Inequity Aversion and Team Incentives*

Pedro Rey Biel†

Abstract

We study optimal contracts when employees are averse to inequity as modelled by Fehr and Schmidt (1999). A "selfish" employer can profitably exploit preferences for equity among his employees by offering contracts which create maximum inequity off-equilibrium and thus, leave employees feeling *envy* or *guilt* when they do not produce the optimal output level. We show how the optimal contract is designed such that the subgame played by the employees is dominance solvable, and thus, a unique optimal level of production is implemented. We also discuss conditions for inequity aversion to affect the optimal output choice. Similar results are obtained for other types of distributional preferences such status-seeking or efficiency concerns.

First draft: June, 2002

This version: July 7, 2004

JEL codes: D23; D63; M52; M54

Keywords: Principal, agent, inequity aversion, team incentives, behavioral contract theory

*I am indebted to Tilman Börgers and Steffen Huck for multiple discussions and to Dirk Engelmann for extensive comments. I am also grateful to Ken Binmore, Martin Bøg and Cloda Jenkins for help and support and to Jordi Brantds, Antonio Cabrales, Antoni Calvó, Armin Falk, Matthew Rabin, Klaus Schmidt and Joel Sobel for comments. I benefited from discussion with seminar participants at Budapest, Bristol, UCL, the SMYE in Leuven (2003), the European Science Days in Steyr (2003) and the EEA-ESEM Joint Meeting in Stockholm (2003). Financial support from Fundación Ramón Areces is gratefully acknowledged.

†Pedro Rey Biel. Department of Economics. University College London (UCL), Gower Street, WC1E 6AB, London. Tel: 00 44 (0)207 679 5842. E-mail: p.biel@ucl.ac.uk.

1 Introduction

In this paper, we study how managers should structure reward schemes when their employees care for the distribution of payoffs among their co-workers. We discuss how contracts can exploit this externality in preferences to the manager's advantage.

One of the most striking results from interview studies with firm managers (Agell and Lundborg (1999), Bewley (1999), Blinder and Choi (1990), Campbell and Kamlani (1997)) is that employees report to care for the well being of co-workers and not only for their own. In particular, they compare co-workers' salaries and performance in the firm with their own. Bewley (1999) shows that 69% of real firm's managers interviewed offer formal pay structures because it creates internal equity. Asked why internal equity is relevant, 78% of managers answered that it was important for morale and internal harmony and 49% responded that internal equity was key for job performance. Our paper aims to capture these stylized facts in a very simple model. We show that when agents are distributionally concerned it is optimal to offer contracts which create more equity when managers' demands are met than when they are not. The reason is that equity affects the employees' incentives to work hard and thus, it affects job performance. Following Holmstrom and Milgrom's (1991) seminal paper, optimal contracts must account for everything employees care about. Here we explore how the optimal contract changes when agents care for equity as modelled by Fehr and Schmidt (1999).[1]

In prominent experimental work, F&S (2000) have argued that fairness considerations lead agents to write incomplete contracts which implement less severe incentives than conventional theory would predict. We develop a simple model in which a principal has to design a reward scheme for two agents who dislike inequity in the way envisaged by F&S. However, contrary to F&S, our principal is not distributional concerned and agents do not care for the principal's welfare, but only for the other agents' and their own. Our main result is that a "selfish" principal can devise schemes which exploit agents' preference for equity by offering them equitable outcomes in situations where they put in the desired effort, and threaten with highly unequal outcomes if agents shirk. Such schemes might, for example, offer extremely unequal rewards in the case that one employee works harder than another. By constructing such schemes, the principal can elicit the desired effort levels offering lower rewards than would have been possible had the agents not been inequity averse. When agents are inequity averse the principal has two instruments at his disposal: rewards and equity. By offering more equity when employees perform the effort level desired by the manager than when they do not, the manager does not need to create as much incentives for employees to meet his demands and thus, he can pay lower rewards. To minimize equilibrium rewards paid, the principal offers rewards such as inequity is maximized off-equilibrium. Finally, because it may be cheaper to provide incentives for agents to work the optimal level of production might change. Moreover, in our simple setting with no informational problems, the principal never loses by accounting for inequity aversion in the design of the contracts, even when faced with standard agents.

Our research is parallel to F&S (2000) in that, even if we are dealing with inequity aversion in a

[1] We use F&S in the following to refer to these authors.

Principal-Agent setting, both the principal and the agents have different preferences than in the F&S papers. That is, in F&S the comparison of utilities among individuals is vertical (employers compare their welfare to their employees' and vice-versa) while in this paper it is horizontal (employees compare their welfare only among themselves and the Principal only cares for his own payoffs).

Horizontal comparisons among agents seem intuitive. It is natural to assume that welfare comparisons are enhanced by repeated interaction and that employees at the same hierarchical level interact more frequently among themselves than with their superiors. Additionally, it could be argued that employees performing the same task have better information about each agent's cost of effort and find it easier to learn about co-workers' rewards than those of their superiors. Finally, sociologists have argued that individuals rarely have altruistic feelings for others that have direct authority over their actions.[2] Thus, utility comparisons seem more meaningful among employees on the same hierarchical level than on different levels.[3] This motivates our research.

We have chosen the F&S utility function as a reduced form of social preferences due to its prominence and because with simple parameter transformations we can obtain similar results for other types of distributional preferences which might be relevant in the workplace.[4] We later discuss distributional preferences like Status and Efficiency Seeking. Notice that we do not discuss more complicated forms of social preferences which include reciprocal behavior and intentions.[5] These preferences could play a role in optimal contract design if we studied repeated interactions in the context of the firm. However, with reciprocal preferences it would be crucial to study the reaction by agents to threats of inequity by the principal. But this reaction would imply that employees care for the intentions of the employer, meaning that vertical considerations would play a role from which we want to abstract.

Our model is very stylized. First, we focus on incentive compatibility, not on participation. We assume that the participation constraint does not bind and thus both agents work for the firm. We normalize the utility of being in the firm to zero and we assume that the utility derived from not being in the firm is below this value. As we do not explicitly model an outside option its utility could take any value. We simply assume it is lower than inside the firm. This could be justified for different reasons: search costs of finding a different job, good matching with employers, specific human capital, disutility of unemployment or the existence of minimum rewards. But in particular, notice that if agents are still inequity averse when taking the outside option, utility when leaving the firm could be lower than inside the firm. As the reference group in the outside option is unclear and it is probably context dependent, we simplify the analysis of the participation constraints by assuming they do not bind. Our results are thus limited to this case. Another possible interpretation of our model is that the rewards in the model are not agents' wages but a bonus offered to perform an extra activity. Thus,

[2] See Homans (1950) for a summary.

[3] For example, Dufwenberg and Kirchsteiger (2000) express doubts on which variables would be used to compare employees and employer's utilities. In particular, they ask how meaningful is to compare employees' salaries with firm's profits or the value of the firm's shares.

[4] In particular, our main result would hold for the models proposed by Bolton and Ockenfels (2000), Bazerman, Loewenstein and Thompson (1989), Andreoni and Miller (1998), Cox and Firedman (2002), and the simplification without intentions by Charness and Rabin (2002).

[5] For good surveys on social preferences see Sobel (2000) and Fehr and Schmidt (2002b).

while the wage would take care of the participation constraint, the extra bonus provides incentives to perform an extra effort. In that view, our results should be interpreted as saying that bonuses might not need to cover employees' cost of doing that extra effort when they feel envy or guilt towards their peers. This interpretation is close to empirical effects observed under real team and relative performance contracts (Bandiera et al. (2004)).

Second, we do not consider an uncertain production environment. In our model output is deterministic and perfectly informative about the effort level performed by each agent. We want to show how inequity aversion in itself changes the optimal contract, without adding uncertainty. In a paper independently written at the same time as this one, Itoh (2004) uses a model where output is uncertain and shows that inequity aversion calls for optimal contracts to specify both agents' rewards under all possible circumstances, which is very similar to our claim. However, Itoh's mechanism is different from ours. In his model, each agent undertakes a different project and the principal writes the contract such that both agents always perform high effort. More equal (or more unequal) rewards are used in Itoh's paper to compensate for the risk of one of the agents' projects failing. In our paper, inequity aversion determines whether it is optimal or not to form teams in which both agents perform high effort and we show how unequal rewards must be offered off-equilibrium to optimally exploit inequity aversion. Both approaches are complementary.

Other papers have simultaneously studied optimal contracts with both vertical and horizontal welfare comparisons. Englmaier and Wambach (2002) study the interaction between an inequity averse agent who compares himself with a *selfish* principal. Grund and Sliwka (2002) study the horizontal comparison in a tournament context. Cabrales and Calvó-Armengol (2002) use inequity aversion only among employees to justify skill segregation. Huck and Rey Biel (2002) look at teams formed by inequity averse agents when there is no principal.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 shows the optimal contract when agents have standard preferences. Section 4 shows the optimal contract when agents are inequity averse. Section 5 discusses optimal contracts when distributional preferences take other forms, such status seeking and efficiency concerns. Section 6 concludes. Appendix A contains the proofs while Appendix B shows two relevant examples.

2 The Model

There are a Principal and two agents $i, j = \{1, 2\}$ with $i \neq j$. Agents can either work or not work. If both agents work, production is normalized to 1 (joint production). If only agent i works, production is q_i, where $0 < q_i < 1$ (individual production by agent i). If no agent works, production is 0. Output is observable. Effort is verifiable and contractible.

The cost for each agent of working is $c_i > 0$. The cost of not working is normalized to 0. A complete contract specifies the rewards offered to both agents for all possible output levels. In order to standardize notation, assume the principal offers rewards $\{w_1, w_2\}$ to agents 1 and 2 respectively when both agents work, $\{w_1^1, w_2^1\}$ when agent 1 individually works and $\{w_1^2, w_2^2\}$ when agent 2 individually works. If no agent works, rewards are zero.[6]

The structure of the game is as follows: the Principal offers rewards for all possible production levels, agents decide simultaneously whether or not to work and, once production is realized, promised rewards for the output level produced are paid. Following Mookherjee (1984) and Moore and Repullo (1988), we look at the contract such that the implemented production level is the unique equilibrium in pure strategies of the subgame played by the agents.

The Principal seeks to maximize its profit, that is, production minus rewards paid. Given the minimum rewards needed to be paid in equilibrium to implement each production level and the productivity parameters (q_i and c_i), the Principal designs the contract that maximizes its profit. Two different specifications for the agents' utility functions will be considered in Sections 3 and 4. These specifications will be explained later.

The structure of the game is common knowledge and, in particular, both the Principal and the agents know the rewards offered, the production level each agent achieves if working individually and each agents' cost of effort. Agents cannot communicate among themselves.

Assume the following.

