The DF Structure Models for Options Pricing On the Dividend-Paying and Capital-Splitting

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Abstract.

Based on the DF structure models for option pricing (F. Dai, 2005), this paper discusses further the DF structure models on three cases, i.e., the underlying stock being dividend-paid, capital-split or dividend-paid and capital-split. These three cases are discussed separately, and are integrated to the general models for call or put. Finally, the examples are given to compare the options prices calculated by the DF formulas and Black-Scholes formulas, and they show, as a whole, that the DF formulas are not inferior to Black-Scholes formulas. DF formula is useful to traders in financial market because it is convenient to adjust along with the trading time.

Key Words.

DF structure model, options pricing, dividend-paying, capital-splitting

1 Introduction

After the options, a kinds of important derivative products, is occurred, the researchers its pricing model are paid attention very much, and especially the American put option. In the studies of option pricing, there have been many significant results (F. Black, M. Scholes, 1973; R.C. Merton, 1976; W.F. Sharpe, 1978), and approximation methods for American put option (H.E. Johnson, 1983; R. Geske, and H.E. Johnson, 1984; L.W. MacMillan, 1986; R. S.Stapleton and M.G. Subrahmanyam, 1997). No exact analytic formula has ever been produced for the value of an American put option until 2005. The structure model for options pricing is given by F. Dai [A]. It could price any of options, include American put option. The new problems are the way how the options be priced after dividend-paying and capital–splitting on its underlings, including European and American call or put options, because the structure model for options pricing (F. Dai, 2005) is on a non-dividend-paying stock.

Based on the structure model for options pricing, this paper will give the ways to price the options after dividend-paying and capital–splitting on its underlings.

Finally, some of the empirical examples are offered. By these examples, we shall see the models given in this paper are suitable to price the options on underlings with dividend-paying and capital–splitting, and very convenient in application.

2 The basic assumptions about stock price

The basic assumptions we use to define an underlying (stock and stock indices) price, regarded as the basis of the discussion in this paper are as follows:

- There are prices (the cost price and the market price) to an underlying assert. The cost price means the average value of all the prices paid by the market traders to buy an underlying asset and the market price is the current exchange price of an underlying asset.
- The prices (cost price and market price) have been fluctuating with time. Any price is non-negative, and the fluctuation range (i.e., the variance) of price is positive.
- Both the cost price and the fluctuation of cost price of an underlying are the basic elements which determine the market prices of the underlying stock. The market prices come into being on the market exchange.
- The possibilities that the market price of underlying is much lower than the cost price, or is much higher than the cost price, will be very small.
- There are no transaction costs or taxes in trading.
- The cost price need not to be continuous, and the market prices are distributed continuously

We shall prefer referring to stock instead of asset, at the same time, the stock can also be replaced by stock indices, and we mention no more in the following discussions.

3 Partial distribution and DF structure

First, we give some basic definitions as follow:

Definition 1 (The Partial Distribution). Let S be a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx & x\geq 0 \\ 0 & x<0 \end{cases} \tag{1}$$

then S is said to satisfy the partial distribution, and denotes $S\in P(\mu,\ \sigma^2)$. The partial distribution is a kind of truncated normal distribution.

Definition 2 (The Partial Process) If stochastic variable S is related to time, i.e. $\forall\ t\in[0,\infty)$, we have $S(t)\in P(\mu(t),\sigma^2(t))$, then the $\{S(t),\ t\in[0,\infty)\}$ is called a partial process.

In general, the stock price changes along with the time, therefore we have

Assumption 1 Let $\mu(t)$ be the cost price of stock at the time t, and $\sigma^2(t)$ be the variance of cost price at the time t. If the market prices of stock follow the basic assumptions in section 2, thus suppose that $S(t)$, the market price variable, follows the partial distribution at time t, and denotes $S(t)\in P(\mu(t),\ \sigma^2(t))$.

$S(t)$ can be regard as a stock or the market price of the stock. From [16], we have the following theorem 1 and theorem 2:

Theorem 1 Let S, the market price variable of a stock, follow the partial distribution $P(\mu,\ \sigma^2)$, thus

1) The expected value $E(S)$ of S, the average of market price, is as follows

$$E(S)=\mu+\sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{\sigma^2}}dx \tag{2}$$

where, $R(S)=\sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^\infty e^{-\frac{(x-\mu)^2}{2\sigma^2}}dx$ is the average trading profit.

