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Bidding Strategies in Internet Yankee Auctions®*

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Abstract

A bidding strategy commonly observed in Internet auctions, though not frequently in live auctions, is that of "jump-bidding," or entering a bid larger than necessary to be a current high bidder. In this paper, we argue that the cost associated with entering on-line bids and the uncertainty concerning bidding competition -- both of which distinguish Internet from live auctions -- can explain this phenomenon. We present a simple theoretical model that accounts for the preceding characteristics, and derive the conditions under which jump-bidding constitutes an equilibrium strategy in a format commonly used for on-line trading, the Yankee Auction®. We then present evidence recorded from hundreds of Internet auctions that is consistent with the basic predictions from our model. We find that jump-bidding is more likely earlier in an Internet auction, when jumping has a larger strategic value, and that the incentives to jump bid increase as bidder competition becomes stronger. Several of our results have implications for starting bid and minimum bid increment rules set by Internet auction houses. We also discuss possible means of reducing bidding costs, and evidence that Internet auctioneers are pursuing this goal.

1. Introduction

The last few years have witnessed a tremendous proliferation of various forms of electronic transactions. In a recent issue of *U.S. News and World Report* it is reported that a minimum of $7 billion worth of transactions took place using electronic commerce during 1998. [1] More importantly, this figure is expected to continue to grow at a high rate over the next few years.

[1] This figure could actually be as high as $13 billion (see Holnstein *et. al.* 1998).

Forecasters predict that by the year 2003, the number of U.S. households that shop on line will be in the order of 40 million or higher, compared to less than 10 million in 1998.

Although the vast majority of electronic transactions are based on posted-prices, auctions have recently become a very popular means of selling commodities on the Internet. Lucking-Reiley (1998) reports that about $15 million worth of goods are sold in Internet auctions each week. [2] The spectrum of goods sold at these auctions has increased considerably in the last few months. These range from computer and electronics (new and refurbished), to collectibles, toys, trading cards, books, music, and many others. More recently amazon.com and the Home Shopping Network, among others, have opened their own auction site where a broad range of consumer products is offered every day.

There are two basic types of Internet auction sites: merchant sites and listing sites. In merchant sites (e.g., Onsale, Ubid, Firstauction, Surplusauction), the items offered come directly from manufacturers. In this case, the auction site plays the role that a regular retailer would play. Typically, these sites offer more than one unit of the same item for sale in open bid simultaneous auctions. In contrast, listing sites (e.g., eBay, Yahoo! Auctions, City Auction), facilitate person-to-person transactions by creating virtual marketplaces where private bidders and sellers can get together. Most auctions in listing sites are for single items.

A popular format in all of the merchant sites is the "Yankee Auction®." [3] This format is a variation of the multi-unit ascending auction described by Vickrey (1962), and Ortega-Reichert (1968). The features of this auction are: (a) there are several identical units for sale; (b) each

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[2] This estimate is as of May of 1998. Currently, this amount has more than doubled (Lucking-Reiley, private conversation, 1999).

[3] This name is a trademark of Onsale.com.

bidder may purchase more than one unit, but all units must be demanded at the same price; (c) bidding takes place in a progressive fashion until a predetermined time allotted for the auction expires; [4] (d) all winning bidders pay their own prices; and (e) ties are broken on a quantity first then time basis. [5] Each auction also specifies minimum starting bids and bid increments.

An important feature of Yankee-type auctions, and most other Internet auctions, is that the cost of participating in a given auction is different from its counterpart in live auctions. In live auctions, there may be a fixed cost of attending (opportunity cost plus possible entry fees), but once you are at the auction the cost of entering a bid is zero. Thus, in a live Yankee-type auction with private values, [6] bidders have a dominant strategy of "ratchet" bidding, i.e., increase bids by the minimum possible increment until the price reaches one's value. On the other hand, Internet auctions take place over a relatively long period of time. Thus, it would be difficult for bidders to costlessly monitor all of the bidding progression and enter ratchet-type bids in any given auction. Furthermore, bidders are likely to incur a cost every time they enter a bid in an Internet auction. This is because (a) there is an Internet connectivity cost, and (b) there is an opportunity cost associated with logging on to the site, filling the bidding form (and confirming

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[4] This is typically at least 24 hours. Some sites have soft deadlines and apply an activity rule at the end of the auction. This rule usually states that the auction will be closed after five minutes have gone by since the last bid was made.

[5] This means that if two bidders enter the same bid price, the one that bids for more units ranks higher. If both bidders bid for the same number of units, the one that placed the earliest first bid ranks higher.

[6] A private value is one that is idiosyncratic to each bidder. For modeling purposes, each bidder is assumed to know her value but faces uncertainty about other bidders' values.

it), and entering the bid itself.[7] Most of the auction sites spare bidders some monitoring costs by sending e-mail notices whenever a bidder has been displaced from the set of winners.

In this paper we develop a simple model that shows that the presence of bidding costs may induce bidders to move away from ratchet strategies in Yankee-type auctions. In this model, we derive conditions on the bidding cost that would cause bidders to enter jump bids (bid increments that are larger than the minimum) in equilibrium. These jump bids play the role of signals sent to potential subsequent bidders in the auction. Signaling bids are credible because jumping is potentially costly: some money may be left on the table if subsequent bidders are weak.[8] It is conceivable that both the sender and the recipient of the signal may be better-off in a jump-bidding equilibrium. The sender because she saves herself bidding costs by deterring potential competition, and the receiver because he saves himself the costs of fruitlessly bidding against a strong opponent. We analyze an example where bidders draw their valuations from a uniform distribution, and show that the jump-bidding equilibrium may be either of the pooling or separating variety, depending on the parameter configurations of the model.

Our model is strongly influenced by the work of Daniel and Hirshleifer (1998), who present an exhaustive equilibrium analysis of a costly sequential bidding model with two bidders. Using takeover bidding as the main motivation, they establish that the presence of bidding costs[9]

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[7] Both authors have experienced these costs on many occasions. Depending on site congestion, it may take as long as 10 to 15 minutes to complete the process of entering a single bid.

[8] Hereafter, the terms signaling bid and jump bid are used interchangeably.

[9] The source of the bidding costs in Daniel and Hirshleifer's model is mainly related to preparing and announcing the bid. However, their formulation is general enough to accommodate other sources as well.

may induce bidding delays and jump bids in "spontaneous" ascending auctions, i.e., auctions without a predetermined bidding increment. While our approach borrows significantly from theirs, their focus is on the effect of bidding costs on the participation decision and bidding delays. In our model, the participation cost is sunk, and instead we highlight the effect of bidding costs on the decision of whether to use a ratchet or jump bidding strategy. Another related model is by Avery (1998), who shows that high-valuing bidders can effectively use jump-bidding as a signal in common-value auctions. [10] The goal of this signal in that model is to let low-valuing bidders know that an attempt to compete with them may result in a high probability of incurring the winner's curse. Because of the common-value setting, no costs are necessary to obtain jump-bidding equilibria in that case.

