MARKET DYNAMICS IN EDGEWORTH EXCHANGE

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Abstract Edgeworth exchange is the fundamental general equilibrium model, yet equilibrium predications and theories of price adjustment for this model remain untested. This paper reports an experimental test of Edgeworth exchange which demonstrates that prices and allocations converge sharply to the competitive equilibrium. Price convergence is evaluated with the tatonnement model, interpreted as a disequilibrium model of across-period price adjustment. Subsequently, the extent of within-period adjustment is compared to that of across-period adjustment. Since most observed price adjustment occurs within trading periods, price adjustment data is evaluated with two disequilibrium models of within-period trades. These models are the Geometric Mean model, which is formulated in this paper, and the Hahn process (Hahn and Negishi [1962]). Price dynamics from experiment sessions fit the Geometric Mean model better than the Hahn process, and in addition, the Geometric Mean model provides direction for development of an Edgeworth exchange bargaining model.

Keywords: Competitive equilibrium, disequilibrium dynamics, double auction, Edgeworth exchange, experimental economics, exchange economy, Hahn process, market dynamics

1 Introduction

Although Edgeworth exchange is the fundamental general equilibrium model, competitive equilibrium and price dynamic predictions for the model remain untested. The purpose of Edgeworth exchange experiments is to determine whether the competitive equilibrium of the model is attained, and if so, to evaluate the convergence process. Experiments support the competitive equilibrium prediction. Market dynamics are evaluated with three price adjustment models. Across-period adjustment is evaluated with the tatonnement model, interpreted as a disequilibrium model. In the tatonnement model, a price is announced by the 'Walrasian auctioneer' and net demands are reported by agents to the auctioneer, who then adjusts price to reduce the magnitude of excess demand.[1] Exchange in this model takes place only when an equilibrium price is announced. Tatonnement, interpreted as a disequilibrium model, is used to evaluate convergence across market replications (or market 'periods'), with the change in average price between periods t and $t+1$ equal to a constant times the excess demand at the average price in period t. Tatonnement though neglects the substantial price adjustment that occurs within trading periods. In the "Hahn process" (Hahn and Negishi [1962]), price adjusts in response to excess demand as in tatonnement, but disequilibrium trades can occur at each announced price, so this model is better suited to analysis of Edgeworth exchange price data generated with the double auction institution. The primary limitation of the Hahn process for the evaluation of disequilibrium dynamics is conceptual: the model relies upon price adjustment in response to excess demand, which is unobserved by traders in the double auction. In view of these limitations of the tatonnement model and of the Hahn process, an alternative adjustment model – called the Geometric Mean model – is formulated in this paper. The Hahn process and the Geometric Mean model are evaluated by comparing their predicted price paths for each trading period to the observed within-period price paths from experiment sessions. This paper demonstrates that the Geometric Mean model provides a better fit than the Hahn process to the price paths observed in Edgeworth exchange experiments.

In order to develop an experimental test of the competitive equilibrium prediction and price dynamics of Edgeworth exchange, a utility function over two commodities is induced (as in Smith [1976, 1982]) for each of six sellers and each of six buyers in an experiment session. Each agent also has an endowment of one of the two commodities. In order to facilitate the emergence of stable terms of trade, one of these two commodities is treated as the numeraire commodity; prices for the non-numeraire commodity are stated in terms of the numeraire. Buyers initially hold only the numeraire commodity in their endowment, and sellers initially hold only the non-numeraire commodity in their endowment.[2] In each

[1] For a description of the tatonnement model and stability results for the model, see Arrow and Hurwicz [1958] and Arrow, Block, and Hurwicz [1959].

[2] The position of each subject in the experiment as either a buyer or a seller is of course a fiction: those labelled 'buyer' in the experiment because they purchase units of the non-numeraire commodity are also sellers of the numeraire commodity, and those labelled 'seller' purchase the numeraire commodity.

experiment session, the group of six sellers and six buyers engage in a series of either 12 or 15 identical trading periods. In each of these trading periods, the sellers post offer prices and the buyers post bid prices for the exchange of one unit of the commodity for the number of units of the numeraire commodity specified in the offer or bid. Sellers and buyers may (and typically do) participate in several trades in each trading period.

By the final period in four of the five sessions, allocations and prices were very near the competitive equilibrium allocation and price: the per capita allocation differed from the competitive equilibrium allocation by less than 3% and the difference between the average price and the equilibrium price was less than 3.1% in each of these four sessions. In addition to the confirmation of the equilibrium model for Edgeworth exchange, these experiment sessions provide a unique opportunity to assess price dynamics in Edgeworth exchange. In the double auction trades are negotiated directly between sellers and buyers, trades typically occur at a variety of prices even within a single trading period, and most trades are out of equilibrium. These characteristics of the trading process though, rather than hindering evaluation of price dynamics, facilitate evaluation of the Hahn process and the Geometric Mean model of within-period price dynamics.

The Geometric Mean model has two primary elements: price adjustment within each period, and price adjustment across periods. The crucial element is within-period adjustment. In the model, price adjusts in the direction of a weighted geometric mean between the sellers' and buyers' marginal rates of substitution at the current allocation. This within-period price adjustment model has several beneficial characteristics: it is based on the individual incentives of sellers and buyers rather than on the market aggregate of excess demand, it can account for and adapt to disequilibrium trades, it converges to a Pareto optimal allocation in each period, and in many cases it tracks within-period price movements extremely well.

This paper progresses though the steps outlined above. The economic environment utilized in the experiment sessions is described in Section 2. The double auction mechanism is described for the Edgeworth exchange context in Section 3. Experiment data is evaluated for across-period convergence in Section 4. Section 5 describes the Hahn process and develops the Geometric Mean model of within-period price dynamics. Predictions of these two models are compared to experiment data in Section 6. Conclusions are drawn in Section 7.

2 Economic environment

At the beginning of each trading period, each buyer is endowed with 1800 units of the numeraire commodity (X), which is described to the buyer as currency. Buyers can use this numeraire to purchase units of the commodity (Y). Each seller is endowed with eighteen units of the commodity, which can be sold individually to acquire units of the numeraire commodity. Following the technique developed by Smith [1976, 1982], utility functions for each buyer and for each seller are induced with a payment that is determined by evaluation of a utility function at the final allocation of currency (the numeraire commodity X) and the commodity (Y). Each buyer and each seller has a constant elasticity of substitution (CES) utility function $u_i(x, y) = c_i\,((a_i\,x_i)^{r_i} + (b_i\,y_i)^{r_i})^{1/r_i}$. The parameter c_i does not affect the competitive equilibrium price or allocation, but it does affect the utility level attained by subject i, and is therefore a useful element of the utility inducement technique, since it permits the experimenter to rescale a subject's payoff with a single parameter.

In each period of an experiment session, each of six buyers (agent type A) had the same utility function $u_A(x_A, y_A)$ and endowment $\omega_A = (1800, 0)$; each of six sellers (agent type B) had the same utility function $u_B(x_B, y_B)$ and endowment $\omega_B = (0, 18)$. Table 1 shows the CES utility function parameters and the endowments of sellers and buyers.

		c_i	a_i	b_i	r_i	ω_i
Sellers' parameters	$(i = 2, 4, \ldots, 12)$	0.2560	2.982	109.89	-1	(1800, 0)
Buyers' parameters	$(i = 1, 3, \ldots, 11)$	0.6950	0.362	109.89	-1	(0, 18)

Table 1: Sellers' and buyers' parameters.

Equilibrium

The equilibrium price p_e is calculated by setting aggregate excess demand in the market for the commodity (Y) equal to zero and solving for the equilibrium price. In equilibrium, the market for the numeraire commodity (X) then clears by Walras' law. Equilibrium allocations for buyers and for sellers are determined by substituting the equilibrium price p_e into the individual excess demand equations. Each agent has the excess demand function

$$Z_i^Y(\,p\,|\,\omega_i) = \frac{b_i^{\rho_i}\,(\omega_i^X + p\,\omega_i^Y)}{p\,\left(b_i^{\rho_i} + p^{\rho_i}\,a_i^{\rho_i}\right)} - \omega_i^Y \tag{1}$$

for the commodity Y, where $\rho_i = \frac{r_i}{1-r_i}$. Market excess demand for Y is

$$\begin{aligned} Z^{Y}(p\,|\,\omega) &= \textstyle\sum_{i=1}^{12} Z_i^{Y}(p\,|\,\omega_i) \\ &= 6\,Z_A^{Y}(p\,|\,\omega_A) + 6\,Z_B^{Y}(p\,|\,\omega_B), \end{aligned} \tag{2}$$

where $Z_A^{Y}(p\,|\,\omega_A)$ is the excess demand for an individual buyer and $Z_B^{Y}(p\,|\,\omega_B)$ is the excess demand for each seller.[3] When parameters from table 1 for buyers and sellers are substituted into equation (1), and the resulting individual excess demand functions are substituted into equation (2), the equilibrium price p_e is obtained by setting aggregate excess demand equal to zero. Then the equilibrium price p_e is substituted into equation (1) to determine equilibrium allocations. With sellers identical to one another, and buyers identical to each other, the economic environment can be displayed in an Edgeworth diagram, as in figure 1. The equilibrium price for the parameter set used in the experiment sessions is $p_e \doteq 91$. Table 2 shows sellers' and buyers' equilibrium allocations and equilibrium utility levels.

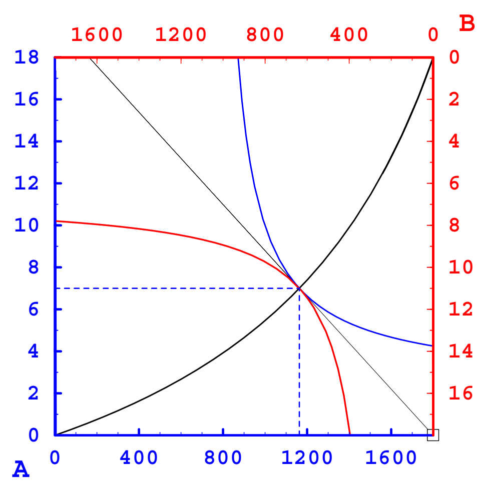


Figure 1: Edgeworth diagrams for experiment sessions.

In order to implement a test of Edgeworth exchange, experiment subjects need to be provided with both a detailed specification of their objectives and of the exchange institution. The next section describes the double auction institution in the context of Edgeworth exchange and describes the representation of the utility inducement technique to subjects.

[3] Dependence of excess demand on endowments ω_A and ω_B or on the current allocations (x^A, y^A) and (x^B, y^B) is indicated because in the Hahn process, price adjusts after each trade in response to excess demand at the current allocation. In Section 5.1, which defines the Hahn process adjustment rule, the excess demand for Y is written $Z^{Y}(p\,|\,(x, y)) = Z_A^{Y}(p\,|\,(x^A, y^A)) + Z_B^{Y}(p\,|\,(x^B, y^B))$ on a per capita basis.

	Equilibrium Allocation	Equilibrium Utility
Sellers	(1163, 7)	189
Buyers	(637, 11)	189

Table 2: Equilibrium allocations and equilibrium utility levels for sellers and buyers.

3 The double auction

In the double auction mechanism, any seller may submit an ask at any time during a trading period. An ask, which is the seller's current report of the fewest units of the numeraire commodity that he is willing to accept for a unit of the commodity, is entered in the area labelled "Enter Ask" on the seller's screen display, as in figure 2. Similarly, a buyer's bid, which represents her current report of the most units of the numeraire commodity that she is willing to pay for a unit of the commodity, may be submitted at any time. If an ask is placed that is at or below the current high bid, a trade results with the price equal to the bid. If a bid is placed that meets or exceeds the current low ask, a trade occurs with the price equal to the ask. A seller may make any number of asks, and may trade any number of units that is consistent with his commodity endowment. Similarly, a buyer may make any number of bids, and may trade any number of units that is consistent with her endowment of the numeraire commodity. Several specific rules are implemented in the version of the double auction used to conduct the experiment sessions reported here. Of these, the most important is the "spread reduction rule," which requires that each new ask is made at a value that is below the current low ask and each new bid is placed at a higher value than the current high bid. Sellers and buyers have the option to remove any ask or bid that they have previously made, provided the request to remove the ask or bid is received before it results in a trade. Each seller is permitted a single ask in the market queue at any time and any new ask by a seller replaces his previous ask if he has one in the queue. Each ask is the unit price offered by the seller for a single unit: multiple unit trades are not permitted. Similar restrictions apply to each buyer's bids.