2) The variance of the market price variable S, the risk of the market price, is as follows

$$D(S)=\sigma^2+E(S)[\mu-E(S)] \tag{3}$$

Theorem 2 For any $x\in[0,\infty]$, the following equations are correct approximately:

1) $\int_0^x e^{-\frac{t^2}{2}}dt=\sqrt{\frac{\pi}{2}(1-e^{-\frac{2}{\pi}x^2})}$.

2) $\int_0^x e^{-\frac{(u-\mu)^2}{2\sigma^2}}du=\sqrt{\frac{\pi}{2}}\sigma\times(\sqrt{1-e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}}+\mathrm{sgn}(x-\mu)\sqrt{1-e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}})$.

where, $\mathrm{sgn}(x)=\begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$;

Essentially, the partial distribution describes stock prices in its distribution construction.

Definition 3 (DF process) If $\{\xi(t),\ t\in[0,\infty)\}$ is a stochastic process, and $\forall\ t\in[0,\infty)$

$$\xi(t)\in P(\mu(t),\ \sigma^2(t)t)$$

then $\{\xi(t),t\in[0,\infty)\}$ is called a DF process.

From reference [17], we have

Definition 4 Let a and b be non-negative constants, If $a>0,b=0$, we define:

$$e^{-\frac{a}{b}}=\lim_{z\to 0^+}e^{-\frac{a}{z}}=0.$$

Definition 5 (DF structure) Let X be the value of an asset related to stock $S(t)\in P(\mu(t),\ \sigma^2(t))$, if $\forall\ t\in[0,\infty)$ and $T>t$, $X_S(t,\ T)\in P(X,\ D[S(t)](T-t))$, i.e. $X_S(t,\ T)$ follows the density distributed as

$$f_{X_S}(x)=\begin{cases} e^{-\frac{(x-X)^2}{2D(S)(T-t)}} \Big/ \int_0^\infty e^{-\frac{(x-X)^2}{2D(S)(T-t)}}dx & x\geq 0 \\ 0 & x<0 \end{cases}$$

then we call $X_S(t, \mathrm{T})$ the DF stochastic structure of X on $S(t)$. $X_S(t, \mathrm{T})$ is called a DF structure of X for short.

When t=T, $X_S(t, \mathrm{T})=X$. So the DF structure means a stochastic value which will be equal to the cash asset X in the future time T under no interest rate.

Although the stock $S(t)$ has certain connections with *DF* structure $X_S(t, \mathrm{T})$ in variance, their stochastic movements may have no inevitable relation, so we have

Assumption 2 Let $X_S(t, \mathrm{T})$ be the DF structure of X on $S(t)$, suppose that $X_S(t, \mathrm{T})$ and $S(t)$ are independent one with another.

4 The prices distribution of underlying assets after dividend-paying and capital-splitting

We have a stock $S(t)\in P(\mu(t), \sigma(t))$, the stock is paid dividends or expanded capital per share at the time τ.

4.1 The prices distribution of stock on dividend-paying and non-capital-splitting

If paying dividends ν at time τ, then stock price is

$S_\nu(t)\in P(\mu_\nu(t),\sigma^2(t))$

where,

$$\mu_\nu(t) = \mu(t) - \nu,\quad \sigma_\nu^2(t) = \sigma^2(t),\ t\geq\tau.$$

After paying dividends, the stock price is lower, but the fluctuation spread (i.e. the variance) of stock price is invariable.

4.2 The prices distribution of stock on non-dividend-paying and capital-splitting

If non-dividend-paying and capital-splitting at τ, then stock price is

$$S_\nu(t) \in P(\mu_\nu(t), \sigma_\nu^2(t))$$

where, $\mu_\nu(t) = k\mu(t)$, $\sigma_\nu^2(t) = k^2\sigma^2(t)$, and $k=\frac{m}{m_1}$, m is the shares before capital-splitting, and m_1 is the shares after capital-splitting, $t\geq\tau$.