In addition to developing a model that explains jump-bidding, we also present evidence of systematic jump-bidding in real-life on-line auctions. We use data recorded from over 200 Yankee-type auctions from two Internet merchant sites to test some of the predictions of our model. Our basic results indicate that, conforming to predictions, jump-bidding is a very common phenomenon in these auctions. In our sample, over 40 % of the bidders place jump bids, and these bids are much more likely early in the auction, when signaling has a larger strategic value. In addition, we find that the relative size of jump bids increases as the number of bidders relative to units increases. This arises from the fact that higher signaled valuations are needed to deter competitors as the competition gets more intense. Finally, we find that the

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[10] A common value is one that is common to all bidders. For modeling purposes, each bidder is assumed to form an estimate of this common value using her private information. If a bidder wins the auction but pays more than the object's true value, that bidder has fallen prey to the "winner's curse."

relative size of jump bids decreases with the value of the item. Since bidding costs are likely to be independent of the item's value, the downside of jump-bidding increases as objects become more valuable. We also test for other predictions related to the institutional setup of the auctions.

2. Model

There are three identical bidders who will potentially compete to buy two homogeneous units at a Yankee-type auction. [11] Each bidder holds a positive private valuation for one unit denoted v_i (i=1,2,3). Bidders draw their valuations from the common distribution $f(\cdot)$ with support $[0, \bar{v}]$.

There is demand uncertainty in the following sense: Two bidders will participate in the auction with certainty, but an exogenous and commonly known two-point distribution determines whether or not a third bidder will participate in the auction. The probability of the third bidder not participating is q, while she participates with probability (1-q). [12]

Bidders arrive at the auction site in a predetermined order. Let us call the bidder that arrives and bids first bidder 1 (B1), the bidder that arrives and bids second bidder 2 (B2), and if a third bidder arrives and bids at all she will be labeled as bidder 3 (B3). The value of each bidder's current high bid is denoted b_i (i=1,2,3). The two highest bidders at the end of the auction will take the two units. Because valuations are drawn from a continuous distribution, ties occur with probability zero.

We also assume for simplicity that the seller's reservation price equals zero and that the cost of browsing, finding an auction, registering, and entering the first bid is sunk. This implies

[11] This is the simplest setup we found to obtain interesting predictions for this type of auction. A model with more bidders and/or units would be considerably less tractable.

[12] For a general formulation of an auction model with a stochastic number of bidders, see McAfee and McMillan (1987).

that as long as her valuation is non-negative, any bidder will potentially find it profitable to enter an initial bid for one of the items. [13]

Finally, bidders are free to revise their bids upwards as many times as they want, [14] and incur a positive cost c every time they revise their current bid. This cost is common to all bidders and does not change as the auction progresses.

2.1. Ratcheting Equilibrium

As shown in many previous studies, if bidding costs are negligible, i.e., c = 0, the auction we just described is an ordinary "progressive" auction (Vickrey 1962, Ortega-Reichert 1968). In our model, if only two bidders enter the auction each one will take a unit at the minimum price (equal to zero in this case). Alternatively, if all three bidders enter, bidding will progress until the price reaches the valuation of the lowest-valuing bidder. In any case, it is a dominant strategy for each bidder to start at the minimum feasible bid and continue bidding up to the point where the price reaches her own valuation.

The expected payoff for B1 and B2 under this equilibrium is:

$$EP_i = v_i q + E\left[\left(v_i - v_2^m\right) \mid v_i > v_2^m\right](1-q), i = 1, 2, \tag{1}$$

where $v_2^m = \text{Min}\left\{v_{-i}\right\}$, and v_{-i} is the vector of the valuations of all bidders other than i.

On the other hand, should B3 enter the auction, her expected payoff would be:

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[13] It can be easily shown that a positive reservation bid actually endogenizes the decision to participate in these auctions, i.e., there will be a subset of low-valuing bidders that will find it optimal not to enter any bid at all. This subset grows larger as the cost of bidding increases. See Daniel and Hirshleifer (1998).

[14] This can be interpreted as having a "soft" bidding deadline.

$$EP_3 = E\left[\left(v_3 - v_2^m\right) \middle| v_3 > v_2^m\right], \tag{2}$$

where v_2^m has the same meaning as above.

Finally, the seller's expected profit is:

$$E\pi = (1 - q)Ev_3^m, \tag{3}$$

where v_3^m is the order statistic of the minimum of the three valuations.

2.2 Jump-Bidding Equilibrium

If bidders incur a positive cost every time they revise their current bid, it may no longer be optimal for them to use the ratcheting strategy. Suppose that B1, upon arriving to the auction site, faces the choice of bidding the minimum price (ratcheting) or entering a bid that truthfully signals her valuation v_1.[15]

If B1 decides to enter a signaling bid, her expected payoff is:

$$EP_1^S = \left[v_1 - b_1^S(v_1)\right]q + E\left[\left(v_1 - b_1^S(v_1)\right) \middle| \hat{v}_1 > v_2^m\right](1 - q), \tag{4}$$

where $v_2^m = \text{Min}\{v_{-i}\}$, b_1^S is a bid that signals a valuation v_1, and $\hat{v}_1$ is the inverse of b_1^S.

In the above case, B1 wins a unit if B3 does not enter the auction. If B3 enters, on the other hand, she wins only if her valuation exceeds the lowest of her two opponents' valuations.

Alternatively, suppose that B1 chooses to enter a ratchet bid. In this case her expected payoff is:

$$EP_1^R = v_1 q + E\left[\left(v_1 - b_1^S(v_1) - c\right) \middle| \hat{v}_1 > v_2^m\right](1 - q), \tag{5}$$

where v_2^m, b_1^S and $\hat{v}_1$ have the same meaning as above.

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[15] There is no loss of generality in concentrating on truthfully revealing strategies (see Daniel and Hirshleifer, 1998).

In the above case, B1 will take a unit for the minimum price (zero) if B3 does not participate. But, should B3 enter, B1 will have to necessarily revise her bid upwards and incur the bidding cost. As shown elsewhere (Daniel and Hirshleifer, 1998), in this instance B1's optimal "defection" strategy involves revising her ratchet bid upwards, and thus incurring the bidding cost, only once. As a result, B1's second and final bid will be a truthfully signaling bid.

Maximizing (4) yields the optimal signaling bid:

$$b_1^S(v_1) = \frac{1}{\left[\frac{q}{1-q} + H(v_1)\right]} \int_0^{v_1} s\, h(s)\, ds, \qquad (6)$$

where $H(\cdot)$ is the distribution of v_2^m.

The above function is the standard equilibrium bid function for a multi-unit discriminating auction with unit demands, but adjusted for the demand uncertainty factor. Note that if q = 0, (6) reduces to the equilibrium strategy in a three-bidder two-unit auction where each winning bidder pays her own bid. Conversely, as q approaches 1, the optimal bid approaches zero, confirming the optimality of ratchet-bidding when B3 is not expected to enter.