(C) *The sum of working agents' costs of effort is smaller than the output produced.*

$$0 \leq c_1 < q_1,$$

$$0 \leq c_2 < q_2,$$

$$c_1 + c_2 < 1.$$

(R1) *Agents' Limited liability: Negative rewards are not possible.*

$$\begin{aligned} w_1, w_1^1, w_2^2 &\geq 0, \\ w_2, w_2^1, w_2^2 &\geq 0. \end{aligned}$$

(R2) *Rewards are paid from output produced.*

$$\begin{aligned} w_1 + w_2 &\leq 1, \\ w_1^1 + w_2^1 &\leq q_1, \\ w_1^2 + w_2^2 &\leq q_2. \end{aligned}$$

Assumption (C) implies that there always exists a surplus above the cost of effort performed. Assumption (R1) is a limited liability constraint ruling out that the Principal can monetarily punish agents for not performing effort.[7] Assumption (R2) is a budget constraint for the Principal. Notice that for contracts to be credible, assumption (R2) must also hold for rewards offered off the equilibrium of the subgame.[8]

[6] This is implied by assumptions (R1) and (R2) below.

[7] As we see below, the key is that even if the Principal is restricted in the monetary punishments he can use, he has another instrument, inequity, to punish agents when his demands are not met.

[8] As it will be clear below, we impose budget constraints off-equilibrium to show the interesting interplay between

3 Optimal contract with standard agents

In this section we derive the optimal contract when agents are standard. Standard agents maximize their "direct utility" which is equal to the reward offered minus the cost of effort performed. Below we show each agents' direct utility in the subgame depending on the action taken and the rewards offered by the Principal.

		Agent 2	
		Work	Not Work
Agent 1	Work	$w_1 - c_1$, $w_2 - c_2$	$w_1^1 - c_1$, w_2^1
	Not Work	w_1^2 , $w_2^2 - c_2$	0 , 0

We first solve for the optimal contract necessary to implement each production level and then, given the optimal rewards, we derive conditions for each production level to be optimal. Although the solution of this problem is straightforward, we solve it here as reference for the following subsection.

3.1 Optimal contract to implement individual production with standard agents

We here find the optimal contract to implement individual production by agent 1 as the unique equilibrium of the subgame played by the agents.[9] The problem is the following:

-The Principal maximizes its profit:

$$Max\ q_1 - w_1^1$$

subject to:

-Assumptions (C), (R1) and (R2).

- Agent 1 prefers to work than not to work when agent 2 works: $w_1^1 - c_1 \geq 0$,

- Agent 2 prefers not to work than work when agent 1 works: $w_2^1 > w_2 - c_2$.

For the subgame to be dominance solvable, the following constraints are also necessary:

- Agent 1 prefers to work than not to work when agent 2 works: $w_1 - c_1 > w_1^2$,

- Agent 2 prefers to work than not to work when agent 1 does not work: $w_2^2 - c_2 > 0$.

creating inequity off-equilibrium via *envy* or *guilt*. Without budget constraints, the Principal would always offer infinite rewards to one agent off-equilibrium, maximizing the other agent's *envy* when not performing the optimal production level.

[9] The optimal contract to implement individual production by agent 2 is symmetric.

The objective function and the restrictions are linear. Thus, the solution is straightforward:

$$\begin{array}{lll} w_1^1 & = & c_1 \qquad\qquad w_2^1 = 0, \\ w_1 & \in & (c_1, 1 - w_2] \qquad w_2 \in [0, c_2), \\ w_1^2 & \in & [0, w_1) \qquad\qquad w_2^2 \in (c_2, q_2 - w_1^2]. \end{array}$$

The optimal contract is such that in equilibrium, the agent who individually works is exactly compensated for his cost of effort ($w_1^1 = c_1$) while the agent who does not work is paid no reward ($w_2^1 = 0$). The Principal's profit in the unique equilibrium of the subgame is then equal to $q_1 - c_1$. Off-equilibrium rewards do not affect the Principal's profits and thus, they can take any value in the intervals shown.

3.2 Optimal contract to implement joint production with standard agents

We here find the optimal contract to implement joint production as the unique equilibrium of the subgame played by the agents. The problem is the following:

-The Principal maximizes its profit:

$$Max\ 1 - w_1 - w_2$$

subject to:

-Assumptions (C), (R1) and (R2).

- Agent i prefers to work than not to work when agent j works: $w_i - c_i \geq w_i^j$.

For the subgame to be dominance solvable, the following constraints are also necessary:

- Agent i prefers to work than not to work when agent j does not work while agent j prefers not to work when agent i does not work:

For $w_i - c_i > w_i^j$ and $w_j - c_j \geq w_j^i$ then $w_i^i - c_i < 0$ and $w_j^j - c_j > 0$.

Again, the objective function and the restrictions are linear so the solution is straightforward:

$$\begin{array}{lll} w_i & = & c_i + \varepsilon \qquad w_j = c_j, \\ w_i^i & \in & [0, c_i) \qquad w_j^i = 0, \\ w_i^j & = & 0 \qquad\qquad w_j^j \in (c_j, q_j - w_i^j]. \end{array}$$

for $i, j = 1, 2,\ i \neq j$.

Notice that for joint production to be the unique equilibrium, it is necessary to add a negligible positive quantity $\varepsilon \to 0$ to one of the agents' equilibrium rewards. As it happened with individual production, in an equilibrium with joint production agents are exactly compensated for their cost of effort.[10] Rewards offered off the equilibrium of the subgame are such that agents do not deviate from

[10] We assume ε to be small enough such that profits and conditions for joint production to be optimal are not affected.

the unique level of production the Principal finds optimal to implement.[11] The Principal's profits in the unique equilibrium of the subgame are equal to $1 - c_1 - c_2$.

3.3 Optimal production level with standard agents

Given that in equilibrium agents are paid a reward exactly equal to their cost of effort when they work, the Principal decides the optimal production level by comparing its profits when joint production is implemented $(1 - c_1 - c_2)$ with its profits with individual production by the agent with highest productivity net of his cost ($q_i - c_i$ for $q_i - c_i \geq q_j - c_j$ for $i, j = 1, 2, i \neq j$). The conditions for each level of production to be optimal are:

- Individual Production by agent 1 if and only if $q_1 - c_1 \geq q_2 - c_2$ and $q_1 \geq 1 - c_2$,
- Individual Production by agent 2 if and only if $q_1 - c_1 < q_2 - c_2$ and $q_2 \geq 1 - c_1$,
-Joint Production if and only if $q_1 < 1 - c_2$ and $q_2 < 1 - c_1$.

Figure 1 draws these conditions.

Figure 1: Optimal production level with standard agents.

4 Optimal contract with inequity averse agents

In this section we derive the optimal contracts when agents are inequity averse. We follow F&S's (1999) model of inequity aversion by adapting their utility function to our context with two agents. Inequity averse agents' utility function in our context is U_i^{FS} where:

$$U_i^{FS} = U_i - \alpha \max\left[U_j - U_i, 0\right] - \beta \max\left[U_i - U_j, 0\right] \quad \text{for} \quad i, j = 1, 2, \quad i \neq j.$$

[11] Notice that the "most natural" contract, offering no reward to an agent who does not work, does not implement a unique equilibrium in the subgame.

where, as before, U_i is each agents' "direct utility" and is equal to rewards offered minus the cost of effort performed.[12]

Assume the following:

(U1) *Agents dislike inequity:*

$$\alpha \geq 0,$$

$$\beta \geq 0.$$

(U2) *Agents care more for their own direct utility than for inequity:*

$$\alpha < 1 \text{ and } \beta \leq \frac{1}{2}.$$

Assumption (U1) imposes inequity *aversion.* Agents derive disutility from direct utilities being unequal. In the following, α refers to *negative inequity aversion* or *envy (* dislike to being worse off than your peers*)*, while β refers to *positive inequity aversion* or *guilt (* dislike to being better off than your peers*)*. We assume that both agents have the same α and the same β for simplicity.[13] Assumption (U2) implies that agents care more for their own direct utilities than for the comparison with other agents' direct utilities. F&S allow for $\alpha > 1$. We assume $\alpha \leq 1$ to show that even if inequity aversion is not dominant, its effects on the optimal contract design can still be substantial. Notice that $\beta \leq \frac{1}{2}$ is also necessary for own direct utility to be dominant. Otherwise, agents would be willing to transfer rewards to the other agent ex-post. Additionally, F&S impose $\beta \leq \alpha$, which we do not for generality.

Below we show each agents' utility in the subgame depending on the action taken and the rewards offered by the Principal to them and to the other agent. Notice than when agents are inequity averse, agent's i direct utility is an externality in agent's j utility.

		Agent 2	
		Work	Not Work
Agent 1	Work	$w_1 - c_1 - \text{ámax}[w_2 - c_2 - w_1 + c_1, 0] - \text{âmax}[w_1 - c_1 - w_2 + c_2, 0]$, $w_2 - c_2 - \text{ámax}[w_1 - c_1 - w_2 + c_2, 0] - \text{âmax}[w_2 - c_2 - w_1 + c_1, 0]$	$w_1^1 - c_1 - \text{ámax}[w_2^1 - w_1^1 + c_1, 0] - \text{âmax}[w_1^1 - c_1 - w_2^1, 0]$, $w_2^1 - \text{ámax}[w_1^1 - c_1 - w_2^1, 0] - \text{âmax}[w_2^1 - c_2 - w_1^1, 0]$
	Not Work	$w_1^2 - \text{ámax}[w_2^2 - c_2 - w_1^2, 0] - \text{âmax}[w_1^2 - w_2^2 + c_2, 0]$, $w_2^2 - c_2 - \text{ámax}[w_1^2 - w_2^2 + c_2, 0] - \text{âmax}[w_2^2 - c_2 - w_1^2, 0]$	0 , 0

[12] While F&S's original formulation refers to agents comparing "payoffs", other authors using their preferences in our context assume that only wages enter into welfare comparisons but not the costs of effort (Grund and Sliwka (2002), Itoh (2004)). Our results hold with this alternative specification although contract design is different and more interesting issues appear when costs of effort enter the comparison. Ultimately, this is an empirical question that may be context dependent. A first experimental study of this issue is Königstein (2000) who confirms that welfare comparisons are context dependent.

[13] We focus on asymmetries in productivity parameters instead than on social preferences because they should be more easily observable and measurable.

In the following subsections we study how the Principal can employ this externality to its advantage. We proceed as before, first solving the optimal contract for each production level and then discussing the conditions for each production level to be optimal.

4.1 Optimal contract to implement individual production with inequity averse agents

As in the previous section, we derive the optimal contract which implements individual production by agent i when both agents are inequity averse. Define (ICC_i^{ind}) as agent's i incentive compatibility constraint for individual production by agent i to be an equilibrium of the subgame (not necessarily unique) and (ICC_i^{indU}) as the necessary constraints for the equilibrium to be unique. The problem is the following:

-The Principal maximizes its profit:

$$Max\ q_i - w_i^i$$

subject to:

-Assumptions (C), (R1), (R2), (U1) and (U2).

$$- (ICC_i^{ind}):\ w_i^i - c_i - \alpha \max[w_j^i - w_i^i + c_i, 0] - \beta \max[w_i^i - c_i - w_j^i, 0] \geq 0,$$

$$\begin{aligned} - (ICC_j^{ind}):\ & w_j^i - \alpha \max[w_i^i - c_i - w_j^i, 0] - \beta \max[w_j^i - w_i^i + c_i, 0] > \\ & w_j - c_j - \alpha \max[w_i - c_i - w_j + c_j, 0] - \beta \max[w_j - c_j - w_i + c_i, 0]. \end{aligned}$$

For the subgame to be dominance solvable, the following constraints are also necessary:

$$\begin{aligned} - (ICC_i^{indU}):\ & w_i - c_i - \alpha \max[w_j - c_j - w_i + c_i, 0] - \beta \max[w_i - c_i - w_j + c_j, 0] > \\ & w_i^j - \alpha \max[w_j^j - c_j - w_i^j, 0] - \beta \max[w_i^j - w_j^j + c_j, 0], \\ - (ICC_j^{indU}):\ & w_j^j - c_j - \alpha \max[w_i^j - w_j^j + c_j, 0] - \beta \max[w_j^j - c_j - w_i^j, 0] > 0. \end{aligned}$$

We solve this problem in the following two *Proposition*s.