After splitting capital, the stock price is lower according to rate of capital-splitting, and the variance of stock price cuts down according to the rate of capital-splitting.

4.3 The prices distribution of stock on dividend-paying and capital-splitting

If paying dividends ν and capital-splitting at τ, then stock price is

$$S_\nu(t) \in P(\mu_\nu(t), \sigma_\nu^2(t))$$

where, $\mu_\nu(t) = k(\mu(t) - \nu)$, $\sigma_\nu^2(t) = k^2\sigma^2(t)$,

ν is the Quantity of dividends paying per share, $k=\frac{m}{m_1}$, m is the shares before capital-splitting, m_1 is the shares after capital-splitting, $t\geq\tau$.

After paying dividends and splitting capitals, the stock price is obviously lower, and further the variance of stock price cuts down according to rate of capital-splitting.

5 The DF Structure Models For Options Pricing On A Non-Dividend-Paying and Non-Capital-Splitting Stock

According to reference [17], we have the following notations:

$S(t)$—the market price of stock at the current time t

X—the strike price of option.

$X_S(t, \mathrm{T})$—DF stochastic structure of X on $S(t)$.

t—current time.

T—time of expiration of option.
r—risk-free rate of interest to maturity T.
$S(t)e^{r(T-t)}$—forward value of $S(t)$ ($\hat{E}(S(\mathrm{T}))$, the expected value in a risk-neutral).
$X_S(t, \mathrm{T})$—DF stochastic structure of X on $S(t)e^{r(T-t)}$.
$C_S(t)$—value of call option to buy one share.
$P_S(t)$—value of put option to sell one share.

If $S(t)\in P(\mu(t),\sigma^2(t))$ and $X_S(t, \mathrm{T})\in P(X, D[S(t)e^{r(T-t)}](\mathrm{T}-t))$, we have the DF structure models of options pricing (DF model for short) as follows:

5.1 The model for call option pricing

The price of call option at time t is

$$C_S(t) = e^{-r(\mathrm{T}-t)} E[\max(S(t)e^{r(\mathrm{T}-t)} - X_S(t,\mathrm{T}),0)]$$

$$= e^{-r(\mathrm{T}-t)} \int_0^{S(t)e^{r(\mathrm{T}-t)}} [S(t)e^{r(\mathrm{T}-t)} - x] f_{X_s}(x)dx$$

$$= (S(t) - Xe^{-r(T-t)}) \times \left[\frac{\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(\mathrm{T}-t)})^2}{D[S(t)](\mathrm{T}-t)}}} + \mathrm{sgn}(S(t)e^{r(\mathrm{T}-t)} - X)\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Xe^{-r(\mathrm{T}-t)})^2}{D[S(t)](\mathrm{T}-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{X^2}{D[S(t)](T-t)}}}} \right]$$

$$+ \sqrt{\frac{2D[S(t)](\mathrm{T}-t)}{\pi}} \left[\frac{e^{-\frac{(S(t)-Xe^{-r(\mathrm{T}-t)})^2}{2D[S(t)](\mathrm{T}-t)}} - e^{-\frac{(Xe^{-r(\mathrm{T}-t)})^2}{2D[S(t)](\mathrm{T}-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(\mathrm{T}-t)})^2}{D[S(t)](\mathrm{T}-t)}}}} \right] \tag{4}$$

When the call option is brought forward to execute at any time $\tau\in[t,T]$, the price of underlying stock, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. Accord- ing to (4) and definition 4, the current value of the option is

$C_S(\tau)=S(\tau)-Xe^{-r(\mathrm{T}-\tau)}$, if $S(\tau)>Xe^{-r(\mathrm{T}-\tau)}$;
$C_S(\tau)=0$, if $S(\tau)\le Xe^{-r(\mathrm{T}-\tau)}$;

i.e. $C_S(\tau)=max\{S(\tau)-Xe^{-r(\mathrm{T}-\tau)}, 0\}$. At this time, the intrinsic value of the call option is

