Option Exercise with Temptation*

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Abstract

This paper analyzes an agent's option exercise decision under uncertainty. The agent decides whether and when to do an irreversible activity. He is tempted by immediate gratification and suffers from self-control problems. This paper adopts the Gul and Pensendorfer self-control utility model. Unlike the time inconsistent hyperbolic discounting model, it provides an explanation of procrastination and preproperation based on time consistency. When applied to the investment and exit problems, it is shown that (i) if the project value is immediate, an investor may invest in negative NPV projects; (ii) if the production cost is immediate, a firm may exit even if it makes positive net profits; and (iii) if both rewards and costs are immediate, an agent may simply follow the myopic rule which compares only the current period benefit and cost.

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1 Introduction

Suppose you have a referee report to write today. You feel writing the referee report is unpleasant and prefer to put off and do it tomorrow. But when tomorrow comes, you tend to delay again. This phenomenon is often referred to as procrastination – wait when you should do it. Suppose you have a coupon to see one movie over the next several weeks, and your allowance does not permit you to pay for a movie. You tend to see a movie in an earlier week even though there may be a better movie in a later week. This phenomenon is often referred to as preproperation – do it when you should wait.[1] Procrastination and preproperation are often explained by the combination of time inconsistent preferences (or the hyperbolic discounting model) and a temporal gap between the costs and rewards associated to an action (O'Donoghue and Rabin (1999a)).[2] The key intuition is that an agent with hyperbolic discounting preferences has a present bias; that is, he discounts short-term events more heavily than long-term events.

The purpose of this paper is to provide an alternative explanation of these phenomena in a general environment where an agent with *time-consistent preferences* makes irreversible binary choices under uncertainty and possibly infinite horizon. I adopt the Gul and Pesendorfer (2001, 2004) self-control utility model and interpret those phenomena as an agent's struggling with temptations.[3] In this model, preferences are defined over a domain of sets of alternatives or decision problems. Utility depends on the decision problem from which current consumption is chosen. The interpretation is that temptation has to do with not just what the agent has consumed, but also what he could have consumed. The agent also seeks immediate gratification because an immediate benefit constitutes a temptation to the agent, but not because it has a higher relative weight. The agent may either succumb to temptations or exercise costly self-control to resist temptations.

The Gul-Pesendorfer model is time consistent because utility satisfies recursivity. Thus, the standard recursive methods such as backward induction and dynamic programming can be applied. In addition to its tractability, the Gul-Pesendorfer model has clear welfare implications because it is based on the standard revealed preference principle. This is in contrast to the hyperbolic discounting model where there is no generally agreed welfare criterion. This is because

[1] These examples and the term "preproperation" are borrowed from O'Donoghue and Rabin (1999a).

[2] Strotz (1956) first studies time-inconsistent preferences in economics. Akerlof (1991) analyzes procrastination, but frames his discussion very differently. The O'Donoghue and Rabin model has been generalized by a number of papers, e.g., O'Donoghue and Rabin (1999b, 2001), Brocas and Carrillo (2001a, 2001b). The hyperbolic discounting model has been applied to study consumption/saving (Laibson (1994, 1997), job search (DellaVigna and Paserman (2004)), social security (Imrohoroglu et al (2003)), and retirement (Diamond and Koszegi (2003)).

[3] The Gul-Pesendorfer model has been applied to study taxation (Krusell and Smith (2001)), asset pricing (Krusell et al (2002), DeJong and Ripoll (2003)), and nonlinear pricing (Esteban et al (2003)).

the agent at different dates is treated as a separate self. An alternative or a policy may be preferred by some selves, while it may make other selves worse off. The Pareto efficiency criterion and the long-run ex ante utility criterion are often adopted.

In section 2, I model an agent's irreversible binary choice problem under uncertainty as an option exercise problem, or more technically, an optimal stopping problem. Irreversibility and uncertainty are important in many binary choice problems such as entry, exit, default, liquidation, project investment, and job search. According to the standard theory (see Dixit and Pindyck (1994)), all these problems can be viewed as a problem where agents decide when to exercise an "option" analogous to a financial call option – it has the right but not the obligation to buy an asset at some future time of its choosing. This real options approach emphasizes the positive option value of waiting.

Unlike the standard theory, I make the distinction according to whether rewards and costs are immediate or delayed, as in O'Donoghue and Rabin (1999a). This distinction is important to explain procrastination and preproperation in the hyperbolic discounting model since it makes present bias critical. This distinction is also important in the present model since it makes immediate temptation critical. After stating the model setup and assumptions, I present the self-control utility model developed by Gul and Pesendorfer (2001, 2004) and compare it with the hyperbolic discounting model. I then provide a few three-period examples in both deterministic and stochastic environments. These examples illustrate the key intuition behind the reason why an agent with self-control preferences may procrastinate or preproperate. I show that, when the agent does not have enough self-control, he cannot resist temptations and thus procrastinates or preproperates. Following these examples, I present propositions to characterize the optimal stopping rules for the general infinite-horizon model when the agent has self-control preferences. I describe the optimal stopping rules as a trigger policy whereby the agent stops the first time the state process hits a threshold value. I also explain the impact of temptation and self-control on the optimal stopping rules. In particular, I show that the cost of self-control may lower the benefit from both stopping and continuation. Moreover, it may outweigh the option value of waiting. Unlike the finite-horizon examples, procrastination and preproperation can occur as long as there is a temptation.

In Section 3, I apply the results in Section 2 to investment and exit problems when the decision maker has self-control preferences. I also conduct welfare analysis. The investment and exit problems represent two different classes of option exercise problems. The project investment decision is an example where an agent decides whether or not to exercise an option to pursue upside potentials. Entry and job search are similar problems. I show the following: When the investment cost is immediate, the investor is tempted to delay investment. Thus, he procrastinates and the welfare loss is the forgone project value, which is equal to the cost of self-control. When the project value is immediate, the investor is tempted to invest early. Thus, he preproperates and the welfare loss is the forgone option value of waiting. If his level of self-control is sufficiently low, the investor may invest in negative net present value (NPV) projects. This reflects the trade-off between investing now but incurring financial losses and waiting but incurring self-control costs. When both the project value and investment cost are immediate, the investor also preproperates and the welfare loss is the forgone option value of waiting. In this case, he never invests in negative NPV projects. If his level of self-control is sufficiently low, he invests according to the myopic rule which compares only the current period benefit and cost.

After analyzing the investment problem, I turn to the exit problem, in which an owner/manager with self-control preferences decides when and if to shut down a firm. This problem represents an example where an agent decides whether or not to exercise an option to avoid downside potentials. Other examples include default and liquidation decisions. I show the following: When the profits are immediate, the owner is tempted to stay even when he should exit. Thus, he procrastinates to exit, even though he incurs substantial losses. The welfare loss is equal to the cost of self-control. By contrast, when the fixed cost of stay in business is immediate, the owner is tempted to avoid this cost and preproperates to exit, even though he may make positive net profits. The welfare loss is the forgone current and future profit opportunities. When both the cost and profit are immediate, the owner also preproperates, but never exits at a time when he makes a negative net profits. If the owner's level of self-control is sufficiently low, the firm exits according to the myopic rule.

Section 4 concludes. Technical details are relegated to an appendix.

2 The Model

I model an agent's option exercise decisions as an optimal stopping problem. Specifically, consider a discrete time and infinite horizon environment. In each period, the agent decides whether stopping a process and taking a termination payoff, or continuing for one more period, and making the same decision in the future. The decision is irreversible in the sense that if the agent chooses to stop, he makes no further choices. Formally, time is denoted by $t = 1, 2, ...$, and uncertainty is generated by a state process $(x_t)_{t \geq 1}$. For simplicity, I assume that x_t is drawn identically and independently from a distribution F on $[0, A]$. Continuation at date t generates a payoff $\pi(x_t)$ and incurs a cost c_c, while stopping at date t yields a payoff $\Omega(x_t)$ and incurs a cost c_s, where π and Ω are continuous and increasing functions. I will provide extensive examples below to illustrate that this simple model covers a wide range of economic applications.

As in O'Donoghue and Rabin (1999a), I make an important distinction according to whether costs and rewards are obtained immediately or delayed. The term of immediate costs is used to refer to the situation where the cost is incurred immediately while the reward is delayed. The term of immediate rewards is used to refer to the situation where the reward is incurred immediately while the cost is delayed. For simplicity, I consider the case of one period delay only. In addition, I also consider the case where both costs and rewards are immediate, which is not explicitly analyzed in O'Donoghue and Rabin (1999a). O'Donoghue and Rabin (1999a) give many examples to illustrate that the preceding distinction is meaningful in reality. Moreover, this distinction is important to generate procrastination and preproperation.

Unlike O'Donoghue and Rabin (1999a), I also consider uncertainty and infinite horizon. Uncertainty is prevalent in intertemporal choices and infinite horizon is necessary to analyze long-run stationary decision problems. These two elements are building blocks in many economic models, especially in macroeconomics and finance. Incorporating them allow me to study some interesting applications in macroeconomics and finance, as illustrated in Section 3.