Consider now B2's problem. Since we are interested in signaling equilibria, assume that B1 found it optimal to enter a signaling bid. This means that, upon entering the auction, B2 finds an outstanding high bid that is larger than zero. We will later verify the necessary condition under which B1 would indeed enter this signaling bid.

For purposes of deriving B2's strategy, it is important to note that there is a discontinuity in B2's problem. Since B1 truthfully signaled her valuation, B2 will know how her valuation compares to B1's. If $v_2 < v_1$, the decision of ratchet versus signal that B2 faces is similar to the one B1 faced before, with the difference that B2's signal will be targeted to B3 rather than to the minimum of two valuations. If $v_2 > v_1$, B2 will be guaranteed to win a unit, but because of demand uncertainty, i.e. uncertainty about B3's entry, she still faces the ratchet versus signal decision. However, because she is already assured of a unit, she may not signal her true valuation in equilibrium. It would suffice for enter a bid equal to v_1 to signal that she has a higher valuation than B1's. Thus, the relative ranking of valuations of B1 and B2 changes the shape of B2's bidding strategy.

Let us focus on the case where $v_2 < v_1$, where v_1 is B1's signaled value. [16] Here, B2 faces the same decision that B1 faced before, i.e., enter a signaling bid to discourage B3 from entering the bidding contest, or enter a ratchet bid to exploit the possible absence of B3.

If B2 enters a signaling bid, her expected payoff is:

$$EP_2^S = \left\{ \left[v_2 - b_2^S(v_2) \right] q + E\left[\left(v_2 - b_2^S(v_2) \right) \middle| \hat{v}_2 > v_3 \right] (1 - q) \right\} P(v_1 > v_2), \tag{7}$$

where b_2^S is a bid that signals a valuation v_2, and $\hat{v}_2$ is the inverse of b_2^S.

Alternatively, if B2 enters a ratchet bid, i.e., $b_2 = 0$, she gets a unit for free if she happens to be the final bidder, but she potentially incurs bidding costs if B3 enters the auction. As with B1, B2's best response to any bid by B3 such that $b_3 < v_2$, is to immediately enter a signaling bid and minimize expected costs. Thus, B2's expected payoff from ratchet bidding is:

$$EP_2^R = \left\{ v_2 q + E\left[\left(v_2 - b_2^S(v_2) - c \right) \middle| \hat{v}_2 > v_3 \right] (1 - q) \right\} P(v_1 > v_2), \tag{8}$$

where b_2^s and $\hat{v}_2$ remain as defined above.

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[16] The case where $v_1 < v_2$ yields equilibrium conditions that are qualitatively similar to the present case. Since the mathematics are somewhat more involved, we present that case in the Appendix.

Maximizing (7) with respect to b_2^s yields:

$$b_2^S(v_2) = \frac{1}{\left[\frac{q}{1-q} + F(v_2)\right]} \int_0^{v_2} s\, f(s)\, ds, \tag{9}$$

where $f(\cdot)$ and $F(\cdot)$ are the PDF and CDF of B3's valuation, respectively.

As was the case with B1, B2's optimal bid is analogous to the two-bidder single unit case, with the added adjustment for demand uncertainty.

We are now ready to state our main result:

PROPOSITION 1. (a) There exists a pair of bidding costs (c_1*, c_2*) such that, for $v_1 > v_2$, if $c >$ Max {c_1*, c_2*}, both B1 and B2 will enter signaling bids (b_1^s, b_2^s) in equilibrium. (b) B3's best response to these signaling bids is to bid $b_3 = \text{Min}\{v_1, v_2\}$ if $v_3 > \text{Min}\{v_1, v_2\}$, and refrain from bidding otherwise.

PROOF. (a) The critical cost for B1, c_1*, follows from subtracting (5) from (4) and combining the resulting expression with (6), which results in the following condition:

$$EP_1^S > EP_1^R \text{ if } c > c_1^* = \frac{\int_0^{v_1} s\, h(s)\, ds}{\left[1 + \left(\frac{1-q}{q}\right) + H(v_1)\right] H(v_1)} \tag{10}$$

Similarly, the critical cost for B2, c_2*, follows from subtracting (8) from (7) and combining with (9). This yields:

$$EP_2^S > EP_2^R \text{ if } c > c_2^* = \frac{\int_0^{v_2} s\, f(s)\, ds}{\left[1 + \left(\frac{1-q}{q}\right) + F(v_2)\right] F(v_2)} \tag{11}$$

Part (b) follows from the fact that both B1 and B2 entered signaling bids. ÿ

COROLLARY. The bidding cost that induces either bidder (1 or 2) to enter a signaling bid increases with the probability that B3 does not enter the auction. As this probability approaches 1, the critical bidding cost approaches infinity.

Observe that both critical costs result from similar expressions ((10) and (11)). Thus, it suffices to show that this claim holds for c_1*. Differentiating c_1* with respect to q yields,

$$\frac{\partial c_1^*}{\partial q} = \left[\frac{-c_1^*}{\left[1 + \left(\frac{1-q}{q} \right) H(v_1) \right] H(v_1)} \right] \left[-\frac{1}{q^2} \right] > 0 \tag{12}$$

The intuition for this result is simple: If the first two bidders believe that there is a strong chance that B3 will not enter the auction, signaling loses most of its strategic value because it is very likely that a signal recipient will not exist. In this case, for any given c, B1 and B2 are better-off taking their chances and entering a ratchet bid.

The slope of the relationship between B2's signaling decision and her valuation cannot be evaluated unambiguously. To gain some insight on this issue we next analyze the special case where bidders draw their valuations from a uniform distribution.

3. An Example: Uniform Distribution of Valuations

Suppose an individual bidder's valuation is distributed uniformly on [0, 1]. In this case, from (10) and (11), the bidding costs that make B1 and B2 indifferent between signaling and ratcheting are:

$$c_1^* = \frac{v_1 \left(1 - \frac{2v_1}{3} \right) (1-q) q}{[q + (1-q) v_1 (2 - v_1)][(1-q)(2-v_1)]}, \tag{13}$$

and

$$c_2^* = \frac{v_2 q}{2\left[q + v_2\left(1 - q\right)\right]} \qquad (14)$$

In Figure 1 we plot c_i* against v_i (i = 1, 2) for given values of q, the probability of B3 not entering the contest. As we see in this figure, there is an increasing relationship between a bidder's type and the bidding cost that makes her indifferent between jumping and ratcheting. This arises from the fact that higher-valuing bidders are more able to absorb the bidding cost, which implies that higher costs will be required to discourage these bidders from ratcheting.

