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Capital Accumulation and Deposit Pricing in Mutual Financial Institutions

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This paper presents a multiperiod model of a deposit-type mutual financial institution and analyzes the manager's capital accumulation and deposit pricing decisions under uncertainty. The model is meant to describe a mutual savings and loan association (SLA) or a credit union in which there are no stockholders and nominal ownership is widely diffused (see [12] and [14]). The resulting paucity of monitoring gives rise to an agency problem (see [13]) in which the manager maximizes his multiperiod expected discounted utility with minimal intervention from nominal owners.

In each period, the manager diverts a portion of the firm's profits toward personal emoluments, i.e., to pecuniary and nonpecuniary benefits in excess of the contractual wage; net profits less emoluments then constitute institutional net worth accumulation. In addition, the manager selects a rate of interest to offer depositors. Initially, deposit insurance is assumed to be incomplete; hence, the supply of deposits to the institution depends on its accumulated net worth as well as the deposit interest rate offered. Although the institution has only one asset (one-period loans) and one liability (one-period deposits), profits are uncertain owing to deposit withdrawal risk and loan default risk. The model permits a characterization of the manager's optimal intertemporal capital accumulation and deposit pricing decisions as functions of the firm's uncertain financial position in each period.

Although the literature on SLAs is extensive (see, for example, [6], [10],

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[11], [16], [18], [19], and [26]), previous analyses fail to examine dynamic decisions under uncertainty. Pyle [23] stresses the need for such models, but cites Fevang [3] who found them to be usually intractable. Goldfeld and Jaffee [4] suggest that the stickiness of SLA deposit interest rates is a consequence of the inherited mortgage portfolio, indicating the need for a dynamic treatment, whereas Stigum [26] stresses the importance of treating uncertainty explicitly. The dynamic aspects of the present model also set it apart from extant treatments of the commercial banking firm, as, for instance, in [1], [15], [20], and [22].

Section I establishes notation, states assumptions, and formulates the manager's decision problem. Optimal deposit pricing, profit diversion, and capital accumulation decisions are characterized in Section II. Section III discusses implications of the model for stockholder-owned institutions, examines the case of complete deposit insurance coverage, and presents a summary and extensions.

I. The Model

The absence of stockholders distinguishes a mutual financial institution from the usual corporate form for which shareholder wealth maximization is the appropriate managerial objective. Also, given governmental deposit insurance and the conventions restricting depositor returns, depositors and borrowers neither bear the risks nor receive the normal rewards of ownership. Consequently, although depositors and borrowers have voting rights, they have little incentive to exercise them and they often carelessly assign their proxies to management (see [14]). In view of the negligible pressures exerted by these nominal owners, personal utility maximization on the manager's part seems to be the more appropriate behavioral assumption.

The focus of the analysis is, therefore, the manager's problem of maximizing his total expected discounted utility over an N-period planning horizon, given a fixed contractual wage per period, w. However, the dissipation of control by nominal owners permits the manager also to divert in each period, n, an additional part of the institution's resources, C_n, to personal emoluments. Unlike Jensen and Meckling [13], these additional benefits are measured in money equivalents, so that the manager's total earnings in period n are $(w+C_n)$, where C_n is a choice variable. The wage plus profit diversions provide the manager with one-period utility denoted by $U(w+C_n)$ and the manager's utility function is assumed to be nondecreasing and concave so that

$$U(0) = 0,\ U' \geq 0,\ U'' \leq 0. \tag{1}$$

In permitting the manager to earn $(w+C_n)$, it is implicitly assumed that labor markets are imperfect, which seems plausible in light of observed practices in the SLA industry (see, for example, [12] and [18]). Also, the specification of C_n as the manager's choice variable is consonant with the modern view of the owner-manager relationship as an agency problem (see [24]), especially in light of the above-mentioned absence of stockholders and the restriction of returns to nominal owners resulting from governmental deposit insurance and public regulation. Although public regulators can be expected to monitor the manager's behavior, it is often impossible to distinguish between legitimate business expenses and questionable perquisites without sustaining the substantial costs of becoming intimately involved in the routine operations of the firm. Moreover, regulators are often primarily concerned with preserving the SLA's solvency and as long as the manager pursues policies consistent with this objective (as will be shown to be the case), regulators can be expected to tolerate limited resource diversions. Finally, empirical evidence indicates significantly higher operating expenses among mutual than among stock SLAs (see [28]), and these inflated expenses may be taken to reflect managerial diversions of the firm's resources.