During each period, a queue on the seller's screen displays all current asks and bids (shown as the "Market Queue" in figure 2); each buyer's screen also displays both queues. When a seller successfully enters an ask into the ask queue, he receives a confirmation

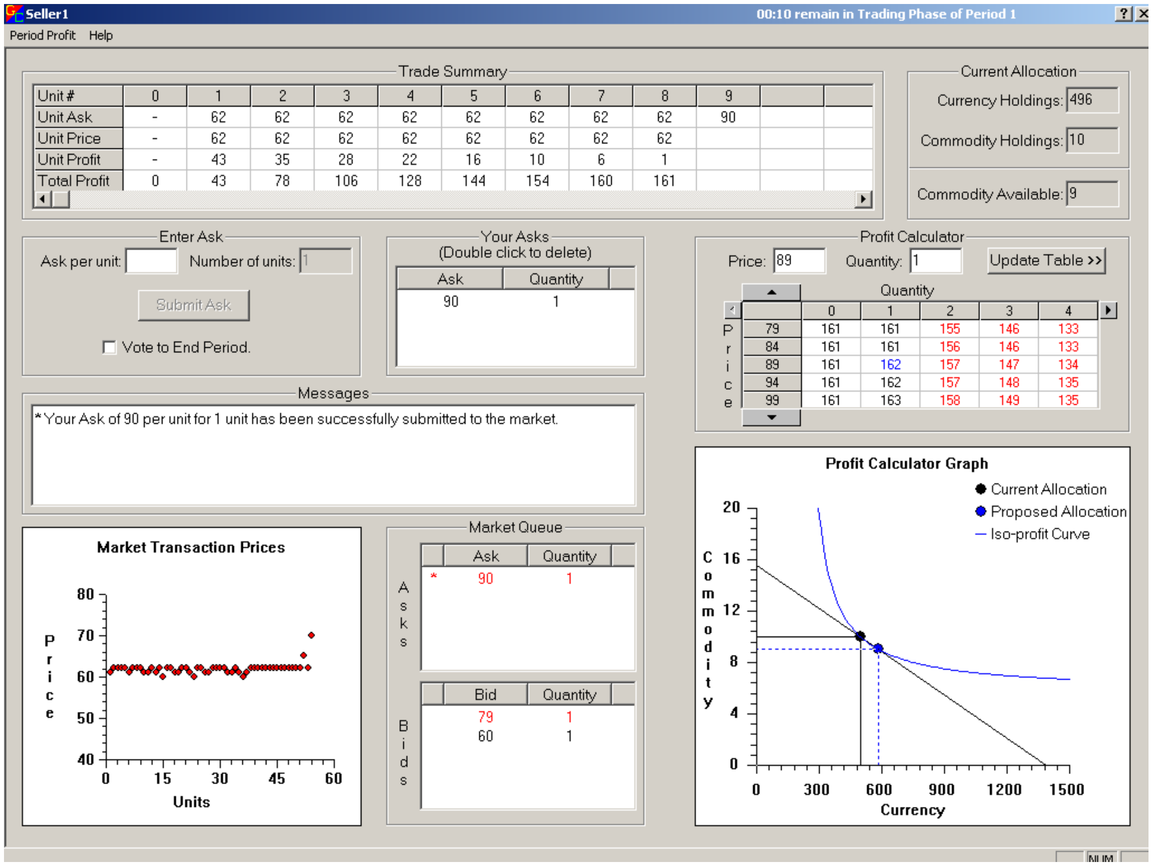
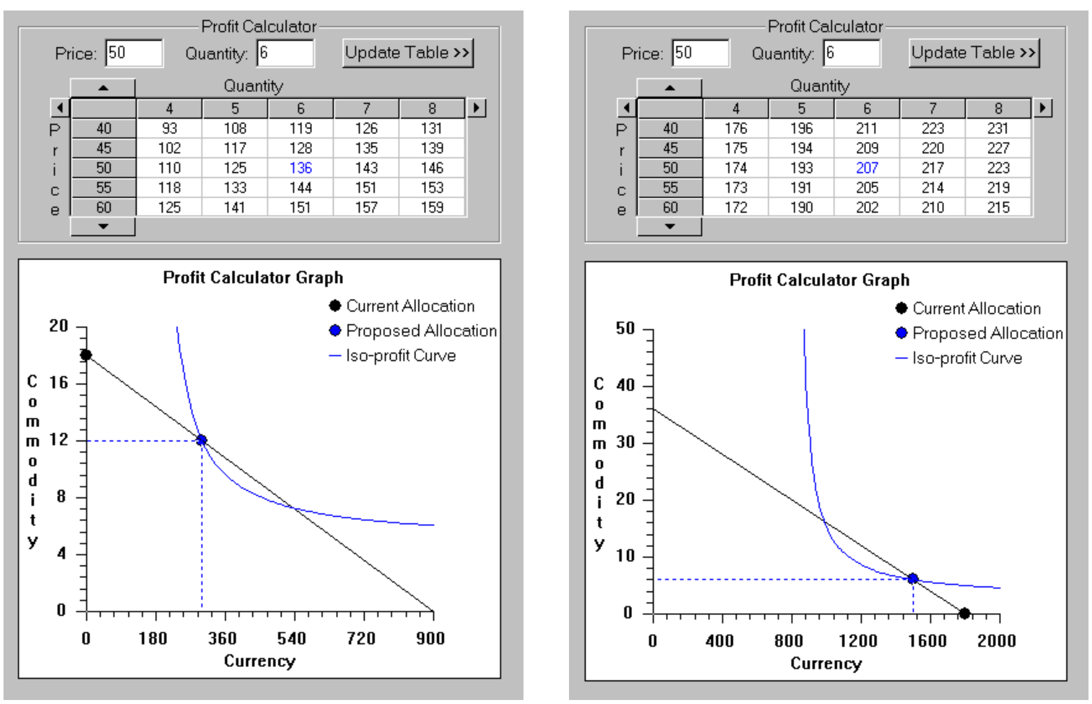


Figure 2: Seller screen with elements of market institution.

message in the "Messages" area of the screen display. This ask also appears in the "Unit Ask" row of the "Trade Summary" table, in the column that corresponds to the unit the seller has offered for sale. Similarly, a buyer receives a confirmation message when she enters a bid into the bid queue, and also sees an update to the appropriate cell in her Trade Summary table. When a seller and buyer complete a trade, they both receive a confirmation message, "Unit Price," "Unit Profit," and "Total Profit" figures are recorded in their Trade Summary tables, and the price appears in a display of all trade prices from the current period, shown as the "Market Transaction Prices" graph. The trading phase of each period lasts 180 seconds. Subjects know the length of the trading phase of each market period, and a clock at the top of the screen of each seller and each buyer shows the time remaining in the current phase. Each of these elements of the market institution appears on a seller's trading screen, as in figure 2. Buyers have a similar trading screen.

Profit (or utility) representation

Each market period is separated into three phases. During the preview phase, which lasts 60 seconds, and during the 180 second trading phase, a seller is able to enter “Price” and “Quantity” into a “Profit Calculator,” which appears on the lower right hand side of the seller screen in figure 2. (The final phase is the 30 second review phase, when sellers and buyers have an opportunity to examine the results of the trading phase.) This Profit Calculator displays both a tabular and a graphical representation of the utility level that would result from a proposed trade. Examples of the Profit Calculator are shown for a seller and for a buyer in figure 3, starting from the seller’s and the buyer’s initial endowments. When a subject enters data into the Price and Quantity boxes in this calculator and clicks “Update Table,” the profit (or utility) level is displayed in the center of the table for the allocation that would result from the proposed exchange. Profit levels are also displayed for prices above and below the proposed price, and for quantities above and below the proposed quantity. In addition, the graph represents the “Current Allocation,” the “Proposed Allocation” which would result from the proposed Price and Quantity, and the “Iso-profit Curve” (or indifference curve) that passes through the Proposed Allocation.



Price	4	5	6	7	8
40	93	108	119	126	131
45	102	117	128	135	139
50	110	125	136	143	146
55	118	133	144	151	153
60	125	141	151	157	159

Price	4	5	6	7	8
40	176	196	211	223	231
45	175	194	209	220	227
50	174	193	207	217	223
55	173	191	205	214	219
60	172	190	202	210	215

Figure 3: Profit calculator table and graph for seller (left) and buyer (right).

Instructions

These elements of the double auction mechanism and of the representation of profit (or utility) levels are explained to subjects in a detailed interactive instruction set. Instructions describe each element of the seller's (or buyer's) trading screen independently, and then describe how elements relate to one another. There are points in the instruction set at which the seller or buyer is prompted for inputs, and there are eight interactive questions that must be answered correctly in order to proceed through the instructions. The sequence of steps through the instructions is outlined in Appendix A.

Experiment sessions and experience

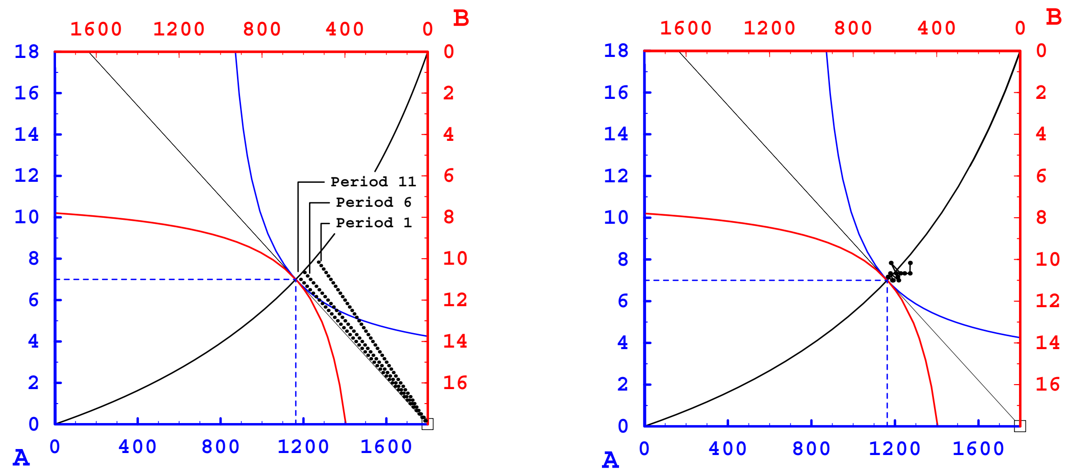
Three experiment sessions were conducted at the University of York in the U.K. on June 7, 2001. The fourth session was conducted at the University of Arizona on November 16, 2001, and the final session was conducted in York on December 14, 2001. Subjects in all Edgeworth exchange sessions had double auction experience in a session with a list of unit costs for each seller and a list of unit values for each buyer, rather than a utility function.

4 Convergence

This section assesses convergence in the five experiment sessions. Section 4.1 assesses convergence of per capita allocations to the competitive equilibrium allocation. Final per capita allocations in each trading period are shown in an Edgeworth diagram for each of the five experiment sessions. In Section 4.2, tatonnement is interpreted as a model of across-period convergence of mean prices, and the adjustment factor from the tatonnement model is estimated for each experiment session. Section 4.3 compares the extent of across-period price convergence to the extent of within-period price convergence.

4.1 Convergence of allocations across periods

Figure 4 (a) shows the reallocations of units that result from all trades in periods 1, 6, and 11 of session IU-CES1-DAs-010607a. The series of dots for Period 1 indicate the per capita allocation of currency and commodity after each trade during the period. The dots for periods 6 and 11 are interpreted similarly. Figure 4 (b) shows the final per capita allocation for each of 12 periods in the same experiment session. (A more detailed view of the final allocations for this session is shown in Figure 9 (b) on page 25.) Figure 5 shows



(a) Trades within periods

(b) Final allocations across periods

Figure 4: Trades (left) and final allocations (right) in session IU-CES1-DAs-010607a.

final allocations by period for each of the other four sessions. There is a significant level of convergence across periods in four of five sessions, and convergence is almost immediate in the first period of the session shown in figure 5 (a). Only the session shown in figure 5 (d) does not converge to the competitive equilibrium allocation.

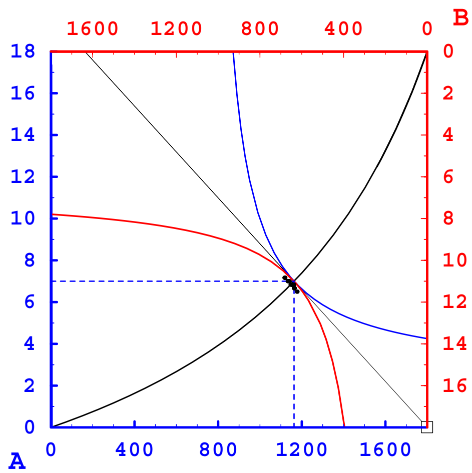
In order to examine the rate of convergence of allocations across periods, the distance of the final allocation from the competitive equilibrium allocation is determined for each period. The metric used to measure the distance between the final allocation $(x_t^A,\, y_t^A)$ observed in period t and the equilibrium allocation $(x_e^A,\, y_e^A)$ is

$$d((x_t^A, y_t^A),\, (x_e^A, y_e^A)) = \frac{1}{\sqrt{2}} \left(\left(\frac{1}{91}\,(x_t^A - x_e^A) \right)^2 + (y_t^A - y_e^A)^2 \right)^{0.5}.$$

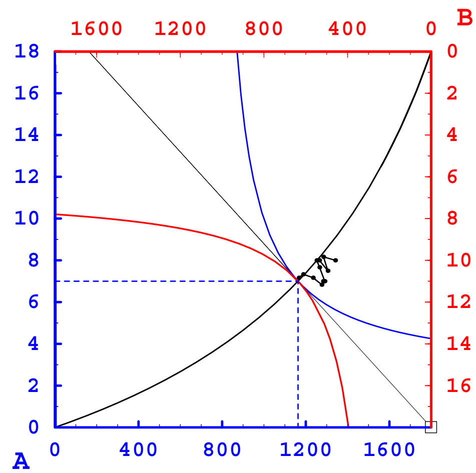
The rationale for this distance metric is straightforward: if trades take place at the equilibrium price $p_e = 91$ and trade falls short of or exceeds the equilibrium allocation by α units, then the distance between the allocation and the equilibrium allocation is α. For example, if in period t all trades take place at the equilibrium price $p_e = 91$ and one unit per capita is not traded that should be to reach the equilibrium allocation $(x_e^A,\, y_e^A) = (1163, 7)$ for agent type A, then $(x_t^A, y_t^A) = (1163 + 91,\, 7 - 1)$. The distance between $(x_t^A,\, y_t^A)$ and $(x_e^A,\, y_e^A)$ in this situation is then $d((x_t^A, y_t^A),\, (x_e^A, y_e^A)) = 1$.