Proposition 1 *To implement individual production when agents are inequity averse rewards paid in the equilibrium of the subgame are the same as with standard agents ($w_i^i = c_i$ and $w_j^i = 0$).*

Intuitively, the agent who individually works in the equilibrium of the subgame must prefer to work than not work, given that the other agent is not working. Due to budget constraints (R2), agents are not paid when they both not work and thus, the utility of both agents when they both do not work is the same and equal to zero. Inequity generates disutility and because there is no inequity when both agents do not work, it is optimal not to create inequity when only one agent works ($w_i^i - c_i = w_j^i$). Given that rewards cannot be negative, (Assumption (R1)), the minimum rewards needed to be paid such that agent 1 prefers to individually work than not to work are $w_i^i = c_i$ and $w_j^i = 0$ and there is no inequity in equilibrium. Notice that we must still check that (ICC_j^{ind}) holds. We do so in the following *Proposition.*

Proposition 2 *For individual production by the inequity averse agent i to be the unique equilibrium of the subgame, off-equilibrium rewards offered are defined by the following constraints:*

a) If $w_j^j - c_j \geq w_i^j$ then: $w_j^j - c_j > \frac{-\beta}{1-\beta} w_i^j$ and

a_1) If $w_j - c_j \geq w_i - c_i$ then: $w_i - c_i > w_i^j + \frac{\alpha}{1+\alpha}(w_j - w_j^j)$ and $w_j - c_j < \frac{\beta}{1-\beta}(c_i - w_i)$,

a_2) If $w_j - c_j < w_i - c_i$ then: $w_i - c_i > \frac{1}{1-\beta}[(1+\alpha)w_i^j - \beta(w_j - c_j) - \alpha(w_j^j - c_j)]$ and

$$w_j - c_j < \frac{\alpha}{1+\alpha}(w_i - c_i).$$

b) If $w_i^j > w_j^j - c_j$ then: $w_1 - c_1 > w_i^j + \frac{\beta}{1-\beta}(w_j^j - c_j)$, $w_j - c_j < \frac{\alpha}{1+\alpha}(w_i - c_i)$ and

$$w_j^j > c_j + \frac{\alpha}{1+\alpha} w_i^j.$$

Proofs are in the Appendix. The main result of this subsection is that optimal rewards paid in equilibrium are equal to the ones paid with standard agents and thus, the Principal cannot use the externalities caused by inequity aversion to implement individual production in equilibrium under a lower total reward cost than with standard agents (*Proposition 1*). The additional restrictions in *Proposition 2* are needed to ensure that the optimal contract implements a unique equilibrium in the subgame played by the agents. In the following subsection, we check how the Principal can exploit inequity aversion to its advantage to implement joint production.

4.2 Optimal contract to implement joint production with inequity averse agents

Define (ICC_i^{JP}) as agent's i incentive compatibility constraint for joint production to be an equilibrium of the subgame (not necessarily unique) and (ICC_i^{JPU}) as the necessary constraints for the equilibrium of the subgame to be unique. The problem is the following:

-The Principal maximizes its profit:

$$Max\ 1 - w_i - w_j$$

subject to:

-Assumptions (C), (R1), (R2), (U1) and (U2).

$$\text{- } (ICC_i^{JP}):\ w_i - c_i - \alpha \max[w_j - c_j - w_i + c_i, 0] - \beta \max[w_i - c_i - w_j + c_j, 0] > w_i^j - \alpha \max[w_j^j - c_j - w_i^j] - \beta \max[w_i^j - w_j^j + c_j].$$

For the subgame to be dominance solvable, the constraints below are necessary.

Either:

$$\begin{aligned}
&\text{a) - } (ICC_i^{JPU}):\ w_i^i - c_i - \alpha \max[w_j^i - w_i^i + c_i, 0] - \beta \max[w_i^i - c_i - w_j^i, 0] > 0,\\
&\quad\text{- } (ICC_j^{JPU}):\ w_j^j - c_j - \alpha \max[w_i^j - w_j^j + c_j, 0] - \beta \max[w_j^j - c_j - w_i^j, 0] > 0,\\
&\text{b) - } (ICC_i^{JPU}):\ w_i^i - c_i - \alpha \max[w_j^i - w_i^i + c_i, 0] - \beta \max[w_i^i - c_i - w_j^i, 0] \leq 0,\\
&\quad\text{- } (ICC_j^{JPU}):\ w_j^j - c_j - \alpha \max[w_i^j - w_j^j + c_j, 0] - \beta \max[w_j^j - c_j - w_i^j, 0] > 0.
\end{aligned}$$

We solve this problem in the following two *Propositions*. First we find the optimal rewards offered off the equilibrium of the subgame. The idea is to design these off equilibrium rewards such that they create the maximum possible inequities between agents' utilities. By maximizing inequity off equilibrium, agents' (ICC_i^{JP})s can hold with minimum rewards paid in equilibrium.

Proposition 3 *To implement joint production when agents are inequity averse it is optimal to offer zero rewards to the agent who does not work while the other agent individually works* ($w_i^j = 0$)*.*

The intuition behind this result is that if the Principal implements joint production, in equilibrium, conditional on the other agent working, both agents must prefer to work than not to work. Therefore, the Principal designs the rewards such that both agents obtain the highest possible disutility when they shirk, given that the other agent works. Due to limited liability constraints (R1) rewards offered cannot be negative, and due to (U2) agents care more for their direct utility than for the comparison with the other agent, thus the disutility of an agent shirking is maximized when he is offered no reward.

We now look at the reward offered to the agent who individually works off the equilibrium of the subgame. The following *Proposition* states a general result for joint production to be an equilibrium. We thus focus on conditions (ICC_i^{JP}). We discuss uniqueness conditions below.

Proposition 4 *To implement joint production when agents are inequity averse it is optimal to offer extreme rewards to the agent who works off the equilibrium of the subgame (agent i). If the potential effect of envy on the shirking agent (j) is relatively high ($\alpha(q_i - c_i) \geq \beta c_i$), agent i must be offered all the output he individually produces ($w_i^i = q_i$). If, in contrast, the potential effect of guilt is relatively high ($\alpha(q_i - c_i) < \beta c_i$), agent i must be offered no reward off the equilibrium of the subgame ($w_i^i = 0$).*

Intuitively, extreme rewards are used to maximize the disutility from inequity aversion off the equilibrium of the subgame. The reward offered to the agent who individually works off-equilibrium (i) appears in the other agent's (j) condition for joint production to be an equilibrium (ICC_j^{JP}). Thus, the Principal chooses this reward such that it maximizes the disutility of agent j when he does not work and agent i works. Agent j derives disutility from both *envy* and *guilt,* but not from both at the same time. If, given productivity parameters, the potential to exploit agent's j *envy* is higher than the potential to exploit agent's j *guilt*, $(\alpha(q_i - c_i) \geq \beta c_i)$, then the reward offered must be the one that maximizes *envy.* To do so, the principal offers all available production (q_i) to agent i when he individually works. Thus, the *envious* agent j minimizes his utility when he does not work and agent i works because not only he does not get any reward, but experiences *envy* as agent i is paid the maximum available reward.

If, on the contrary, the potential to exploit agent's j *guilt* is higher than the potential to exploit agent's j *envy* $(\alpha(q_i - c_i) < \beta c_i)$, then agent i is offered no reward when he individually works. By doing so, agent's j utility is minimized when he does not work because he is not only paid a reward equal to zero but he also experiences *guilt* because agent i is performing a costly effort and is paid the lowest possible reward, which given (R1) it is zero. Notice that without budget constraints and limited liability, the potential to maximize the effects of *envy* and *guilt* would be unlimited. The

Principal could threat and agent who does not work with offering the other agent an infinite reward when the other agent individually works (to maximize *envy*) or offer an infinite monetary punishment (to maximize *guilt*). We assume (R1) and (R2) to restrict attention to limited and credible threats of inequity.

In the conditions that determine whether *envy* or *guilt* have more potential to harm the shirking agent, not only do the inequity aversion parameters (α and β) enter, but also the costs of effort relatively to productivity. Thus, it is easy to reinterpret these conditions in terms of the costs of effort. Intuitively, if the cost of effort of the agent individually working off the equilibrium is low ($c_i \longrightarrow 0$), the potential to harm the shirking agent due to *guilt* is low. Agent j does not feel very *guilt*y for leaving agent i to work individually because working is not very costly for agent i. But, in contrast, agent j would feel very *envious* if agent i is offered a high reward when he individually works, as the net effect after subtracting the low cost of effort would be high. By rewarding individual work as high as possible (limited by the amount of total output produced) the Principal maximizes *envy*. In contrast, if the cost of effort is high ($c_i \longrightarrow q_i$), the potential for the Principal to exploit *guilt* by offering no reward to the agent who individually works is high, and thus it is optimal to offer no reward at all to the agent who works off the equilibrium path.

We finally look at the equilibrium rewards paid when joint production is implemented. The following two *Proposition*s are the main result of this paper and show optimal rewards for all output levels when joint production is implemented as the unique equilibrium of the subgame.

Proposition 5 *To implement joint production when agents are inequity averse it is optimal to pay the following rewards in equilibrium:*

- *If* $\alpha(q_i - c_i) \geq \beta c_i$ *and* $\alpha(q_j - c_j) \geq \beta c_j$*:*

  For $q_j - c_j \geq q_i - c_i$ *then:* $w_i = c_i - \frac{\alpha(\beta-1)(q_j-c_j)-\alpha^2(q_i-c_i)}{\beta-1-\alpha}$ *and* $w_j = c_j - \frac{\alpha\beta(q_j-c_j)-\alpha(1+\alpha)(q_i-c_i)}{\beta-1-\alpha}$.

- *If* $\alpha(q_i - c_i) < \beta c_i$ *and* $\alpha(q_j - c_j) \geq \beta c_j$*:*

  For $\alpha(q_j - c_j) \geq \beta c_i$ *then:* $w_i = c_i - \frac{\alpha\beta c_i+\alpha(1-\beta)(q_j-c_j)}{1+\alpha-\beta}$ *and* $w_j = c_j - \frac{\beta(1+\alpha)c_i-\alpha\beta(q_j-c_j)}{1+\alpha-\beta}$.