In keeping with the idea that the public regulator exercises some control, the manager's personal emoluments in each period are assumed to be constrained by the firm's profit in that period. Let π_n denote the uncertain (possibly negative) profit earned during the interval $(n-1,n)$, which becomes available at time n $(\pi_0 \equiv 0)$. Then it will be assumed that C_n cannot exceed π_n and if $\pi_n \leqslant 0$, then $C_n = 0$. Thus,

$$(2) \qquad 0 \leqslant C_n \leqslant \pi_n \vee 0 \equiv \text{Max}\ \{\pi_n, 0\}.$$

The constraint on C_n precludes the pathological case in which the manager absconds with the entire net worth of the firm. Otherwise, with complete capital markets, the manager might find it optimal to liquidate the firm and invest the proceeds in a portfolio of securities that replicates the firm's return stream. However, the availability of such a return stream would imply redundance of the intermediary, in which case the problem under consideration would vanish. This issue is avoided in [13] by shifting the agency costs arising from the manager's diversion of the firm's resources back to the manager via the equity market. In the present model, the absence of stockholders precludes an equity market discipline. Furthermore, depositors have virtually no incentive to discipline managers and, although public regulators have an incentive to do so, the observability problem forces them to employ imperfect or noisy monitors.[1] It is, therefore, assumed that monitoring will be limited to ensuring that resource diversions will not result in reported losses, period-by-period.

Let K_n denote the firm's accumulated net worth at time $n \leq N$, consisting of the initial net worth K_0, plus the firm's profits less any managerial profit diversions over n periods, so that $K_n = K_0 + \sum_{j=1}^{n} (\pi_j - C_j)$, i.e.,

$$K_n = K_{n-1} + (\pi_n - C_n), \qquad n = 1,2,\ldots N. \tag{3}$$

Thus, given the net worth K_{n-1} at time (n-1) and the profit π_n earned during (n-1, n), the manager determines $C_n \varepsilon [0, \pi_n \vee 0]$, enjoys the utility $U(W+C_n)$, and the firm's net worth changes according to equation (3).

Assume that the process terminates at N, or upon insolvency (i.e., whenever $K_n \leq 0$), whichever comes first. Upon termination of the firm, the manager receives a terminal reward R(K), defined in utility terms, that depends upon the firm's terminal net worth, K. Insolvency of the firm is assumed to result in disutility to the manager, and the marginal disutility increases in the magnitude of insolvency. Similarly, it is assumed that with positive net worth, terminal utility is positive and increasing with net worth, but at a diminishing rate. Thus,

$$R(0) = 0,\ R'(K) \geq 0,\ R''(K) \leq 0,\ -\infty < K < \infty. \tag{4}$$

This assumption may be justified on grounds that the manager's reputation and ability to find alternative employment upon termination of the firm presumably depend upon the firm's performance. The terminal reward (penalty) provides the manager with an incentive to protect the solvency and thereby promote the growth of the firm. (The firm's growth will also be shown to be in the manager's interest because it enhances future profit diversion opportunities.)

Let $\alpha \varepsilon (0,1)$ denote the manager's one-period discount factor (the present value of a unit of utility realized one period hence), and suppose the manager maximizes the total expected discounted utility over the life of the firm. Denoting the termination (stopping) time by

$$T = \text{Min}\ \{N,\ \text{Min}\ \{n;\ K_n \leq 0\}\}, \tag{5}$$

[1]The key to any interesting agency problem is the presumed inability of the principal to observe the agent's actions without error *ex post*. Otherwise, as shown by Harris and Raviv [9], "forcing contracts" can be employed to eliminate the moral hazard problem.

the manager's problem is to select a sequence of profit diversions $\{ C_n, n = 1,\ldots T \}$ to maximize

$$E[\sum_{n=1}^{T-1} \alpha^n U(w+C_n) + \alpha^T R(K_T)]$$

subject to

$$0 \leq C_n \leq \pi_n \vee 0, \quad n = 1,\ldots T,$$

and

$$K_n = K_{n-1} + \pi_n - C_n, \quad n = 1,\ldots T.$$

Now, the sequence of one-period profits $\{\pi_n, n = 1,2,\ldots\}$, and hence the sequence of net worths $\{K_n, n = 1, 2,\ldots\}$ and the terminal date T are random variables due to uncertainties in both deposit supply and loan default losses. Suppose that while default losses are not directly controllable,[2] deposit supply can be stochastically controlled by varying the interest rate offered depositors, r_D.