(**Figure 1 about here)**

A second point we can infer from Figure 1 is that, depending on the actual value of the bidding cost, the equilibrium may be of the pooling or separating type. For example, in the region where $c \geq 0.05$ and $q \leq 0.1$ all B1 and B2 types find it optimal to enter jump bids. Thus we have a pooling equilibrium resulting from the combination of low costs and the high probability of further competition. On the other hand, in the case where c = 0.15 and q = 0.5 about 55% of all bidder types choose to enter jump bids. This is a separating equilibrium wherein only the lower-valuing bidders are induced to jump-bid because of the relatively high bidding cost and the likelihood of facing further competition.

An important lesson we extract from this example is that for most reasonable values of q, a relatively small cost will induce a large fraction of bidders to enter jump bids. [17] In the two scenarios we discussed above, either all or more than half of all bidder types choose to place signaling bids when bidding costs are in the order of 5% or 15% of the upper support of the valuation distribution, respectively. We find it quite plausible that many real-life bidders may be subject to bidding costs of such magnitude, especially when bidding for relatively small objects.

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[17] This is consistent with the examples presented in Daniel and Hirshleifer (1998).

4. Empirical Analysis

4.1 Predictions

The basic empirical predictions that are consistent with the simple theoretical model we presented in the previous two sections are:

(a) *Bidders that bid early in the auction are more likely to enter jump bids*. There are two reasons for this. First, an early bidder is more likely to anticipate further competition ahead in the auction. Thus jump bids have a greater signaling value. Second, later bidders have more information about the level of a ratcheting bid that would make it certain that she wins. The last bidder at an auction will use a ratchet-bid strategy with virtual certainty.

(b) *Any given bidder is more likely to enter a jump bid earlier in her bidding*. The intuition for this is closely related to (a). Signaling has a higher value earlier in the auction. If a bidder failed to signal effectively early in the auction (by entering too low a jump bid), she may still make an adjustment later on. Given bidding costs, it seems very unlikely that any bidder will enter more than two jump bids (or two jump bids followed by one or more ratcheting bids).

(c) *Jump bids are more likely in auctions with more bidders relative to units*. This follows for two reasons. First, as we emphasized before, having a large number of bidders increases the signaling value of jump-bidding. Second, signal bids are targeted to bidders that would be just excluded from the set of winners. As the number of bidders relative to units increases, the expected valuation of the highest excluded bidder increases (by properties of order statistics). Thus the signaling bid intended to discourage the marginal bidder also increases.

(d) *The relative size of the jump is likely to decrease with the value of the item*. This arises from the fact that the cost of bidding is, most likely, independent of the value of the item. Thus, bidders should face stronger incentives to jump bid in auctions where this cost is a meaningful fraction of the item's value. An implication of this is that a normalized measure of jump size should become smaller as the value of the item increases.

There are two additional predictions that are more related to the institutional structure of the auction than to the strategic behavior of bidders. These are:

(e) *Jump bids are more likely in auctions with a low starting bid relative to item value*.

(f) *Jump bids are more likely in auctions with a low bid increment relative to item value*.

4.2 Data

Data were recorded from 241 separate auctions at two different Internet auction sites, Onsale.com and Ubid.com, using a computer program that ran twenty-four hours a day. The two auction sites specialize to a large extent in consumer electronic and computer products, and offer both new and refurbished items, usually with the item's manufacturer as the seller. Since the items are auctioned for original manufacturers or distributors, and not for individuals, any effect of uncertainty about delivery and warranty on prices should be minimized.

All auctions in our sample ran approximately 24 hours. The data collection program collected publicly posted winning bid information for each auction every 15 minutes, and every few minutes during the last half-hour of the auction, where bidding activity is most concentrated, until the auction was declared closed. Each new set of bid data was compared to the previous round to see if any change had occurred in bid values or winning bidders. If so, the data were kept, and counted as a new round of bidding. If there was no change, the data were discarded as redundant. On average, 77% of the collected data were discarded due to no change in the winning bids, leaving us reasonably confident that we did not miss significant bidding activity.

The auctions sometimes attracted bidders interested in a large number of units, who were often judged to be dealers by other bidders based on comments attached to their bids. Frequently, these bidders would appear early in an auction, and disappear when prices reached certain level. In order to reduce the distortions the sustained presence of such bidders might present, we discarded all auctions in which a winner purchased more than 2 units, or a bidder bid for more than 2 units in the late rounds. [18] This resulted in the elimination of 39 auctions from the sample.

Table 1 presents some descriptive statistics of the 202 auctions that remain in our sample. Although no single auction matches this exactly, the typical auction we observed would offer 10 units and last 25 rounds, with 52 bids placed by 34 bidders. Examining some of the extreme cases provides a sense of the range of bidding activity captured in the sample. In one auction 17 units of a pager were offered, but only 3 were sold, to 3 bidders, in 3 rounds, each paying the minimum starting price of $29. In another auction H-P offered 73 units of a personal computer, attracting 191 bidders. In yet another case, 97 rounds of bidding were recorded for an auction of 23 units of a different H-P personal computer model, attracting 152 bidders who placed 286 bids.

(**Table 1 about here)**

4.3. Results

In analyzing all bids recorded in our sample, we define a jump bid as a bid which is larger, by at least one bid increment, than the minimum bid required to secure the item in that round of bidding. This will undercount jump bids slightly since it assumes that it is always necessary to strictly beat the current minimum winning bid to win the item in that round. This is not true for

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[18] There are several such possible distortions. First, the presence of multi-unit demand bidders is likely to introduce asymmetries into the bidding contest. Second, if these bidders are dealers, their values may be correlated rather than private. Finally, since these bidders' have positive valuations for several units, demand-reduction strategies may be admissible (see Ausubel and Cramton 1998, Tenorio 1997).

all bidders by typical Yankee-type auction rules, which will break a tie in bids by favoring the bidder who placed her first bid earliest. We ignore this rule in assessing jump bids, so we do not count as jump bids cases where a bidder could have won the item by placing a tying bid, but instead bid up by one increment. In addition, we define a large jump bid as a bid which is at least two bid increments larger than the minimum bid required to secure the item in that round of bidding. This latter measure permits us to test the sensitivity of our results to our definition of jump-bidding, and provides some insight in analyzing some outliers in the data.

Table 2 presents the number of jump bids observed per quartile of bidding rounds. 91% (94%) of all (large) jump bids occur in the first quartile of rounds, and 97% (98%) of all (large) jump bids occur in the first half of the rounds. It is clear that bidders who bid early in the auction, when a jump bid has greater signaling value, are more likely to enter jump bids, as asserted in prediction (a). [19] Note too that less than 0.5% of jump bids are placed in the last quartile of bidding, when ratchet bidding is almost certainly a better strategy.