With the distance $d((x_t^A, y_t^A),\, (x_e^A,\, y_e^A))$ denoted d_t, convergence is evaluated with the regression equation $d_t = d_1\, e^{-r\, \ln t}\, \epsilon_t$. This model can be expressed as the linear model

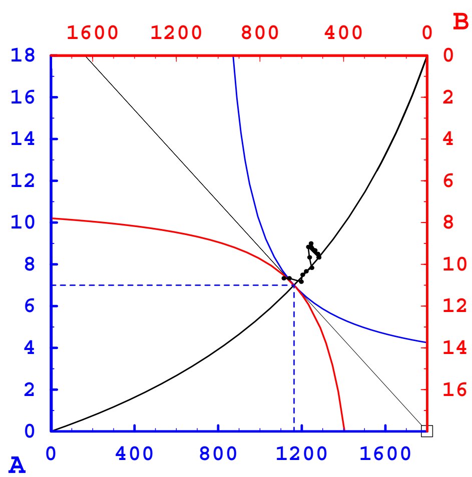
$$\ln d_t = \ln d_1 - r\, \ln t + \ln \epsilon_t. \tag{3}$$



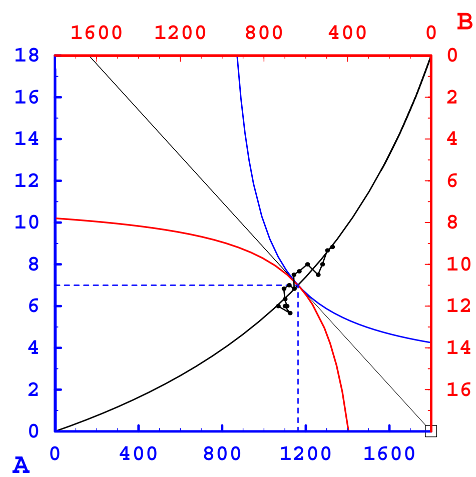
(a) IU-CES1-DAs-010607b



(b) IU-CES1-DAs-010607c



(c) IU-CES1-DAs-011116b



(d) IU-CES1-DAs-011214b

Figure 5: Edgeworth diagrams for IU-CES1-DAs sessions.

Figure 6 shows the convergence of the distance between final allocations in each period and the equilibrium allocation, together with the estimated curve from the model in equation (3) for session IU-CES1-DAs-010607a. Although a linear model can be fit to this data, the exponential model in the equation is more appropriate, since distances cannot be negative. Table 3 summarizes convergence estimates for each of the five sessions. For each session, the table includes the estimate $\widehat{d}_1$ of the initial distance from equilibrium, the estimate of the rate of convergence $\widehat{r}$, the *p-value* for the hypothesis test that $r > 0$, the R^2 statistic for the model, the distance d_T between the allocation in the last period and the equilibrium allocation that is implied by the estimates $\widehat{r}$ and $\widehat{d}_1$ (where $d_T = \widehat{d}_1\, e^{-\widehat{r}\ln T}$), and the number of trading periods T in the session. Strong convergence occurs in four of the five sessions (as measured by the estimate $\widehat{r}$ and its *p-value*).[4,5]

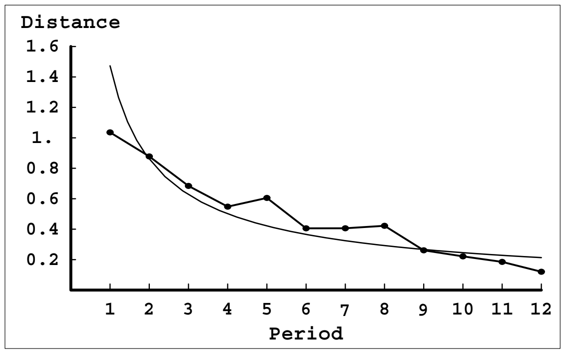


Figure 6: Convergence of allocations in session IU-CES1-DAs-010607a.

	Periods	$\widehat{d}_1$	$\widehat{r}$	*p-value* ($\widehat{r} > 0$)	R^2	d_T
IU-CES1-DAs-010607a	1 - 12	1.473	0.778	0.000	0.817	0.213
IU-CES1-DAs-010607b	1 - 12	0.238	0.066	0.349	0.016	0.202
	2 - 12	0.392	0.348	0.008	0.490	0.170
IU-CES1-DAs-010607c	1 - 12	2.269	0.654	0.006	0.482	0.446
IU-CES1-DAs-011116b	1 - 15	2.424	0.540	0.002	0.489	0.561
IU-CES2-DAs-011214b	1 - 15	0.728	0.009	0.483	0.000	0.710

Table 3: Regression coefficients and statistics from the convergence model.

The regression model in equation (3) can be reformulated to account for subjects' decision errors. In two sessions, sellers frequently sold units at a loss. The regression model is reformulated with the additional variable $e_t\left(1 - 2\, I_{\{\bar{p}_t > p_e\}}\right)$, where e_t is the difference

[4] Fourteen price errors were corrected from the five sessions for the estimates reported here and in Section 6. In the five sessions there were 2932 trades, so the errors represent less than one in 200 trades. Nine of fourteen errors occurred when a seller omitted a digit in his ask.

[5] Two regression estimates are reported in table 3 for session IU-CES1-DAs-010607b. The first estimate is for all 12 periods. The second estimate does not include period 1. The average price in the first period of this session was $\bar{p}_1 = 93.44$, which is very near the competitive equilibrium price $p_e = 91$. The average price moved away from the competitive equilibrium price in period 2 to $\bar{p}_2 = 95.23$. The substantial movement away from the equilibrium allocation between periods 1 and 2 masks the significant restoration toward equilibrium that occurred over the subsequent 11 periods.

between the number of units purchased at a loss by agent type A and the number of units sold at a loss by agent type B. The second factor, $\left(1 - 2\, I_{\{\bar{p}_t > p_e\}}\right)$, is included to account for the fact that errors have different implications for convergence depending on the side of the equilibrium price that trades occur on. For example, if agent type B sells units at a loss, this will depress the price. When the average price is below the equilibrium price, this slows convergence, whereas if the average price is above the equilibrium, this hastens convergence. With the decision error term included, the regression equation is

$$d_t = d_1\, e^{\left(-r + s\, e_t\, \left(1 - 2\, I_{\{\bar{p}_t > p_e\}}\right)\right)\, \ln t}\, \epsilon_t,$$

which can be expressed as the linear model

$$\ln d_t = \ln d_1 - r\, \ln t + s\, e_t\, \left(1 - 2\, I_{\{\bar{p}_t > p_e\}}\right)\, \ln t + \ln \epsilon_t. \tag{4}$$

Table 4 shows parameter estimates for equation (4). The second session had no net decision error in any period, so the estimates reported are from table 3. As noted above, across period convergence in this session was weak, but this resulted from the price movement away from the equilibrium price between the first and second periods. Finally, in the fifth session, the estimate on the convergence rate r is much more consistent with the other sessions when decision errors are included in the regression equation: the convergence rate parameter estimate $\widehat{r}$ is significantly positive at the level $p = 0.10$.

	Periods	$\widehat{d_1}$	$\widehat{r}$	*p-value* ($\widehat{r} > 0$)	$\widehat{s}$	*p-value* ($\widehat{s} > 0$)	R^2
IU-CES1-DAs-010607a	1 - 12	1.347	0.679	0.000	0.194	0.031	0.879
IU-CES1-DAs-010607b	1 - 12	0.238	0.066	0.349	——	——	0.016
IU-CES1-DAs-010607b	2 - 12	0.392	0.348	0.008	——	——	0.490
IU-CES1-DAs-010607c	1 - 12	2.037	0.631	0.010	0.100	0.246	0.510
IU-CES1-DAs-011116b	1 - 15	1.391	0.341	0.003	0.082	0.000	0.823
IU-CES1-DAs-011214b	1 - 15	1.045	0.367	0.074	0.078	0.018	0.319

Table 4: Regression summary for convergence model with decision error term.

4.2 Price convergence across periods: tatonnement

Although the tatonnement model does not involve trade out of equilibrium, the model can be reinterpreted to accommodate disequilibrium trades with repetition of the exchange environment, as in these Edgeworth exchange experiment sessions. The tatonnement model predicts price changes of the form $(\Delta\, p^X,\, \Delta\, p^Y) = ((1-c)\, Z^X(p),\, c\, Z^Y(p))$ for commodities

X and Y. The price of the numeraire commodity (X) does not change in the experiment, but the tatonnement model specifies a theoretical change in the price of X, so the predicted price for the numeraire commodity is not normalized. The normalization is reestablished by specifying the (renormalized) predicted price of the non-numeraire commodity Y as the ratio of the predicted non-normalized prices $p^Y = p + c\,Z^Y(p)$ and $p^X = 1 + (1-c)\,Z^X(p)$.

This adjustment rule depends not only on the excess demand for Y, but also on the excess demand for X. Since X is the numeraire commodity, the excess demand for X is stated as a function of the price p of the non-numeraire commodity:

$$Z_i^X(\,p\,|\,\omega_i) = \frac{p^{\rho_i}\,a_i^{\rho_i}(\omega_i^X + p\,\omega_i^Y)}{\left(b_i^{\rho_i} + p^{\rho_i}\,a_i^{\rho_i}\right)} - \omega_i^X. \tag{5}$$

The market excess demand for X is

$$\begin{aligned} Z^X(\,p\,|\,\omega) &= \textstyle\sum_{i=1}^{12} Z_i^X(\,p\,|\,\omega_i) \\ &= 6\,Z_A^X(\,p\,|\,\omega_A) + 6\,Z_B^X(\,p\,|\,\omega_B). \end{aligned} \tag{6}$$

The adjustment rules for the average prices of X and Y between periods t and $t+1$ are $\bar{p}_{t+1}^X = 1 + (1-c)\,Z^X(\,\bar{p}_t)$ and $\bar{p}_{t+1}^Y = \bar{p}_t + c\,Z^Y(\,\bar{p}_t)$. These two equations can be combined to obtained a non-linear regression equation with the predicted average price of Y, $\bar{p}_{t+1}$, on the left hand side of the equation. Since $\bar{p}_{t+1} = \bar{p}_{t+1}^Y / \bar{p}_{t+1}^X$, the regression equation is

$$\bar{p}_{t+1} = \frac{\bar{p}_t + c\,Z^Y(\bar{p}_t)}{1 + (1-c)\,Z^X(\bar{p}_t)} + \epsilon_t. \tag{7}$$

Table 5 shows estimated values of the adjustment rate parameter c from equation (7), asymptotic standard errors, and 95% confidence intervals for the estimates.[6] An estimate of c at or below one indicates adjustment in the direction predicted by the tatonnement model. For all values $c < \bar{c}$ where $\bar{c} > 1$, price adjusts in the direction predicted by the tatonnement

[6] The regression estimate in table 5 for session IU-CES1-DAs-010607b does not include periods 2 and 3. The average price in the first period of this session was $\bar{p}_1 = 93.44$, which is very near the competitive equilibrium price $p_e = 91$. The average price moved away from the competitive equilibrium price in period 2 to $\bar{p}_2 = 95.23$, moved further away in period 3 to $\bar{p}_3 = 95.85$, and then began to slowly adjust back to the equilibrium price. Although the average price movements in these two periods were small, they moved in the opposite direction from the excess demand, which was very small in these two periods. It is remarkable that price adjusted between periods 4 and 12 toward the competitive equilibrium, since the excess demand at the mean price in period 3 was $Z^Y(95.85) = -0.799$, or -0.133 on a per capita basis. The estimate $\widehat{c}$ with periods 2 and 3 included is $\widehat{c} = 1.00012$.

model. The threshold value $\bar{c}$ is determined by setting $\bar{p}_{t+1} = \bar{p}_t$ in equation (7) and solving the resulting equation for $\bar{c}$. For fixed values of the CES utility function parameters and endowments, the equation that determines $\bar{c}$ is only a function of $\bar{p}_t$. The value of $\bar{c}$ listed for each session in table 5 is the maximum value of c that is consistent with the tatonnement adjustment prediction for the average price $\bar{p}_t$ observed in each period of the session. Values of $\bar{c}$ differ by session because the range of observed average period prices differ by session. In the first three sessions, the adjustment prediction of the tatonnement model holds for every value of c in the 95% confidence interval on the estimate $\widehat{c}$. In the fourth session, the adjustment is in the predicted direction for the estimate $\widehat{c}$ and for most of the confidence interval. In the final session, adjustment is in the predicted direction at the estimate, but it is not in the predicted direction for a large part of the confidence interval.