Treating the deposit interest rate as a decision variable warrants a word of explanation. In earlier analyses of SLAs, it was common to treat deposit interest rates as rigid owing to regulatory ceilings or other institutional idiosyncrasies (see [4]). However, among specialized intermediaries such as SLAs where liquidity management is a secondary concern and earning assets are largely of one kind (residential mortgages), suppressing liability management tends to trivialize the management problem. Of course, the asset management problem can be rediscovered by recognizing differences among mortgages and between mortgages and the small quantities of other assets held. But the vigor of marketing efforts directed toward attracting deposits belies the description of these institutions as passive deposit repositories. In fact, with the emergence of money market CDs and other innovative funding instruments, and the eclipse of Regulation Q, recent studies have been according increasing attention to liability management (see, for instance, [16] and [26]). Thus, r_D is viewed here as subsuming all expenditures--advertising, give-aways, branch expenditures, as well as direct interest payments--directed toward the solicitation of deposits. For convenience, r_D is referred to as the interest rate on deposits, even though it represents the cost of a wide

[2] In particular, it is assumed that credit is not rationed as, for instance, in [25].

array of deposit gathering activities.[3]

Given the manager's choice of r_{D_n} for the n^{th} period, potential depositors respond by supplying a deposit level D_n for the interval (n, n+1). A higher interest rate is assumed to attract more deposits, although at a diminishing rate. Also, with incomplete deposit insurance, a higher current net worth, K_n, should elicit higher deposit levels due to the diminished likelihood of the firm's insolvency (see [27]). Finally, it is assumed that K_n and r_{D_n} are substitutes in attracting deposits, so that higher values of either diminish the effectiveness of the other. That is, increases in K_n improve the safety of deposits and, therefore, depositors are willing to maintain given deposit levels at lower r_{D_n}. The deposit level, D_n, is taken to be a random variable with a conditional distribution function $F(\cdot|K_n, r_{D_n})$ and expected value $E(D_n|K_n, r_{D_n})$.

Given K_n and r_{D_n}, the deposit level is assumed to be instantaneously realized at n, at which time D_n plus K_n are immediately used to purchase one-period loans, A_n, at a market determined interest rate, r_A. Thus, after the managerial decisions C_n and r_{D_n} at time n, the firm's balance sheet is

$$(6) \qquad A_n = K_n + D_n, \quad n=0,1,\ldots T.$$

Note that there are no cash reserve or capital requirements and that the entire net worth is available to purchase loans. Including cash and liquid assets would not be difficult, but would add little to the analysis.

The one-period loans, A_n, involve uncertain default losses. Let $\theta \in [0,1]$ be the probability that a given dollar of loans will default. Then the total amount of default loss, L_n, on loans $(K_n + D_n)$ has the binomial distribution with parameters $(K_n + D_n, \theta)$, and the expected default loss is $\theta(K_n + D_n)$.

The profit made during (n, n+1) consists of the revenue on loans minus default losses and the cost of deposits, so that

$$(7) \qquad \pi_{n+1} = r_A(K_n+D_n-L_n) - L_n - r_{D_n} D_n, \qquad n = 0,1,2,\ldots$$

is the profit realized at time n+1, which has the expected value

[3]Without the deposit pricing decision, the manager's profit diversion problem closely resembles the personal consumption and investment problems studied by [5] and [21], and others.

$$(8)\quad E(\pi_{n+1} \mid K_n, r_{D_n}) = K_n[r_A - \theta(1+r_A)] + E(D_n \mid K_n, r_{D_n})\,[r_A - \theta(1+r_A) - r_{D_n}].$$

To ensure that equation (8) is nonnegative, a plausible precondition for engaging in financial intermediation, it is assumed that

$$(9)\qquad 0 \leqslant \theta \leqslant r_A/\,(1+r_A)$$

and

$$(10)\qquad 0 \leqslant r_{D_n} \leqslant r_A - \theta(1+r_A),$$

so that the expected loss on a dollar of loans cannot exceed the gross interest income obtainable, and the interest cost of deposits cannot exceed the corresponsing loan interest income, net of expected default loss.