(**Table 2 about here)**

Prediction (b) concerns a closely related point. A bidder is more likely to place a jump bid early rather than late within her individual bidding. Table 3, Panel A, presents data concerning the timing of the first jump bid by all bidders in the sample. Of the 41% of bidders who placed jump bids, 91% placed their first jump bid as their first bid, around 7% as their second bid, and less than 2% any later than that. In Panel B we see that of the 41% of bidders who place jump bids, 85% place just one jump bid, 15% place two separate jump bids, and no

[19] Although we do not run a formal test, it is obvious that any such test would overwhelmingly support the prediction.

bidders in our sample place more than two jump bids. Once again the evidence is largely consistent with the theoretical prediction.

(**Table 3 about here)**

Figure 2 presents a scatter plot of all 202 auctions, with the horizontal axis representing the ratio of the number of bidders to the number of units available, and the vertical axis representing the percentage of bids which were jump bids of any size. Note that for all ratios under 1 there is no jump-bidding observed. This is to be expected, since there is no advantage to signaling if all bidders can acquire an item for the minimum bid amount. As the ratio of the number of bidders to units increases, there are both more bidders to signal to, and a higher expectation of the realization of the highest valuations, so we expect jump-bidding to increase. Though data are somewhat sparse for higher values of the ratio, the percentage of jump bids occurring increases in Figure 2 (Spearman rank-correlation between these two variables is .29) thus supporting prediction (c). We do not show a plot for the percentage of large jump bids, as it is nearly indistinguishable from Figure 2.

(**Figure 2 about here)**

In Figure 3 we use the minimum winning bid in the final bidding round as a proxy for item value, and plot it against the average of all jump bids relative to this value. Although our data are sparse at high levels of item value, and there are several auctions of low-value items for which there is no jump-bidding at all, it is clear that the size of the relative jump bid decreases as the value of the item increases (rank-correlation of -.45). This shows support for prediction (d). The plot restricted to large jump bids is again nearly identical and is not shown here.

(**Figure 3 about here)**

Figure 4 arrays each of the 202 auctions along the vertical axis using the percentage of bids that are jump bids. The horizontal axis represents the ratio of the minimum starting bid to the item value, again proxied by the overall minimum winning bid at the end of the auction. It is clear that the percentage of jump bids decreases as the required starting bid approaches the value of the item. Spearman rank-correlation here is -.29 (-.31 for large jumps). This supports prediction (e), and has some implications for the auctioneer as well: As long as it is reasonable to expect more bidders than units available, there would appear to be little risk in setting a low minimum starting bid, as jump-bidding is likely to occur in such cases.

One would expect that at a ratio of 1:1, when the minimum starting bid equals the value of the object for the lowest bidding winner, there would be no jump-bidding. We do find one auction, however, where 50% of bidders placed jump bids, though some bidders were successful in obtaining the item for the minimum starting bid. This is not the case, however, with the more conservative large-jump-bid measure, revealing that those jump bids were just one bid increment greater than what was required to win the items.

(**Figure 4 about here)**

Figure 5 again arrays each auction on the vertical axis using the percentage of bids that are jump bids, as in Figure 4. However the auctions are arrayed on the horizontal axis according to the ratio of the bid increment (which was 5, 10, 15 or 20 dollars for nearly all auctions in the sample) to the minimum winning bid in the final round. Here it is clear that jump-bidding in general occurs only when the bid increment is less than 15% of the approximate value of the item (for large jumps, jump bids occur only when the bid increment is set at less than about 8% of the item value). It is also clear that jump-bidding is less likely as the relative value of the bid increment increases, supporting prediction (f). The Spearman rank-correlation of -.61 (-.69 for large jumps) is very high in this case .

While this result is somewhat tautological, given that we define jump bids in terms of bid increments, there are still some implications for auction design here. It appears that there is no penalty for making the bid increment small, since jump-bidding from bidders with higher bidding costs will still move the auction along quickly. Setting the bid increment too high relative to item value might discourage signaling bidders because their calculated signal bid may not appear as a jump, given the large increment. In addition, you may discourage ratchet bidders, who still make up a significant fraction of bidders in many cases.

(**Figure 5 about here)**

5. Discussion and Conclusions

In this paper we have presented simple theoretical arguments to explain why bidders may choose to follow jump-bidding strategies in auctions with bidding costs. Using a multi-unit, stochastic demand extension of a model by Daniel and Hirshleifer (1998) we show that jump bids may play a signaling role in auctions where it is costly for bidders to revise their bids. Our model specifies conditions on the bidding cost that will induce bidders to use jump-bidding strategies in equilibria of progressive-type auctions. We also show that, depending on the parameters of the model, the equilibria may be of the pooling or separating type.

We have also presented extensive evidence from a sample of Internet Yankee-type auctions that is largely consistent with the theoretical predictions. Our analysis shows that a very large fraction of bidders at these auctions enter jump bids and that these bids are mostly entered early in the auction, when they have greater strategic value. We also show that jump-bidding is more likely in auctions where bidder competition is more intense, and that jump bids are relatively larger for objects of low value, resulting from the larger incidence of the bidding cost relative to the object's value.

There are a number of interesting questions that remain open, both on the bidder and seller side of auctions with costly bidding. For instance, on the bidder side we assumed that bidders arrive to the auction site on a predetermined basis. As such, timing is not part of the bidding strategy. Yet, one may argue that a strategy of waiting to bid until the final minutes of the auction may be advantageous. [20] A formal analysis of this claim would probably require an asymmetric model, where higher-cost bidders self-select to bid earlier in the auction and are more likely to enter jump bids. Conversely, bidders with lower costs could afford to keep track of the bidding more closely and be active at the specific times of the day when the auctions are set to end. Of course the downside of these timing strategies would be that bidding deadlines are often firm and, if the site is very congested at the end of an auction, one may lose the opportunity to bid altogether. Other extensions on the bidder side may include more bidders/items, richer structures of uncertainty and valuations, and multi-unit demands.

There are many issues that remain unexplored on the seller side as well. For instance, as shown by Daniel and Hirshleifer (1998), seller expected revenue in auctions with costly bidding is smaller than its costless counterpart by a factor related to the expected bidding costs. As such, it is in the seller's best interest to undertake actions that will reduce the cost of bidding. It seems that real-life Internet auctioneers are perfectly aware of this phenomenon. We have found that sellers have introduced two practices aimed towards reducing bidding costs.

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[20] In fact a large amount of bidding activity is usually observed during the closing minutes of each auction.

The first bid-cost reducing seller practice is "proxy bidding."[21] With proxy bidding, bidders may privately provide their maximum possible bid (reservation price) to a secure computer program which will update their current high bid to the minimum necessary for that bidder to be included in the set of winners. In other words, this mechanism seeks to turn these auctions into Vickrey-like auctions and eliminate the potential costs a bidder will incur when revising her bid. A curious phenomenon is that although many auction sites offer this service as an option, not all bidders choose to use it. It may be that bidders do not trust the privacy of the interaction or the reliability of the computer program, or perhaps they would like to retain the flexibility to revise their reservation price if value correlation is an issue. An additional complication introduced by proxy bidding is the introduction of heterogeneity. How should non-proxy bidders bid against proxy bidders? Is jump-bidding more or less likely in these cases? These are interesting questions for future research.