2.1 Self-Control Preferences

O'Donoghue and Rabin (1999a) explain procrastination and preproperation by adopting the time-inconsistent hyperbolic discounting model proposed by Phelps and Pollak (1968). This model can be described as follows. Let $U_t(c_t, ..., c_T)$ represent an agent's intertemporal preferences from a consumption stream $(c_t, ..., c_T)$ in period t. T could be finite or infinite. The hyperbolic discounting preferences are represented by

$$U_t(c_t, ..., c_T) = u_t(c_t) + \beta E\left[\sum_{k=1}^{T} \delta^k u_{t+k}(c_{t+k})\right],\ t \geq 1,$$

where $0 < \beta, \delta \leq 1$. Here δ represents long-run, time-consistent discounting and β represents a 'bias for the present'. The agent at each point in time is regarded as a separate 'self' who is choosing his current behavior to maximize current preferences, where his future selves will control his future behavior. In this model, an agent must form expectation about his future selves' preferences. Two extreme assumptions are often made. In one extreme, the agent is naive and believe his future selves' preferences will be identical to her current self's, not realizing changing tastes. In the other extreme, the agent is sophisticated and know exactly what his future selves' preferences will be. The solution concept of subgame perfect Nash equilibrium is often adopted. As typical in dynamic games, multiple equilibria may arise.

The time inconsistency model provides an intuitive interpretation of procrastination and preproperation. The key intuition relies on the following feature of the hyperbolic discounting model. When $\beta < 1$, the agent gives more relative weight to period t when he makes a choice in period t than he does when he makes the choice in any period prior to period t. That is, the agent has a time-inconsistent taste for immediate gratification. There seems to be ample experimental evidence on the time-inconsistent behavior.[4] In a typical experiment, subjects choose between a smaller period t reward and a larger period $t+1$ reward. If the choice is made in period t then the smaller earlier reward is chosen. If the choice is made earlier, then the larger later reward is chosen.

Gul and Pesendorfer (2001, 2004) propose an alternative interpretation of this behavior based on time-consistent preferences. Their key insight is that the agent finds immediate rewards tempting. When the agent makes the choice in period t, the period t reward constitutes a temptation to the agent. So he may choose a smaller period t reward rather than a larger period $t+1$ reward. However, if he makes a choice prior to period t, neither period t reward nor period $t+1$ reward can be consumed immediately and hence his decisions are unaffected by temptations.

To capture this intuition, Gul and Pesendorfer (2001, 2004) develop a model of self-control based on a choice theoretic axiomatic foundation.[5] They define self-control preferences over sets of alternative consumption levels or decision problems – a domain different from the usual one. The interpretation is that temptation has to do with not just what the agent chooses, but what he could have chosen. Specifically, let B_t be the agent's period t decision problem and W_t represent his intertemporal utility in period t. Then the self-control preferences are represented by

$$W_t(B_t) = \max_{c_t \in B_t} \{u_t(c_t) + v_t(c_t) + \delta E[W_{t+1}(B_{t+1})]\} - \max_{c_t \in B_t} v_t(c_t), \ t \geq 1. \tag{1}$$

If T is finite, since there is no continuation problem in period T,

$$W_T(B_T) = \max_{c_T \in B_T} \{u_T(c_T) + v_T(c_T)\} - \max_{c_T \in B_T} v_T(c_T). \tag{2}$$

Here $u_t + \delta W_{t+1}$ represents the commitment utility in period t and v_t is the temptation utility in period t. The expression $u_t(c_t) + v_t(c_t) + \delta E[W_{t+1}(B_{t+1})]$ reflects the comprise between commitment and temptation. The agent's optimal choice in period t maximizes this expression. When this choice is identical to the temptation choice in the second maximum, the agent succumbs to the temptation and there is no self-control cost. However, when the two choices do

[4] See, for example, Thaler (1981), Ainslie and Haslam (1992), Kirby and Herrnstein (1995).

[5] See Gul and Pesendorfer (2001, 2004) for detailed axioms. The key axiom is set betweenness. Their model is more general than the one presented in this paper.

not coincide, the agent exercises costly self-control and $v_t(c_t) - \max_{c_t \in B_t} v_t(c_t)$ represents the cost of self-control. If $T = \infty$, I consider a stationary model and drop time subscripts,

$$W(B) = \max_{c \in B} \left\{ u(c) + v(c) + \delta E\left[W\left(B'\right)\right] \right\} - \max_{c \in B} v(c). \tag{3}$$

Here B' denotes the choice problem in the next period and $E[\cdot]$ denotes the expectation operator.

An important feature of the Gul-Pesendorfer model is that it is time consistent since utility in (1)-(3) is defined recursively. Thus, the usual recursive method such as backward induction and dynamic programming can be applied. Importantly, in addition to this tractability, this model has clear welfare implications. That is, this model follows the revealed preference tradition of standard economic models: if the agent chooses one alternative over another, then he is better off with that choice. By contrast, time inconsistent models do not have a universally agreed welfare criterion. Some researchers such as Laibson (1994, 1997) adopt a Pareto efficiency criterion, requiring all period selves weakly prefer one strategy to another. Other researchers such as O'Donoghue and Rabin (1999a) adopt an ex ante long-run utility criterion. The problem of the welfare analysis of the time inconsistent models is that the connection between choice and welfare is broken.

In the present paper, I adopt the Gul-Pesendorfer model to analyze the option exercise problem. In this problem, the set B consists of two elements {stop, continue} since the choice problems are binary. If the agent chooses to stop, then there is no continuation problem so that $B' = \emptyset$ and $W(B') = 0$. If the agent chooses to continue, then he faces the same decision problem in the next period and hence $B' = \{\text{stop, continue}\}$. To simplify exposition, I always assume risk neutrality. That is, $u(c) = c$ and $v(c) = \lambda c$, $\lambda > 0$. Here λ is the self-control parameter. An increases in λ raises the weight on the temptation utility and leads to a decrease in the agent's (instantaneous) self-control.[6] When $\lambda = 0$, the model reduces to the standard time-additive expected utility model with exponential discounting.

2.2 Examples

To understand the intuition behind the results below, I consider six simple examples. In the first three examples, the environment is deterministic and similar to O'Donoghue and Rabin (1999a). Specifically, an agent has an activity to complete. He has $T < \infty$ periods to do it and can do it exactly once. In each period t, the agent must choose either to do it or to wait. If he does the activity in period t, he receives reward v_t and incurs a cost I_t. If he waits, he then

[6] See Gul and Pesendorfer (2004) for the definition and characterization of measures of self-control. To distinguish between differences in impatience and differences in self-control, one should fix intertemporal choices and consider instantaneous self-control only.

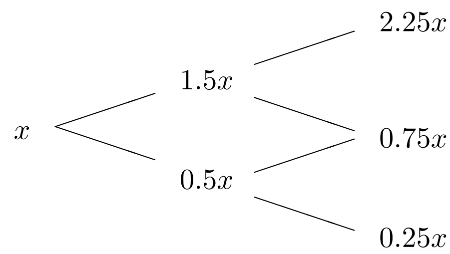

Figure 1: Rewards from the activity

will face the same choice in period $t+1$. I assume that if the agent is indifferent between doing the activity and waiting, the agent does it. I normalize the utility from not doing the activity to be zero. Moreover, there are three periods $T=3$ and there is no discounting, $\delta=1$. Let $\overline{\tau}$ and τ^* denote the period in which the agent does the activity if he has standard preferences or self-control preferences, respectively. Let $\mathbf{v}=\{v_1,...,v_T\}$ and $\mathbf{I}=\{I_1,...,I_T\}$.

The remaining three examples study the same problem. The only difference is that there is uncertainty over rewards. Specifically, I assume that the initial value of the reward is $x=100$. In periods 2 and 3, the value of rewards may go up or down by 50% with equal probabilities (see Figure 1). These events are independent. Moreover, I assume costs are deterministic and equal to I in each period.

Example 1: (Preproperation) Suppose rewards are immediate, $\mathbf{v}=\{2,3,4\}$, and $\mathbf{I}=\{1,1,3\}$. Then, $\overline{\tau}=2$, $\tau^*=1$ if $\lambda \geq 0.5$, and $\tau^*=2$ if $0 \leq \lambda < 0.5$.

Since there is no discounting, an agent with standard preferences chooses the period having the maximal net rewards, no matter when the rewards or costs are received. Consider now an agent with self-control preferences. I solve his decision problem by backward induction. In period 3, the agent solves the following problem

$$W_3=\max\{(1+\lambda)v_3-I_3,0\}-\lambda\max\{v_3,0\}.$$

Since the agent always face the same decision problems if he does not stop before, I omit the argument B in W_3. Note that the cost I_3 is incurred in the next period. It does not affect the current temptation. The agent is tempted to do the activity now and obtain v_3, as illustrated by the last term in the preceding equation. He succumbs to this temptation and there is no self-control cost. In period 2, if the agent does the activity, he has payoff $(1+\lambda)v_2-I_2$ since the cost I_2 does not affect the current temptation utility. If he waits, he has payoff W_3. By the principle of optimality, the agent solves the following problem

$$W_2 = \max\left\{(1+\lambda)\, v_2 - I_2, W_3\right\} - \lambda \max\left\{v_2, 0\right\} = 2.$$

Again the agent is tempted to do the activity now and succumbs to this temptation. Similarly, in period 1, the problem is

$$W_1 = \max\left\{(1+\lambda)\, v_1 - I_1, W_2\right\} - \lambda \max\left\{v_1, 0\right\}.$$

The agent is tempted to do the activity now. To resist this temptation, he must exercise costly self-control. When the temptation is large enough or the agent's level of self-control is low enough, i.e., $\lambda \geq 0.5$, the agent will succumb to the temptation and preproperate to do the activity in period 1. By contrast, if $0 \leq \lambda < 0.5$, the agent is able to resist the temptation. He then does the activity in period 2, just as an agent with standard preferences.