	Periods	$\widehat{c}$	Asymptotic Standard Error	Confidence Interval	$\bar{c}$
IU-CES1-DAs-010607a	2 - 12	0.99931	0.00008	(0.99912, 0.99949)	1.00013
IU-CES1-DAs-010607b	4 - 12	0.99995	0.00007	(0.99979, 1.00010)	1.00011
IU-CES1-DAs-010607c	2 - 12	0.99952	0.00015	(0.99918, 0.99987)	1.00013
IU-CES1-DAs-011116b	2 - 15	0.99983	0.00042	(0.99941, 1.00024)	1.00011
IU-CES2-DAs-011214b	2 - 15	0.99994	0.00030	(0.99930, 1.00058)	1.00007

Table 5: Regression coefficients and statistics from the tatonnement model.

The regression model in equation (7) can be reformulated to account for subjects' decision errors, as was done for the convergence estimates in equation (4). The regression model, reformulated to include the decision error term $e_t \left(1 - 2\, I_{\{\bar{p}_t > p_e\}}\right)$, is

$$\bar{p}_{t+1} = \frac{\bar{p}_t + c\, Z^Y(\bar{p}_t)}{1 + (1-c)\, Z^X(\bar{p}_t)} + d\, e_t \left(1 - 2\, I_{\{\bar{p}_t > p_e\}}\right) + \epsilon_t. \tag{8}$$

No decision error was made in any period of the second session. In each of the other four sessions, the vector of decision errors was non-zero so the model can be estimated with the decision error term. Estimates and confidence intervals for the model with the decision error term are shown in table 6.

In the first and third sessions, there were few net decision errors. Consequently, the estimates $\widehat{c}$ for these two sessions are similar to the estimates from the regression without the decision error term. Asymptotic standard errors for the estimates of $\widehat{d}$ from these two sessions are large due to the infrequency of decision errors. In the second session, as noted

	$\Sigma_{t=1}^{T}\lvert e_t\rvert$	$\widehat{c}$	ASE ($\widehat{c}$)	CI ($\widehat{c}$)	$\widehat{d}$	ASE ($\widehat{d}$)	CI ($\widehat{d}$)
0607a	2	0.9993	0.0001	(0.99916, 0.99950)	0.952	0.658	(-0.536, 2.441)
0607b	0	0.9999	0.0001	(0.99979, 1.00010)	——	——	——
0607c	6	0.9994	0.0002	(0.99900, 0.99980)	1.025	0.792	(-0.766, 2.817)
1116b	40	0.9990	0.0002	(0.99860, 0.99941)	1.254	0.250	(0.709, 1.798)
1214b	29	0.9997	0.0002	(0.99922, 1.00012)	1.797	0.412	(0.899, 2.694)

Table 6: Tatonnement regression model estimates with decision error term.

above, there was no net decision error so the reported estimates are from the model with no decision error term. In the final two sessions, the numbers of decision errors $\left(\Sigma_{t=1}^{T}\lvert e_t\rvert\right)$ were large, and the estimates of $\widehat{c}$ are lower when the decision error term is included. In addition, the entire 95% confidence intervals for the estimates $\widehat{c}$ lie in the set of values of c that imply convergence under tatonnement for the fourth session and most of the 95% confidence interval lies in the region that implies convergence in the fifth session.

4.3 Comparison of across-period and within-period price convergence

Table 3 in Section 4.1 demonstrates that allocations converged to the competitive equilibrium allocation across trading periods in four of five sessions. Table 4 in the same section demonstrates that allocations converged across periods in the final session, at the level $p = 0.10$, when the decision error term is included in the regression equation. Table 5 in Section 4.2 demonstrates that average prices converged to the competitive equilibrium price for the price adjustment process predicted by the tatonnement model, in three of five sessions. Table 6 demonstrates that average prices converged in the final two sessions when decision errors are included in the regression. This section compares the extent of across-period price convergence to the extent of within-period price convergence and demonstrates that across-period price changes are typically smaller than within-period price changes, which motivates the analysis of within-period price adjustment in Sections 5 and 6.

For periods $t = 2, 3, \ldots, T$, across-period price convergence is measured by the ratio

$$R_t^a = \frac{\lvert \bar{p}_t - p_e\rvert}{\lvert \bar{p}_{t-1} - p_e\rvert}. \tag{9}$$

Denote the number of trades in period t by K_t. Within-period price convergence for periods $t = 1, 2, \ldots, T$ is measured as the ratio

$$R_t^w = \frac{\left|\sum_{k=K_t-4}^{K_t} p_{t,k} - p_e\right|}{\left|\sum_{k=1}^{5} p_{t,k} - p_e\right|}. \tag{10}$$

Three cases will help interpret these measures of across-period and within-period convergence. The three cases are ordered from the case with only within-period adjustment, equal within-period and across-period adjustment, and only across-period adjustment.

(1) If price adjusts within each period so that

$$\tfrac{1}{5}\sum_{k=K_t-4}^{K_t}(p_{t,k} - p_e) = \alpha\,\tfrac{1}{5}\sum_{k=1}^{5}(p_{t,k} - p_e),$$

that is, the average difference between the last five prices and the equilibrium price is α times the average difference between the first five prices and the equilibrium price, and if there is no adjustment across periods (so that period t is identical to period $t-1$), then $R_t^a = 1$ for $t = 2, 3, \ldots, T$ and $R_t^w = \alpha$ for $t = 1, 2, \ldots, T$.

(2) If price adjusts linearly within each period so that the difference between the final price in period t and the equilibrium price is α times the difference between the initial price and the equilibrium price, and if the first price in period t is equal to the last price in period $t-1$, then $R_t^a = \alpha$ and $R_t^w \doteq \alpha$.

(3) If price in each period is constant and the difference between the price in period t and the equilibrium price is α times the difference between the price in period $t-1$ and the equilibrium price, then $R_t^a = \alpha$ and $R_t^w = 1$.

The average across-period price adjustment R_t^a for all five sessions was $\bar{R}^a = 0.967$ and the average within-period adjustment R_t^w was $\bar{R}^w = 0.882$. Both the across-period and within-period adjustment factors from periods $8-15$ in session IU-CES1-DAs-011214b were significant outliers, due to the fact that prices in that session diverged from the equilibrium price after period 7. With those periods eliminated, the average across-period adjustment factor was $\bar{R}^a = 0.904$ and the average within-period adjustment factor was $\bar{R}^w = 0.712$.

These measures of observed across-period and within-period adjustment are close to the first benchmark case above: most price adjustment occurs within periods. Since the magnitude of within-period adjustment typically exceeds that of across-period adjustment, it is likely that across-period adjustment occurs in response to price changes observed within periods. In this case, the tatonnement model has limited capacity to explain price adjustment. The next section explores the predictions of two models of within-period price adjustment.

5 Dynamics

In four of the five experiment sessions, prices converge to the competitive equilibrium price, and these prices support allocations that are approximately equal to the competitive equilibrium allocation. In order to assess the mechanics of this convergence, two models of within-period price dynamics are evaluated. Section 5.1 describes the Hahn process (Hahn and Negishi [1962]), which adapts the tatonnement model to incorporate disequilibrium trades. An alternative model of within-period price dynamics – the Geometric Mean (GM) model – is motivated in Section 5.2 and developed in Section 5.3.

The Geometric Mean model is motivated by a simple observation: for any allocation outside of the set of Pareto optimal allocations, there is a set of prices that support Pareto improving trades. Imagine that for the first k trades in period t, each price $p_{t,k}$ is the same until gains from exchange are exhausted at that price for one side of the market. At this point, the price must adjust in order to achieve a Pareto improving trade. The Geometric Mean model generalizes the price adjustment rule from this observation. In the Geometric Mean model, price adjusts after each trade, and the new price is a weighted geometric mean of the marginal rates of substitution for the representative seller and the representative buyer. The price adjustment rule for the Geometric Mean model is described in detail in Section 5.3.

5.1 Hahn process

The Hahn process is conceptually similar to the tatonnement model. In both the tatonnement model and the Hahn process, price adjusts in response to excess demand. In tatonnement, a price is announced, agents submit net demands, and the announced price is adjusted to reduce the magnitude of excess demand. Trade occurs when an equilibrium price is announced. In the Hahn process, trades take place as soon as a price is announced. This initial price is held constant during some time interval $[0, h]$, and subsequently (for $t > h$) price adjusts continually in response to excess demand according to the adjustment rule $\dot{p}_t = c \cdot Z(p_t|(x_t, y_t))$, where (x_t, y_t) is the allocation that results from all trades that have occurred along the price path $(p_s)_{s\in(0,t)}$. Hahn and Negishi [1962] demonstrate that, in the continuous-time version of their model, every limit point of this dynamical system is a competitive equilibrium.

Both the Hahn process and the Geometric Mean model use a price adjustment rule that depends on the allocation reached after each trade. It is useful to represent per capita allocations of sellers and buyers after k trades in period t in terms of initial endowments and the vector of trade prices $P_{t,k} = (p_{t,1}, p_{t,2}, \ldots, p_{t,k})$. The per capita allocation for buyers can be written in terms of $P_{t,k}$ as $\left(x^A_{t,k}(P_{t,k}),\, y^A_{t,k}(P_{t,k})\right) = \left(\omega^A_x - \frac{1}{6}\sum_{i=1}^{k} p_{t,i},\, \omega^A_y + \frac{1}{6}k\right)$. The per capita allocation for sellers is $\left(x^B_{t,k}(P_{t,k}),\, y^B_{t,k}(P_{t,k})\right) = \left(\omega^B_x + \frac{1}{6}\sum_{i=1}^{k} p_{t,i},\, \omega^B_y - \frac{1}{6}k\right)$.

In the discrete version of the Hahn process, the price adjustment rule for the numeraire commodity X after trade k in period t is

$$\begin{aligned} p^{X,H}_{t,k+1} &= 1 + (1-c)\; Z^X\left(p_{t,k}\,\middle|\left(x_{t,k}(P_{t,k}),\, y_{t,k}(P_{t,k})\right)\right) \\ &= 1 + (1-c)\;\Big(Z^X_A\left(p_{t,k}\,\middle|\left(x^A_{t,k}(P_{t,k}),\, y^A_{t,k}(P_{t,k})\right)\right) + \\ &\qquad Z^X_B\left(p_{t,k}\,\middle|\left(x^B_{t,k}(P_{t,k}),\, y^B_{t,k}(P_{t,k})\right)\right)\Big)\,; \end{aligned} \tag{11}$$

the price adjustment rule for the commodity Y is

$$\begin{aligned} p^{Y,H}_{t,k+1} &= p_{t,k} + c\; Z^Y\left(p_{t,k}\,\middle|\left(x_{t,k}(P_{t,k}),\, y_{t,k}(P_{t,k})\right)\right) \\ &= p_{t,k} + c\;\Big(Z^Y_A\left(p_{t,k}\,\middle|\left(x^A_{t,k}(P_{t,k}),\, y^A_{t,k}(P_{t,k})\right)\right) + \\ &\qquad Z^Y_B\left(p_{t,k}\,\middle|\left(x^B_{t,k}(P_{t,k}),\, y^B_{t,k}(P_{t,k})\right)\right)\Big)\,. \end{aligned} \tag{12}$$

The Hahn process adjustment rule is

$$\begin{aligned} p^H_{t,k+1} &= \frac{p^{Y,H}_{t,k+1}}{p^{X,H}_{t,k+1}} \\ &= \frac{p_{t,k} + c\; Z^Y\left(p_{t,k}\,\middle|\left(x_{t,k}(P_{t,k}),\, y_{t,k}(P_{t,k})\right)\right)}{1 + (1-c)\, Z^X\left(p_{t,k}\,\middle|\left(x_{t,k}(P_{t,k}),\, y_{t,k}(P_{t,k})\right)\right)}. \end{aligned} \tag{13}$$

Equation (13) characterizes the Hahn process price path during the course of a trading period, with the free parameter 'c'.

Hahn process example

For the parameters used in the experiment sessions and for $c = 0.995$ in the Hahn process price adjustment rule, discrete-time simulation of the Hahn process price path approximates the continuous version very closely when the discrete grid is fine. For the initial price $p_{t,1} = 60$, when each trade is for 0.0001 unit (so that it requires approximately 70,000 trades to reach the per capita equilibrium allocation (1163, 7)), the average price in the period is 90.9229 whereas the equilibrium price is 90.9406. For $k \in \{1, 2, 3, 4\}$, if each trade is for 10^{-k} unit, $p_{t,1} = 60$, and $c = 0.995$, simulation of the Hahn process produces an average price $\bar{p}_t$ that lies in the interval $\left(\left(1 - 2 \times 10^{-k}\right) p_e,\, p_e\right)$.

