D}(x_H - x)p(x)\frac{\partial\beta}{\partial D}$, we obtain the following first order condition for minimum,

$$\frac{dV}{dD} = c_D \int_{x_L}^{D+c_D} \frac{\partial[1-q(x)]}{\partial D} f(x)dx + c_E \int_{D+c_D}^{x_H} \frac{\partial[1-p(x)]}{\partial D} f(x)dx + c_E \int_{x_L}^{D+c_D} \frac{\partial[q(x)(1-\bar{p}))]}{\partial D} f(x)dx = 0 \tag{B2}$$

where

$$\frac{\partial\beta}{\partial D} = \frac{-\int_{D+c_D}^{x_H} f(x)dx[\int_{x_L}^{D+c_D} c_D f(x)dx + \int_{D+c_D}^{x_H}(x-D)f(x)dx] - \{I - [\int_{x_L}^{D+c}(x-c_D)f(x)dx + \int_{D+c}^{x_H} Df(x)dx]\} \int_{D+c}^{x_H} f(x)dx}{[\int_{x_L}^{D+c_D} c_D f(x)dx + \int_{D+c_D}^{x_H}(x-D)f(x)dx]^2} < 0 \tag{B3}$$

and the second order condition for minimum is $\dfrac{d^2V}{dD^2} > 0$. Equation (B2) and (B3) define D^* implicitly, and hence the optimal capital structure α^* implicitly, since α is a function of D from equation (11).

The **proof of Proposition 5** proceeds as follows. To avoid the triviality of debt being risk-free (in which case it is obvious that the firm would issue some debt), assume that $c_D = x_L$. Then take D to 0 in equations (B2) and (B3) to obtain,

$$\begin{aligned} \frac{dV}{dD}_{D=0} &= c_E \int_{x_L}^{x_H} \tfrac{\partial[1-p(x)]_{D=0}}{\partial D} f(x)dx \\ &= \frac{c_E}{c_D} \int_{x_L}^{x_H} (x_H - x)P(x) \frac{\partial \beta}{\partial D}_{D=0} f(x)dx \end{aligned} \tag{B4}$$

This expression is negative, since $\frac{\partial \beta}{\partial D}_{D=0} = -\frac{Ex-I}{Ex^2} < 0$.

9 Appendix C: alternative contracts

Here we consider the possibility that verification state payoffs can be made conditional on both the true cash flow, x, and the announced cash flow $\tilde{x}$. Specifically, to obtain truth-telling in the cheapest possible way, for any x, we consider the maximum penalty for false reports, which is to punish such that $U(x, \tilde{x}) = 0$ whenever $\tilde{x} \neq x$.

9.1 Debt contracts

We suppose that contracts specify the payoffs to the manager in the verification state as,

$$U(x, \tilde{x}|\text{verification}) = \begin{cases} x - \min(D, x),\ \tilde{x} = x \\ 0, \text{ for } \tilde{x} < x \end{cases} \tag{C1}$$

where $\tilde{x}$ is the reported x. Notice that this contract may imply a payout to the creditor higher than D (in the case where x is sufficiently high, and $\tilde{x} \neq x$).[24] We first consider the incentives for truth-telling for $x \in [x_L, D + c_D]$. The utility from truth-telling becomes simply,

$$U(x) = Q(x)c_D \tag{C2}$$

[24] If the payout to the creditor is limited to D also when a lie is detected, it can easily be shown that the equilibrium accept function is identical to in the original problem.

The utility from announcing $\tilde{x}$, where $\tilde{x} < x$,

$$U(\tilde{x}) = Q(\tilde{x})(x - \tilde{x} + c_D) + (1 - Q(\tilde{x}))0. \tag{C3}$$

To obtain truthful reporting,

$$U(x) - U(\tilde{x}) = Q(x)c_D - Q(\tilde{x})(x - \tilde{x} + c_D) \geq 0, \, \tilde{x} \leq x, \, x \in [x_L, D + c_D] \tag{C4}$$

This is the same expression as in the original setup, and hence we obtain that for truth-telling to occur for x on the interval $[x_L, D+c_D]$ we get the same solution as in the original setup. We now consider the incentives for truth-telling when $x \in [D+c_D, x_H]$, and where the announcement lies below this interval.