Example 2: (Procrastination) Suppose costs are immediate, $\mathbf{v} = \{2,3,4\}$, and $\mathbf{I} = \{1,1,3\}$. Then, $\overline{\tau} = 2$, and $\tau^* = 2$ if $0 \leq \lambda \leq 2$. The agent never does the activity if $\lambda > 2$.

Again I solve this example by backward induction, but write the Bellman equations in the appendix. When costs are immediate, the agent is tempted to delay. One can verify that if $\lambda > 2$, the temptation utility is so high that the agent succumbs to the temptation to delay. He then procrastinates and never completes the activity. By contrast, if $0 \leq \lambda \leq 2$, he is able to resist temptation and exercise self-control.

Example 3: (Preproperation) Suppose both costs and rewards are immediate, $\mathbf{v} = \{2,3,4\}$, and $\mathbf{I} = \{1,1,3\}$. Then, $\overline{\tau} = 2$, $\tau^* = 1$ if $\lambda \geq 1$, and $\tau^* = 2$ if $0 \leq \lambda < 1$.

One can verify the above solution by solving Bellman equations given in the appendix. Because the net rewards from doing the activity is positive, the agent is tempted to do the activity now as in the case of immediate rewards in Example 1. If the temptation is high enough, the agent cannot resist it and preproperates. Because the temptation utilities are smaller than that in the case of immediate rewards, a higher self-control parameter is needed to induce peproperation.

Example 4: (Preproperation) Suppose $x = 100$, $I = 75$ and rewards are immediate.

Because there is uncertainty, one has to use the method of stochastic dynamic programming to solve this example. Because the Bellman equations are different from those in the previous examples, I describe them below:

- In period 3,

$$
\begin{aligned}
W_3(2.25x) &= \max\{(1+\lambda)\,2.25x - I, 0\} - \lambda \max\{2.25x, 0\} = 150, \\
W_3(0.75x) &= \max\{(1+\lambda)\,0.75x - I, 0\} - \lambda \max\{0.75x, 0\} = 0, \\
W_3(0.25x) &= \max\{(1+\lambda)\,0.25x - I, 0\} - \lambda \max\{0.25x, 0\} = 0.
\end{aligned}
$$

- In period 2,

$$
\begin{aligned}
W_2(1.5x) &= \max\left\{(1+\lambda)\,1.5x - I, \frac{1}{2}W_3(2.25x) + \frac{1}{2}W_3(0.75x)\right\} - \lambda \max\{1.5x, 0\}, \\
W_2(0.5x) &= \max\left\{(1+\lambda)\,0.5x - I, \frac{1}{2}W_3(0.25x) + \frac{1}{2}W_3(0.75x)\right\} - \lambda \max\{0.5x, 0\}.
\end{aligned}
$$

- In period 1,

$$
W_1(x) = \max\left\{(1+\lambda)\,x - I, \frac{1}{2}W_2(1.5x) + \frac{1}{2}W_2(0.5x)\right\} - \lambda \max\{x, 0\}.
$$

The interpretation of the preceding equations is similar to that in the case of certainty. The difference is that here the decision problems depend on the states or decision nodes. Thus, I use the states instead of decision problems at those states as the domain for the value functions. Simple calculation reveals the following result. If $0 \leq \lambda < 0.1$, then the agent has enough self-control so that he can resist the temptation to do the activity now. Thus, he waits to do it in period 2 when the rewards go up to $1.5x$. If the rewards go down to $0.5x$, then he waits and do the activity in period 3 when the rewards go up to $0.75x$. This option exercise rule is the same as that for the agent with standard preferences. By contrast, If $\lambda \geq 0.1$, then the temptation utility is so high that the agent cannot resist the temptation. Thus, he succumbs to the temptation and does the activity in period 1.

Example 5: (Procrastination) Suppose $x = 100$, $I = 75$ and costs are immediate. Because there is no discounting, as in example 4, an agent with standard preferences will complete the activity in period 2 when the value of the rewards goes up to $1.5x$. If the value of the rewards goes down to $0.5x$ in period 2, the agent will wait and complete the activity in period 3 when the value of the rewards goes up to $0.75x$.

Consider now an agent with self-control preferences. Solving the Bellman equations given in the appendix reveals the following result. Because the agent incurs immediate costs and obtains delayed rewards when doing the activity, he is tempted to put off doing it. To resist this temptation, he must exercise costly self-control. When $\lambda > 2$, the temptation utility is so high that the agent cannot resist the temptation to delay and will never complete the activity. When $0 < \lambda \leq 2$, the agent has enough self-control to resist the temptation. But he still procrastinates one period and completes the activity in period 3 when the value of the rewards goes up to $2.25x$.

Example 6: (Preproperation) Suppose $x = 100$, $I = 100$ and both costs and rewards are immediate. This example illustrates that when both costs and rewards are immediate, the agent with self-control preferences may preproperate. Since rewards are stochastic, an agent with self-control preferences is tempted to do the activity now in 'good' states and is tempted to wait in 'bad' states. It seems that there is no unambiguous conclusion. However, using the Bellman equations given in the appendix, one can check that if $\lambda \geq 0.25$, the agent with self-control preferences does the activity in period 2 when the reward goes up to $1.5x$. If the ward goes down to $0.5x$ in period 2, then the agent waits and does it in period 3 when the reward goes up to $2.25x$. If $0 \leq \lambda < 0.25$, then the agent has enough self-control to resist temptations. He does the activity in period 3 when the reward goes up to $2.25x$. In this case, the agent does the activity at the same time as an agent with standard preferences.

2.3 Optimal Stopping Rules

I now return to the infinite horizon model described in Section 2.1. I solve the agent's option exercise problem by dynamic programming.[7] The Bellman equations are different for the cases of immediate costs, immediate rewards and immediate costs and rewards. They are described as follows.

1. Immediate Costs

$$\begin{aligned} W(x) &= \max\left\{\delta\Omega(x) - (1+\lambda)\,c_s, \delta\pi(x) - (1+\lambda)\,c_c + \delta\int W(x')\,dF(x')\right\} \\ &\quad -\lambda\max\{-c_c, -c_s\}. \end{aligned} \tag{4}$$

2. Immediate Rewards

$$\begin{aligned} W(x) &= \max\left\{(1+\lambda)\,\Omega(x) - \delta c_s, (1+\lambda)\,\pi(x) - \delta c_c + \delta\int W(x')\,dF(x')\right\} \\ &\quad -\lambda\max\{\pi(x), \Omega(x)\}. \end{aligned} \tag{5}$$

[7] See Stokey and Lucas (1989) and Dixit and Pindyck (1994) for the theory of dynamic programming. The existence of a bounded and continuous value function is guaranteed by the contraction mapping theorem.

3. Immediate Costs and Rewards

$$\begin{aligned} W(x) &= \max\left\{(1+\lambda)(\Omega(x)-c_s),(1+\lambda)(\pi(x)-c_c)+\delta\int W(x')\,dF(x')\right\} \\ &\quad -\lambda\max\{\pi(x)-c_c,\Omega(x)-c_s\}. \end{aligned} \tag{6}$$

I explain (4) in some detail. The interpretations of the other two equations are similar. Suppose costs are immediate. In each period, the agent faces the decision problem of whether to continue or to stop. Stopping incurs an immediate cost c_s and gets a discounted payoff from the next period $\delta\Omega(x)$. After stopping, the agent has no further choice, and hence the continuation value is zero. Because of the compromise between the temptation and the commitment utilities, the total payoff of stopping is $\delta\Omega(x)-(1+\lambda)c_s$, which is the first term in the first bracket in (4). Continuation incurs an immediate cost c_c and gets a discounted payoff from the next period $\delta\pi(x)$. The agent has to make the same choice of whether to stop or to continue in the next period, and hence gets continuation value $\delta\int W(x')\,dF(x')$. Thus, we have the second term in the first bracket in (4). Finally, the agent is tempted by whether to stop now and avoid the cost of continuation c_c or to continue and avoid the cost of stopping c_s. The temptation utility is $\lambda\max\{-c_c,-c_s\}$, which is the last term in (4).

Clearly, continuation is optimal for those values of x for which the maximum in the first line of (4) is attained at the second term in the bracket. Immediate termination is optimal when the opposite is true. Call the corresponding divisions of the range of x the continuation region and the stopping region, respectively. A similar analysis applies to (5) and (6). In general, for arbitrary payoffs $\pi(x)$ and $\Omega(x)$, the continuation and stopping regions could be arbitrary. In most applications, these regions can be easily characterized. In particular, there is a threshold value such that it partitions the state space into a continuation region and a stopping region. Consequently, the optimal stopping rule is characterized by a trigger policy. That is, the agent stops the first time the process $(x_t)_{t\geq 1}$ hits the threshold value. Importantly, depending on the payoff structure, the stopping region could be above the threshold value or below it. The former case describes the problems of pursing upside potentials such as investment and job search. The latter case describes the problems of avoiding downside potentials such as exit and default. Next section will study these problems in detail.

Here I do not provide general conditions for the structure of the continuation and stopping regions.[8] Instead, I provide explicit characterizations of the threshold value for the case where the agent pursues upside potentials. That is, I assume that the continuation region is below the threshold value. In the applications in Section 3, I will impose explicit assumptions and provide

[8] See Dixit and Pindyck (1994) for such conditions for standard preferences.

a more complete and transparent analysis. In what follows, I denote by x^* the threshold value for the agent with self-control preferences and by $\overline{x}$ the threshold value for the agent with standard preferences or preferences without self-control problems. Since the mean value of the option exercise time increases with the threshold value, comparative static analysis for the threshold value reveals properties of the average option exercise time.