In light of the assumptions regarding $E(D_n|K_n,r_{D_n})$, it is reasonable to assume that the expected profit $E(\pi_{n+1}\,|K_n, r_{D_n})$ is concave in (K_n, r_{D_n}) and increasing in K_n. However, marginal improvement in the expected profit due to increasing net worth diminishes with the rate of interest paid on deposits, i.e., $\dfrac{\partial^2 E(\pi_{n+1}|K_n,r_{D_n})}{\partial K_n\,\partial r_{D_n}} \leqslant 0$. Thus, expected profits are assumed to be "substitutable" in (K_n,r_{D_n}). In fact, given the nonlinearities involved, these assumptions must be extended to cover the entire conditional probability distribution of one-period profit rather than just its expected value. Thus, let $H(\bar{\pi} \mid K, r_D)$ be the probability that the one-period profit π_{n+1} will be less than or equal to $\bar{\pi}$, given that $K_n = K$ and $r_{D_n} = r_D$. Here H is obtained by compounding the deposit supply distribution, F, with the binomial distribution of default losses. Letting $\bar{H}(\bar{\pi} \mid K, r_D) = 1 - H(\bar{\pi} \mid K, r_D)$, it will be assumed that $\bar{H}$ satisfies the following conditions for any $K \geqslant 0$, $\bar{\pi} \in (-\infty,\infty)$ and for r_D in a range of values that may depend upon K and which we denote as $[0, r(K)]$:

(11) $\bar{H}\,(\bar{\pi}|K, r_D)$ is nondecreasing in (K, r_D) (stochastic monotonicity);

(12) $\bar{H}\,(\bar{\pi} \mid K, r_D)$ is concave in (K, r_D) (stochastic concavity);

and

(13) $\dfrac{\partial^2\bar{H}(\bar{\pi}|K, r_D)}{\partial K\partial r_D} \leqslant 0$ (stochastic substitutability).

The stochastic evolution of the firm's financial position and the manager's decisions can now be summarized. At any time n, the manager observes the previous period's net worth K_{n-1} and the realized profits, π_n, and selects $C_n \varepsilon [0,\pi_n \vee 0]$, deriving utility $U(w+C_n)$. This decision results in the new net worth $K_n = K_{n-1} + \pi_n - C_n$. The manager then makes a pricing decision, r_{D_n}, which together with K_n determines the random deposit level D_n according to $F(\cdot|K_n, r_{D_n})$. The total amount $A_n = K_n + D_n$ is then invested in one-period loans at the interest rate r_A. The firm sustains a random default loss, L_n (which has the binomial (A_n, θ) distribution), and deposits are redeemed with interest. The resulting random profit π_{n+1} is generated according to the composite distribution $H(\cdot|K_n, r_{D_n})$. At time (n+1), the manager again observes (K_n, π_{n+1}) and selects $(C_{n+1}, r_{D_{n+1}})$ and the process continues. Note that in any period n, the manager's decisions (C_n, r_{D_n}) are made sequentially, permitting the decomposition of his problem into two subproblems. The decision process terminates at T (= N or when $K_n \leq 0$) and the manager receives the terminal reward (penalty) $R(K_T)$.