  For $\alpha(q_j - c_j) < \beta c_i$ *then:* $w_i = c_i - \frac{\alpha(1+\alpha)(q_j-c_j)-\beta^2 c_i}{1+\alpha-\beta}$ *and* $w_j = c_j - \frac{\beta(1-\beta)c_i+\alpha^2(q_j-c_j)}{1+\alpha-\beta}$.

Rewards paid in equilibrium are the result of solving the Principal's problem depending on whether it is optimal to maximize each agent's *envy* or *guilt* off the equilibrium of the subgame. From *Proposition 4,* this is determined by whether $\alpha(q_i - c_i) \gtreqless \beta c_i$. *Proposition 5* covers three cases, first when it is optimal to exploit both agents' *envy* off-equilibrium ($w_i^i = q_i$) and second when it is optimal to exploit one agent's *envy* and the other agent's *guilt* ($w_i^i = q_i$ and $w_j^j = 0$).[14] In equilibrium both agents' (ICC^{JP})s are satisfied with equality. Given the slopes of the indifference curves defined by (U1) and (U2), the Principal maximizes profits at the point where both agents' indifference curves intersect each other. Notice that whether this point is on either side of the 45^o degree line depends on which agent suffers more from inequity aversion when the other agent individually works. In general, the agent who suffers more from inequity aversion off-equilibrium is the agent who obtains less direct utility in equilibrium. The following three graphs show the solution of the Principal's problem for the three possible sub-cases.

[14] The remaining case is studied in *Proposition 6.*

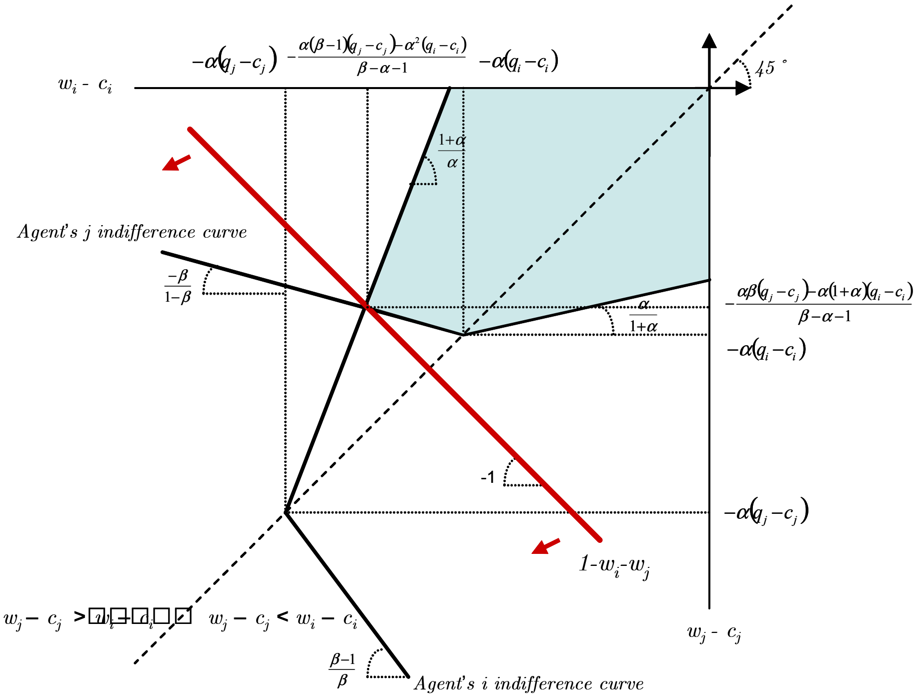

Figure 3: Equilibrium rewards when *envy* dominates for both agents.

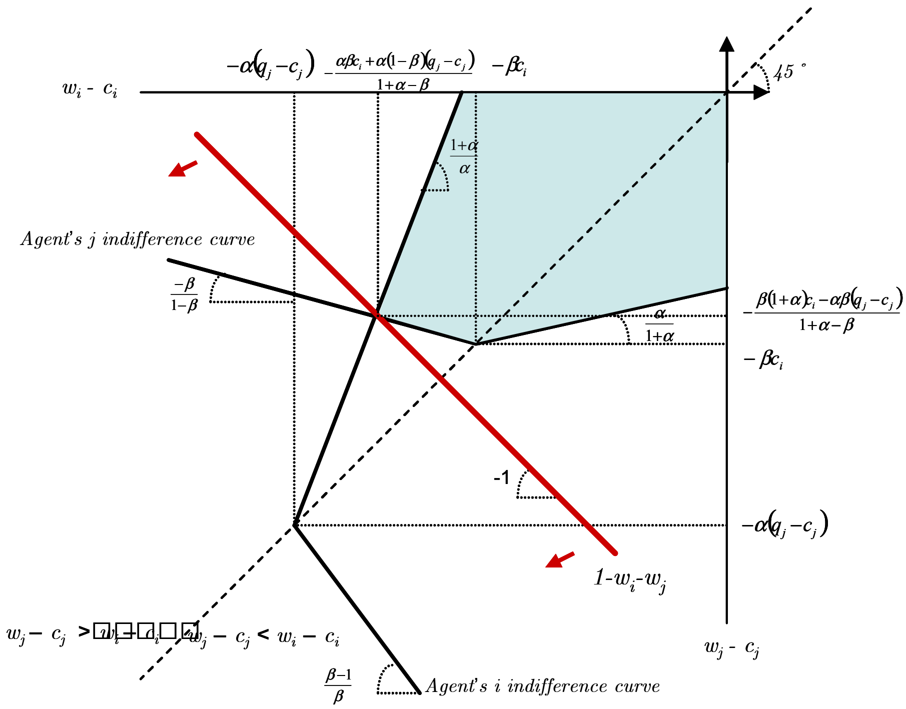

Figure 4: Equilibrium rewards when agent i is *envious* and agent j experiences *guilt* if the other agent works.

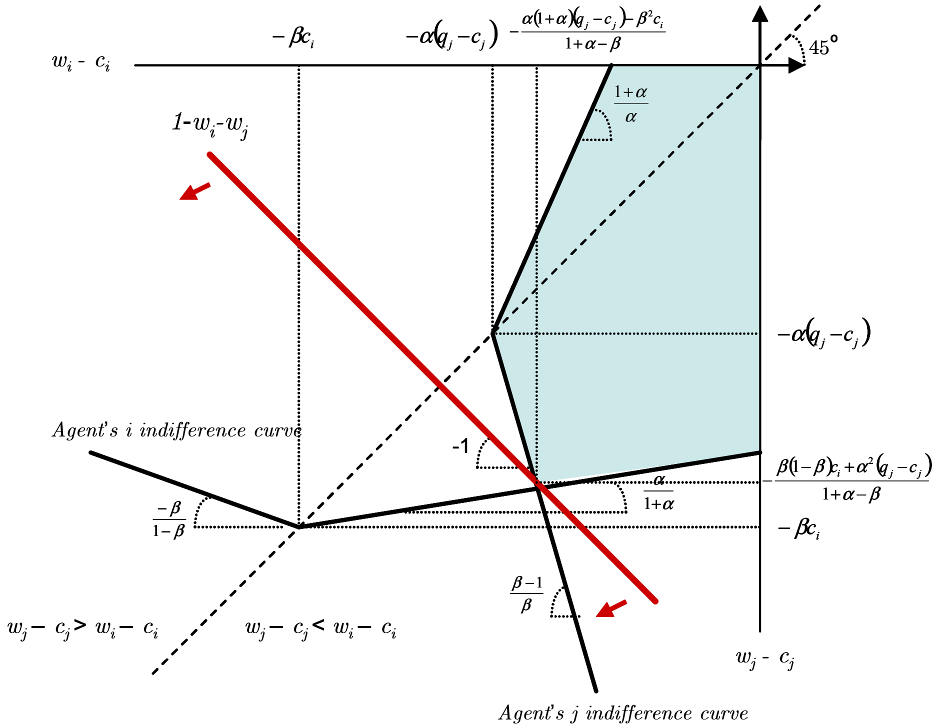

Figure 5: Equilibrium rewards when agent i experiences *guilt* and agent j is *envious* when the other agent works.

We have left out one possible case. When $\alpha(q_i - c_i) < \beta c_i$, the potential effect of *guilt* dominates the potential effect of *envy* for both agents. Thus, it would be optimal to offer both agents a reward equal to zero when they individually work ($w_i^i = 0$). However, notice that this would imply that both agents would prefer not to work when the other agent is also not working, turning no production into an equilibrium of the subgame. As we are interested in uniqueness of the equilibrium of the subgame, such that the contract offered by the Principal implements the optimal level of production, one of the rewards offered to an individually working agent has to be changed. As *Proposition* 6 states, under these circumstances it is optimal to continue offering no reward to one of the agents when he individually works ($w_i^i = 0$) while it is optimal to offer all the available production to the other agent when he individually works ($w_j^j = c_j$).[15] Thus, off equilibrium, one agent will feel *envious* while the other feels *guilty*. *Proposition* 6 shows which agent is optimally offered a reward equal to the available production off-equilibrium and the rewards paid in equilibrium.

Proposition 6 *To implement joint production when agents are inequity averse and guilt dominates for both agents, the optimal rewards are as follows:*

For $\alpha(q_i - c_i) < \beta c_i$, $\alpha(q_j - c_j) < \beta c_j$ *and* $c_j \geq c_i$ *then:*

If $\alpha(q_j - c_j) \geq \beta c_i$

if $(1 - 2\beta)[\alpha(q_j - c_j) - \beta c_j] \geq (1 + 2\alpha)[\alpha(q_i - c_i) - \beta c_i]$,

[15] Notice that when maximum *guilt* cannot be generated due to the equilibrium not being unique, it is optimal to go to other extreme and generate the maximum possible *envy*.

then $w_i^i = 0, w_j^j = q_j$, $w_i = c_i - \frac{\alpha\beta c_i + \alpha(1-\beta)(q_j - c_j)}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\beta(1+\alpha)c_i - \alpha\beta(q_j - c_j)}{1+\alpha-\beta}$,

if $(1-2\beta)[\alpha(q_j - c_j) - \beta c_j] < (1+2\alpha)[\alpha(q_i - c_i) - \beta c_i]$,

then $w_i^i = q_i, w_j^j = 0$, $w_i = c_i - \frac{\beta(1-\beta)c_j + \alpha^2(q_i - c_i)}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\alpha(1+\alpha)(q_i - c_i) - \beta^2 c_j}{1+\alpha-\beta}$.

If $\alpha(q_j - c_j) < \beta c_i$

if $\alpha(1+2\alpha)(q_j - c_j - q_i + c_i) \geq \beta(1-2\beta)(c_j - c_i)$,

then $w_i^i = 0, w_j^j = q_j$, $w_i = c_i - \frac{\alpha(1+\alpha)(q_j - c_j) - \beta^2 c_i}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\beta(1-\beta)c_i + \alpha^2(q_j - c_j)}{1+\alpha-\beta}$,

if $\alpha(1+2\alpha)(q_j - c_j - q_i + c_i) < \beta(1-2\beta)(c_j - c_i)$,

then $w_i^i = q_i, w_j^j = 0$, $w_i = c_i - \frac{\beta(1-\beta)c_j + \alpha^2(q_i - c_i)}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\alpha(1+\alpha)(q_i - c_i) - \beta^2 c_j}{1+\alpha-\beta}$.