$max\ \{S(\tau)-X, 0\}$

thus

$$C_S(\tau)\ge max\{S(\tau)-X, 0\} \tag{5}$$

5.2 The model for put option pricing

The price of put option at time t is

$$P_S(t) = e^{-r(\mathrm{T}-t)} E[\max(X_S(t,\mathrm{T}) - S(t)e^{r(\mathrm{T}-t)},0)]$$

$$= e^{-r(\mathrm{T}-t)} \int_{S(t)e^{r(\mathrm{T}-t)}}^{\infty} [x - S(t)e^{r(\mathrm{T}-t)}] f_{X_s}(x)dx$$

$$= (Xe^{-r(\mathrm{T}-t)} - S(t)) \times \left[\frac{1-\mathrm{sgn}[S(t)e^{r(\mathrm{T}-t)} - X]\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Xe^{-r(\mathrm{T}-t)})^2}{D[S(t)](\mathrm{T}-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(\mathrm{T}-t)})^2}{D[S(t)](\mathrm{T}-t)}}}} \right]$$

$$+ \sqrt{\frac{2D[S(t)](\mathrm{T}-t)}{\pi}} \left[\frac{e^{-\frac{(S(t)-Xe^{-r(\mathrm{T}-t)})^2}{2D[S(t)](\mathrm{T}-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(\mathrm{T}-t)})^2}{D[S(t)](\mathrm{T}-t)}}}} \right] \tag{6}$$

When the put option is brought forward to execute at any time $\tau\in[t, \mathrm{T}]$, the price of underlying stock, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. According to expression (6) and definition 4, the current value of the option is

$P_S(\tau)=Xe^{-r(\mathrm{T}-\tau)}-S(\tau)$, if $S(\tau)<Xe^{-r(\mathrm{T}-\tau)}$
$P_S(\tau)=0$, if $S(\tau)\ge Xe^{-r(\mathrm{T}-\tau)}$

i.e. $P_S(\tau)= max\{Xe^{-r(\mathrm{T}-\tau)}-S(\tau), 0\}$. At this time, the intrinsic value of the call option is

$max\{X-S(\tau), 0\}$

thus

$P_S(\tau) \leq max\{X\text{-}S(\tau), 0\}$ (7)

The expression (4) can, for short, be written as:

$$C_S(t) = (S(t) - Xe^{-r(\mathrm{T}-t)}) \times \frac{\varphi(d_1) + \varphi(d_2)}{\varphi(\infty) + \varphi(d_2)} + \sqrt{D[S(t)](\mathrm{T}-t)}\ \frac{e^{-\frac{d_1^2}{2}} - e^{-\frac{d_2^2}{2}}}{\varphi(\infty) + \varphi(d_2)} \tag{8}$$

And the expression (6) can be written as:

$$P_S(t) = (Xe^{-r(\mathrm{T}-t)} - S(t)) \times \frac{\varphi(\infty) - \varphi(d_1)}{\varphi(\infty) + \varphi(d_2)} + \sqrt{D[S(t)](\mathrm{T}-t)}\ \frac{e^{-\frac{d_1^2}{2}}}{\varphi(\infty) + \varphi(d_2)} \tag{9}$$

where, $\varphi(x) = \int_0^x e^{-\frac{t^2}{2}} dt$, $d_1 = \dfrac{S(t) - Xe^{-r(\mathrm{T}-t)}}{\sqrt{D[S(t)](\mathrm{T}-t)}}$, $d_2 = \dfrac{Xe^{-r(\mathrm{T}-t)}}{\sqrt{D[S(t)](\mathrm{T}-t)}}$.

6 The Structure Models For Options Pricing On A Dividend-Paying and Capital-Splitting Stock

Here, we suppose that the price of original stock is $S(t) \in P(\mu(t), \sigma(t))$.

After paying dividends or splitting capitals, the price of stock is

$$S_v(t) \in P(\mu_v(t), \sigma_v^2(t)) \tag{10}$$

where, $\mu_v(t)$ and $\sigma_v(t)$ could be ones in section 4.1, 4.2 or 4.3.

DF stochastic structure of X on $S_v(t)\, e^{r(T-t)}$ is $X_{S,v}(t, \mathrm{T}) \in P(X, D[S_v(t)e^{r(\mathrm{T}-t)}](\mathrm{T}-t))$.