Proposition 1 *Suppose costs are immediate.*

(i) The threshold value x^* satisfies the equation

$$\begin{aligned} & (1-\delta)\left[\delta\Omega(x^*)-c_s\right]+\lambda(1-\delta)\left[-c_s-\max\left\{-c_c,-c_s\right\}\right] \\ = \; & \delta\pi(x^*)-c_c+\delta\int_{x^*}^{A}\delta\left[\Omega(x')-\Omega(x^*)\right]dF(x')+\delta\int_{0}^{x^*}\delta\left[\pi(x')-\pi(x^*)\right]dF(x') \\ & +\lambda\left[-c_c-\max\left\{-c_c,-c_s\right\}\right] \end{aligned} \tag{7}$$

(ii) If $c_s \geq c_c$, then $x^* \geq \overline{x}$. If $c_s < c_c$, then $x^* < \overline{x}$.

The interpretation of (7) is as follows. The expression on the left side of equation (7) describes the normalized per period benefit from stopping, while the expression on the right side describes the benefit from continuation or the opportunity cost of stopping. The agent optimally stops at the threshold value x^* such that he is indifferent between stopping and continuation.

When $\lambda = 0$, the model reduces to the one with standard preferences. It merits comments on the components of the benefit from continuation. The first term in the second line of equation (7) represents the one period benefit. The other two terms represent the option value of waiting when the agent waits for one more period and gets a better draw $x' > x^*$ and a worse draw $x' < x^*$.

Importantly, all terms containing λ represent the cost of self-control. Specifically, the second term in the first line of (7) represents the normalize per period cost of self-control if the agent chooses to stop. The third line of (7) represents the cost of self-control if the agent chooses to continue.

For part (ii), if the cost of stopping is higher the cost of continuation, i.e., $c_s \geq c_c$, then the agent is tempted to continue. Thus, if the agent chooses to continue, there is no self-control cost so that the term in the third line of (7) vanishes. By contrast, if the agent chooses to stop, then he has to exercise self-control and incurs a cost given in the second term in the first line of (7). Consequently, compared with the standard model, the benefit of stopping is lowered and the agent procrastinates to exercise the option. The interpretation of the other case ($c_s < c_c$) is similar.

Proposition 2 *Suppose rewards are immediate.*

(i) The threshold value x^* *satisfies the equation*

$$
\begin{aligned}
& (1-\delta)\left[\Omega(x^*)-\delta c_s\right]+\lambda(1-\delta)\left[\Omega(x^*)-\max\left\{\pi(x^*),\Omega(x^*)\right\}\right] \qquad (8)\\
= \;& \pi(x^*)-\delta c_c+\delta\int_0^{x^*}\left[\pi(x')-\pi(x^*)\right]dF(x')+\delta\int_{x^*}^{A}\left[\Omega(x')-\Omega(x^*)\right]dF(x')\\
& +\lambda\left[\pi(x^*)-\max\left\{\pi(x^*),\Omega(x^*)\right\}\right]\\
& +\lambda\delta\int_0^{x^*}\left[\pi(x')-\max\left\{\pi(x'),\Omega(x')\right\}-\left(\pi(x^*)-\max\left\{\pi(x^*),\Omega(x^*)\right\}\right)\right]dF(x')\\
& +\lambda\delta\int_{x^*}^{A}\left[\Omega(x')-\max\left\{\pi(x'),\Omega(x')\right\}-\left(\Omega(x^*)-\max\left\{\pi(x^*),\Omega(x^*)\right\}\right)\right]dF(x').
\end{aligned}
$$

(ii) If $\Omega(x)\geq\pi(x)$ *for all* x*, then* $x^*\leq\overline{x}$*.*

The interpretation of (8) is similar to that of (7). The difference is that here self-control not only incurs a current period cost but also has an option value component. The former is represented by the third line of (8). The latter is represented by the last two lines of equation (8). This is because the agent is tempted by stochastic rewards in the next period. In the next period the state may be better $x'>x^*$ or worse $x'<x^*$. Compared to the state at the threshold value x^*, the cost of self-control at x' may be higher or lower. Hence the option value component may be positive or negative.

Consider part (ii). When the rewards from stopping are always higher than the rewards from continuation, $\Omega(x)\geq\pi(x)$, the agent is tempted to stop. Stopping at x^* means the agent succumbs to the temptation and hence there is no cost of self-control. Thus, the second term in the first line of (8) vanishes. If the agent decides to continue at x^*, he has to resist the temptation to stop and hence incurs a cost of self-control represented by the third line of (8). Consider next the option value component of the cost of self-control. In the next period, if $x'>x^*$, the agent stops and succumbs to the temptation. There is no cost of self-control and hence the term in the last line of (8) vanishes. If $x'<x^*$, the agent should continue and incur a cost of self-control in the next period. Compared to the state at x^*, it may be higher or lower and hence the option value component of the cost of self-control represented by the fourth line of (8) may be positive or negative. It turns out that the current period cost of self-control always outweighs the option value component of the self-control cost. This implies that the benefit from continuation is lowered, compared with the standard model. Consequently, the agent preproperates to exercise the option.

Proposition 3 *Suppose both costs and rewards are immediate.*

(i) The threshold value x^* satisfies the equation

$$
\begin{aligned}
& (1-\delta)\left[\Omega(x^*)-c_s\right]+(1-\delta)\lambda\left[\Omega(x^*)-c_s-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right] \qquad (9)\\
= \; & \left(\pi\left(x^*\right)-c_c\right)+\delta\int_0^{x^*}\left[\pi\left(x'\right)-\pi\left(x^*\right)\right]dF\left(x'\right)+\delta\int_{x^*}^{A}\left[\Omega(x')-\Omega(x^*)\right]dF\left(x'\right)\\
& +\lambda\left(\pi\left(x^*\right)-c_c-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right)\\
& +\lambda\delta\int_0^{x^*}\left[\pi\left(x'\right)-c_c-\max\left\{\pi\left(x'\right)-c_c,\Omega\left(x\right)-c_s\right\}\right.\\
& \left.-\left(\pi\left(x^*\right)-c_c-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right)\right]dF\left(x'\right)\\
& +\lambda\delta\int_{x^*}^{A}\left[\Omega(x')-c_s-\max\left\{\pi\left(x'\right)-c_c,\Omega\left(x'\right)-c_s\right\}\right.\\
& \left.-\left(\Omega\left(x^*\right)-c_s-\max\left\{\pi\left(x^*\right)-c_c,\Omega\left(x^*\right)-c_s\right\}\right)\right]dF\left(x'\right).
\end{aligned}
$$

(ii) If $\Omega(x)-c_s\geq\pi(x)-c_c$ for all x, then $x^*\leq\overline{x}$.

The interpretation of this proposition is similar to that of Proposition 2. So I omit it.

Finally, when the continuation region is above the threshold value, the agent tries to avoid downside potentials. This happens in the exit problem as described in the next section. One can provide characterizations for the threshold value similar to Propositions 1-3.

3 Applications

This section applies the results in Section 2 to study investment and exit problems.

3.1 Investment

An important type of option exercise problems is the project investment problem. Consider that an investor decides on whether and when to invest in a project with stochastic values x_t in period t. Investment incurs a sunk cost I. This investment problem can be cast into our framework by setting

$$\Omega(x)=x,\ c_s=I,\ \ \pi(x)=c_c=0.$$

In standard investment problems, costs and benefits come at the same time. In reality, there are many instances where costs and benefits do not arrive at the same time. For example, an important feature of real investment is time to build. It is often the case that it takes time to complete a factory or develop a new product. This is an instance of immediate costs and delayed rewards. As a different example, some firms start investing in a project financed by borrowing.

Debts may be gradually repaid after the firms earn profits. This is an instance of immediate rewards and delayed costs.

I now analyze these different cases by rewriting the Bellman equations (4)-(6) as follows:

1. Immediate Costs
$$W(x) = \max\left\{\delta x - (1+\lambda) I, \delta \int W(x') \, dF(x')\right\} - \lambda \max\{0, -I\}. \tag{10}$$

2. Immediate Rewards
$$W(x) = \max\left\{(1+\lambda) x - \delta I, \delta \int W(x') \, dF(x')\right\} - \lambda \max\{x, 0\}. \tag{11}$$

3. Immediate Costs and Rewards
$$W(x) = \max\left\{(1+\lambda)(x - I), \delta \int W(x') \, dF(x')\right\} - \lambda \max\{x - I, 0\}. \tag{12}$$

From the above equations, the effect of self-control is transparent. When rewards are immediate, the investor is tempted to invest now. He may either succumb to temptation or exercise costly self-control. Self-control acts as if the benefit of waiting is lowered by λx. Thus, the investor has an incentive to preproperate. By contrast, when costs are immediate, the investor is tempted to wait. Self-control acts as if the cost of investment is increased by an amount of λI. This causes the investor to procrastinate. The interesting case is when both costs and rewards are immediate. When $x > I$, the investor is tempted to invest earlier. But when $x < I$, the investor is tempted to wait. Thus, the result seems to be ambiguous. The following proposition formalizes the preceding intuition and characterizes the optimal investment rule for each case.