The second bid-cost reducing seller practice is "quick auctions," also known as "flash auctions" or "express auctions." These are auctions where all of the bidding takes place within a short time period, usually a half-hour or an hour. Presumably, what auctioneers intend to do with these auctions is to attract bidders with low bidding costs, i.e., bidders that can afford to be in front of their computers for the duration of the auction. Our preliminary analysis of a sample of these auctions reveals that no meaningful jump-bidding occurs in them, i.e., they are more like live auctions. Although it is clear that low-cost bidders should be typically attracted to these auctions, the net effect over the auctioneer's revenue would ultimately depend on whether these bidders hold some other special characteristics that could affect the revenue in a well-defined way. For instance, are these bidders more likely to have lower values than the rest of the bidding

[21] Also known as "bid agents", 'bid-makers", "bid elves", or "bid butlers."

population? If so, quick auction revenues could actually be negatively affected by bidder selectivity. We intend to examine this as well as other related issues in future projects.

6. Appendix

This appendix derives the condition under which bidder 2 (B2) will utilize a jump-bidding strategy given that bidder 1 (B1) has truthfully signaled her valuation and $v_2 > v_1$.

If $v_2 > v_1$, B2 is assured to win one of the units. However, should B2 decide to enter a jump bid, all she would need to signal is that $v_2 > v_1$ (and not necessarily her true valuation). To credibly signal this fact, while at the same time winning a unit, she must bid

$$b_2^S = v_1, \tag{A1}$$

which yields expected payoff:

$$EP_2^S = E\left[(v_2 - v_1)\middle|v_2 > v_1\right] \tag{A2}$$

On the other hand, if v_2 uses a ratchet-bidding strategy, her payoff is:

$$EP_2^R = v_2 q P(v_2 > v_1) + E\left[(v_2 - v_2^m - c)\middle|v_2 > v_1\right](1-q) \tag{A3}$$

Subtracting (A3) from (A2) yields the bidding cost condition under which B2 will enter a jump bid:

$$EP_2^S - EP_2^R > 0 \text{ if} \tag{A4}$$

$$c > c_2^{**} = \frac{v_2 q\, P(v_2 > v_1) + E\left[(v_2^m - v_m)\middle|v_2 > v_1\right](1-q) - E\left[(v_2 - v_1)\middle|v_2 > v_1\right]}{(1-q)\, P(v_2 > v_1)}$$

Since $v_2^m = \mathrm{Min}\{v_1, v_3\}$, the numerator of (A4) is necessarily positive, implying that $c_2^{**} > 0$. As a result, conditions (10) and (A4) define the jump-bidding equilibrium when $v_2 > v_1$. ÿ

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Table 1

Data Description

Number of Auctions in Sample: 202

	Number of Bidders	Units Available	Rounds of Bidding	Number of Bids Placed
Total	6894	2051	5103	10565
Minimum	3	2	2	3
Maximum	191	73	97	286
Average per Auction	34	10	25	52

Number of Bidders is the number of bidders for each auction, and may include individual bidders who bid in multiple auctions in the sample.

Units Available is the number of units of an item available in each auction.

Rounds of Bidding is the number of recorded bidding rounds in which a new bid occurred.

Number of Bids Placed is the number of new bids placed.

Table 2

Number of Jump Bids Observed per Quarter of Recorded Bid Rounds

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Total
All Jump Bids	3088	214	75	13	3390
Percent All Jump Bids	91.1%	6.3%	2.2%	0.4%	
Large Jump Bids	1963	96	30	9	2098
Percent Large Jump Bids	93.6%	4.6%	1.4%	0.4%	

All Jump Bids includes any bid at least 1 bid increment greater than the minimum bid required to win at that time.
Large Jump Bids includes any bid at least 2 bid increments greater than the minimum bid required to win at that time.

Table 3

Timing of First Jump Bid and Number of Jump Bids for Each Bidder

Panel A	Bidder's First Jump Bid Occurs				
	As First Bid	As Second Bid	As Later Bid	Never	Total
Number of Bidders	2578	207	48	4061	6894
Percent of All Bidders	37.4%	3.0%	0.7%	58.9%	100%
Percent of Jump Bidders	91.0%	7.3%	1.7%		

Panel B	Bidder Places a Jump Bid				
	Once	Twice	More Often	Never	Total
Number of Bidders	2408	425	0	4061	6894
Percent of All Bidders	34.9%	6.2%	0%	58.9%	100%
Percent of Jump Bidders	85.0%	15.0%	0%		

Figure 1
The Cost of Bidding that Makes B1 and B2 Indifferent between Jump Bidding and Ratchet Bidding as a Function of Valuations for a Uniform Valuation Distribtution on [0,1]