Denoting D_v=$D[S_v(t)]$ and Combining formula (4) and formula (6) with theorem 2, we know, after paying dividends or splitting capitals at τ, the prices of options on stock are

6.1 The pricing model for call option on the dividend-paying and capital-splitting stock

The price of call option at time t ($t > \tau$)

$$C_{S,v}(t) = e^{-r(T-t)} E[\max(S_v(t)e^{r(T-t)} - X_{S,v}(t,T), 0)]$$

$$= e^{-r(T-t)} \int_0^{S_v(t)e^{r(T-t)}} [S_v(t)e^{r(T-t)} - x] f_{X_{S,v}}(x) dx$$

$$= (S_v(t) - Xe^{-r(T-t)}) \times \left[\frac{\sqrt{1 - e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D_v(T-t)}}} + \mathrm{sgn}(S_v(t)e^{r(T-t)} - X)\sqrt{1 - e^{-\frac{2}{\pi}\frac{(S_v(t) - Xe^{-r(T-t)})^2}{D_v(T-t)}}}}{1 + \sqrt{1 - e^{-\frac{2}{\pi}\frac{X^2}{D_v(T-t)}}}} \right]$$

$$+ \sqrt{\frac{2D_v(T-t)}{\pi}} \left[\frac{e^{-\frac{(S_v(t) - Xe^{-r(T-t)})^2}{2D_v(T-t)}} - e^{-\frac{(Xe^{-r(T-t)})^2}{2D_v(T-t)}}}{1 + \sqrt{1 - e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D_v(T-t)}}}} \right] \tag{11}$$

6.2 The pricing model for put option on the dividend-paying and capital-splitting stock

The price of put option at time t ($t > \tau$)

$$P_{S,v}(t) = e^{-r(T-t)} E[\max(X_{S,v}(t,T) - S_v(t)e^{r(T-t)}, 0)]$$

$$= e^{-r(T-t)} \int_{S_v(t)e^{r(T-t)}}^{\infty} [x - S_v(t)e^{r(T-t)}] f_{X_{S,v}}(x) dx$$

$$= (Xe^{-r(T-t)} - S_v(t)) \times \left[\frac{1 - \text{sgn}[S_v(t)e^{r(T-t)} - X]\sqrt{1 - e^{-\frac{2}{\pi}\frac{(S_v(t) - Xe^{-r(T-t)})^2}{D_v(T-t)}}}}{1 + \sqrt{1 - e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D_v(T-t)}}}} \right] + \sqrt{\frac{2D_v(T-t)}{\pi}} \left[\frac{e^{-\frac{(S_v(t) - Xe^{-r(T-t)})^2}{2D_v(T-t)}}}{1 + \sqrt{1 - e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D_v(T-t)}}}} \right] \quad (12)$$

6.3 The general models for options pricing

Summing up the discussions above and supposing the dividend-paying and capital-splitting stock is at time τ ($0<\tau<\mathrm{T}$), we have the general models for options pricing as follow.

1) The model for call option pricing is

$$C(t) = \begin{cases} C_S(t), & 0 < t < \tau \\ \\ C_{S,v}(t), & \tau < t \le \mathrm{T} \end{cases} \quad (13)$$

where, t is the current time, $C_S(t)$ is determined by formula (4) or (8), $C_{S,v}(t)$ is determined by formula (11).

2) The model for put option pricing is

$$P(t) = \begin{cases} P_S(t), & 0 < t < \tau \\ \\ P_{S,v}(t), & \tau < t \le \mathrm{T} \end{cases} \quad (14)$$

where, t is the current time, $P_S(t)$ is determined by formula (6) or (9), $P_{S,v}(t)$ is determined by formula (12).

7 The empirical Examples

Here we give an example on the stock of MSFT (MICROSOFT CP) and its options in reference [17].

7.1 The fitting of partial distribution for MSFT

We take the close prices of MSFT as sample data.

Time: Jan. 29, 2002 -Dec. 24, 2002.

The estimated results of parameters are as follows:

$\hat{\mu}$ =53.58500013; $\hat{\sigma}^2$ = 24.62632700;

The corresponding histogram, samples foldgram and fitting curve of Partial Distribution are shown in the figure 1.