The choice of which agent to offer all the available output when he individually works depends on the difference between the maximum possible effect of exploiting each agent's *envy* and *guilt*. In particular, it is crucial whether $\alpha(q_j - c_j) \gtreqless \beta c_i$. The Principal, in order to maximize profits, chooses the off equilibrium rewards such that the sum of the rewards paid in equilibrium is the lowest possible. In the two figures below we draw the two points at which the Principal could be maximizing profits. In Figure 6, both points are on the same side of the 45' degree line, meaning that no matter if agent i's *envy* or *guilt* is exploited off equilibrium, agent j obtains more direct utility in the unique equilibrium of the subgame than agent i. Figure 7, draws the case where depending on which agents' *envy* or *guilt* is exploited, one agent would be better off than the other in the equilibrium of the subgame.

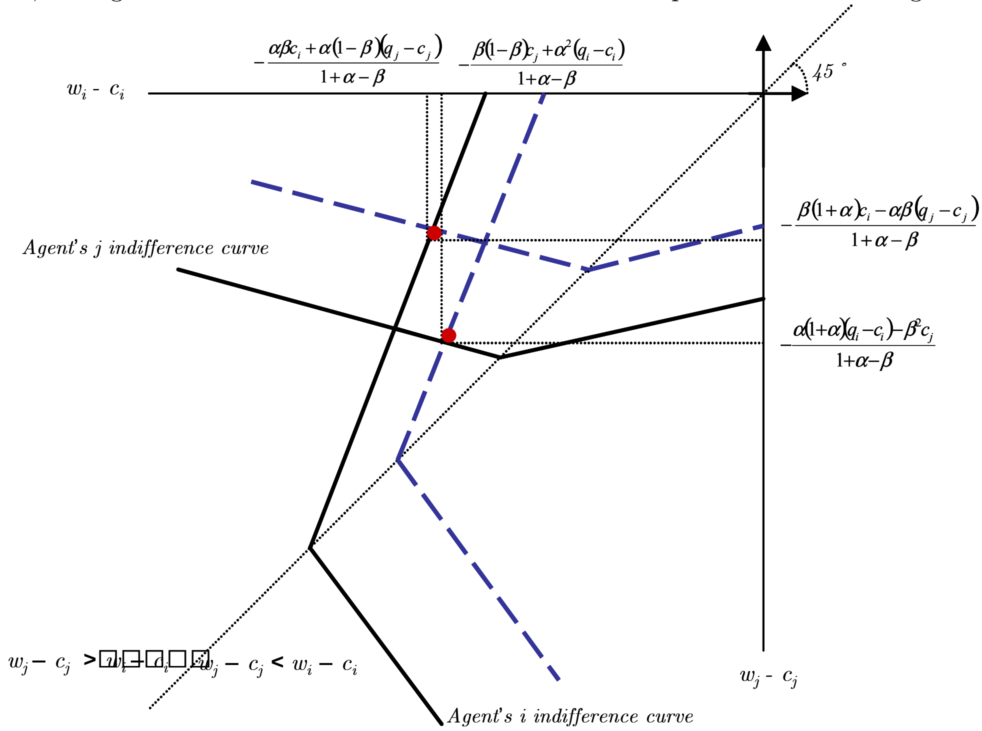

Figure 6: Equilibrium rewards when $w_i - c_i < w_j - c_j$.

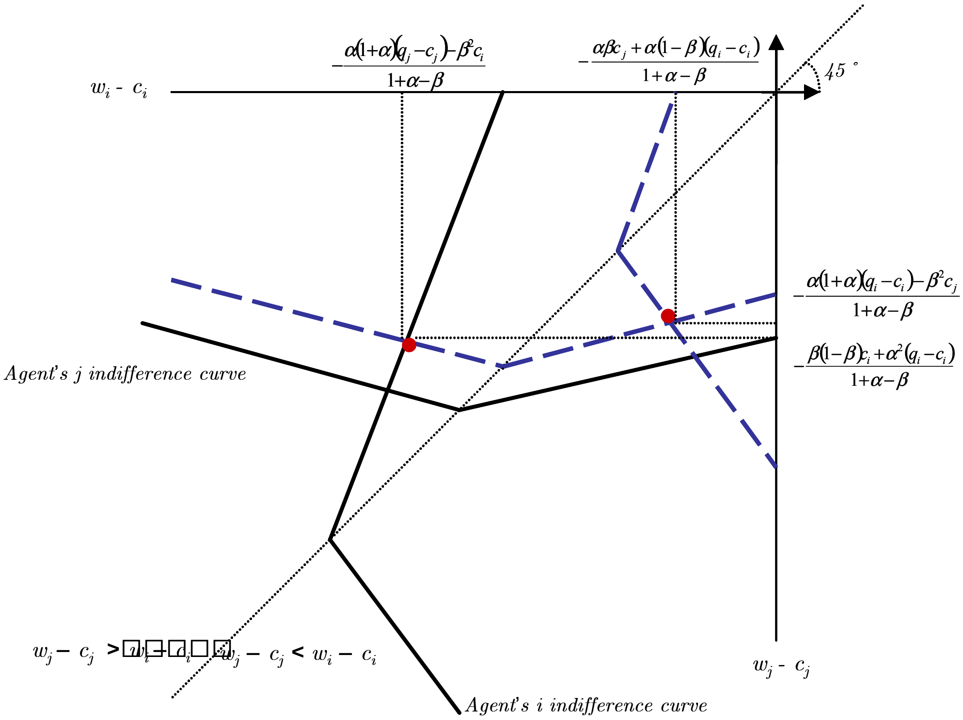

Figure 7: Equilibrium rewards depending on whether $w_i - c_i \gtreqless w_j - c_j$.

Finally. given the results of *Propositions 5* and *6* we can conclude the following:

Corollary 7 *The cost of implementing joint production is lower with inequity averse agents than with standard ones.*

Intuitively, the Principal could always implement an equilibrium in which both agents work by exactly compensating them for their cost of effort when they work, and offering them no reward when they do not work. The reason is that in equilibrium, when both agents are exactly compensated by their costs of effort, there is no inequity and thus, transformed utilities are the same as direct utilities. However, the Principal can do better than exactly compensate the costs of effort, and thus, pay lower rewards. Following *Propositions* 3 to 6, the Principal can create inequity off the equilibrium of the subgame such that inequity averse agents' utilities are lower than standard agents' direct utilities. Thus, paying agents a reward smaller than their cost of effort but maintaining more equity in equilibrium than off-equilibrium, joint production is optimally implemented at a lower total cost for the Principal than with standard preferences.

Notice that this does not mean that equity is maximized when joint production is implemented nor that rewards paid in equilibrium are the same for both agents. Rewards paid just need to be sufficiently close for both (ICC_i^{JP})s to hold at the lowest reward cost in equilibrium for the Principal.

Once we have studied optimal rewards, we need to look at possible changes in the conditions for optimal implementation of individual or joint production.

4.3 Optimal production level with inequity averse agents

Once we have shown the optimal rewards needed to be paid in equilibrium to implement each production level, we look at the conditions for each output level to be optimal. Notice that from previous results it is obvious that whenever the conditions for joint production to be optimal with standard agents are satisfied ($q_i < 1 - c_j$ for $i, j = 1, 2,\ i \neq j$) it is still optimal to implement joint production when agents are inequity averse. The reason is that while the total reward cost needed to be spent in equilibrium to implement individual production is the same with standard an inequity averse agents, from *Corollary 7* the total reward cost needed to implement joint production is lower with inequity averse agents. Thus, it is possible that under same values for the productivity parameters, it may be optimal to implement individual production by standard agents while it is optimal to implement joint production with inequity averse agents. Obviously, changes of equilibrium implemented from individual production by one agent to individual production by the other agent are not possible.

We now show the conditions for the Principal to find optimal to implement joint production under the three possible sets of equilibrium rewards paid when agents are inequity averse:

- If $w_i = c_i - \frac{\alpha(\beta-1)(q_j-c_j)-\alpha^2(q_i-c_i)}{\beta-1-\alpha}$ and $w_j = c_j - \frac{\alpha\beta(q_j-c_j)-\alpha(1+\alpha)(q_i-c_i)}{\beta-1-\alpha}$ then joint production is optimal when $q_i > 1 - c_j + \frac{\alpha(1-2\beta)(q_j-c_j)+\alpha(1+2\alpha)(q_i-c_i)}{\beta-1-\alpha}$.
- If $w_i = c_i - \frac{\alpha\beta c_i+\alpha(1-\beta)(q_j-c_j)}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\beta(1+\alpha)c_i-\alpha\beta(q_j-c_j)}{1+\alpha-\beta}$ then joint production is optimal when $q_i > 1 - c_j - \frac{(1+2\alpha)\beta c_i+\alpha(1-2\beta)(q_j-c_j)}{\beta-1-\alpha}$.
- If $w_i = c_i - \frac{\alpha(1+\alpha)(q_j-c_j)-\beta^2 c_i}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\beta(1-\beta)c_i+\alpha^2(q_j-c_j)}{1+\alpha-\beta}$ then joint production is optimal when $q_i > 1 - c_j - \frac{\alpha(1+2\alpha)(q_i-c_i)+\beta(1-2\beta)c_j}{\beta-1-\alpha}$.

Otherwise, the Principal implements individual production by the agent for which $q_i - c_i$ is highest.

Appendix A contains a numerical example showing how the optimal contract changes when, under same productivity parameters, it is optimal to implement individual production with standard agents and joint production with inequity averse agents. Appendix B contains a second example which shows that even if the optimal production level does not change, the loss in profits the Principal incurs when he does not take into account inequity aversion is far from negligible. This example is symmetric as we assume $q_1 = q_2 = 0.5$ and $c_1 = c_2 = 0.4$. Under those parameter values it is optimal to implement joint production when agents are standard and thus, it will still be optimal to implement joint production when agents are inequity averse. The loss for the Principal is defined as the difference in his profits (production minus rewards paid) between offering an inequity averse contract and a standard contract to inequity averse agents as a proportion of the total output implemented in joint production (equal to 1). This loss is an increasing function in the *envy* (α) and *guilt* (β) parameters. The Principal's loss can be up to 40% of the total output produced.

In summary, in our simple model without uncertainty, accounting for inequity aversion has no extra cost for the Principal in equilibrium, and the Principal can benefit from it to implement joint production under a lower total reward cost, thus obtaining higher profits. Inequity aversion might also change the optimal production level. In the next section we comment on the robustness of our results to other types of distributional preferences.

5 Status and Efficiency seeking Preferences

It can be argued that in some contexts, other types of distributional preferences might be more relevant than inequity aversion. In particular, in very competitive firms, agents might not be averse to inequity but instead they might enjoy it, as long as it is the other agent who is worse off than them. Such agents will not feel *guilt* but *spite* when being better off than their peers, while they will still feel *envious* when being worse off. We call these agents "Status Seeking", interpreting having higher status as being higher in the ranking of agents' welfare, i.e., as being better off than other agents.