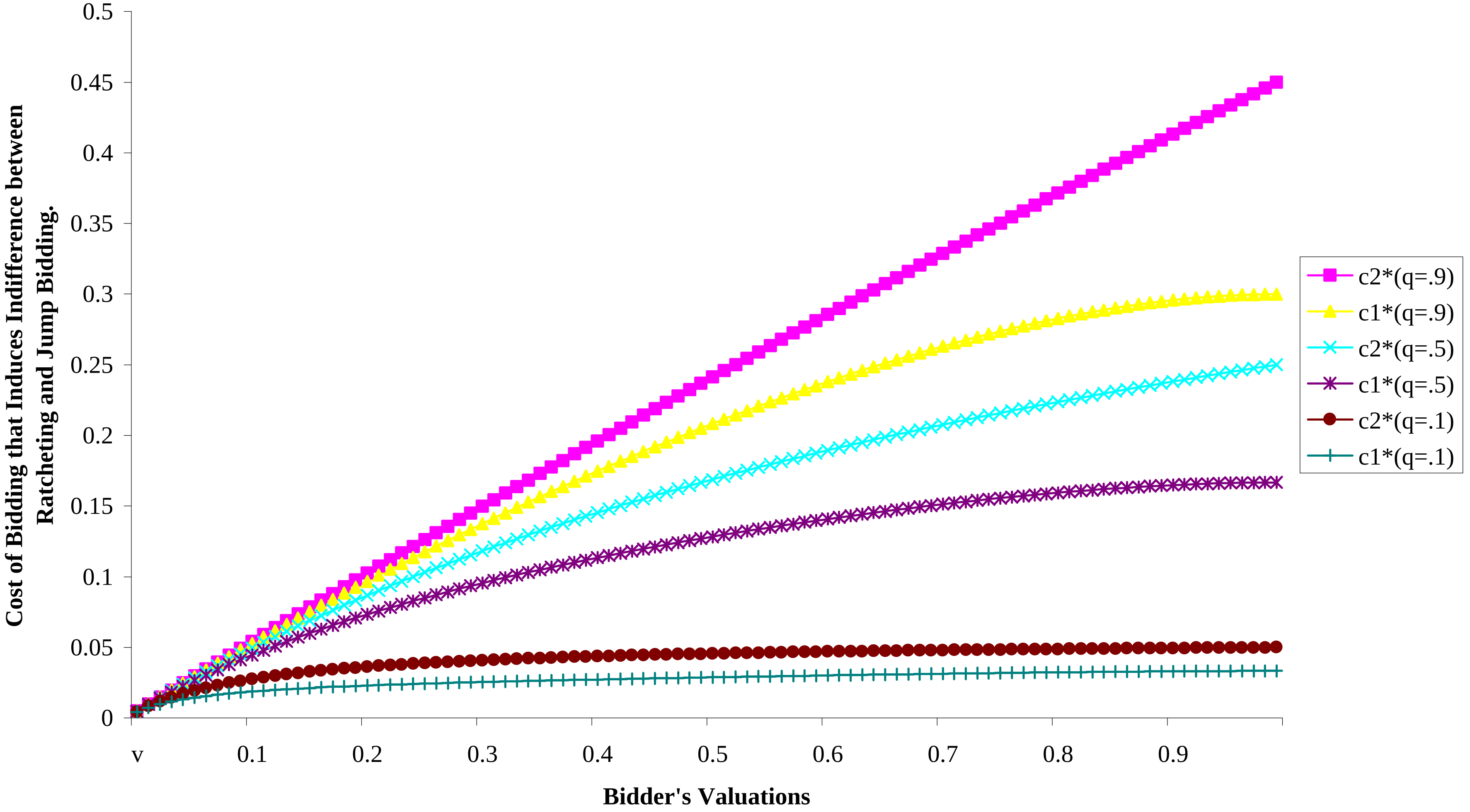

Figure 2
The Percentage of Jump Bids Increases in the Ratio of Bidders to Units Available