Proposition 4 *Under the conditions given in the appendix, there is a unique threshold value* $x^* \in [0, A]$ ($\overline{x} \in [0, A]$) *such that the investor with self-control preferences (standard preferences) invests the first time the process* $(x_t)_{t \geq 1}$ *reaches this value.*

(i) If costs are immediate, then x^* *satisfies*
$$(\delta x^* - I)(1-\delta) - \lambda I (1-\delta) = \delta \int_{x^*}^{A} \delta (x - x^*) \, dF(x), \tag{13}$$
and $\overline{x}$ *is the solution for* $\lambda = 0$. *Moreover,* $x^* > \overline{x} > I/\delta$ *and* x^* *increases with* λ.

(ii) If rewards are immediate, then x^* *satisfies*
$$(x^* - \delta I)(1-\delta) = \delta \int_{x^*}^{A} (x - x^*) \, dF(x) - \lambda x^* + \lambda \delta \int_{0}^{x^*} (x^* - x) \, dF(x), \tag{14}$$

and $\overline{x}$ is the solution for $\lambda = 0$. Moreover, $x^* < \overline{x}$, $\overline{x} > \delta I$ and x^* decreases with λ.

(iii) If both costs and rewards are immediate then x^* satisfies

$$\begin{aligned}(x^* - I)(1 - \delta) &= \delta \int_{x^*}^{A} (x - x^*)\, dF(x) \\ &\quad -\lambda (x^* - I) + \lambda\delta \int_0^{x^*} [(x^* - I) - \max(0, x - I)]\, dF(x),\end{aligned} \tag{15}$$

and $\overline{x}$ is the solution for $\lambda = 0$. Moreover, $I \leq x^* < \overline{x}$ and x^* decreases with λ.

I first discuss briefly the solution for the standard model corresponding to $\lambda = 0$. As is well known, because of irreversibility and uncertainty, waiting has positive option value. The option value for each case is represented by the first term on the right side of the corresponding equations (13)-(15). The investor with standard preferences invests at the time the threshold value is higher than the cost. That is, at the time of investment the project has positive net present value (NPV).

I next turn to the case with self-control. If costs are immediate, the investor is tempted to wait. To resist this temptation, investing now must incur a self-control cost $\lambda I (1 - \delta)$, this lowers the benefit from investment. Thus, the investor chooses to procrastinate, compared with the standard model. Since x^* increases with the self-control parameter λ, the agent delays further as the self-control parameter becomes larger.[9] When λ is sufficiently large, x^* may exceed the upper bound A so that the investor never undertakes the investment project.

By contrast, if rewards are immediate, the investor is tempted to invest now. Waiting incurs a direct current self-control cost λx^*. Importantly, self-control adds a positive option value of waiting to invest, $\lambda\delta \int_0^{x^*} (x^* - x)\, dF(x)$. This is because, when the investor waits for one more period and gets a worse draw $x < x^*$, he saves the cost of self-control.[10] It turns out that the former cost dominates the latter positive value. Thus, compared with the standard model, the benefit from waiting is lowered and the investor chooses to preproperate.

Since x^* decreases with the self-control parameter λ, as λ gets larger and larger, the investor invests sooner and sooner. When λ is sufficiently large, the investor invests at a threshold value lower than that prescribed by the NPV rule, whose threshold value is δI. This implies that the investor may incur negative NPV at the time of investment. This seems counter-intuitive. In fact, tempted by investing now, the investor may reason, "If I invest now, I get a reward and

[9] The mean value of the waiting time of investment is given by $(1 - F(x^*))^{-1}$. It is increasing in the threshold value x^*.

[10] When he gets a better draw $x > x^*$, the investor succumbs to the temptation of investing so that there is no self-control cost.

incur a cost in the future. If I do not invest now, I have to exercise costly self-control. The cost of self-control may outweigh the option value of waiting. Thus, I prefer to invest now even though I get negative NPV." In reality, we do observe the phenomena that investors rush to embark on investments with negative NPV. For example, Rook (1987) finds empirical evidence that the presence of credit opportunities results in present-oriented, unplanned, and impulse buying.

I now consider the case where both costs and rewards are immediate. It is important to observe that the investor would never invest at a project value less than the cost; that is, x^* cannot be less than I. This is because when $x^* < I$, the investor has no temptation to invest and can choose costlessly not to invest, thereby obtaining the outside value zero. Given $x^* \geq I$, at the threshold value x^* the investor is tempted to invest. There is no self-control cost of investing at x^* so that the last term in the first line of (9) vanishes. Consider now the self-control cost of waiting. Waiting incurs a direct self-control cost $\lambda (x^* - I)$. Waiting also saves the cost of self-control by an amount represented by the last term in (15) when the investor gets a worse draw $x < x^*$. When he gets a better draw $x > x^*$, the investor succumbs to temptation of investing so that there is no self-control cost. It turns out the direct self-control cost dominates so that the benefit from waiting is lowered. Thus, compared with the standard model, the investor preproperates. Note that as in the case of immediate rewards, x^* decreases with the self-control parameter λ. As λ is sufficiently large, the threshold value approaches the value I under the myopic rule.

I now turn to welfare implications. I ask the question: How severely does the self-control problem hurt a person? I compute the utility loss from investment for an investor with self-control preferences, compared with an investor with standard preferences. Let $V(x)$ be the value function for the investor with standard preferences corresponding to $\lambda = 0$. The utility loss from self-control problems is measured as $V(x^*) - W(x^*)$. The following proposition gives the utility loss.

Proposition 5 *Let x^* and $\overline{x}$ be given in Proposition 4. When costs are immediate, the utility loss from investment is given by λI. When rewards are immediate or both costs and rewards are immediate, the utility loss from investment is given by $\overline{x} - x^*$.*

By Proposition 4, when costs are immediate, the investor with self-control preferences procrastinates – he waits when he should invest if he had standard preferences. The utility loss is the forgone project value. This loss is increasing in the self-control parameter λ. However, it does not increase with λ without bound. This is because as λ approaches the value $\delta A/I - 1$, the investment threshold x^* approaches the upper bound of the project value A. When λ is

	λ	Threshold	Waiting Time	Utility Loss
Immediate Costs	0	0.96	22.5	
	0.3	0.97	36.0	0.40
	0.6	0.99	90.0	0.77
Immediate Rewards	0	0.74	4.0	
	0.3	0.49	2.0	0.87
	0.6	0.32	1.5	1.44
Immediate Costs and Rewards	0	0.76	4.2	
	0.3	0.69	3.2	0.29
	0.6	0.65	2.8	0.44

Table 1: This table presents solutions for the investment threshold values, the mean waiting time until investment and the utility loss. The utility loss is measured as $(V(x^*) - W(x^*))/V(x^*)$.

increased further, no investment is ever made and the agent gets zero. Thus, the upper bound of the utility loss is $\delta A - I$. When rewards are immediate or both costs and reward are immediate, the investor with self-control preferences preproperates – he invests when he should wait if he had standard preferences. The utility loss is then the forgone option value of waiting.

The following example illustrates Propositions 4-5 numerically.

Example 7: Let $A = 1$, $\delta = 0.9$, $I = 0.5$, and $F(x) = x$. Table 1 reports the solution. It reveals that even for small self-control problems, i.e. small λ, the utility loss could be quite large. For example, when costs are immediate and $\lambda = 0.6$, the investor procrastinates about 70 periods to invest. The utility loss accounts for 77% of the project value. When rewards are immediate and $\lambda = 0.6$, the investor preproperates and invests in negative NPV projects since the investment threshold 0.32 is less than that $\delta I = 0.45$ according to the NPV rule. The utility loss accounts for 144% of the option value. When both costs and rewards are immediate, the investor also preproperates. But the utility loss is less than that in the case of immediate rewards.

3.2 Exit

For the firm exit problem, the process $(x_t)_{t\geq 1}$ could be interpreted as the stochastic profit flows. Stay in business incurs a fixed cost $c_f > 0$. Let the scrapping value of the firm be zero. The owner/manager decides when and if to shut down the firm and exit from business. This problem fits into our framework by setting

$$\Omega(x) = 0,\ c_s = 0,\ \pi(x) = x,\ c_c = c_f.$$

As in the investment problem described in the preceding subsection, there are many instances that profits and costs may not come at the same time. Thus, I consider various cases and rewrite the Bellman equations (4)-(6) as follows:

1. Immediate Costs
$$W(x) = \max\left\{0, \delta x - (1+\lambda) c_f + \delta \int W(x') \, dF(x')\right\} - \lambda \max(0, -c_f). \tag{16}$$

2. Immediate Rewards
$$W(x) = \max\left\{0, (1+\lambda) x - \delta c_f + \delta \int W(x') \, dF(x')\right\} - \lambda \max(x, 0). \tag{17}$$

3. Immediate Costs and Rewards
$$W(x) = \max\left\{0, (1+\lambda)(x - c_f) + \delta \int W(x') \, dF(x')\right\} - \lambda \max\{x - c_f, 0\}. \tag{18}$$

The following proposition characterizes the solution.

Proposition 6 *Under the conditions given in the appendix, there is a unique threshold value* $x^* \in [0, A]$ $(\overline{x} \in [0, A])$ *such that the owner with self-control preferences (standard preferences) shuts down the firm the first time the process* $(x_t)_{t\geq 1}$ *falls below this value.*

(i) If costs are immediate, then x^* *satisfies*

$$0 = \delta x^* - c_f + \delta \int_{x^*}^{A} \delta (x - x^*) \, dF(x) - \lambda c_f, \tag{19}$$

and $\overline{x}$ *is the solution for* $\lambda = 0$. *Moreover,* $x^* > \overline{x}$, $\overline{x} < c_f/\delta$ *and* x^* *increases with* λ.