7.2 The comparative analysis for MSFT

Time: Dec. 25, 2002.

Product: The option contract on MSFT.

Maturity: Expires After: Fri,16-Jan-04.

Underlying: Close price of MSFT at current date, 53.39$.

In table1, there are the prices on Dec.24, 2002, which were the closing prices traded actually in the United States option market (TP), the call and put options prices calculated by DF structure formulas, (DF) ,and the call and put options prices calculated by Black-Scholes formulas, (B-S).

According to the data from table 1, it is difficult to know whether the DF formula is better than B-S formula or not, we should do further empirical research. Taking the strike price, X=60, and T=212 for

Table 1. Contrast of options prices of **MSFT**

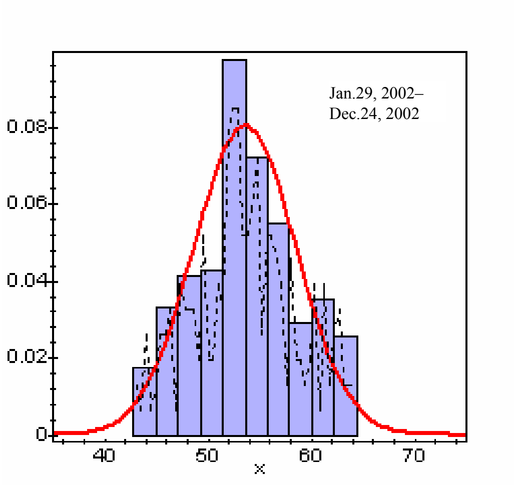
Strike prices	Call options prices			Put options prices		
	TP	DF	B-S	TP	DF	B-S
50.0	11.40	5.406	5.369	7.70	.1399	.1028
55.0	9.20	1.702	1.706	9.90	1.248	1.252
60.0	6.80	.2213	.2466	12.50	4.580	4.605
65.0	5.10	.0212	.0151	15.90	9.192	9.186
70.0	3.80	.0007	.0004	19.50	13.98	13.98

example, the variety of call and put option prices calculated by DF formulas and B-S formulas are respectively shown in figure 2(a) and (b).

7.3 The comparisons between options prices before and after dividend-paying and capital-splitting

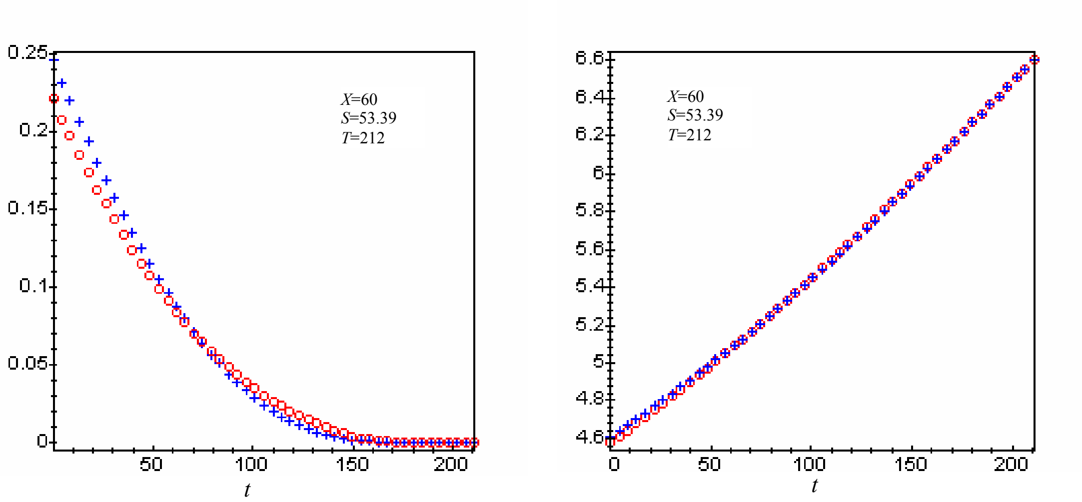
Here are the three examples about comparisons between options prices before and after their underlying stock are dividend-paid and capital-split. The current price of stock is the close price on Dec. 25, 2002, i.e. S=53.39\$. Combining the three cases in section 3.1, 3.2, 3.3 separately, we have

1) Comparisons between options prices before and after the underlying stock being dividend-paid. Suppose, strike price of option is X=60, dividend-paying is v=1.6\$ and completes on τ=60(exchange day). Thus, Comparisons between options prices before and after their underlying stock being dividend-paid are drawn in figure 3(a) and figure 3(b).