In other contexts in which each agent contributes a lot to total production, agents might feel disutility when shrinking because the total amount of output, and thus, the total amount of rewards available to be distributed among agents, gets smaller when they shirk. We call these agents "Efficiency Seeking", interpreting efficiency as the sum of agents' welfare net of the costs of effort.

These distributional concerns have been captured by other forms of utility functions.[16] However, it is worth noticing that by simply changing the range of values parameters α and β in the F&S utility function can take, it is possible to look at the array of possible purely distributional concerns in a unified model. We now use this re-parametrization of the model to explore the consequences in optimal contract design. Notice that the problems we solve are the same as in Section 4, although solutions change when we allow for different parameter values.

5.1 Reward Design with Status Seeking Preferences

Assume now that $\alpha \in [0, 1)$, $\beta < 0$ and $|\beta| \leq 1$. This means that agents are still averse to disadvantageous inequity but like advantageous inequity. The following two *Proposition*s cover the key issues of contract design when agents are status seeking.

Proposition 8 *To implement individual production when agents are status seeking, rewards paid in the equilibrium of the subgame are the same as with standard agents ($w_i^i = c_i$ and $w_j^i = 0$).*

As it happened with inequity averse agents, the optimal contract to implement individual production implies paying the agent who individually works (i) a reward exactly equal to his cost of effort (c_i) and paying no reward to the shirking agent. The reason is that in the right hand side (RHS) of (ICC_i^{ind}) there is no production and thus, both agents are paid zero and no agent is ahead. One could argue that since agent i likes being better off than his peer, it would be easier to provide incentives to agent i to work by making him better off than agent j when agent i individually works. However, given that it is still optimal to pay no reward to agent j when he does not work (due to (R1) the Principal cannot pay him less), the only way to use that agent i is status seeking is by making him better off than agent j. But this would imply paying agent i above his cost of effort, which cannot be optimal. Thus, as it happened with inequity averse agents, status seeking preferences cannot be used to implement individual production with lower rewards than with standard preferences.

[16] See Charness and Rabin (2002) for a summary.

Proposition 9 *To implement joint production when agents are status seeking the optimal contract is as follows:*

$$\begin{aligned} w_i &= c_i - \frac{\alpha(\beta-1)(q_j-c_j)-\alpha^2(q_i-c_i)}{\beta-1-\alpha} & \quad w_j &= c_j - \frac{\alpha\beta(q_j-c_j)-\alpha(1+\alpha)(q_i-c_i)}{\beta-1-\alpha}, \\ w_i^i &= q_i & w_j^i &= 0, \\ w_i^j &= 0 & w_j^j &= q_j, \end{aligned}$$

for $\alpha(q_i - c_i) \geq \beta c_i$ *and* $i, j = 1, 2, i \neq j$.

Notice that to implement joint production, the only way to create inequity off the equilibrium of the subgame is by generating disutility via *envy* on the agent who shirks, and thus it is optimal to offer no reward to the agent who shirks and all individual output to the agent who individually works. Therefore, only *envy* is used in this case. The reason is that *spite* provides utility to the shirking agent, making his (ICC_i^{JP}) more difficult to hold. Rewards paid in the equilibrium of the subgame are defined by the first expression in *Proposition* 5 .The graphic representation is the same as Figure 5. Notice that rewards paid are not necessarily equal when productivities are asymmetric and thus, in equilibrium still one agent could be better off than the other, although with a smaller difference than off the equilibrium of the subgame. Now things are even better for the Principal. As agents like to be better off than each other, the Principal needs to pay even less in equilibrium to the agent who is best off. As agents' (ICC_i^{JP})s bind, the agent who suffers more from *envy* off-equilibrium is the one who will optimally be better off in the equilibrium, i.e., if $q_i - c_i \geq q_j - c_j$ then it is optimal to choose w_i and w_j such that $w_j - c_j \geq w_i - c_i$.

Following results in section 4.3, the Principal finds optimal to implement joint production when $q_i > 1 - c_j + \frac{\alpha(1-2\beta)(q_j-c_j)+\alpha(1+2\alpha)(q_i-c_i)}{\beta-1-\alpha}$ for $i, j = 1, 2$, $i \neq j$. Otherwise, the Principal implements individual production by the agent for which $q_i - c_i$ is highest.

5.2 Reward Design with Efficiency Seeking Preferences

Assume now that $\alpha < 0$, $\beta \in [0, 1/2)$, and $|\alpha| \leq |\beta|$. This implies that agents always prefer higher rewards for themselves and the other agent, but are more in favor of getting higher rewards for themselves when they are worse off than the other agent than when they are better off. Agents' concern for efficiency means that they care the total amount of rewards offered by the Principal. They always prefer a higher total amount of rewards, no matter if the extra rewards are all offered to the other agent. This leaves the possibility for the Principal to exploit efficiency seeking preferences.[17] The following two *Propositions* cover the key issues of contract design when agents are efficiency seeking.

Proposition 10 *To optimally implement individual production when agents are efficiency seeking, the sum of rewards paid in the equilibrium of the subgame is the same as with standard agents* ($w_i^i + w_j^i = c_i$)

[17] We define efficiency from the point of view of the agents, i.e., as the sum of agents' direct utilities.

Intuitively, the agent who individually works in equilibrium (agent i) must prefer to work than not to work given that agent j does not work. When both agents do not work rewards are zero and thus, agent i should obtain positive utility when working for his (ICC_i^{ind}) to hold. However, the only way to use that agent i is efficiency concerned to implement an equilibrium in which he individually works and paying a lower reward than his cost of effort is by offering "more efficient rewards". i.e., a total sum of rewards that adds to more agent's i cost of effort. This is a contradiction. Thus, individual production cannot be implemented with a total sum of rewards paid in equilibrium lower than the cost of effort of the agent who individually works. Notice that equilibrium rewards are not necessarily equal to the cost of effort of the agent who individually works, although the sum of rewards paid must be equal to it.

Proposition 11 *To implement joint production when agents are efficiency seeking the optimal contract is as follows:*

$$
\begin{aligned}
w_i &= c_i + \frac{\beta^2}{1+\alpha-\beta}c_i \qquad & w_j = c_j + \frac{\beta(\beta-1)}{1+\alpha-\beta}, \\
w_i^i &= 0 \qquad & w_j^i = 0, \\
w_i^j &= 0 \qquad & w_j^j = c_j,
\end{aligned}
$$

for $c_i > c_j$.

Notice that, contrary to previous sections, now extreme rewards (all production or no production at all) are not offered to all agents off the equilibrium of the subgame. In particular, agent j receives an offer equal to his cost of effort when he individually works ($w_j^j = c_{j,}$). The reason is that offering the most inefficient rewards off equilibrium, i.e., no reward to all agents off equilibrium, the equilibrium of the subgame would not be unique. Notice that if $w_i^i = w_j^i = w_i^j = w_j^j = 0$, then no production is clearly an equilibrium of the subgame, as agents obtain the same rewards when they both do not work than when they individually work and, as no agent performs effort, there is more efficiency when they both do not work. To obtain uniqueness, it is necessary to offer a reward that compensates one agent for his cost of effort, in order for him to prefer to individually work than not to work, given that the other agent is not working. The choice of which agent is offered a reward equal to his effort cost when individually working is determined by agents' costs of effort. Notice that *Proposition 11* says that the agent who has a smaller cost of effort (agent j) is the one that must be offered a reward equal to his cost of effort. The reason is that, by offering a reward equal to zero to the agent with highest cost ($w_i^i = 0$), the Principal creates more inefficiency off equilibrium and thus, he can implement joint production as the unique equilibrium of the subgame with the lowest possible total sum of rewards paid. Also notice that the agent with the highest cost is paid in equilibrium a reward higher than his cost of effort ($w_i = c_i + \frac{\beta^2}{1+\alpha-\beta}c_i > c_i$ as $\beta < \frac{1}{2}, \left|\alpha < \frac{1}{2}\right|$), while the other agent is paid a reward sufficiently lower than his cost of effort ($w_j = c_j + \frac{\beta(\beta-1)}{1+\alpha-\beta}c_i < c_j$), such that the total sum of rewards paid in equilibrium is lower than the sum of both agents' cost of effort.

Finally, it is optimal to implement joint production when agents are efficiency concerned whenever $1 - c_j + \frac{2\beta(\beta-1)}{1+\alpha-\beta}c_i > q_i$ and $1 - c_i + \frac{2\beta(\beta-1)}{1+\alpha-\beta}c_i > q_j$ for $c_i > c_j$. If these conditions are not satisfied, the Principal implements individual production by the agent for which $q_i - c_i$ is highest.

6 Discussion

We have shown how distributional preferences change optimal contracts in a simple Principal-Agent setting where agents have already entered the firm. Optimal rewards paid to implement joint production are lower than with standard agents and the optimal level of production can change. Finally, we have shown that accounting for distributional preferences is beneficial for the Principal and has no drawbacks.

Despite its simplicity, our model provides a new rationale for team and relative performance contracts in contexts with no informational asymmetries. In both these types of contracts, agents are threaten with welfare inequities when some employees work harder than others. In team contracts, if a member of the team shirks, the team's performance is going to be less successful and thus, other members of the team who have worked hard will not see their efforts rewarded, for which the shrinking agent might feel guilty. Therefore, agents might decide not to shirk even if rewards are low, to avoid feeling guilty for the members of the tam who work hard. In a relative performance contract, an agent who does not work hard will be ranked low, and thus, will be worse off than higher ranked agents, for which he may actually feel envious. Thus in competitive contexts it may not be necessary to offer such high rewards if agents are envious of each other and compete not to be ranked lower than their peers. Thus, welfare comparisons among peers can be used by the employer to provide incentives to work hard. We just show, depending on how employees compare to their peers, when it is optimal to use each type of contract.

Our model highlights how Behavioral Contract Theory could be useful to study issues of organization in the firm. Both the Human Resources Literature and the Personnel Economics Literature have studied these issues before.[18] The contribution of our paper is that it highlights that comparisons among agents are important and can be affected by the design of the contract. Our model suggests that optimal contracts depend on the strength of welfare comparisons. If that is the case, it may be possible to affect the strength of those comparisons in the workplace. We have here assumed everything was common knowledge. However, in real firms the employer might be able to influence which information is easily available to his employees, once it has been clarified which variables enter employees welfare comparisons in different contexts. In particular, decisions such as whether to make salaries publicly available to co-workers or not, or the allocation of office space (which might affect the observability of effort by co-workers) could be illuminated by issues here discussed. Although in many firms rewards are kept secret[19] and employees work in separate and closed offices, we have provided one of the factors that might affect these decisions.

Finally, notice that by using subgame perfect implementation our results are collusive proof as the subgame is dominance solvable and thus, the equilibrium is unique. Equilibrium Uniqueness is important since exploiting distributional preferences to the Principal's advantage implies that both agents would be worse off when they both work than when they do not. Thus, if the equilibrium was

[18] See Lazear (1995).

[19] Even if Bewley (1999) reports that 87% of managers interviewed think that their employees know each others' wages.

not unique, agents could coordinate on not working to avoid being exploited by the Principal. Notice also that in our model agents do not have incentives to transfer rewards to the other agent to reduce inequity, as they care more for own rewards than for equity. Therefore, we have shown, contrary to the gift-exchange idea discussed in the labor literature, how under some circumstances distributional preferences may be used profitably to provide incentives for employees to work.