The utility from truth-telling becomes,

$$U(x) = x - D. \tag{C5}$$

The utility from announcing $\tilde{x}$, where $\tilde{x} < D + c_D$,

$$U(\tilde{x}) = Q(\tilde{x})(x - \tilde{x} + c_D) + (1 - Q(\tilde{x}))0. \tag{C6}$$

To obtain truthful reporting,

$$U(x) - U(\tilde{x}) = x - D - Q(\tilde{x})(x - \tilde{x} + c_D) \geq 0, \, \tilde{x} < D + c_D < x \tag{C7}$$

Solving for $Q(\tilde{x})$ we obtain,

$$Q(\tilde{x}) \leq \frac{x - D}{x - \tilde{x} + c_D}, \, \tilde{x} < D + c_D < x \tag{C8}$$

For every x, this equation defines the set of accept probabilities consistent with truth-telling. The maximum accept probability (which is the relevant to ensure truth-telling in the cheapest possible way) for each x is hence defined as,

$$\frac{x - D}{x - \tilde{x} + c_D}, \, \tilde{x} < D + c_D < \; x \tag{C9}$$

As can easily be verified, this function is minimized for $x = x_H$ (for every $\tilde{x}$).[25] Hence, for truth-telling to occur in the cheapest possible way,

$$Q(\tilde{x}) = \frac{x - D}{x - \tilde{x} + c_D},\ \tilde{x} < D + c_D < \ x \tag{C10}$$

Using the corner condition $Q(D + c_D) = 1$, we obtain the equilibrium accept probability function,

$$Q(x) = \begin{cases} \frac{x_H - D}{x_H - x + c_D},\ x \leq D + c_D \\ 1, \text{ for } x > D + c_D \end{cases} \tag{C11}$$

This function is continuous, increasing, and convex, and takes on the value 1 for $x = D + c_D$. In other words it has the same qualitative properties as the $Q(.)$, function derived in the main text.

9.2 Equity contracts

We consider contracts that are linear in x conditional on truth-telling, but yields $U(x) = 0$ in the event of false reports. More specifically, suppose that contracts specify,

$$U(x, \tilde{x}|\text{verification}) = \begin{cases} (1 - \beta)x,\ \tilde{x} = x \\ 0, \text{ for } \tilde{x} < x \end{cases} \tag{C12}$$

We now derive the accept probability in this case. The utility from truth-telling becomes,

$$U(x) = P(x)[x - \beta x + c_E] + [1 - P(x)][x - \beta x] = (1 - \beta)x + P(x)c_E. \tag{C13}$$

The utility from announcing $\tilde{x}$, where $\tilde{x} < x$,

$$U(\tilde{x}) = P(\tilde{x})(x - \beta\tilde{x} + c_E) + (1 - P(\tilde{x}))0 = P(\tilde{x})(x - \beta\tilde{x} + c_E). \tag{C14}$$

For announcing truthfully to be incentive compatible, it must be the case that,

$$U(x) - U(\tilde{x}) = (1 - \beta)x + P(x)c_E - P(\tilde{x})(x - \beta\tilde{x} + c_E) \geq 0,\ \tilde{x} < x. \tag{C15}$$

[25] Formally, $\frac{x_H - D}{x_H - k + c_D} < \frac{x - D}{x - k + c_D}$, $k < D + c_D < x, x_H$.

Rearranging,

$$P(x) \geq \frac{P(\tilde{x})(x - \beta\tilde{x} + c_E) - (1-\beta)x}{c_E}. \tag{C16}$$

For every x, this equation defines the set of accept probabilities consistent with truth-telling. Suppose that truth-telling is hardest to obtain for $x = x_H$ (i.e., has the lowest maximum value of $P(\tilde{x})$ consistent with truth-telling). Then, imposing the corner condition $P(x_H) = 1$ yields,

$$P(\tilde{x}) \leq \frac{(1-\beta)x_H + c_E}{x_H - \beta\tilde{x} + c_E} \tag{C17}$$

Hence, for truth-telling to occur in the cheapest possible way,

$$P(\tilde{x}) = \frac{(1-\beta)x_H + c_E}{x_H - \beta\tilde{x} + c_E} \tag{C18}$$

In that case, we get an equilibrium accept probability function which equals,

$$P(x) = \frac{(1-\beta)x_H + c_E}{x_H - \beta x + c_E},\ x \in [x_L, x_H] \tag{C19}$$

Notice that this function is increasing and convex, and takes on the value 1 for $x = x_H$. In other words it has the same qualitative properties as the $P(.)$ function derived in the main text.[26]

The question is now under which conditions the $P(.)$ function defined in (C19) ensures truth-telling for *all* x (or in other words when truth-telling is hardest to obtain for $x = x_H$). In that case $P(.)$ in (C19) is a solution to the problem. We have the following result.[27]

Proposition 6 *For sufficiently small c_E, the $P(.)$ function given by (C19) ensures truth-telling in equilibrium for all x.*

[26] Not surprisingly, the $P(.)$ function defined here induces a lower verification cost than the $P(.)$ function derived in the main text. However, we have not taken into account that making announcements verifiable to courts may have some cost.