Curve of Partial Distribution
Curve of sample flodgram

Figure 1 Partial Distribution fitting for MCFT



(a) Comparison of call option prices (b) Comparison of put option prices

○○○○○○ Curve of DF price
++++++ Curve of B-S price

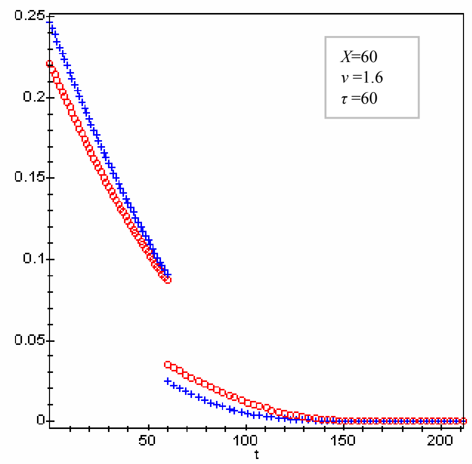
Figure 2 Comparison of options prices on MSFT

2) Comparisons between options prices before and after the underlying stock being capital-split. Suppose, strike price of option is X=50, ratio of capital-splitting is $k = m/m_1$ =0.96 and completes on τ=80(exchange day). Thus, Comparisons between options prices before and after their underlying stock being capital-split are drawn in figure 4 (a) and figure 4(b).

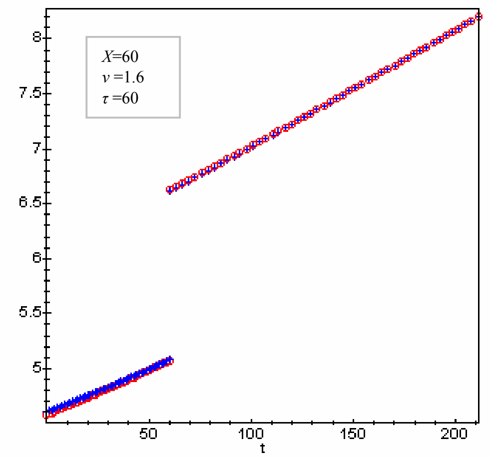
3) Comparisons between options prices before and after the underlying stock being dividend-paid and capital-split. Suppose, strike price of option is X=65, dividend-paying is v=1.6\$, ratio of capital-splitting is $k= m/m_1$ =0.96, and completes on τ=90(exchange day). Thus, Comparisons between options prices before and after their underlying stock being dividend-paid and capital-split are drawn in figure 5(a) and figure 5(b).

We see from the examples above, for any case, the call option should be executed before the underlying stock being dividend-paid or capital-split if needed, and the put option should be executed after the underlying stock being dividend-paid or capital-split if needed.

Now, we have discussed the models and methods of option pricing when underlying stock is dividend-paid or capital-split for one time. The discussion is similar for many times.