For coarser price adjustments – such as those used in the experiment session in which each trade is for 1/6 unit on a per capita basis – the discrete version of the Hahn process is a coarse approximation to the competitive equilibrium, but this is an advantage for model evaluation, since the discrete version of the Hahn process then predicts a price path that differs significantly from the equilibrium price path. Once a value for c and an initial price $p_{t,1}$ for the period are specified, the price path predicted by the Hahn process is determined. For example, if the initial price is $p_{t,1} = 60$, $c = 0.995$, and each trade is for 1/6 unit, the Hahn process prediction for the last trade price of the period is $p_{t,45} \doteq 85$. In the analysis of experiment data, values of the adjustment rate parameter c are obtained by regressing observed prices from each period of each experiment session on a weighted average of the price path predicted by the Hahn process and the first trade price for the period, with the value of c selected to optimize the fit of the regression model, in a sense described in Section 6.1.

5.2 Geometric Mean model: motivation

Imagine that in trading period t an initial price $p_{t,1}$ is established, and subsequent prices $p_{t,k}$ are equal to $p_{t,1}$ so long as the trade quantity is below the minimum of the quantity that buyers want to purchase at $p_{t,1}$ and the quantity the sellers make available at $p_{t,1}$. At this point, either the demand of the buyers or the supply of the sellers is exhausted.

The Edgeworth diagram on the left side of figure 7 shows $k = 54$ trades at $p_{t,1} = 37$ and the allocation $(x_{t,k},\, y_{t,k})$ that results from these trades. For the price $p_{t,1} = 37$ shown in the diagram, the buyers (agent type A) want to purchase over 12 units of the commodity per capita; the sellers (agent type B) are willing to sell 9 units per capita at $p_{t,1}$. After k trades have occurred at the price $p_{t,1}$ that lead to the per capita allocation $(x_{t,k},\, y_{t,k})$, the price $p_{t,1}$ is equal to the marginal rate of substitution for agent type B, so agent type B no longer offers units for sale at $p_{t,1}$. In the Hahn process, the excess demand at $p_{t,1}$, which is about 3.5 units per capita, leads to an upward price adjustment.

In the Geometric Mean model the new price lies in the cone of prices that support Pareto improving trades. The Edgeworth diagram on the right side of figure 7 shows the situation once the allocation reaches $(x_{t,k},\, y_{t,k})$. Agent type B is no longer willing to sell

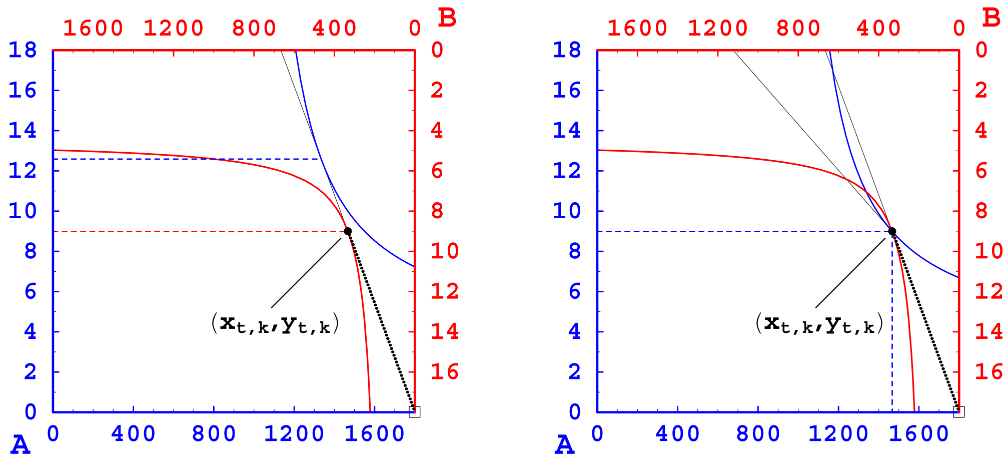


Figure 7: Trades from the endowment (left) and from the allocation $(x_{t,k}, y_{t,k})$ (right).

units to agent type A at the price $p_{t,1} = 37$, but there is a cone of prices that support allocations that are Pareto improvements over the allocation $(x_{t,k},\, y_{t,k})$. The original price $p_{t,1} = 37$ is a lower bound on the prices that support Pareto improving trades; the upper bound $\tilde{p} = 87$, which is also shown in the diagram, is determined from the tangency to the indifference curve of agent type A at the allocation $(x_{t,k},\, y_{t,k})$. For the example shown in the Edgeworth diagram, with the original price $p_{t,1}$ below the equilibrium price, the prices $p \in (p_{t,1},\, \tilde{p})$ that support Pareto improving trades are above $p_{t,1}$. If trades occur at a new price $p_{t,k+1} \in (p_{t,1},\, \tilde{p})$, then further gains from exchange – beyond those achieved at the allocation $(x_{t,k},\, y_{t,k})$ – can be realized. In the subsequent period, if the initial price is drawn from a uniform distribution on the interval $(p_{t,1},\, p_{t,k+1})$, then price also adjusts across periods in the direction of the competitive equilibrium price. The Geometric Mean model generalizes this example by expanding the set of events that trigger a price change and by specifying the within- and across-period price adjustment rules.

5.3 Geometric Mean (GM) model: formulation

The GM model includes specifications of both within-period and across-period adjustment. The key model element, within-period adjustment, is developed in this section and estimated in Section 6.2. A simple model of across-period adjustment is added to the description of the GM model, and the resulting model of both within-period and across-period price adjustment is simulated to demonstrate the path of final allocations across periods for the

GM model. The simulated path of final allocations is then compared in the example to the path of across-period final allocations observed in an experiment session.

Within-period adjustment

Price adjustment in the Geometric Mean model is similar to price adjustment in the motivating example, but in the Geometric Mean model, price adjusts after each exchange. The marginal rate of substitution for the representative buyer is

$$\begin{aligned} MRS_A\left(x^A_{t,k},\, y^A_{t,k}\right) &= \left(\frac{b_A}{a_A}\right)^{r_A} \left(\frac{y^A_{t,k}}{x^A_{t,k}}\right)^{r_A - 1} \\ &= \frac{a_A}{b_A} \left(\frac{x^A_{t,k}}{y^A_{t,k}}\right)^2 . \end{aligned}$$

As described in Section 5.1, after k trades in period t at prices $P_{t,k} = (p_{t,1}, p_{t,2}, \ldots, p_{t,k})$, the per capita allocation is $\left(x^A_{t,k}(P_{t,k}),\, y^A_{t,k}(P_{t,k})\right) = \left(1800 - \frac{1}{6} \sum_{i=1}^{k} p_{t,i},\, \frac{1}{6}\, k\right)$ for the representative buyer. As a function of the observed prices $P_{t,k}$, the per capita marginal rate of substitution for the representative buyer is therefore

$$MRS_A\left(x^A_{t,k}(P_{t,k}),\, y^A_{t,k}(P_{t,k})\right) = \frac{a_A}{b_A} \left(\frac{1800 - \frac{1}{6} \sum_{i=1}^{k} p_{t,i}}{\frac{1}{6}\, k}\right)^2 .$$

The marginal rate of substitution for the representative seller is

$$MRS_B\left(x^B_{t,k},\, y^B_{t,k}\right) = \frac{a_B}{b_B} \left(\frac{x^B_{t,k}}{y^B_{t,k}}\right)^2 .$$

For the representative seller, $\left(x^B_{t,k}(P_{t,k}),\, y^B_{t,k}(P_{t,k})\right) = \left(\frac{1}{6} \sum_{i=1}^{k} p_{t,i},\, 18 - \frac{1}{6}\, k\right)$ is the per capita allocation after k trades in period t. The per capita marginal rate of substitution for the sellers is therefore

$$MRS_B\left(x^B_{t,k}(P_{t,k}),\, y^B_{t,k}(P_{t,k})\right) = \frac{a_B}{b_B} \left(\frac{\frac{1}{6} \sum_{i=1}^{k} p_{t,i}}{18 - \frac{1}{6}\, k}\right)^2 .$$

These two marginal rates of substitution define lower and upper bounds on the prices that can support Pareto improving trades. Price in the GM model is determined as a weighted geometric mean of $MRS_A\left(x^A_{t,k},\, y^A_{t,k}\right)$ and $MRS_B\left(x^B_{t,k},\, y^B_{t,k}\right)$. This weighted geometric mean price is

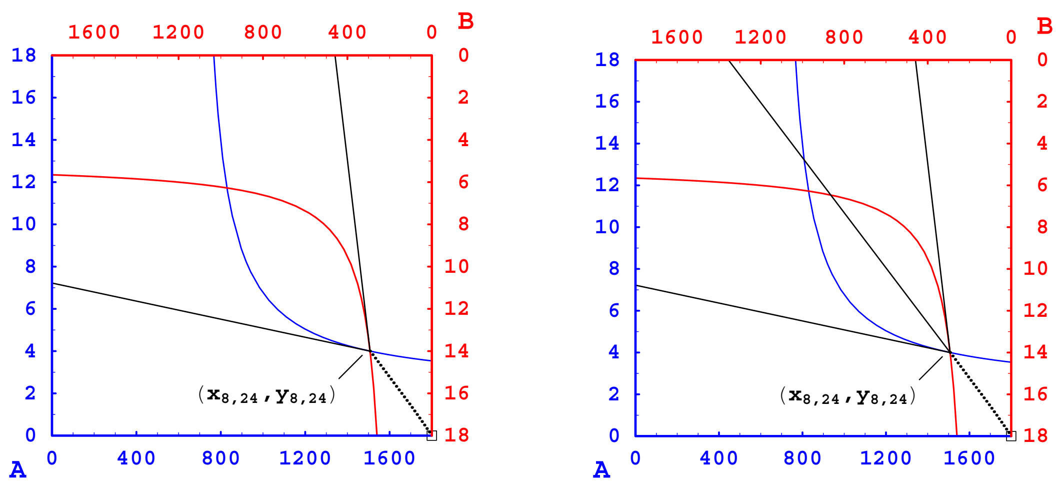
$$p^G_{t,k+1}\left(x^A_{t,k},\, y^A_{t,k}\right) = MRS_A\left(x^A_{t,k},\, y^A_{t,k}\right)^{\frac{1}{2}(1-\phi)} \times MRS_B\left(x^B_{t,k},\, y^B_{t,k}\right)^{\frac{1}{2}(1+\phi)} .$$

The weighted geometric mean price after k trades can be written in terms of utility function parameters, endowments, the vector of trades prices $P_{t,k}$, and ϕ as

$$p^G_{t,k+1}(P_{t,k}) = \left(\frac{a_A}{b_A}\right)^{\frac{1}{2}(1-\phi)} \left(\frac{10800 - \sum_{i=1}^{k} p_{t,i}}{k}\right)^{1-\phi} \left(\frac{a_B}{b_B}\right)^{\frac{1}{2}(1+\phi)} \left(\frac{\sum_{i=1}^{k} p_{t,i}}{108-k}\right)^{1+\phi}. \quad (14)$$

The parameter ϕ can be interpreted as a proxy for differences between the bargaining capability of sellers and buyers. When $\phi = 0$ agent types A and B are treated symmetrically. The bargaining power of buyers relative to sellers is an increasing function of ϕ. For example, if $\phi = 1$ then $p^G_{t,k+1}$ is equal to the marginal rate of substitution for the sellers, so that all of the gains from exchange go to the buyers.

Equation (14) completely characterizes within-period GM model price adjustment. The price adjustment rule depends only on the slopes of the representative seller's and the representative buyer's utility functions at their current per capita allocations, and on the parameter ϕ. Trade continues until a Pareto optimal allocation is reached.



(a) Marginal rates of substitution

(b) Geometric mean price

Figure 8: Marginal rates of substitution and geometric mean after 24 trades.

Figure 8 (a) shows the first 24 trades in period 8 from session IU-CES1-DAs-010607c, and in addition, for the representative buyer and the representative seller it shows the marginal rates of substitution at the per capita allocation that results from these trades. Figure 8 (b) shows the same information as figure 8 (a), and in addition, it shows the weighted geometric mean price $p^G_{8,24}(P_{8,24})$ after 24 trades. (Trades from period 8 in this session were selected for graphical representation because, as discussed in Section 6.2 below, in this period the Geometric Mean model was closest to the median fit across all 66 periods from the five sessions.)

Across-period adjustment

The initial price in period $t+1$ is assumed to be drawn from a uniform distribution on the range of prices defined by the first price ($p_{t,1}$) and the last price (p_{t,K_t}) from period t:

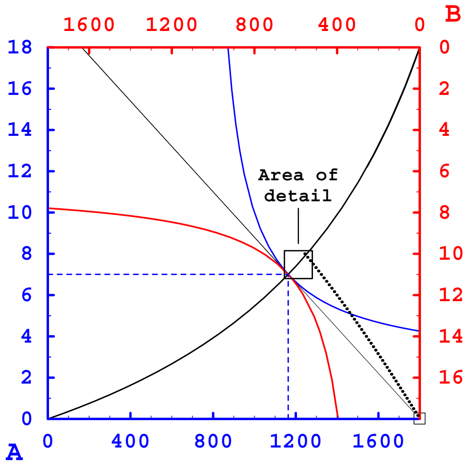
$$p_{t+1,1} \sim U\left([\min\{p_{t,1}, p_{t,K_t}\}, \max\{p_{t,1}, p_{t,K_t}\}]\right). \tag{15}$$

If the first trade price in period one is determined randomly according to some distribution, trades within each period are determined by equation (14), and the initial prices in periods $t = 2, 3, \ldots, T$ are determined by the distributions in (15), then an adjustment process is completely specified, both within and across periods.