(ii) If rewards are immediate, then x^* *satisfies*

$$-\lambda x^* (1-\delta) = x^* - \delta c_f + \delta \int_{x^*}^{A} (x - x^*) \, dF(x) - \lambda \delta \int_{0}^{x^*} (x - x^*) \, dF(x), \tag{20}$$

and $\overline{x}$ is the solution for $\lambda = 0$. Moreover, $x^* < \overline{x} < \delta c_f$ and x^* decreases with λ.

(iii) If both costs and rewards are immediate, then x^* satisfies

$$\begin{aligned} 0 &= x^* - c_f + \delta \int_{x^*}^{A} (x - x^*) \, dF(x) \\ &\quad + \lambda (x^* - c_f) + \lambda \delta \int_{x^*}^{A} (x - x^* - \max\{x - c_f, 0\}) \, dF(x), \end{aligned} \tag{21}$$

and $\overline{x}$ is the solution for $\lambda = 0$. Moreover, $c_f \geq x^* > \overline{x}$ and x^* increases with λ.

In the standard model ($\lambda = 0$), because of irreversibility and uncertainty, there is a positive option value of waiting in the hope of getting better shocks. The firm will not exit as soon as it incurs loss since keeping it alive has an option value. The option value from stay in business for each case is represented by the second term in the right side of equations (19)-(21), respectively. Only when the loss is large enough, the firm exits.

I now discuss the case where the owner has self-control preferences. When costs are immediate, the owner is tempted to exit. Exercising self-control is costly. The cost of self control is represented by the last term in (19). It lowers the benefit from stay in the business. Thus, he preproperates and exits earlier. Note that the exit threshold x^* is increasing in the self-control parameter λ.[11] When λ is large enough, x^* approaches the upper bound of profits A. In this case, the owner succumbs to temptation and shuts down the firm right away even if the firm can still make positive net profits.

When rewards are immediate, the owner is tempted to stay in business. He procrastinates to shut down the firm. There are two effects in force. First, exit incurs a direct self-control cost given by the expression in the left side of (20). This cost lowers the benefit from exit. Second, stay in business has a positive option value from self-control. This is because when the firm gets a bad shock, it can save self-control costs. These two effects imply that the exit threshold x^* is lower than $\overline{x}$ so that the firm stays longer in business. When λ is large enough, x^* approach zero and the owner still keeps the firm alive even though he suffers substantial losses.

Consider the case where both costs and rewards are immediate. Since profits are stochastic, the owner is tempted to stay if profits are higher than the fixed cost and is tempted to exit if profits are lower than the fixed cost. It seems that there is no unambiguous conclusion. However, it is important to note that the firm will never exit at the profit level higher than the fixed cost. Otherwise, at that profit level the owner has no temptation to exit. Thus, stay for one more period incurs no self-control cost and the firm can still make positive profits. Because of this

[11] The mean value of the exit time is given by $F(x^*)^{-1}$, which decreases with the threshold value.

fact, at the exit threshold, the owner cannot make positive profits and has a temptation to exit. Exercising self-control is costly, which lowers the benefit from stay. The cost of self-control is represented by the last two terms in equation (21). Thus, the firm preproperates and exits earlier than the model with standard preferences. Part (iii) of Proposition 6 also implies that the exit threshold x^* increases with λ. Thus, when λ is sufficiently large, x^* approaches the fixed cost c_f so that the firm exits according to the myopic rule. The cost of self-control erodes completely away the option value of waiting.

I finally consider the welfare implications. Similarly to Proposition 5, the following proposition gives the utility loss due to self-control problems.

Proposition 7 *Let x^* and $\overline{x}$ be given in Proposition 6. When costs are immediate, the utility loss is given by $\delta x^* - \delta \overline{x}$. When rewards are immediate, the utility loss is given by λx^*. When both rewards and costs are immediate, the utility loss is given by $x^* - \overline{x}$.*

By Proposition 6, when costs are immediate or both rewards and costs are immediate, the owner preproperates – he exits when he should stay if he had standard preferences. The utility loss is the forgone profit opportunities. When rewards are immediate, the owner procrastinates – he stays when he should exits if he had standard preferences. The utility loss is the cost of self-control incurred from resisting the temptation to stay. As in the investment model, this cost does not increases with λ without bound. When λ is sufficiently large, the exit threshold approaches zero and the owner never shuts down the firm. The maximal utility loss from keeping the firm alive is $\delta (c_f - Ex) / (1 - \delta)$, which is the absolute value of the NPV of profits and is positive by the assumption in the appendix.

The following example illustrates Propositions 6-7 numerically.

Example 8: Let $A = 1, \delta = 0.9, c_f = 0.6$, and $F(x) = x$. Table 2 reports the solution. It reveals the following: When costs are immediate, the owner shuts down the firm too early even if he suffers from very small self-control problems, i.e., $\lambda = 0.2$. The firm exits at the profit level 0.78, which is bigger than the fixed cost 0.6. If the owner had standard preferences, he should exit at the value 0.59 less than the fixed cost because of the option value of waiting. The utility loss accounts for 24% of the profits. When rewards are immediate, the owner procrastinates to exit. He suffers from larger and larger losses if he has less and less self-control since the profit level at exit becomes smaller and smaller. The utility loss is proportional to λ since it is equal to $\lambda x^*/x^* = \lambda$. When both costs and rewards are immediate, the owner preproperates. However, the utility loss is less than that when costs are immediate.

	λ	Threshold	Waiting Time	Utility Loss
Immediate Costs	0	0.59	1.7	
	0.2	0.78	1.3	0.24
	0.4	0.93	1.1	0.36
Immediate Rewards	0	0.35	2.9	
	0.2	0.31	3.2	0.2
	0.4	0.28	3.5	0.4
Immediate Costs and Rewards	0	0.48	2.1	
	0.2	0.50	2.0	0.04
	0.4	0.51	1.9	0.07

Table 2: This table presents solutions for the exit threshold values, the mean waiting time until exit and the utility loss. The utility loss is measured as the fraction of the profits at exit.

4 Conclusion

This paper analyzes the option exercise problem for an agent who is tempted by immediate gratification and suffers from self-control problems. I adopt the Gul-Pesendorfer self-control utility model and provide an explanation for procrastination and preproperation based on time consistency. Unlike the time-inconsistency approach which depends on the expectations about future selves' preferences, there is no multiplicity of predictions. When applied to the investment and exit problems, the present model has a number of testable implications. For example, the present model implies that overinvestment, excess entry, procrastination to terminate a project or shut down a firm may be the rational choices of those investors/managers/entrepreneurs having self-control preferences, who are tempted by immediate profit opportunities. On the other hand, the opposite phenomena can be caused by such decision makers who are tempted to avoid immediate costs. Further, when both costs and rewards are immediate, the myopic option exercise rule may be optimal for such decision makers, who have sufficiently low levels of self-control.

In terms of future research, it should be important to conduct empirical or experimental analyses to differentiate between the time-consistency approach and the usual time-inconsistency approach.

Appendix

A Solutions to the Examples in Section 2.2

Example 2: I describe the agent's decision problem by the following Bellman equations:

$$\begin{aligned} W_3 &= \max\{v_3 - (1+\lambda) I_3, 0\} - \lambda \max\{-I_3, 0\}, \\ W_2 &= \max\{v_2 - (1+\lambda) I_2, W_3\} - \lambda \max\{-I_2, 0\}, \\ W_1 &= \max\{v_1 - (1+\lambda) I_1, W_2\} - \lambda \max\{-I_1, 0\}. \end{aligned}$$

One can verify the solution using these Bellman equations.

Example 3: One can verify the solution using the following Bellman equations:

$$\begin{aligned} W_3 &= \max\{(1+\lambda)(v_3 - I_3), 0\} - \lambda \max\{v_3 - I_3, 0\}, \\ W_2 &= \max\{(1+\lambda)(v_2 - I_2), W_3\} - \lambda \max\{v_2 - I_2, 0\}, \\ W_1 &= \max\{(1+\lambda)(v_1 - I_1), W_2\} - \lambda \max\{v_1 - I_1, 0\}. \end{aligned}$$

Example 5: One can verify the solution using the following Bellman equations:

- In period 3,

$$\begin{aligned} W_3(2.25x) &= \max\{2.25x - (1+\lambda) I, 0\} - \lambda \max\{-I, 0\}, \\ W_3(0.75x) &= \max\{0.75x - (1+\lambda) I, 0\} - \lambda \max\{-I, 0\}, \\ W_3(0.25x) &= \max\{0.25x - (1+\lambda) I, 0\} - \lambda \max\{-I, 0\}. \end{aligned}$$

- In period 2,

$$\begin{aligned} W_2(1.5x) &= \max\left\{1.5x - (1+\lambda) I, \frac{1}{2} W_3(2.25x) + \frac{1}{2} W_3(0.75x)\right\} - \lambda \max\{-I, 0\}, \\ W_2(0.5x) &= \max\left\{0.5x - (1+\lambda) I, \frac{1}{2} W_3(0.25x) + \frac{1}{2} W_3(0.75x)\right\} - \lambda \max\{-I, 0\}. \end{aligned}$$

- In period 1,

$$W_1(x) = \max\left\{x - (1+\lambda) I, \frac{1}{2} W_2(1.5x) + \frac{1}{2} W_2(0.5x)\right\} - \lambda \max\{-I, 0\}.$$

Example 6: One can verify the solution using the following Bellman equations:

- In period 3,

$$\begin{aligned} W_3(2.25x) &= \max\{(1+\lambda)(2.25x-I),0\} - \lambda\max\{2.25x-I,0\}, \\ W_3(0.75x) &= \max\{(1+\lambda)(0.75x-I),0\} - \lambda\max\{0.75x-I,0\}, \\ W_3(0.25x) &= \max\{(1+\lambda)(0.25x-I),0\} - \lambda\max\{0.25x-I,0\}. \end{aligned}$$

- In period 2,

$$\begin{aligned} W_2(1.5x) &= \max\left\{(1+\lambda)(1.5x-I), \frac{1}{2}W_3(2.25x) + \frac{1}{2}W_3(0.75x)\right\} \\ &\quad -\lambda\max\{1.5x-I,0\}, \\ W_2(0.5x) &= \max\left\{(1+\lambda)(0.5x-I), \frac{1}{2}W_3(0.25x) + \frac{1}{2}W_3(0.75x)\right\} \\ &\quad -\lambda\max\{0.5x-I,0\}. \end{aligned}$$

- In period 1,

$$W_1(x) = \max\left\{(1+\lambda)(x-I), \frac{1}{2}W_2(1.5x) + \frac{1}{2}W_2(0.5x)\right\} - \lambda\max\{x-I,0\}.$$

B Proofs

Proof of Proposition 1: (i) The value function W satisfies

$$W(x) = \begin{cases} \delta\Omega(x) - (1+\lambda)c_s - \lambda\max\{-c_c,-c_s\} & \text{if } x \geq x^*, \\ \delta\pi(x) - (1+\lambda)c_c + \delta\int W(x')\,dF(x') - \lambda\max\{-c_c,-c_s\} & \text{if } x < x^*. \end{cases} \tag{B.1}$$

Since $W(x)$ is continuous at the threshold value x^*,

$$\begin{aligned} &\delta\Omega(x^*) - (1+\lambda)c_s \\ = {}& \delta\int W(x')\,dF(x') + \delta\pi(x^*) - (1+\lambda)c_c \qquad \text{(B.2)} \\ = {}& \delta\int_0^{x^*} W(x')\,dF(x') + \delta\int_{x^*}^{A} W(x')\,dF(x') + \delta\pi(x^*) - (1+\lambda)c_c. \end{aligned}$$

Substituting (B.1) into this equation yields

$$\begin{aligned} &\delta\Omega(x^*) - (1+\lambda)c_s \\ = {}& \delta\int_0^{x^*}\left\{\delta\int W(x)\,dF(x) + \delta\pi(x') - (1+\lambda)c_c - \lambda\max\{-c_c,-c_s\}\right\}dF(x') \\ &+\delta\int_{x^*}^{A}\left[\delta\Omega(x') - (1+\lambda)c_s - \lambda\max\{-c_c,-c_s\}\right]dF(x') + \delta\pi(x^*) - (1+\lambda)c_c \end{aligned}$$

Using (B.2) to substitute $\delta \int W(x)\, dF(x)$ delivers

$$\begin{aligned}
& \delta\Omega(x^*) - (1+\lambda)\, c_s \\
= \; & \delta \int_0^{x^*} \left[\delta\Omega(x^*) - (1+\lambda)\, c_s - \delta\pi(x^*) + \delta\pi\left(x'\right) - \lambda \max\{-c_c, -c_s\}\right] dF\left(x'\right) \\
& + \delta \int_{x^*}^{A} \left[\delta\Omega(x') - (1+\lambda)\, c_s - \lambda \max\{-c_c, -c_s\}\right] dF\left(x'\right) \\
& + \delta\pi(x^*) - (1+\lambda)\, c_c.
\end{aligned}$$

Subtracting $[\delta\Omega(x^*) - (1+\lambda)\, c_s]\, \delta F(x^*)$ on each side of the above equation yields

$$\begin{aligned}
& [\delta\Omega(x^*) - (1+\lambda)\, c_s]\, [1 - \delta F(x^*)] \\
= \; & \delta \int_0^{x^*} \left[\delta\pi\left(x'\right) - \delta\pi(x^*) - \lambda \max\{-c_c, -c_s\}\right] dF\left(x'\right) \\
& + \delta \int_{x^*}^{A} \left[\delta\Omega(x') - (1+\lambda)\, c_s - \lambda \max\{-c_c, -c_s\}\right] dF\left(x'\right) \\
& + \delta\pi(x^*) - (1+\lambda)\, c_c.
\end{aligned}$$

Subtracting $[\delta\Omega(x^*) - (1+\lambda)\, c_s]\, \delta\, [1 - F(x^*)]$ on each side of the above equation and simplifying yield the desired result.

(ii) If $c_s \geq c_c$, then

$$\lambda\, [c_s (1-\delta) - \delta \max\{-c_c, -c_s\} - c_c] = \lambda\, (1-\delta)\, (c_s - c_c) \geq 0.$$

If $c_s < c_c$, then

$$\lambda\, [c_s (1-\delta) - \delta \max\{-c_c, -c_s\} - c_c] = \lambda\, (c_s - c_c) < 0.$$

Proof of Proposition 2: (i) The value function W satisfies

$$W(x) = \begin{cases} (1+\lambda)\, \Omega(x) - \delta c_s - \lambda \max\{\pi(x), \Omega(x)\} & \text{if } x \geq x^*, \\ (1+\lambda)\, \pi(x) - \delta c_c + \delta \int W(x')\, dF(x') - \lambda \max\{\pi(x), \Omega(x)\} & \text{if } x < x^*. \end{cases} \tag{B.3}$$

Since $W(x)$ is continuous at the threshold value x^*, it follows that

$$\begin{aligned}
& (1+\lambda)\, \Omega(x^*) - \delta c_s \\
= \; & (1+\lambda)\, \pi(x^*) - \delta c_c + \delta \int W\left(x'\right) dF\left(x'\right) \qquad \text{(B.4)} \\
= \; & \delta \int_0^{x^*} W\left(x'\right) dF\left(x'\right) + \delta \int_{x^*}^{A} W\left(x'\right) dF\left(x'\right) + (1+\lambda)\, \pi(x^*) - \delta c_c
\end{aligned}$$

Substitute (B.3) into this equation to deduce

$$\begin{aligned}
&(1+\lambda)\,\Omega(x^*)-\delta c_s\\
=\;&\delta\int_0^{x^*}\left\{(1+\lambda)\,\pi(x)-\delta c_c+\delta\int W(x')\,dF(x')-\lambda\max\{\pi(x),\Omega(x)\}\right\}dF(x)\\
&+\delta\int_{x^*}^{A}\left[(1+\lambda)\,\Omega(x)-\delta c_s-\lambda\max\{\pi(x),\Omega(x)\}\right]dF(x)+(1+\lambda)\,\pi(x^*)-\delta c_c
\end{aligned}$$

Using (B.4) to substitute $\delta\int W(x')\,dF(x')$ yields

$$\begin{aligned}
&(1+\lambda)\,\Omega(x^*)-\delta c_s\\
=\;&\delta\int_0^{x^*}\{(1+\lambda)\,\pi(x)+(1+\lambda)\,\Omega(x^*)-\delta c_s-(1+\lambda)\,\pi(x^*)-\lambda\max\{\pi(x),\Omega(x)\}\}\,dF(x)\\
&+\delta\int_{x^*}^{A}\{(1+\lambda)\,\Omega(x)-\delta c_s-\lambda\max\{\pi(x),\Omega(x)\}\}\,dF(x)+(1+\lambda)\,\pi(x^*)-\delta c_c
\end{aligned}$$

Subtract $[(1+\lambda)\,\Omega(x^*)-\delta c_s]\,\delta F(x^*)$ on each side of the above equation to derive

$$\begin{aligned}
&[(1+\lambda)\,\Omega(x^*)-\delta c_s]\,[1-\delta F(x^*)]\\
=\;&\delta\int_0^{x^*}\{(1+\lambda)\,\pi(x)-(1+\lambda)\,\pi(x^*)\}\,dF(x')-\lambda\delta\int\max\{\pi(x),\Omega(x)\}\,dF(x)\\
&+\delta\int_{x^*}^{A}\left\{(1+\lambda)\,\Omega(x')-\delta c_s\right\}dF(x')+(1+\lambda)\,\pi(x^*)-\delta c_c.
\end{aligned}$$

Finally, subtract $[(1+\lambda)\,\Omega(x^*)-\delta c_s]\,\delta\,[1-F(x^*)]$ on each side of the above equation and rearrange to deduce

$$\begin{aligned}
&[(1+\lambda)\,\Omega(x^*)-\delta c_s]\,(1-\delta)\\
=\;&\delta\int_0^{x^*}(1+\lambda)\,[\pi(x)-\pi(x^*)]\,dF(x')+\delta\int_{x^*}^{A}(1+\lambda)\left[\Omega(x')-\Omega(x^*)\right]dF(x')\\
&+\pi(x^*)-\delta c_c+\lambda\pi(x^*)-\lambda\delta\int\max\{\pi(x),\Omega(x)\}\,dF(x).
\end{aligned}$$

Rearranging yields the desired result.