The manager's problem is to select an optimal sequence of decisions (C_n, r_{D_n}), n = 1,2,...T, given the uncertainties regarding (D_n, L_n), to maximize his total expected discounted utility over T periods. This problem can be formulated in the dynamic programming framework as follows. At time n, if $K_{n-1} = K$ and $\pi_n = \pi$, let $V_n(K,\pi)$ denote the manager's maximum total expected discounted utility over periods n, n+1,...T corresponding to his optimal profit diversion and pricing decisions $\{(C^*_m, r^*_{D_m}), m=n, n+1,...T\}$. When n = T, the decision process terminates and $V_n(k,\pi) = R(K+\pi)$ is the manager's (dis)utility. If n < T, the manager selects an optimal $C^*_n \varepsilon [0, \pi \vee 0]$ and his immediate utility is $U(w+C^*_n)$. Given the firm's new net worth, $K_n = K' = K + \pi - C^*_n$, the manager selects an optimal deposit interest rate $r^*_{D_n}$, which together with K' results in profit $\pi_{n+1} = \pi'$ according to the distribution $H(\cdot|K', r^*_{D_n})$. At time (n+1), with the new state (K', π'), the manager's maximum total expected discounted utility from n+1 to T will be $V_{n+1}(K', \pi')$ which is discounted back to time n by the factor α. Therefore, given K', the manager's choice of $r^*_{D_n}$ should maximize the expected future discounted utility $E[V_{n+1}(K', \pi')|K',r_D]$, yielding $v_{n+1}(K') \equiv E[V_{n+1}(K', \pi')|K', r^*_{D_n}]$. Similarly, given (K,π), the choice of C^*_n should maximize the sum of the current utility $U(w+C)$ and the discounted value of the maximum expected future utility, $v_{n+1}(K + \pi-C)$. This analysis yields the following dynamic programming recursion satisfied by the optimal value function $V_n(K,\pi)$ for all n=0,1,2,...T-1, $K \geq 0$ and π,

$$(14) \qquad V_n(K,\pi) = \underset{C \,\varepsilon\, [0,\, \pi \vee 0]}{\text{Max}} \{U(w+C) + \alpha\, v_{n+1}(K + \pi - C)\}$$

where, upon writing $K' = K+\pi-C$,

$$(15) \qquad v_{n+1}(K') = \underset{r_D \,\varepsilon\, [0,\, r(K')]}{\text{Max}} E[V_{n+1}(K',\pi')\,|\,K',r_D]$$

with the insolvency (or terminal) condition

$$(16) \qquad V_T(K,\pi) = R(K + \pi).$$

Given any (K,π) at time n, the optimal $C_n^*(K,\pi)$ is the one yielding the maximum in equation (14) and for the resulting $K'=K+\pi-C_n^*(K,\pi)$, the optimal $r_{D_n}^*(K,\pi)$ is the one yielding the maximum in equation (15). In determining C_n, the manager considers the current utility generated by $(w+C_n)$ against the present value of his expected future utility under the assumption that he will continue to make optimal profit diversion and pricing decisions. A high current C_n implies a lower K' which adversely affects π' and hence the entire stream of future profit diversion opportunities.

II. Optimal Capital Accumulation and Deposit Pricing

In this section, the properties of the manager's multiperiod optimal expected discounted utility function $V_n(K,\pi)$ are derived in order to characterize optimal profit diversion $C_n^*(K_n, \pi_n)$ and deposit pricing $r_{D_n}^*(K_n,\pi_n)$ decisions as functions of the firm's financial positions (K_n, π_n) for all n = 1,2...T.[4] The results are stated in the form of three propositions each followed by interpretation. Proofs are relegated to the Appendix.

The first result is that under the assumptions of the model, the manager's long-term utility function reflects risk aversion and substitutability between the net worth and profit.

[4]This problem can be reformulated with an infinite time horizon by letting $N \to \infty$ wherein all the properties of the optimal choice variables remain unchanged.

Proposition 1

Given assumptions (1), (4), (11), (12), and (13), the optimal value function $V_n(K,\pi)$ defined by equations (14) through (16) is nondecreasing, concave, and substitutable in (K,π) for all $n = 1,2,...T$.

Proposition 1 implies that both current net worth and profits of the firm benefit the manager; they enhance immediate as well as future profit diversion opportunities. Since the maximum expected discounted utility increases with the net worth of the firm, the manager has an incentive to accumulate net worth, i.e., to limit the diversion of profit and promote growth of the firm. Concavity of V_n in (K,π) means that the marginal gain to the manager in terms of the optimal multiperiod utility declines with increasing (K,π). Finally, substitutability of V in (K,π), i.e., $\frac{\partial^2 V}{\partial K \partial \pi} \leq 0$, means that from the manager's long-run viewpoint, the firm's current net worth and the profit earned in the previous period serve as substitutes for each other. Thus, Proposition 1 provides an understanding of the manager's multiperiod utility function even though it is obtained by a complicated process of backward induction. These properties of V_n permit a characterization of the manager's profit diversion, capital accumulation, and deposit pricing decisions in the next two results.