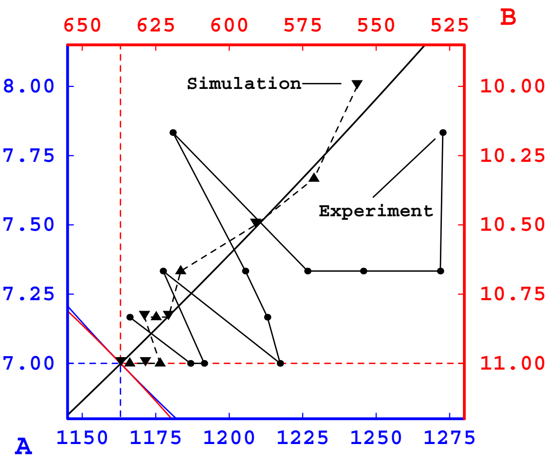
GM model simulation example

Figure 9 (a) shows all first period trades generated by the Geometric Mean model in equation (14) (with $\phi = -0.01$ starting from an initial price $p_{1,1} = 60$). The triangles connected by the dashed line in figure 9 (b) show the final allocations across periods from a simulation of this model with the initial condition $p_{1,1} = 60$. The dots connected by the solid line show final allocations in each period of session IU-CES1-DAs-010607a, which also had the initial trade price $p_{1,1} = 60$ in period 1. The primary difference between the series of final allocations from the model and from the experiment session is that in the experiment sessions, there are frequently final allocations that are not Pareto optimal, whereas in the model, by definition the final allocation is Pareto optimal (relative to the discrete grid of allocations possible in the experiment). Final allocations generated by the model could be matched more closely to final allocations from each period in the experiment session by including a positive probability that trades which generate low incremental payoffs are not completed, and trades which lead to small losses are completed. In this way, final allocations on either side of the Pareto set would be observed in the model. Since the focus of the empirical assessment of the adjustment model is on within-period price adjustment, this feature has not been added to the model, to keep the model parsimonious.

There is one important caveat with regard to this convergence example that should be noted. The parameter ϕ in equation (14) is necessary in this example to obtain convergence to the competitive equilibrium price and allocation. To illustrate, if $\tilde{\phi} = 0$ (rather than the weighted geometric mean with $\phi = -0.01$), then the fixed point for the price adjustment rule



(a) Geometric mean model trades



(b) End of period allocations

Figure 9: Within-period trades (left) and across-period final allocations (right).

in equation (14), with the across-period adjustments specified by the distributions in (15), is $p = 85$. From the model simulation shown in figure 9 (b) it is clear that with $\phi = -0.01$ in equation (14) and the across-period adjustments determined by the distributions in (15), the average price in the model converges to the equilibrium price $p_e = 91$ and the equilibrium allocation is reached. The next section demonstrates that, in addition to reaching the competitive equilibrium of this exchange economy in simulations, the model generally fits the within-period price data well, and in many cases fits the data remarkably well.

6 Model estimation

Both the Hahn process and the Geometric Mean model are estimated by assuming that prices follow paths that are weighted averages of the initial price in the trading period and the path predicted by the Hahn process or the Geometric Mean model. This section specifies the estimation procedure for both models and compares estimates of the weight on the price path predicted by each model.

Section 4.1 demonstrates that decision errors significantly affect convergence of allocations in the final session. Section 4.2 demonstrates that decision errors significantly affect price convergence across periods in the final two sessions. Sixty-nine of seventy-seven decision errors occurred in the final two sessions. (See column 1 of table 6.) In the fourth session, IU-CES1-DAs-011116b, there were 33 decision errors in the first eight periods and only seven errors in the final seven periods. In the fifth session, IU-CES1-DAs-012114b, there were three decision errors in the first eight periods and 26 decision errors in the final seven periods. Fifteen periods from the final two sessions include a large fraction (59 of 77) of the decision errors. Given the large number of decision errors in these periods, there were a number of subjects whose behavior indicates that they had different objectives (or a different understanding of the induced objectives) from those in the experimental design.

The analysis in the remainder of this section focuses primarily on the 51 periods in which the number of decision errors was small. Specifically, the analysis focuses on periods $1-12$ of the first three sessions, periods $9-15$ of the fourth session, and periods $1-8$ of the fifth session. These periods produce much more consistent parameter estimates for the Hahn process and the Geometric Mean model than the full data set does. Estimates from the full data set are reported for completeness, but the 51 periods in which the large majority of trades led to Pareto improving allocations provide a clearer picture of the price adjustment process in Edgeworth exchange.

6.1 Hahn process estimation

The Hahn process is evaluated by regressing the observed price path $P_{t,k} = (p_{t,k})_{k=1}^{K_t}$ in period t on a weighted average of the initial price $p_{t,1}$ and the price path $P_{t,k}^H$ predicted by the Hahn process. Equation (13) shows that the predicted price $p_{t,k+1}^H$ depends on the most recent price $p_{t,k}$, on the excess demands for X and Y at the current allocation $(x_{t,k}, y_{t,k})$ and the most recent price $p_{t,k}$, and on the adjustment rate parameter c. In the experiment, the allocation $(x_{t,k}, y_{t,k})$ after k trades is observed, the price $p_{t,k}$ is observed, and excess demand is known as a result of the utility inducing technique. The procedure used to select an adjustment rate parameter c for the estimation of the regression equation is explained after the regression equation is defined.

For a fixed value of c, a predicted price path is determined starting from the initial price $p_{t,1}$. The predicted second price $p_{t,2}^H$ is determined based on the observed first price and the adjustment rule in equation (13). Subsequent predicted prices are determined iteratively based on the observed initial price $p_{t,1}$ and the predicted prices $p_{t,2}^H, p_{t,3}^H, \ldots, p_{t,k}^H$, again following equation (13). Denote the observed first price and the subsequent predicted prices, through k trades, by $P_{t,k}^H = (p_{t,1}, p_{t,2}^H, p_{t,3}^H, \ldots, p_{t,k}^H)$. The price prediction $p_{t,k+1}^H$ is determined from equation (13), with the excess demand evaluated at the allocation predicted after k trades. With the arbitrary price path $P_{t,k}$ in equation (13) replaced by the predicted price path $P^{H}_{t,k}$ (given the initial price $p_{t,1}$), successive elements of the predicted price path are determined iteratively:

$$p^{H}_{t,k+1} = \frac{p^{H}_{t,k} + c\, Z^{Y}\left(p_{t,k} \,\middle|\, \left(x_{t,k}\left(P^{H}_{t,k}\right),\, y_{t,k}\left(P^{H}_{t,k}\right)\right)\right)}{1 + (1-c)\, Z^{X}\left(p_{t,k} \,\middle|\, \left(x_{t,k}\left(P^{H}_{t,k}\right),\, y_{t,k}\left(P^{H}_{t,k}\right)\right)\right)}. \tag{16}$$

Equation (16) is the Hahn process analog of equation (7), which defines the adjustment rule for the tatonnement model of across-period adjustment. For $k = 1, 2, 3, \ldots, K_t - 2$, the regression equation is $p_{t,k+1} = \alpha_t\, p^{H}_{t,k+1} + (1-\alpha_t)\, p_{t,1} + \epsilon_{t,k+1}$, which can be expressed as

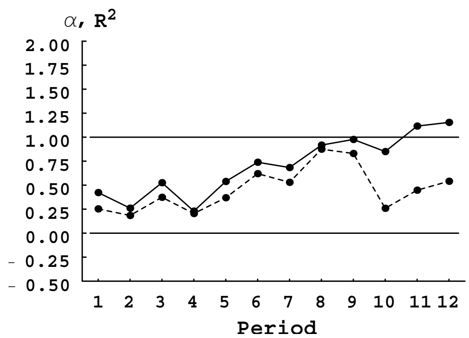
$$p_{t,k+1} - p_{t,1} = \alpha_t\, (p^{H}_{t,k+1} - p_{t,1}) + \epsilon_{t,k+1}. \tag{17}$$

Estimates of α_t from equation (17) depend on the adjustment rate parameter c in equation (16). An estimate of $\widehat{\alpha}_t(c)$ for period t that is near one with R^2 near one would indicate that the price path over the course of period t is very similar to the path predicted by the Hahn process, when the initial condition for the predicted Hahn process price path is the observed first price $p_{t,1}$. The objective of this section is to compare the Hahn process and the Geometric Mean model. The comparison of the estimates from the two models that is carried out in the remainder of this section demonstrates that the Geometric Mean model outperforms the Hahn process model. For this reason, the adjustment parameter c has been selected to minimize $S(\widehat{\alpha}(c)) \equiv T^{-1} \sum_{t=1}^{T} (\widehat{\alpha}_t(c) - 1)^2$, which is the average squared distance between the estimates $\widehat{\alpha}_t(c)$ and one, so that the best possible fit for the Hahn process model can be compared to the fit for the Geometric Mean model. Table 7 shows the values of c^* that are obtained when $S(\widehat{\alpha}(c))$ is minimized separately for each session.

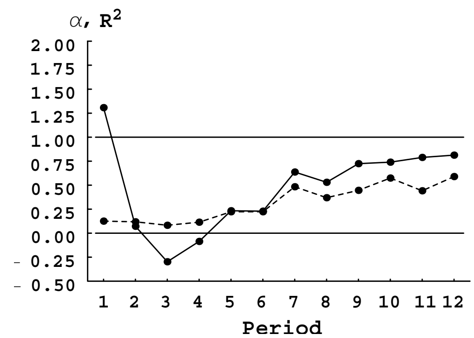
Session	Periods	c^*	$S(\widehat{\alpha}_t(c^*))$
IU-CES1-DAs-010607a	1 - 12	1.000024	0.15972
IU-CES1-DAs-010607b	1 - 12	1.000036	0.45619
IU-CES1-DAs-010607c	1 - 12	1.000013	0.54311
IU-CES1-DAs-011116b	9 - 15	0.999891	0.45706
IU-CES2-DAs-011214b	1 - 8	0.999956	0.19429

Table 7: Adjustment rate c^* and average squared deviations $S(\widehat{\alpha}_t(c^*))$.

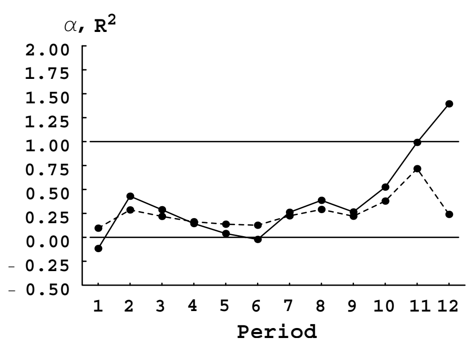
For the 51 trading periods included in table 7, the value c^* of the adjustment rate parameter that minimizes $S(\widehat{\alpha}(c)) = T^{-1} \sum_{t=1}^{T} (\widehat{\alpha}_t(c) - 1)^2$, pooled across these fifty-one periods, is $c^* = 0.999982$. This value of c^* has been used to obtain estimates of the parameters $\alpha_t(c)$ in equation (17) for periods $1 - 12$ in the first three sessions, for periods $9 - 15$ from the fourth session, and for periods $1 - 8$ from the fifth session. Over these 51 trading periods, $S(\widehat{\alpha}_t(c^*))$, which is the average squared difference between the estimates $\widehat{\alpha}_t(c^*)$ and one, is 0.383. Estimates $\widehat{\alpha}(c^*)$ for each period in each of the first three sessions, and for periods $2 - 15$ in the fourth session are shown in figure 10.



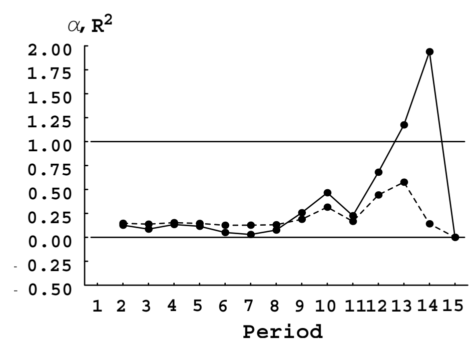
(a) IU-CES1-DAs-010607a



(b) IU-CES1-DAs-010607b



(c) IU-CES1-DAs-010607c



(d) IU-CES1-DAs-011116b

Figure 10: Estimates $\widehat{\alpha}_t(c^*)$ and R^2 from Hahn process regressions of equation (17).