7 References

Agell, J., Lundborg, P., (1999) “Theories of Pay and Unemployment: Survey Evidence from Swedish Manufacturing Firms”. *Scandinavian Journal of Economics 97, 295-307.*

Andreoni, J., Miller, J., (1998). ”Analyzing choice with revealed preference : is altruism rational?,” Working papers 14, Wisconsin Madison - Social Systems.

Bandiera, O., Barankay, I., Ramsul, I., (2004). ”Cooperation in the Workplace: Evidence form the Field”. Mimeo.

Bazerman, M. , Loewenstein, G. and Thompson, L., (1989). ”Social utility and decision making in interpersonal contexts”. *Journal of Personality & Social Psychology*, 57, 426-441

Bewley, T., (1999) “Why rewards don’t fall during a Recession”. Harvard University Press.

Blinder, A., Choi, D., (1990) “A Shred of Evidence on Theories of Reward Stickiness”. *Quarterly journal of Economics 105, 1003-1015.*

Bolton, G., Ockenfels, A., (2000), “ERC: A Theory of Equity, Reciprocity and Competition”. *American Economic Review.* Vol 90.

Cabrales, A., Calvó-Armengol, A., (2002) “Social Preferences and Skill Segregation”. Universitat Pompeu Fabra and Universidad Carlos III. Mimeo.

Campbell, C.M., Kamlani, K.S, (1997) “The reasons for reward rigidity: Evidence from a Survey of Firms”. *Quarterly journal of Economics 112, 759-789.*

Charness, G., Rabin, M., (2002), “Understanding Social Preferences with Simple Tests” *Quarterly Journal of Economics.* Forthcoming.

Cox, J., Friedman, D., (2002). “A Tractable Model of Reciprocity and Fairness”. University of Arizona. Working Paper.

Dufwenberg, M., Kirchsteiger, G., (2000), “Reciprocity and reward Undercutting”. *European Economic Review, 44, 1069-1078.*

Englmaier, F., Wambach, A., (2002), “Contracts and Inequity Aversion”. University of Munich. Mimeo.

Fehr, E., Schmidt, K., (1999), “A Theory of Fairness, Incentives and Contractual Choices ”. *Quarterly Journal of Economics,* 114, 817-868.

Fehr, E., Schmidt, K., (2000), “Fairness, Incentives and Contractual Choices”, *European Economic Review,* 44, 1057-1068.

Fehr, E., Schmidt, K., (2000b), “Theories of Fairness and Reciprocity: Evidence and Economic Applications”, forthcoming in *Advances in Economic Theory. Eight World Congress of the Econometric Society.* Cambridge University Press.

Grund, C., Sliwka, D., (2002), "Envy and Guilt in Tournaments". Bonn Graduate School of Economics. Discussion Paper 32/2002.

Itoh, H., (2004), "Moral Hazard and Other-Regarding Preferences". *The Japanese Economic Review,* vol 55 (1).

Holmstrom, B. and Milgrom, P., (1991), "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership and Job Design", *Journal of Law, Economics and Organization,* 7, 24-52.

Homans, g., (1950), The Human Group. New York. Harcourt.

Huck, S., Rey Biel, P., (2002), "Inequity Aversion and the Timing of Team Production". University College London. Mimeo.

Kögnistein, M., (2000), "Equity, Efficiency and Evolutionary Stability in Bargaining Games with Joint Production". Lecture Notes in Economics and Mathematical Systems. Springer.

Lazear, E., (1995), Personnel Economics, MIT Press.

Mookherjee, D., (1984), "Optimal Incentive Schemes with Many Agents". *Review of Economic Studies*, 51, 433-446.

Moore, J., Repullo, R., (1988), "Subgame Perfect Implementation". *Econometrica,* 56, 1191-1220.

Sobel, J., (2000) "Social Preferences and Reciprocity". Mimeo.

8 Appendix A

Proof of *Proposition* 1

Rewards paid in the equilibrium of the subgame (w_i^i, w_j^i) appear in (ICC_i^{ind}) and (ICC_j^{ind}). By (R2) the value of the Right Hand Side (RHS) of (ICC_i^{ind}) is zero and both agents obtain the same utility when they both do not work. By (U1) α and β are positive. By choosing $w_i^i - c_i = w_j^i$, the terms that compare direct utilities in (ICC_i^{ind}) are equal to zero and do not subtract utility in the Left Hand Side (LHS) of the condition. The Principal's objective is to Maximize $q_i - w_i^i$, thus Minimize w_i^i. By setting $w_i^i = c_i$ and $w_j^i = 0$ the Principal maximizes profits with (ICC_i^{ind}) holding. Although the optimal value of w_i and w_j is determined in *Proposition 2* below, notice that when $w_i^i = c_i$ and $w_j^i = 0$, condition (ICC_j^{ind}) holds more easily. *Proposition 2* shows the optimal values of w_i and w_j in order for *(ICC_j^{ind})* to hold, and thus for individual production to be an equilibrium.

Proof of *Proposition* 2

The restrictions are the result of rearranging conditions (ICC_j^{ind}), (ICC_i^{indU}) and (ICC_j^{indU}) and simplifying the terms that compare direct utilities. There are four cases depending on whether $w_i - c_i \lesseqgtr w_j - c_j$ and $w_i^j \lesseqgtr w_j^j - c_j$. Of these four cases, the combination $w_i - c_i < w_j - c_j$ and $w_i^j > w_j^j - c_j$ violates (ICC_j^{ind}) if (ICC_i^{indU}) and (ICC_j^{indU}) hold and thus, this case is removed.

Proof of *Proposition* 3

The Principal decides the optimal w_i^j to minimize agent's i utility when he does not work, given that agent j works (the right hand side of (ICC_i^{JP})):

$$w_i^j - \alpha \max\left[w_j^j - c_j - w_i^j, 0\right] - \beta \max\left[w_i^j - w_j^j + c_j, 0\right].$$

Notice that inequity aversion imposes that an agent obtains disutility either from being better off or worse off than the other agent, but not from both at the same time.

a) If agent i is worse off than agent j, the effect of *envy* dominates and $w_j^j - c_j - w_i^j \geq 0$. Thus, to minimize the utility of agent i when not working, $w_i^j = 0$, as the derivative of agent's i utility with respect to the reward offered to agent j equals $1 + \alpha > 0$ by assumption (U1).

b) If agent i is better off than agent j, the effect of *guilt* dominates and $w_i^j - w_j^j + c_j \geq 0$. Thus, to minimize the utility of agent i when he does not work, $w_i^j = 0$, as the derivative of agent's i utility with respect to the reward offered to agent j equals $1 - \beta > 0$, by assumption (U2).

Proof of *Proposition* 4

Assume agent i individually works off the equilibrium of the subgame.

The reward offered to agent i when he individually works (w_i^i) only appears in agent j's no deviation condition (ICC_j^{JP}). The Principal seeks to minimize agent j's utility when he does not work.

By *Proposition 1*, the optimal reward offered to agent j when agent i individually works is $w_j^i = 0$.

The utility of agent j when agent i individually works is thus:

$$-\alpha \max\left[w_i^i - c_i, 0\right] - \beta \max\left[-w_i^i + c_i, 0\right]$$

where by (R1) and (R2),

$$w_i^i \in [0, q_i],$$

and by (C),

$$0 \leq c_i \leq q_i.$$

Thus, minimizing agent j's utility implies:

$$w_i^i = q_i \qquad \text{if} \quad \alpha(q_i - c_i) \geq \beta c_i$$

and

$$w_i^i = 0 \qquad \text{if} \quad \alpha(q_i - c_i) < \beta c_i.$$

Proof of *Proposition* 5

If $\alpha(q_i - c_i) \geq \beta c_i$ for $i = 1, 2$, it is optimal to choose $w_i^i = q_i$. Conditions (ICC^{JPU})s hold using results in *Proposition* 3. The Principal maximizes $1 - w_1 - w_2$ subject to both (ICC^{JP})s. Using the slopes of the indifference curves given by (U1) and (U2), the conditions optimally hold with equality and profits are maximized at the unique point at which indifference curves intersect.

If $\alpha(q_i - c_i) < \beta c_i$ and $\alpha(q_j - c_j) \geq \beta c_j$ for $i, j = 1, 2$, $i \neq j$, it is optimal to choose $w_i^i = 0$ and $w_j^j = q_j$. The two cases are created by whether the intersection of both indifference curves occurs at a point where $w_i - c_i \lesseqgtr w_j - c_j$.

Proof of *Proposition* 6

If $\alpha(q_i - c_i) < \beta c_i$ for $i = 1, 2$, inequity off equilibrium would be maximized by setting $w_i^i = 0$ and $w_j^j = q_j$. However the equilibrium of the subgame played by the agents would not be unique. Inequity has to be maximum off equilibrium and thus, it is optimal to offer off equilibrium rewards such that one agent's *envy* is maximized and the other agent's *guilt* is maximized. Thus, $w_i^i = 0$ and $w_j^j = q_j$ for $i, j = 1, 2$, $i \neq j$. Therefore, one of the indifference curve of the agents' is not satisfied with equality. The optimal rewards are obtained at the intersection of the indifference curve which holds with equality and the parallel indifference curve that does not hold with equality (The discontinued lines in Figure 6 and 7). The conditions indicate for which of the four possible cases, profits are maximized.

Proof of Corollary 7

From *Propositions* 5 and 6, there are three possible cases:

- If $w_i = c_i - \frac{\alpha(\beta-1)(q_j-c_j)-\alpha^2(q_i-c_i)}{\beta-1-\alpha}$ and $w_j = c_j - \frac{\alpha\beta(q_j-c_j)-\alpha(1+\alpha)(q_i-c_i)}{\beta-1-\alpha}$ then
  $w_i + w_j = c_i + c_j - \frac{\alpha(1-2\beta)(q_j-c_j)+\alpha(1+2\alpha)(q_i-c_i)}{\beta-1-\alpha} < c_i + c_j$ by (C), (U1) and (U2).
- If $w_i = c_i - \frac{\alpha\beta c_i+\alpha(1-\beta)(q_j-c_j)}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\beta(1+\alpha)c_i-\alpha\beta(q_j-c_j)}{1+\alpha-\beta}$ then
  $w_i + w_j = c_i + c_j + \frac{(1+2\alpha)\beta c_i+\alpha(1-2\beta)(q_j-c_j)}{\beta-1-\alpha} < c_i + c_j$ by (C), (U1) and (U2).
- If $w_i = c_i - \frac{\alpha(1+\alpha)(q_j-c_j)-\beta^2 c_i}{1+\alpha-\beta}$ and $w_j = c_j - \frac{\beta(1-\beta)c_i+\alpha^2(q_j-c_j)}{1+\alpha-\beta}$ then
  $w_i + w_j = c_i + c_j + \frac{\alpha(1+2\alpha)(q_i-c_i)+\beta(1-2\beta)c_j}{\beta-1-\alpha} < c_i + c_j$ by (C), (U1) and (U2).