[27] The problem with generalizing this result to hold for all c_E is that for sufficiently high c_E the function defined by (C19) will not induce truth-telling for all values of x. In particular, there will exist $x << x_H$ such that lying yields a higher payoff then truth-telling. We conjecture that a $P(.)$ function can be defined such that there always exists (truth-telling) equilibria, but this is a rather complex variational calculus problem that lies beyond the reach of the present paper.

Proof. We need to show that for sufficiently small (but positive) c_E, the $P(.)$ function defined by (C19) ensures truth-telling for all x. Letting c_E go to zero in (C15), we obtain that to ensure truth-telling,

$$(1-\beta)x - P(\tilde{x})(x-\beta\tilde{x}) \geq 0 \tag{C20}$$

substituting in for $P(.)$ implies that,

$$(1-\beta)x - \frac{(1-\beta)x_H}{x_H-\beta\tilde{x}}(x-\beta\tilde{x}) \tag{C21}$$

$$= x - \frac{(x-\beta\tilde{x})x_H}{x_H-\beta\tilde{x}} = \beta\tilde{x}\frac{x_H-x}{x_H-\beta\tilde{x}} > 0; \forall x < x_H, \tilde{x} < x$$

By the continuity of $\frac{x_H-x}{x_H-\beta\tilde{x}}$, there exists a strictly positive constant ε, such that $\frac{x_H-x}{x_H-\beta\tilde{x}} > 0$ for $c_E \in [0,\varepsilon]$. ■

We can notice that the (expected) verification cost functions for both types of financing in this case is convex, so not surprisingly it can be shown that a mixed capital structure can indeed be optimal also in this modified setup.

10 Appendix D: $\tilde{D}$ is observable to equity holders

In the main text, we assumed that $\tilde{D}$ was unobservable to equity holders. In this appendix, we consider the case where $\tilde{D}$ is observable to the equity holders, so that the investor's accept probability function becomes a function of both $\tilde{E}$ and $\tilde{D}$.

For a truth-telling equilibrium to exist, as before it must as before be the case that,

$$\tilde{D}(x) = \begin{cases} x - c_D \text{ for } x \in [x_L, D + c_D] \\ D, \text{ for } x \in [D + c_D, x_H] \end{cases} \tag{D1}$$

There are two cases of interest, $\tilde{D} = D$ and $\tilde{D} < D$. When $\tilde{D} = D$, the true cash-flow is not fully revealed, and we enter the equity subgame, with the same solution as before, i.e.,

$$P(\tilde{x}) = \mathbf{e}^{-\frac{\beta(x_H - \tilde{x})}{c_E}} \tag{D2}$$

The second case occurs when $\tilde{D} < D$, in which case there remains c_D of the cash-flow after the creditor is paid, which is known to the outside equity holder. Since the verification payoff to the equity holder equals $\beta c_D - c_E$ with certainty, the manager can ensure *acceptance with probability 1* by offering $\tilde{E} = \beta c_D - c_E + \varepsilon$, where ε is positive but small. Hence in the equity subgame that follows an accepted offer of $\tilde{D} < D$ to the creditor, the manager offers $\beta c_D - c_E$ (or arbitrarily close) and the equity holder accepts with probability 1.