The values c^* in table 7 that minimizes $S(\widehat{\alpha}(c))$ for each of the first four sessions, and the value $c^* = 0.999982$ that minimizes $S(\widehat{\alpha}(c))$ pooled across 51 periods are measures of the rate at which price adjusts from one transaction to the next. The estimates $\widehat{c}$ of across-period adjustment rate parameters obtained for the tatonnement model in table 5 of Section 4.2 are all between $\widehat{c} = 0.99931$ and $\widehat{c} = 0.99995$. These adjustment parameter estimates are all below the value $c^* = 0.999982$ for within-period adjustment. For values of $c \in (0.996, 1.000)$ adjustment is faster for smaller values of c. If $c < 0.996$, adjustment is faster yet, but the price overshoots the equilibrium with the discrete adjustments of the experiment design. This is the main reason that the estimates of c in Section 4.1 and the values of c^* in this section fall in a narrow range near one. Although the estimates $\widehat{c}$ of across-period adjustment in the tatonnement model from Section 4.1 are all below $c^* = 0.999982$, the within-period adjustment rate applies to each trade. The total within-period adjustment implied by the value $c^* = 0.999982$ exceeds the adjustment implied by the across-period estimates in table 5 by a large margin. Figure 11 displays the predicted and observed levels of across-period adjustment and the predicted and observed levels of within-period adjustment for session IU-CES1-DAs-010607a. For each period in the session, the figure shows the level of across-period adjustment predicted by the tatonnement model price adjustment rule as the series of dots connected by the dashed line. The tatonnement price adjustment rule used is from equation (7) with the estimated value $\widehat{c}$ for session IU-CES1-DAs-010607a from table 5. The graph shows the observed across-period adjustment ratio from equation (9) as the series of dots connected with a solid line. Within-period adjustment implied by the Hahn process is defined as

$$R_t^{w,H} = \frac{\widehat{\alpha}_t(c^*)\, p^H_{t,K_t} + (1 - \widehat{\alpha}_t(c^*))\, p_{t,1} - p_e}{p_{t,1} - p_e}, \tag{18}$$

where p^H_{t,K_t} is the predicted final price from the Hahn process path from equation (16) and $\widehat{\alpha}_t(c^*)$ is the estimate on the Hahn path from the regression equation (17). The level of within-period price adjustment implied by the Hahn process is shown as the series of squares connected by the dashed line. The level of observed within-period adjustment, as measured by equation (10) is shown as the series of squares connected with the solid line. The fact that within-period adjustment exceeds adjustment across periods is apparent from the graph. This comparison of adjustment parameter estimates from the tatonnement model and the Hahn process is consistent with the direct comparison of within-period and across-period adjustment levels in Section 4.3.

Evidence from experiments on the Scarf example of global instability under the tatonnement dynamic (Scarf [1960]) by Anderson, Granat, Plott, and Shimomura [2003] also supports the importance of within-period price movements. Anderson et al. examine the number of price changes that move in the same direction as the prediction of the Hahn process (which they call the "instantaneous excess demand model"). They find that in the convergent treatment 60.3% of price changes move in the same direction as the instanta-

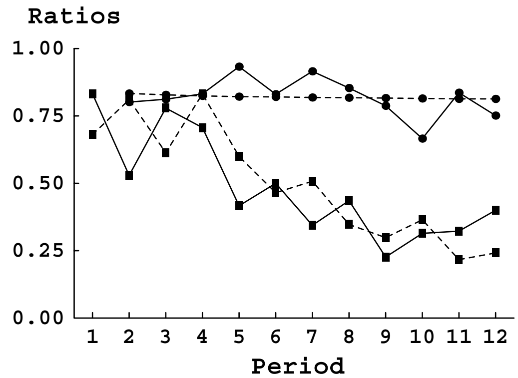


Figure 11: Predicted and observed across-period and within-period adjustment.

neous excess demand model or Hahn process. In the Edgeworth experiment sessions, 869 of 1538 or 56.5% of price changes are in the predicted direction. Suppose that price changes follow the process $p_{k+1} = a + b\,p_k + \epsilon_k$, where $\epsilon_k \sim N(0,\, \sigma^2)$. Assume without loss of generality that $b > 0$. For $b > 0$, the observed price movement agrees with the predicted price movement when $p_{k+1} > p_k$. The probability of this event is $P[\,p_{k+1} > p_k] = P[\,\epsilon_{k+1} - \epsilon_k > b\,]$. Since $\epsilon_{k+1} - \epsilon_k \sim N(0,\, 2\,\sigma^2)$, price movements in the direction of predicted price movements will occur 56.5% of the time for $\frac{b}{2\,\sigma^2} = 0.164$, or $b = 0.164\,\sigma^2$. The average variance of prices from the Edgeworth experiments around the predicted Hahn process price paths was 4.22. If the variance of price around a linear trend is similar, then the value of b that would result in 56.5% of observed price changes in the direction of predicted price changes is $b \doteq 0.69$. With an average of 43 trades per period, the resulting difference between the price at the beginning of a period and the price at the end of a period would be $|p_{K_t} - p_1| = 29.75$. Relative to the equilibrium price level of $p_e = 91$, this is a large within-period price change. In Anderson et al., frequencies of observed price movements that are consistent with the prediction of the Hahn process are all similar to or greater than the frequency of 56.5% that was observed in the Edgeworth experiment.

Anderson et al. conclude that across-period price adjustment is more important than within-period adjustment. They base this conclusion on the fact that the frequency of within-period price changes that are consistent with the tatonnement adjustment predictions exceeds the frequency of within-period price changes that are consistent with the Hahn process predictions. Anderson et al. use predictions of both the across-period adjustment predicted by tatonnement and the within-period adjustment predicted by the Hahn process to assess within-period adjustment. The observed frequencies of within-period price changes that are consistent with both models can only occur if there is a substantial level of within-period price adjustment.

6.2 GM model estimation

The Geometric Mean model is tested by regressing observed prices $p_{t,k+1}$ in period t on a weighted average of the initial price $p_{t,1}$ in period t and on the prices $p^G_{t,k+1}(P_{t,k})$ predicted by the Geometric Mean model from equation (14). For $k = 1, 2, 3, \ldots, K_t - 2$ the regression equation is $p_{t,k+1} = \alpha_t\, p^G_{t,k+1} + (1 - \alpha_t)\, p_{t,1} + \eta_{t,k+1}$ which can be expressed as

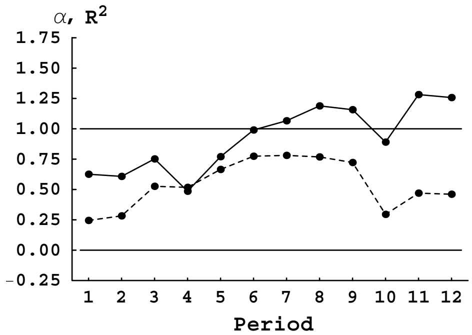
$$p_{t,k+1} - p_{t,1} = \alpha_t\, (p^G_{t,k+1} - p_{t,1}) + \eta_{t,k+1}. \tag{19}$$

Since the initial endowment is on the boundary, the geometric mean price isn't defined until after the first trade. Then the second price ($p_{t,2}$) can be predicted as a function of the geometric mean price after the first trade ($p^G_{t,2}(P_{t,1})$) and so forth. Regressions are run on the data from each period separately. As with the Hahn process model, estimates of α_t near one and R^2 near one would constitute strong confirmation of the model.

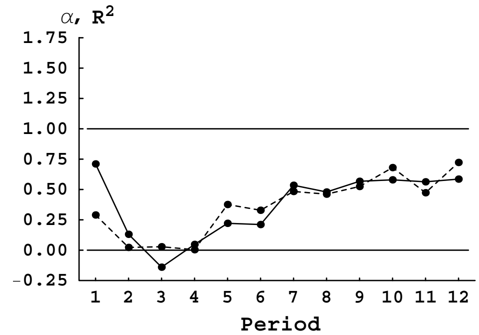
Regression results are summarized for each period of each of the first four sessions in figure 12. The dots connected with the solid line show the estimates of $\widehat{\alpha}_t$ for each period. The dots connected by the dashed line show the R^2 statistics from each period for the model in equation (19).

Since each value of $\widehat{\alpha}_t$ in figures 12 (a) through (d) is a summary statistic from an entire period of trading, it is valuable to compare observed price paths to the price paths predicted by the Geometric Mean model for several representative trading periods. Several important patterns of the predicted price paths and the observed price paths from the experiment sessions are depicted in figure 13. Each graph in figure 13 shows the observed prices for one trading period (as a solid line) and the value of the weighted geometric mean price $p^G_{t,k}$ after each trade during the period (as a dashed line).

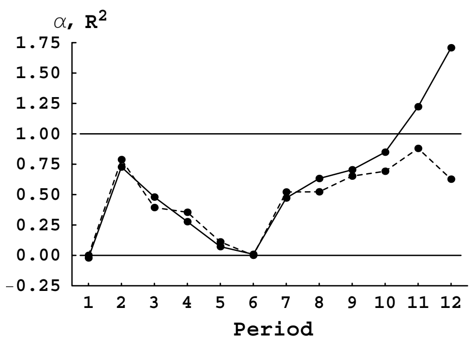
The estimate of $\widehat{\alpha}_t$ closest to zero occurred in period 6 of session IU-CES1-DAs-010607c. The prices from this session are shown in figure 13 (a) (as the solid line), along with the price $p^G_{t,k}$ predicted by the GM model after each trade during the period (as the dashed line). For the period shown in figure 13 (a), the estimate $\widehat{\alpha}_t$ from equation (19) was 0.009 and $R^2 = 0.001$. Throughout this period, prices were nearly constant. Clearly from the figure, the prices did not follow the Geometric Mean model price prediction, but figure 12 (c),



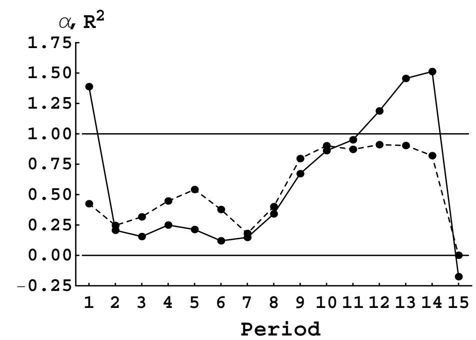
(a) IU-CES1-DAs-010607a



(b) IU-CES1-DAs-010607b



(c) IU-CES1-DAs-010607c



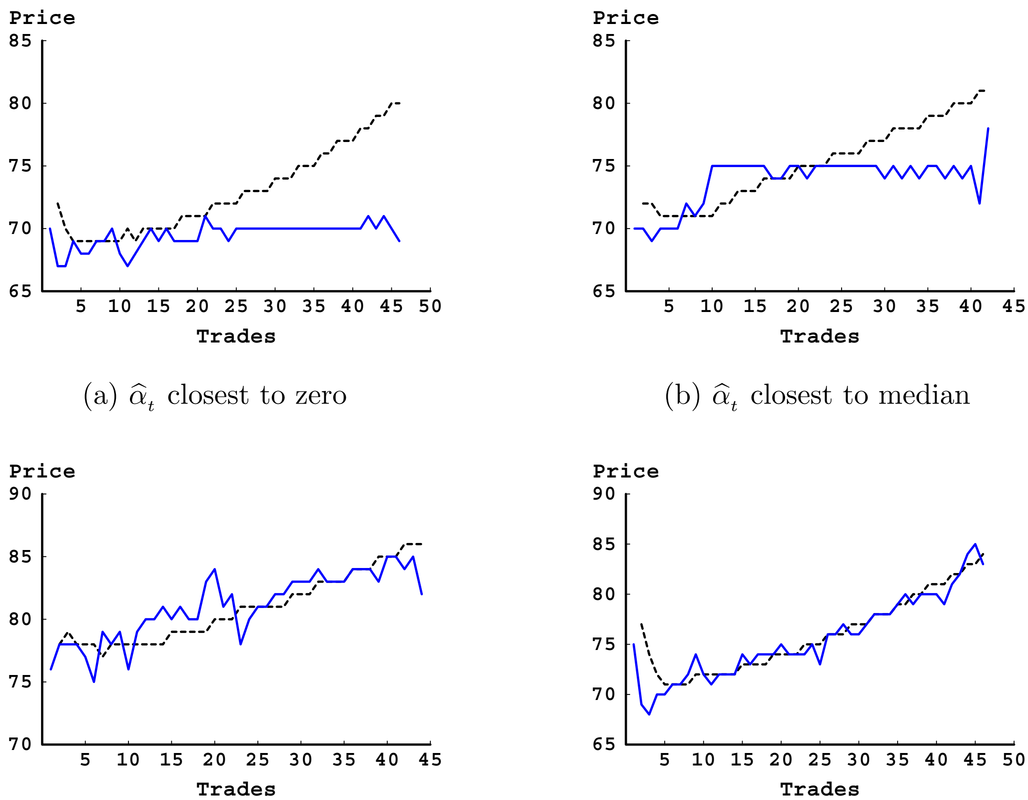
(d) IU-CES1-DAs-011116b

Figure 12: Alpha coefficients and R^2 from GM model regressions of equation (19).

which shows the estimates of $\widehat{\alpha}_t$ for each period in the session, demonstrates that prices did begin to follow the Geometric Mean model prediction starting in period 7 and continued to do so until the end of the session. This pattern – stable prices in early periods followed by prices that tracked the Geometric Mean model price prediction – also occurred in session IU-CES1-DAs-011116b. (Compare figures 12 (c) and (d).)

The median value of $\widehat{\alpha}_t$ across 66 periods in five sessions was 0.639. The value $\widehat{\alpha}_t = 0.633$ in period 8 from the third session, IU-CES1-DAs-010607c, was closest to this median value. The R^2 statistic for the model was $R^2 = 0.523$ in this period. Figure 13 (b) shows the trade prices in that period (as a solid line) and the value of $p^G_{t,k}$ after each trade during the period (as the dashed line).