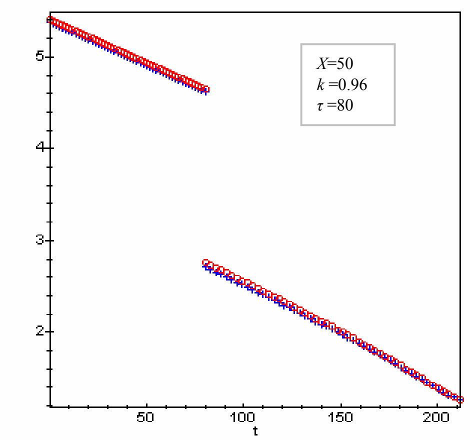
(a) Comparison of call option prices



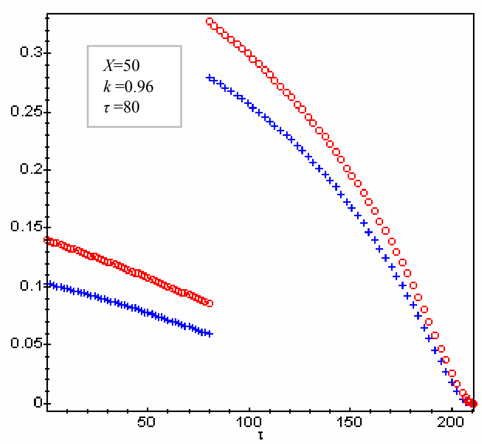
(b) Comparison of put option prices

○○○○○○ Curve of DF price
++++++ Curve of B-S price

Figure 3 Comparisons of options prices before and after MSFT being dividend-paid



(a) Comparison of call option prices



(b) Comparison of put option prices

○○○○○○ Curve of DF price
++++++ Curve of B-S price

Figure 4 Comparisons of options prices before and after MSFT being capital-split

8 Conclusions

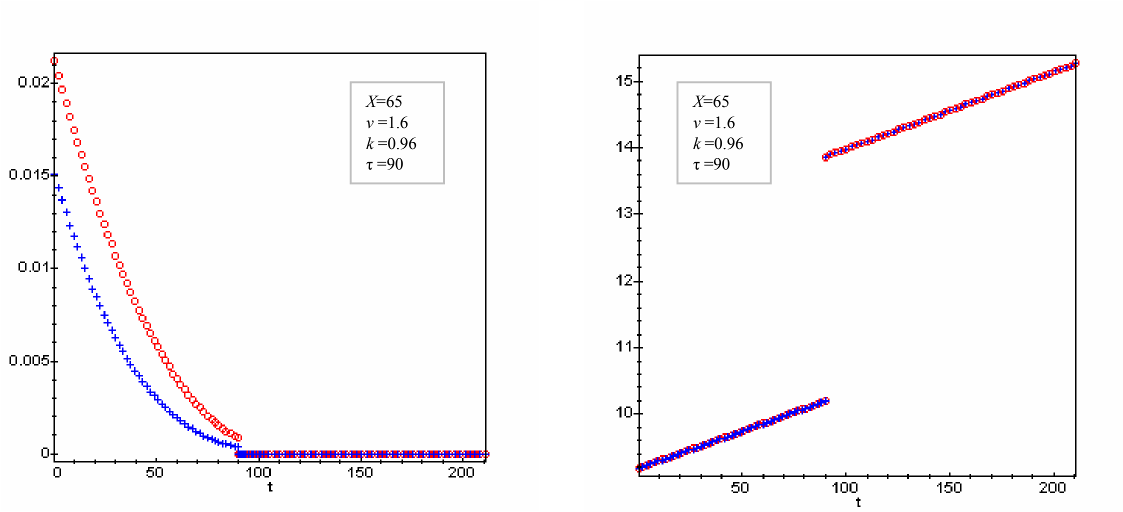
Based on the structure model for option pricing [17], this paper gives the structure models for options pricing on three cases which underlying stock is dividend-paid, capital-split, and dividend-paid and capital-split. These models are suitable to American or European option.

"The call option should be executed before the underlying stock being dividend-paid or capital-split if needed, and the put option should be executed after the underlying stock being dividend-paid or capital-split if needed.” is a valuable conclusion. It is inosculated to the practice in financial trading and useful to traders in financial market.

For original DF structure model or it on dividend-paying and capital-splitting, there is an assumption in section 2, i.e., there are no transaction costs or taxes in trading. In fact, the pricing formulas (4), (6), (11) and (12) are also right when the transaction costs or taxes in trading are subtracted from them.

Finally, the examples about options pricing on stock of MSFT are given, which include that the stock of MSFT is dividend-paid, capital-split, and dividend-paid and capital-split. Comparing the prices on DF structure models and those on Black-Scholes models, the prices on DF structure models are rational and better.

So the researches in this paper support the DF structure model for options pricing further.



(a) Comparison of call option prices

(b) Comparison of put option prices

oooooo Curve of DF price
++++++ Curve of B-S price

Figure 5 Comparisons of options prices before and after MSFT being dividend-paid and capital-split

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