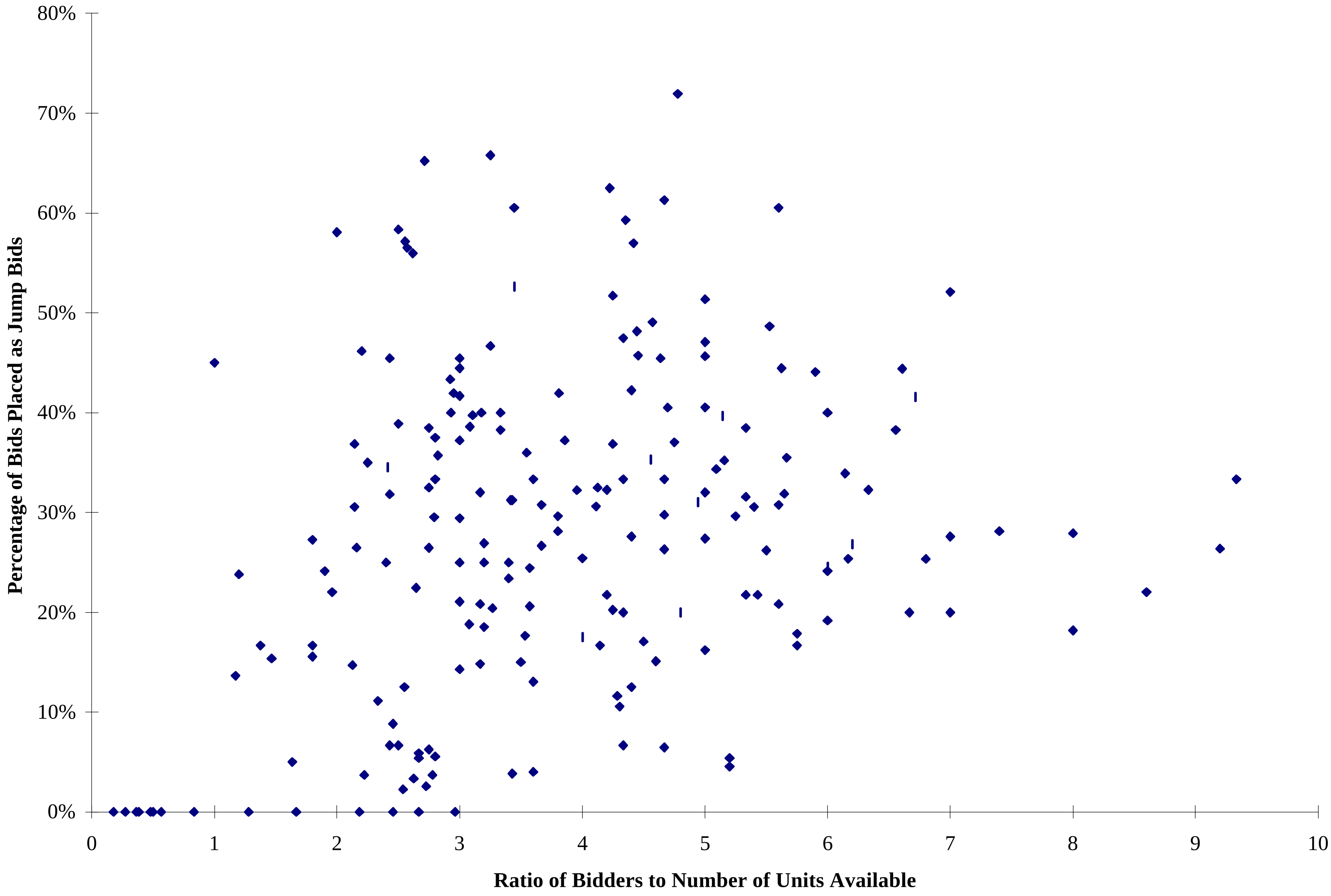

Figure 3
Relative Size of Jump Bids Decreases in the Value of Item

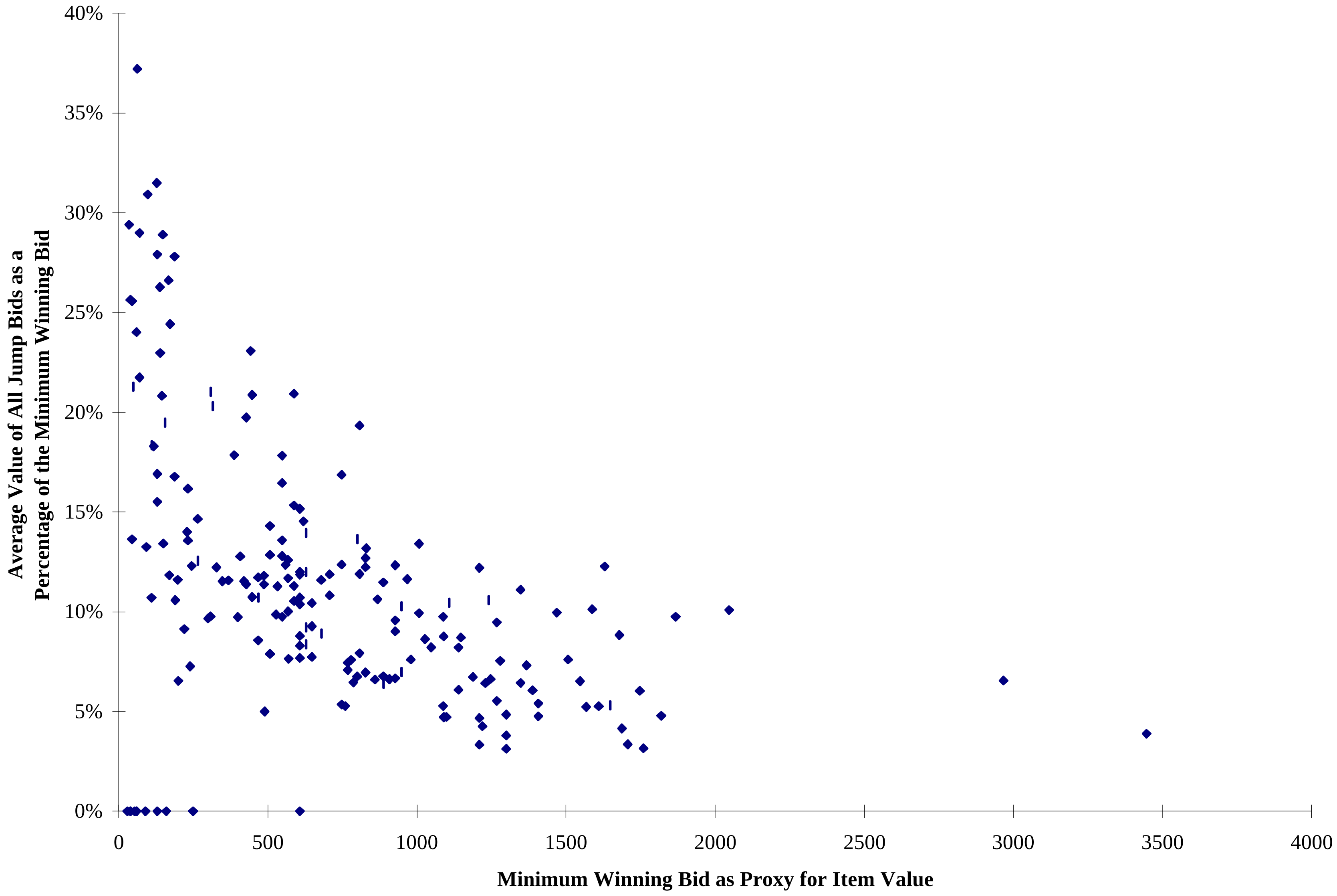

Figure 4
The Percentage of Jump Bids Decreases
in the Size of the Minimum Starting Bid Relative to Item Value

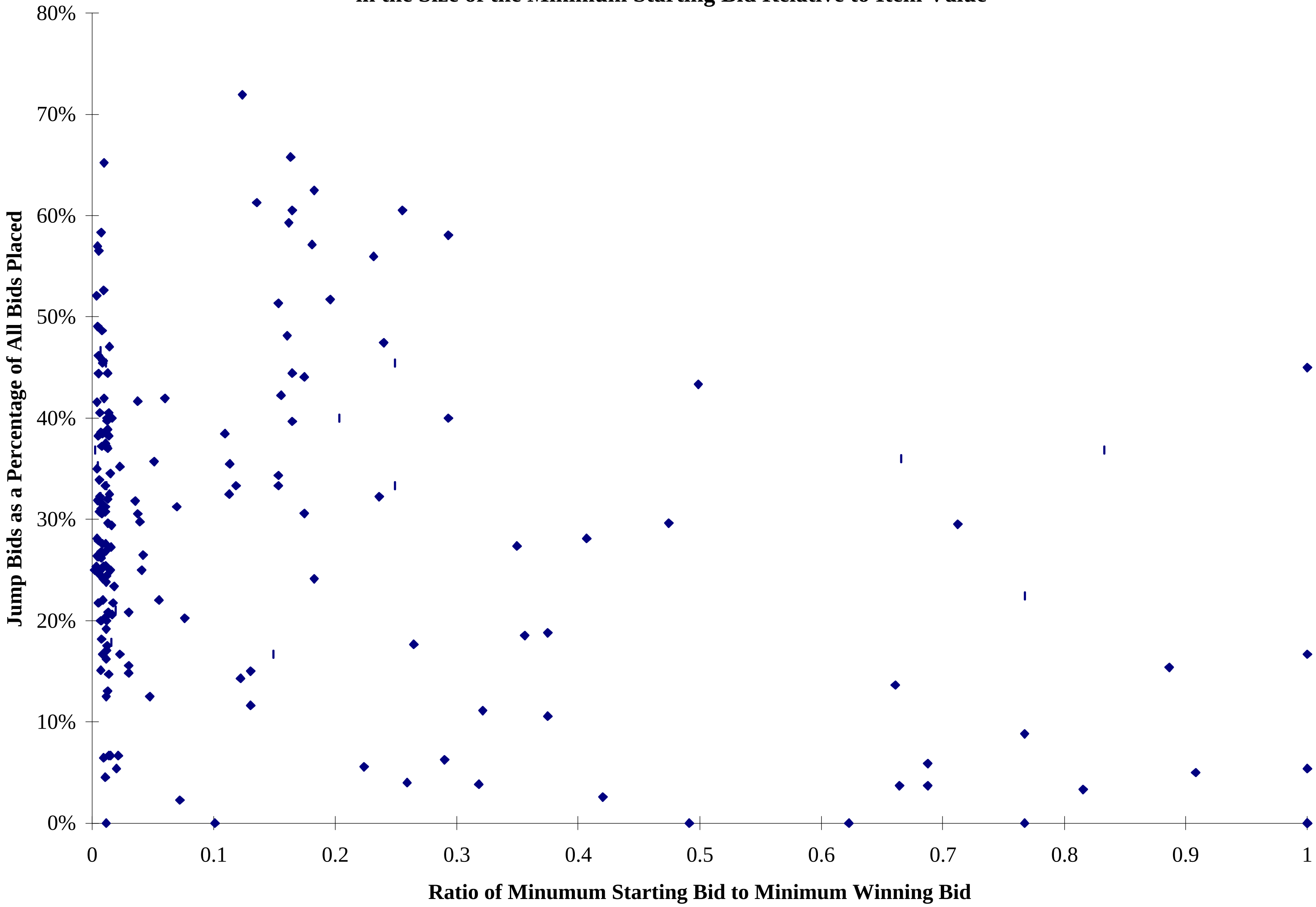

Figure 5
The Percentage of Jump Bids Decreases
in the Size of the Bid Increment Relative to Item Value