The estimate of $\widehat{\alpha}_t$ from all of the 66 trading periods that was closest to one occurred in period 6 from session IU-CES1-DAs-010607a. In that period, $\widehat{\alpha}_t = 0.978$ and $R^2 = 0.774$. Figure 13 (c) shows the trade prices and the value of $p^G_{t,k}$ after each trade during the period.



(a) $\widehat{\alpha}_t$ closest to zero

(b) $\widehat{\alpha}_t$ closest to median

(c) $\widehat{\alpha}_t$ closest to one

(d) $\widehat{\alpha}_t$ and R^2 close to one

Figure 13: Prices, equilibrium prices, and geometric mean prices from four periods.

Finally, figure 13 (d) shows the prices and the predicted prices $p^G_{t,k}$ from the Geometric Mean model for period 11 from session IU-CES1-DAs-011116b. The predicted prices in this period were close to the actual prices both in terms of the estimated value $\widehat{\alpha}_t = 0.951$ of the coefficient on $p^G_{t,k}$ in the regression model from equation (19) and also in terms of the R^2 statistic from the regression equation, which was $R^2 = 0.872$.

The Geometric Mean model fit is superior to that of the Hahn process. The median value of the estimates $\widehat{\alpha}_t$ across sixty-six periods in the five sessions for the GM model was 0.639. For the Hahn process, the median value of $\widehat{\alpha}_t(c^*)$ was 0.448 across all sixty-six periods. The median value of R^2 for the Geometric Mean model was 0.522; for the Hahn process the median value of R^2 was 0.224. Another important measure of the fit for these models is $S(\widehat{\alpha})$ (or $S(\widehat{\alpha}(c^*))$ for the Hahn process). If the average squared deviation between the parameter estimates $\widehat{\alpha}_t$ and one is small, then price paths from the experiment sessions are predicted well by the model. With the sums taken over the fifty-one periods evaluated in the previous sections, $S(\widehat{\alpha}) = 0.354$ for the Geometric Mean model and $S(\widehat{\alpha}(c^*)) = 0.383$ for the Hahn process model. If the sums are taken over all sixty-six periods, $S(\widehat{\alpha}) = 0.509$ for the Geometric Mean model, and $S(\widehat{\alpha}(c^*)) = 5.772$ for the Hahn process. Although the average fit of the Geometric Mean model is imperfect, the fit improves dramatically for the last four periods of the four convergent sessions. In these 16 periods, the mean value of the coefficient $\widehat{\alpha}_t$ is 0.959, the median value is 0.951, and the mean squared difference between the estimates $\widehat{\alpha}_t$ and one is 0.130. Subjects in the experiments had no experience with markets that involved induced utility. With experience from early periods, strategies began to respond to differences in profits for the two sides of the market at disequilibrium prices, and hence trade prices tracked changing conditions more effectively in later periods. Consequently, it is reasonable to assume that Geometric Mean model price paths predictions – which track observed price paths closely in these later periods – reflect the price adjustment process in Edgeworth exchange accurately.

7 Conclusions

Edgeworth exchange is the fundamental model in the neoclassical theory of value. Empirical assessment of the Axioms of Revealed Preference, as in Cox [1997], demonstrates that choices over a small number of commodities are consistent with the existence of a regular preference ordering or utility function. The experiments reported in this paper demonstrate that when agents with (induced) regular utility functions engage in exchange, the result is consistent with the Edgeworth model. Consequently, the two key assumptions in the neoclassical theory of value – that (1) agents have consistent preference orderings, and (2) exchange by agents with consistent preference orderings (or utility) leads to a competitive equilibrium allocation – are consistent with experimental evidence, at least when these two assumptions are evaluated separately.

In addition to confirmation of the static predictions of the Edgeworth model, this paper addresses important aspects of the dynamics of convergence. Edgeworth exchange experiments exhibit both across-period and within-period convergence. Comparison of price adjustment across periods and price adjustment within periods demonstrates that most price adjustment occurs within each trading period. Consequently, models that predict price adjustment within trading periods are essential in order to understand the dynamics of convergence in general equilibrium models. Comparison of the Hahn process to the Geometric Mean model – both of which provide price path predictions within trading periods – demonstrates that the Geometric Mean model produces a better prediction of the within-period price adjustment process.

In addition to the superior fit of the Geometric Mean model of price adjustment, the model links to important local characteristics of sellers and buyers. The price adjustment rule in the Geometric Mean model is based on marginal rates of substitution of sellers and buyers. Since prices predicted by the GM model balance the buyers' and sellers' local characteristics, this analysis points in the direction of a model of the interactions among individual sellers and buyers. One possible direction for development of such models would be to extend the heuristic bargaining model in Gjerstad and Dickhaut [1998] (GD98) and Gjerstad [2003] from the partial equilibrium case in which sellers have a list of marginal costs and buyers have a list of marginal values to the case of Edgeworth exchange. Agents' marginal conditions in the form of values and costs play a prominent role in the model of double auction bargaining in GD98 and in Gjerstad [2003]. The fact that a price adjustment model that balances marginal conditions of sellers and buyers also predicts price movements well in an Edgeworth exchange economy suggests that a model analogous to the model in GD98 could provide a basis for the price adjustment posited in this paper. In this way, the observation that price adjustment is predicted well by a simple function of marginal conditions might form the basis for development of a model of the bargaining process that underlies general equilibrium. By exploiting the role of marginal conditions in the price adjustment process that is demonstrated in this paper, it is likely that models will be developed that demonstrate how competitive equilibrium outcomes are achieved in general equilibrium models.

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Appendix A

Prior to the start of an experiment session, a seller goes through an instruction set that describes each of the elements of the seller screen and their operations. These elements include: (1) the seller's endowment and current allocation of currency and commodity, (2) the seller's profit (or utility) function, (3) input that the seller provides during the session, (4) processing of input (by the double auction mechanism) from the seller, as well as from other sellers and from buyers, and (5) the determination of the seller's profit, which is based on the seller's allocation of currency and commodity. Buyers' instructions are analogous to sellers' instructions. The most important principle adhered to in our implementation of instructions is that inputs of asks by sellers (or bids by buyers) are initiated by the seller (or by the buyer) in order to avoid creation of a reference point effect that would influence asks or bids once the session begins.

Each seller and each buyer views a total of 33 screens, and makes inputs on five of these screens. There are eight interactive questions that each seller and each buyer must answer correctly in order to proceed. Although there are 33 screens, most of the screen space is devoted to the actual screen display that the subject views and uses during the experiment session. During the instructions, a text box describes elements of the seller's screen display. The total length of the sellers' instructions is 3250 words, which is equivalent to approximately six pages of text. Subjects typically complete instructions in 15 to 35 minutes.

The sequence of steps through the screens is described below. The instruction summary below refers frequently to elements of the seller's screen, which is shown in figure 2 (p. 7). The instructions for a buyer are similar. Direct experimenter interaction with subjects was kept to a minimum whenever possible, including sign-in, seating, and payment.

Screen 1: The subject's earnings are based on subject's decisions and the decisions of other participants.

Screen 2: The subject is a seller throughout an experiment session that lasts for 12 trading periods (in the first three of the five sessions) or 15 periods (in the last two sessions).

Screen 3: The subject should not communicate with or distract others. The subject's data is anonymous.

Screen 4: Payment is made at an exchange rate of £0.01 per unit of experiment currency accumulated. (This rate was $0.008 in session 4 at the University of Arizona.) Payment is made anonymously in cash at the conclusion of the experiment session.

Screen 5: There is a set of interactive instructions that follow this screen. The seller will know that he either has or has not completed instructions based on the status message at the top of his screen.

Screen 6: Each period of the session consists of a 60 second "Preview Phase", a "Trading Phase" of 180 seconds and a "Review Phase" of 30 seconds. A clock at the top of the screen ticks down to the end of each phase.

Screen 7: The seller begins each period with eighteen units of the commodity. The current balance of both currency and commodity are shown throughout each trading period in the Current Allocation box on the seller's screen.

Screen 8: The location and purpose of the "Profit Calculator" is described to the seller.

Screen 9: The seller is prompted to enter a Quantity and a Price into the Profit Calculator. The profit that would result if the seller sold the proposed quantity at the proposed price is shown in the Profit

Calculator table, and the Current Allocation and Proposed Allocation are shown in the Profit Calculator Graph. (See figure 3 on page 9 for a view of the Profit Calculator for a seller and for a buyer.)

Screen 10: The Profit Calculator table is described to the seller. Rows correspond to different prices and columns correspond to different numbers of units sold. The profit level for the price and quantity that the seller entered on Screen 9 is displayed in the center of the table.

Question 1: For the quantity 'q' and the price 'p' that the seller entered, he is asked to state what his profit would be if he were to sell '$q + 1$' units at price 'p'.

Screen 11: The Profit Calculator Graph is described. The graph shows the seller's Current Allocation, the seller's Proposed Allocation (which is the allocation that would result if the seller exchanges q units of the commodity for $p\,q$ units of the numeraire commodity), and the Iso-profit Curve that passes through the Proposed Allocation.

Screen 12: The seller is reminded that he begins each period with eighteen units of the commodity, and is asked to use the Profit Calculator to determine what his total profit would be if he were to sell all eighteen units of the commodity at the price that he entered on Screen 9.

Question 2: The seller is asked to enter the profit that he would obtain if he were to sell all 18 units from his commodity endowment at the price that he entered on screen 9.

Screen 13: The Profit Calculator includes left and right arrows that can be used to decrease or increase the quantity proposal and up and down arrows that can be used to decrease or increase the price proposal. The seller is encouraged to test these features.

Screen 14: An ask is entered in the "Enter Ask" area of the seller screen. The subject enters an ask at this point.

Screen 15: The ask entered by the subject appears in the Unit Ask row of the Trade Summary display. Commodity Holding remains unchanged, but Commodity Available is reduced by one unit. The ask also appears in the "Your Asks" display area.

Screen 16: Asks by all sellers and bids by all buyers appear in the "Market Queue" display.

Screen 17: A new ask is generated randomly to simulate an action by another seller. The new ask is at a higher amount than the subject's ask, to illustrate the Ask Improvement rule. (The new ask is generated randomly from one to ten above the seller's own ask, though the subject is not informed of the process that generates the new ask.) The subject is also informed that if he makes a new ask at this point, the new ask will replace his current ask.

Question 3: The seller has a quiz question appear at this point. The seller, after seeing a description of the Ask Improvement rule, is prompted to state what is the highest ask that can be submitted at this point.

Question 4: After the subject answers Question 1 correctly, he is asked to state the number of asks that will be in the Market Queue if he submits a new ask.

Screen 18: The subject is asked to enter a new ask that improves on the ask submitted by the simulated seller.

Screen 19: The new ask replaces the seller's previous ask. The "Messages" display is shown, and updates to the Trade Summary display are described.

Screen 20: The subject is prompted to remove his ask by double clicking on the ask in the "Your Asks" display.

Screen 21: Changes to the seller screen that result from the removal of his ask are described, including updates to the Commodity Available, Your Asks, Market Queue, Trade Summary, and Messages displays.

Screen 22: The seller is prompted to enter a new ask to replace the ask that he just removed.

Screen 23: Updates to the seller's screen that result from the new ask are reviewed.

Screen 24: A trade occurs when a seller's ask is at or below the current best bid, or a buyer's bid meets or exceeds the best ask. The seller is informed that on the next screen, a bid from a simulated buyer will be generated that will meet the seller's ask, so that the seller will trade with the simulated buyer.

Screen 25: Updates to the seller's Unit Price, Unit Profit, and Total Profit entries in his Trade Summary table that result from the most recent trade are described. Changes to the Currency Holdings and Commodity Holdings areas are described. The Market Transaction Prices graph update is described.

Screen 26: The price determination rule is reviewed and the seller is informed that after two questions, a bid will be simulated that results in a trade with the current low ask.

Question 5: The seller is asked whether a trade will result if a buyer now submits a bid that is below the current low ask.

Question 6: The subject is asked what the trade price will be if a bid is submitted that meets or exceeds the current low ask.

Screen 27: A bid is simulated that generates another trade. (This trade is between a simulated seller and a simulated buyer, so the subject only sees public information regarding the trade, i.e., its price on the Market Transaction Prices graph.)

Screen 28: A new simulated bid appears in the Market Queue. The price determination rule is reviewed once more and the subject is asked two more questions.

Question 7: The seller is asked whether a trade will result if he submits an ask that exceeds the current high bid.

Question 8: The seller is asked what the trade price will be if he submits an ask that is below the current high bid.

Screen 29: The seller is prompted to enter an ask that is at or below the current high bid, in order to produce a new trade.

Screen 30: Changes to the Current Allocation, Trade Summary, and other screen displays that result from the most recent trade are reviewed.

Screen 31: The seller is informed of the Vote to End Period option, and the unanimity rule that triggers an early end to the trading phase of the current period.

Screen 32: A Period Profit window appears during the review phase of each period.

Screen 33: Subjects are cautioned that the ask by another seller and the bids by other buyers in the instructions were simulated and that these may not be similar to the responses by buyers and by other sellers during the experiment. The subject is informed that he has now completed the instructions and trading will begin when all subjects have completed their instructions.