Proof of *Proposition* 8

Rewards paid in the equilibrium of the subgame (w_i^i, w_j^j) appear in (ICC_i^{ind}) and (ICC_j^{ind}). By (R2) the value of the Right Hand Side (RHS) of (ICC_i^{ind}) is zero and both agents obtain the same utility when they both do not work. As $\alpha > 0$, the only possible way to make condition (ICC_i^{ind}) hold under a lower total reward cost is by setting $w_i^i - c_i \geq w_j^i$. However, by (R1) $w_j^i \geq 0$, and thus, $w_i^i \geq c_i$. The minimum reward needed to be paid in equilibrium are thus $w_i^i = c_i$ and $w_j^i = 0$.

Proof of *Proposition* 9

As $\beta < 0$, agents only obtain disutility from *envy.* To maximize the effect of *envy* off the joint production equilibrium, the agent who does not work off equilibrium is offered no reward ($w_i^j = 0$ for $i, j = 1, 2$ and $i \neq j$) and the agent who works is offered all available production ($w_i^i = q_i$ for $i = 1, 2$). The expression for the equilibrium rewards paid follows calculations in *Proposition* 5.

Proof of *Proposition* 10

Assume agent i individually works off the equilibrium of the subgame. For (ICC_i^{ind}) to hold, agent i must obtain positive utility when he works given that agent j does not work. However, given that $\alpha < 0$, the only possible way to implement individual production by agent i with $w_i^i < c_i$ is by paying agent j a reward that gives him more direct utility than agent i, i.e., $w_j^i - w_i^i - c_i > 0$. Thus, $w_j^i > w_i^i + c_i$, which, given (R1), implies $w_i^i + w_j^i \geq c_i$ The minimum sum of rewards thus needed to be paid to implement individual production by agent i is $w_i^i + w_j^i = c_i$.

Proof of *Proposition* 11

To maximize the effect of inefficiency off equilibrium, all agents should be offered no reward off equilibrium, no matter if they work or they do not. However, by doing so, no production would be an equilibrium as (ICC_i^{JPU}) for $i = 1, 2$ would not hold. Thus, it is necessary that one agent prefers to individually work than not to work, given that the other agent does not work. Assume agent i is the agent who prefers to work than not to work. For (ICC_i^{JPU}) to hold with the maximum possible inefficiency when agent i individually works, it is optimal to set $w_i^i = c_i$ and $w_j^i = 0$. When agent j individually works, maximum inefficiency is generated by setting $w_i^j = w_j^j = 0$. The remaining two equilibrium rewards are obtained at the intersection between both (ICC_i^{JP}) for $i = 1, 2$:

$$w_i - c_i - \beta(w_i - c_i - w_j + c_j) \geq 0$$
$$w_j - c_j - \alpha(w_i - c_i - w_j + c_j) \geq -\beta c_i,$$

which yields: $w_i = c_i + \frac{\beta^2}{1+\alpha-\beta} c_i$ and $w_j = c_j + \frac{\beta(\beta-1)}{1+\alpha-\beta} c_i$.

Notice that the sum of rewards paid in equilibrium equals $w_i + w_j = c_i + c_j + \frac{2\beta(\beta-1)}{1+\alpha-\beta} c_i$. As $\alpha < 0$, $\beta \in [0, 1/2)$, and $|\alpha| \leq |\beta|$ then $\frac{2\beta(\beta-1)}{1+\alpha-\beta} < 0$ and thus, it is optimal to set $w_i^i = c_i$ for the agent for which the cost of effort is highest, i.e., for $c_i > c_j$ and $i, j = 1, 2$, $i \neq j$.

9 Appendix B: Numerical examples

9.1 Change of optimal production level

Assume $\alpha = 0.9, \beta = 0.1, q_1 = 0.7, c_1 = 0.5, q_2 = 0.5$ and $c_2 = 0.4$.

The conditions for individual production by agent 1 to be optimal, $1 - c_2 \leq q_1 \quad if \quad (q_1 - c_1) > (q_2 - c_2)$, holds as $1 - 0.4 \leq 0.7$ with $(0.7 - 0.5) > (0.5 - 0.4)$. Therefore, in the equilibrium of the subgame when agents are standard rewards paid are $w_1^1 = 0.5$ and $w_2^1 = 0$, and profits $(q_1 - w_1^1)$ are equal to 0.2.

Now we look at joint production with inequity averse agents. From *Proposition* 3, it is optimal to offer $w_1^2 = w_2^1 = 0$ to the agent who does not work when the other agent individually works. Notice also that $\alpha(q_i - c_i) > \beta c_i$ for $i = 1, 2$, as:

$$0.9(0.7 - 0.5) > 0.1(0.5)$$
$$0.9(0.5 - 0.4) > 0.1(0.4).$$

Thus, it is optimal to offer all output to the agent who individually works off equilibrium: $w_1^1 = q_1 = 0.7$ and $w_2^2 = q_2 = 0.5$.

Finally, notice that $\alpha(q_1 - c_1) > \alpha(q_2 - c_2)$ as $0.18 > 0.09$. Thus, in equilibrium $w_1 - c_1 > w_2 - c_2$ and the (ICC_i^{JP})s are:

$$w_1 - 0.5 - 0.1(w_1 - 0.5 - w_2 + 0.4) \geq -0.09$$
$$w_2 - 0.4 - 0.9(w_1 - 0.5 - w_2 + 0.4) \geq -0.18.$$

Solving these two inequalities with equality, we obtain the optimal equilibrium rewards for joint production, $w_1 = 0.415$ and $w_2 = 0.265$.Thus, profits when joint production is implemented are equal to $1 - w_1 - w_2 = 0.32$, which are higher than profits with individual production by agent 1 as $0.32 > 0.2$. There, joint production is optimal when agents are inequity averse while individual production by agent 1 is optimal when agents are standard.

9.2 Principal's loss when joint production is not optimally implemented

Assume $q_1 = q_2 = 0.5$ and $c_1 = c_2 = 0.4$.

The condition for joint production to be optimal when agents are standard, $1-q_1 \geq c_2$ if $(q_1-c_1) \geq (q_2 - c_2)$ holds, as $1 - 0.5 \geq 0.4$ with $(0.5 - 0.4) \geq (0.5 - 0.4)$.

Thus, with standard preferences the total cost of implementing joint production equals the sum of both agents' costs of effort: $w_1 + w_2 = c_1 + c_2 = 0.8$.

When agents are inequity averse, the agent who individually works off equilibrium is offered a reward equal to total individual production, $w_i^i = q_i$ if $\alpha(q_j - c_j) \geq \beta c_j$ for $i, j = 1, 2, i \neq j$. Thus, there are two cases:

$$\text{a) If } \alpha(0.5 - 0.4) \geq \beta(0.4) \Rightarrow \alpha \geq 4\beta \text{ then: } w_1^1 = w_2^2 = 0.5,$$
$$\text{b) If } \alpha(0.5 - 0.4) < \beta(0.4) \Rightarrow \alpha < 4\beta \text{ then: } w_1^1 = w_2^2 = 0.$$

a) Assume $\alpha \geq 4\beta$. The no deviation conditions for each agent to work when the other agent works are:

$$w_1 - 0.4 - \alpha \max[w_2 - 0.4 - w_1 + 0.4, 0] - \beta \max[w_1 - 0.4 - w_2 + 0.4, 0] \geq -\alpha[0.5 - 0.4],$$
$$w_2 - 0.4 - \alpha \max[w_1 - 0.4 - w_2 + 0.4, 0] - \beta \max[w_2 - 0.4 - w_1 + 0.4, 0] \geq -\alpha[0.5 - 0.4].$$

As the productivity parameters are the same for both agents, in equilibrium there is no inequity and equilibrium rewards are:

$$w_1 = w_2 = 0.4 - 0.1\alpha.$$

b) Assume $\alpha < 4\beta$. The no deviation conditions for each agent to work when the other agent works are:

$$w_1 - 0.4 - \alpha \max[w_2 - 0.4 - w_1 + 0.4, 0] - \beta \max[w_1 - 0.4 - w_2 + 0.4, 0] \geq -\beta(0.4),$$
$$w_2 - 0.4 - \alpha \max[w_1 - 0.4 - w_2 + 0.4, 0] - \beta \max[w_2 - 0.4 - w_1 + 0.4, 0] \geq -\beta(0.4).$$

As the productivity parameters are the same for both agents, in equilibrium there is no inequity and equilibrium rewards are:

$$w_1 = w_2 = 0.4(1 - \beta).$$

We calculate the Principal's possible loss as the difference between the Principal's profits (production minus rewards) with and without inequity aversion. As production when both agents work is normalized to 1, this loss is expressed in terms of the total production exerted.

Thus, the loss function is

$$[1 - 2(0.4 - 0.1\alpha)] - [1 - 0.8] \qquad \text{when} \quad \alpha \geq 4\beta,$$

$$[1 - 2(0.4)(1 - \beta)] - [1 - 0.8] \qquad \text{when} \quad \alpha < 4\beta.$$

Figure 8 displays this loss function for $\alpha \in [0, 1)$ and $\beta \in [0, \frac{1}{2}]$.

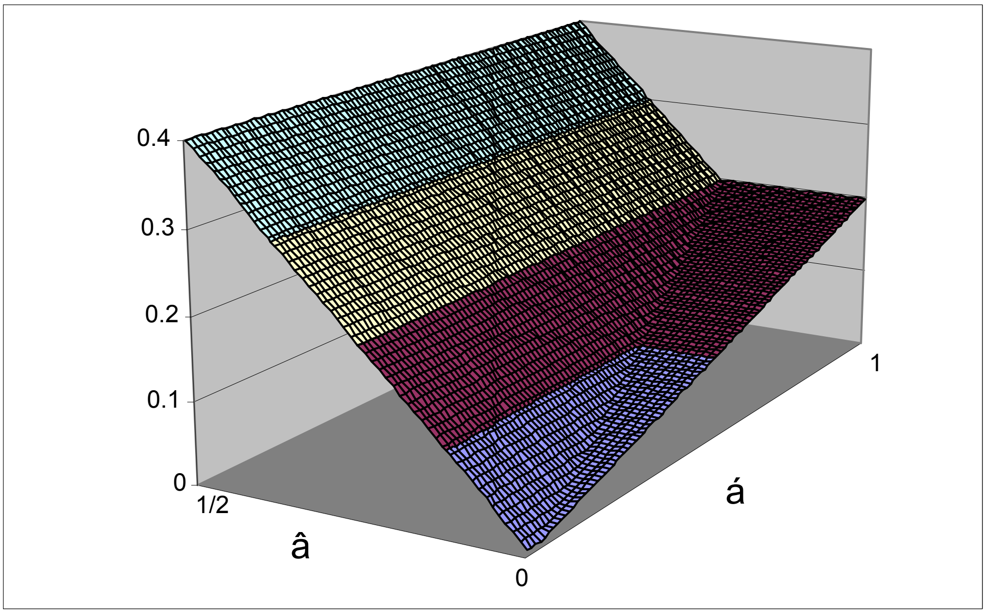

Figure 8: Principal's loss when $q_1 = q_2 = 0.5$ and $c_1 = c_2 = 0.4$.