(ii) If $\Omega(x)\geq\pi(x)$ for all x, then

$$\begin{aligned}
&\lambda\delta\int_0^{x^*}[\pi(x)-\max\{\pi(x),\Omega(x)\}]\,dF(x)\\
&+\lambda\delta\int_{x^*}^{A}[\Omega(x)-\max\{\pi(x),\Omega(x)\}]\,dF(x)\\
&-\lambda\,[1-\delta F(x^*)]\,[\Omega(x^*)-\pi(x^*)]\\
=\;&\lambda\delta\int_0^{x^*}[\pi(x)-\Omega(x)]\,dF(x)-\lambda\,[1-\delta F(x^*)]\,[\Omega(x^*)-\pi(x^*)]<0.
\end{aligned}$$

Thus, $x^*\leq\overline{x}$.

Proof of Proposition 3: (i) Rewrite the value function as follows

$$W(x) = \begin{cases} (1+\lambda)(\Omega(x) - c_s) - \lambda \max\{\pi(x) - c_c, \Omega(x) - c_s\} & \text{if } x \geq x^*, \\ (1+\lambda)(\pi(x) - c_c) + \delta \int W(x') \, dF(x') - \lambda \max\{\pi(x) - c_c, \Omega(x) - c_s\} & \text{if } x < x^*. \end{cases} \tag{B.5}$$

Since $W(x)$ is continuous at the threshold value x^*, it follows that

$$\begin{aligned} & (1+\lambda)(\Omega(x^*) - c_s) \\ = \ & (1+\lambda)(\pi(x^*) - c_c) + \delta \int W(x') \, dF(x') \\ = \ & \delta \int_0^{x^*} W(x') \, dF(x') + \delta \int_{x^*}^A W(x') \, dF(x') + (1+\lambda)(\pi(x^*) - c_c) \end{aligned} \tag{B.6}$$

Substitute (B.5) into this equation to deduce

$$\begin{aligned} & (1+\lambda)(\Omega(x^*) - c_s) \\ = \ & \delta \int_0^{x^*} \left\{ (1+\lambda)(\pi(x) - c_c) + \delta \int W(x') \, dF(x') - \lambda \max\{\pi(x) - c_c, \Omega(x) - c_s\} \right\} dF(x) \\ & + \delta \int_{x^*}^A \{(1+\lambda)(\Omega(x) - c_s) - \lambda \max\{\pi(x) - c_c, \Omega(x) - c_s\}\} \, dF(x) + (1+\lambda)(\pi(x^*) - c_c) \end{aligned}$$

Use (B.6) to substitute $\delta \int W(x') \, dF(x')$ to deduce

$$\begin{aligned} & (1+\lambda)(\Omega(x^*) - c_s) \\ = \ & \delta \int_0^{x^*} \{(1+\lambda)(\pi(x) - \pi(x^*)) + (1+\lambda)(\Omega(x^*) - c_s) - \lambda \max\{\pi(x) - c_c, \Omega(x) - c_s\}\} \, dF(x') \\ & + \delta \int_{x^*}^A \{(1+\lambda)(\Omega(x) - c_s) - \lambda \max\{\pi(x) - c_c, \Omega(x) - c_s\}\} \, dF(x) + (1+\lambda)(\pi(x^*) - c_c) \end{aligned}$$

Subtract $(1+\lambda)(\Omega(x^*) - c_s) \, \delta F(x^*)$ on each side of the above equation to derive

$$\begin{aligned} & (1+\lambda)(\Omega(x^*) - c_s)[1 - \delta F(x^*)] \\ = \ & \delta \int_0^{x^*} \{(1+\lambda)(\pi(x) - \pi(x^*))\} \, dF(x) - \lambda \delta \int \max\{\pi(x) - c_c, \Omega(x) - c_s\} \, dF(x) \\ & + \delta \int_{x^*}^A \{(1+\lambda)(\Omega(x) - c_s)\} \, dF(x) + (1+\lambda)(\pi(x^*) - c_c). \end{aligned}$$

Finally, subtract $[(1+\lambda)\Omega(x^*) - c_s] \, \delta [1 - F(x^*)]$ on each side of the above equation and rearrange to deduce

$$\begin{aligned} & (1+\lambda)(\Omega(x^*) - c_s)(1 - \delta) \\ = \ & \delta \int_0^{x^*} (1+\lambda)[\pi(x) - \pi(x^*)] \, dF(x) + \delta \int_{x^*}^A (1+\lambda)[\Omega(x) - \Omega(x^*)] \, dF(x) \\ & + (1+\lambda)(\pi(x^*) - c_c) - \lambda \delta \int \max\{\pi(x) - c_c, \Omega(x) - c_s\} \, dF(x). \end{aligned}$$

Rearranging yields the desired result.

(ii) The proof is similar to that of Proposition 3.

Proof of Proposition 4: The equations determining x^* are derived from Propositions 1-3. One can verify that the left sides of these equations are increasing functions of x^*, while the right sides are decreasing functions of x^*. To show that there is a unique solution to these equations, one need only show that there is a unique intersection point for each pair of curves implied by those functions. To guarantee this, the following conditions are necessary and sufficient:

- For part (i), $\delta A \geq (1+\lambda) I$.
- For part (ii),
$$(1-\delta)(A-\delta I) \geq -\lambda A + \lambda\delta \int_0^A (A-x)\, dF(x),$$
- For part (iii),
$$(1-\delta)(A-I) \geq -\lambda(A-I) + \lambda\delta \int_0^A (A-I-\max(0, x-I))\, dF(x).$$

Figure 2 illustrates part (i). As λ is increased, the curve implied by the left side of (13) shifts down so that x^* increases. Parts (ii) and (iii) are proved similarly.

Proof of Proposition 5: When costs are immediate, the agent with standard preferences has already made the investment at x^* since his investment threshold $\overline{x} < x^*$. Thus, $V(x^*) = \delta x^* - I$ and the welfare loss is $V(x^*) - W(x^*) = (\delta x^* - I) - (\delta x^* - (1+\lambda) I) = \lambda I$. When rewards are immediate, the agent with standard preferences does not invest at x^* since his investment threshold $\overline{x} > x^*$. Thus, $V(x^*) = \delta \int V(x')\, dF(x') = \overline{x} - \delta I$ and the welfare loss is $V(x^*) - W(x^*) = (\overline{x} - \delta I) - (x^* - \delta I) = \overline{x} - x^*$. The case with both immediate costs and rewards is similar to the case with immediate rewards.

Proof of Proposition 6: The proof is similar to that of Proposition 4. The conditions for the existence and uniqueness of the threshold value are given below:

- For part (i), $\delta A \geq (1+\lambda) c_f \geq \delta \int_0^A \delta x dF(x)$,
- For part (ii),
$$\int_0^A x dF(x) \leq c_f \leq A(1+\lambda(1-\delta))/\delta - \lambda \int_0^A (x-A)\, dF(x)$$

- For part (iii),

$$\frac{\delta \int_0^A x dF(x) + \lambda \delta \int_0^{c_f} x dF(x)}{1 + \lambda [1 - \delta (1 - F(c_f))]} \leq c_f \leq A.$$

Proof of Proposition 7: The proof is similar to that of Proposition 5. When costs are immediate, since $x^* > \overline{x}$, an agent with standard preferences will not exit at x^*. $V(x^*) = \delta x^* - c_f + \delta \int V(x') dF(x') = \delta x^* - c_f - (\delta \overline{x} - c_f) = \delta x^* - \delta \overline{x}$. Since $W(x^*) = 0$, the utility loss is $V(x^*) - W(x^*) = \delta x^* - \delta \overline{x}$. When rewards are immediate, since $x^* < \overline{x}$, $V(x^*) = 0$. Since $W(x^*) = -\lambda x^*$, $V(x^*) - W(x^*) = \lambda x^*$. Finally, when both costs and rewards are immediate, $V(x^*) = x^* - c_f + \delta \int V(x') dF(x') = x^* - c_f - (\overline{x} - c_f) = x^* - \overline{x}$. Since $W(x^*) = 0$, $V(x^*) - W(x^*) = x^* - \overline{x}$.

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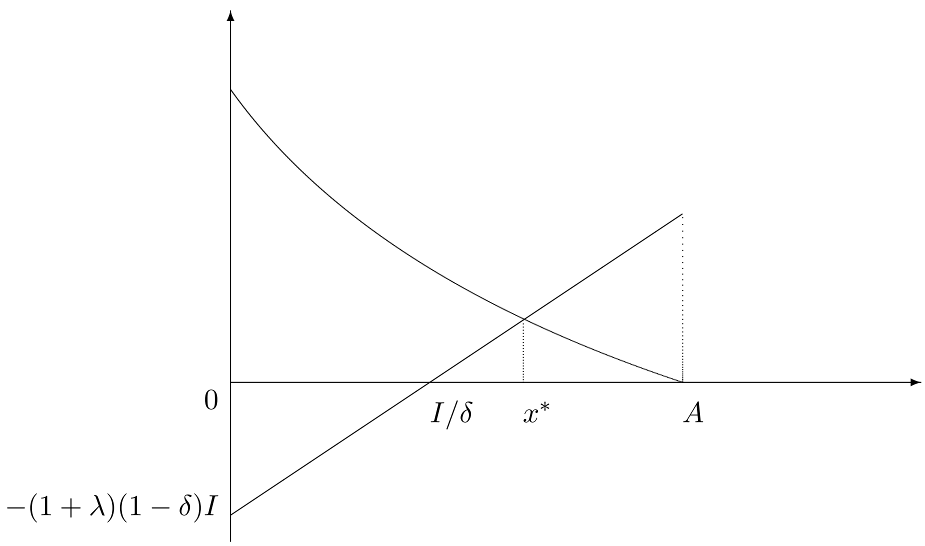

Figure 2: **The determination of the threshold value x^*.**