Let us find the $q(.)$ function that induces truth-telling given $x, \tilde{x} \in [x_L, D + c_D)$. If the manager announces $\tilde{x}$, the surplus of the manager will be,

$$U(\tilde{x}) = q(\tilde{x})[c_D(1-\beta) + c_E + d] \tag{D3}$$

Differentiating this expression with respect to $\tilde{x}$ and setting $\tilde{x} = x$, we obtain the differential equation,

$$q(x) - \frac{dq(x)}{dx}[c_D(1-\beta) + c_E] = 0 \tag{D4}$$

which yields the solution,

$$Ke^{\frac{x}{c_D(1-\beta)+c_E}} \tag{D5}$$

We must now determine the integration constant K. To induce truth-telling, the payoff from truth-telling must be continuous in the point $x = D + c_D$ (if not, the manager would have incentives to under- or overreport),

$$\lim_{x \to (D+c_D)^-} U(x) = \lim_{x \to (D+c_D)^+} U(x) \tag{D6}$$

which implies that,

$$\lim_{x \to (D+c_D)^-} \{q(x)[c_D(1-\beta) + c_E]\} = \lim_{x \to (D+c_D)^+} \{p(x) + (x - D)(1-\beta)\}$$

As argued above, $\lim_{x \to (D+c_D)^+}\{p(x)\} = p(D + c_D)$. Denote $\lim_{x \to (D+c_D)^-}\{q(x)\}$ by $q(D+$

$c_D)^-$. We then substitute into (D6) to obtain,

$$q(D+c_D)^-[c_D(1-\beta)+c_E] = p(D+c_D)c_E + c_D(1-\beta) \tag{D7}$$

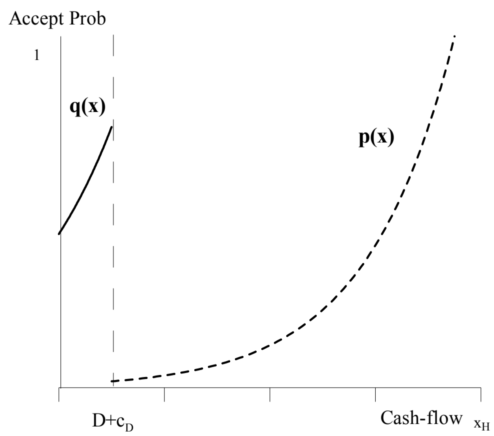
Substituting in for $q(D+c_D)^-$ and $p(D+c_D)$, we can then determine K,

$$K\mathbf{e}^{\frac{D+c_D}{c_D(1-\beta)+c_E}}[c_D(1-\beta)+c_E] = \mathbf{e}^{-\frac{\beta(x_H-D-c_D)}{c_E}}c_E + (1-\beta)c_D \tag{D8}$$

which gives,

$$K = \mathbf{e}^{-\frac{D+c_D}{c_D(1-\beta)+c_E}}\frac{\mathbf{e}^{-\frac{\beta(x_H-D-c_D)}{c_E}}c_E + (1-\beta)c_D}{c_D(1-\beta)+c_E}$$

Notice that this implies that $q(D+c_D)^- < 1 \neq q(D+c_D) = 1$, in other words the accept function of the creditor is discontinuous in the point $x = D+c_D$, i.e., a small default will imply a discontinuous jump (down) in accept probability from 1. The equilibrium then has the following structure, illustrated in a figure.



For a low x, there is a positive probability of the creditor monitoring, while the probability of the investor monitoring (conditional on the creditor not monitoring) is zero. For a higher x, there is a zero probability of the creditor verifying, and a positive (and decreasing) probability of the investor verifying. Hence the monitoring responsibility is completely specialized in equilibrium; the creditor has the role of disciplining the entrepreneur in bad states, and the outside investor has the role of disciplining the manager in good states. We can notice that the probability of the creditor verifying is discontinuous in the point $x = D + c_D$, as in the original setup of Townsend (1979) but now without any assumed commitment power by the creditor.[28]

Priority violations occur in equilibrium, since in the region $x \in [x_L, D + c_D]$ the lender will accept payments less than D without demanding a verification (with probability $q(x)$), and at the same time the repayment to the investor is strictly positive. In fact, since it is known that only c_D remains after the creditor is paid out, the equity holder will accept any offer higher than $\beta c_D - c_E$ with probability 1. As before, strategic defaults occur in equilibrium for $x \in [D, D + c_D]$, by which the manager defaults even though the firm has sufficient cash on hand to pay out the full debt value D. An example with exactly the same qualitative properties as Example 1 can easily be constructed and is skipped for brevity.

[28] A somewhat puzzling implication is that the "total" intervention probability decreases in the point $D + c_D$. The intuition for this goes as follows. First note that the manager is essentially the residual claimant in the financing game, after the verification costs have been paid. He is therefore less anxious about equity intervening than debt intervening, because c_E is lower than c_D. Hence to obtain truth-telling around the point $D + c_D$ we need the equity holder to be more lenient with the manager, so that the intervention probability must drop in the point $D + c_D$.