Intuitively, one should expect the manager's optimal consumption decision in any period to be influenced by the realized profit in that period as well as the institution's existing net worth. The precise direction of this effect and the manner in which it impinges on the intertemporal accumulation of net worth are described in Proposition 2.

Proposition 2

In any period n, the manager's optimal consumption $C_n^*(K,\pi)$ is nondecreasing in (K,π). Moreover, the firm's resulting net worth, $[K + \pi - C_n^*(K,\pi)]$, is also nondecreasing in the (K,π) prevailing at the beginning of the period.

Thus, a higher net worth or a higher profit in any period induces the manager to divert more towards personal emoluments. However, profit diversion increases are always less than profit increases so that the resulting net worth of the firm is also nondecreasing in (K,π). Thus, it is optimal for the manager to accumulate capital. Although the manager diverts profits to himself, he also seeks to protect the solvency and promote the growth of the firm. The firm warrants protection because it promises future benefits. Capital accumulation not only enhances the magnitude of potential future returns to the manager, but it also increases the probability that future returns will continue to be realized. Thus, even in the case of extreme ownership diffusion, profit diversions are self-limiting and growth in net worth is predictable. The mutual's behavior thus accords qualitatively with that usually associated with stockholder-controlled institutions.

Since the supply of deposits to the institution is predicated upon both its net worth and the rate of interest it offers depositors, the manager's optimal interest rate policy can be expected to vary with the net worth he or she observes. This notion is formalized in the final proposition.

Proposition 3

In any period n, the optimal deposit interest rate $r^*_{D_n}(K')$ is nonincreasing in the firm's current net worth $K_n = K'$.

Proposition 3 indicates that a firm with a greater net worth will tend to offer a lower interest rate to depositors. With incomplete deposit insurance, depositors discern lower default risk with greater net worth and a smaller risk premium is, therefore, required to attract deposits. Thus, one way in which K increases future profits is through its "indirect source of funds" function. Capital serves as a substitute for deposit interest payments. This indicates one of the sources of the so-called capital adequacy problem (see [17]). As deposit insurance coverage becomes increasingly complete, net worth loses its deposit-attracting potency. Hence, management's incentive to accumulate capital is diminished, increasing the incentive to divert profits to personal emoluments.

III. Interpretations and Extensions

In this section, two extensions of the above analysis are discussed. The first subsection examines how the model could be modified to represent a stockholder-owned institution and how the behavior of the manager of such an institution would differ from that of the mutual manager. The second subsection considers the completion of deposit insurance coverage along with a concomitant regulatory capital requirement. Since these issues are not readily amenable to treatment in the context of the formal model developed in the previous section, most of the discussion is qualitative. The third and final subsection concludes with suggestions for future research.

A. Stock versus Mutuals

Stockholder-owned institutions can augment their capital by selling new equity, but mutuals cannot. Since a smaller opportunity set implies higher risk, this distinction may be summarized in terms of a lower discount factor, α, for the mutual. Further, since stockholders can diversify their risks across firms, they should be neutral toward a given institution's idiosyncratic risk. To promote efficient risk sharing, stockholders should, therefore, insure the manager against such risk (see [9]) and this again implies a lower overall risk and a higher α for the manager of the stockholder-owned institution. From equations (14) and (15), the C^* chosen by the mutual manager should then be higher, *ceteris paribus*. Consequently, a stockholder-owned institution should display a higher rate of capital accumulation through time, on an expected value basis, than an otherwise comparable mutual. Note, however, that impounded in the *ceteris paribus* is the absence of dividend payments.

A second distinction between stock and mutual SLAs involves monitoring. By virtue of their less diffused ownership, stockholder-owned institutions can be expected to expend greater resources on monitoring, at least part of which can be expected to be drawn directly from the firm. Although these activities are not explicitly modeled here, it is conjectured that their impact should reduce C^* for two reasons. First, increased surveillance should directly reduce profit diversions and, second, the costs of monitoring will reduce both profits and net worth. In order for monitoring to be rational, the net effect should be an increased growth in capital (unless dividend payments are increased).

These conclusions have considerable empirical support. Hester [10] found that the assets of stockholder-owned associations grew faster than those of comparable mutuals, and Verbrugge and Goldstein [28] discovered significantly higher operating expenses among mutuals than among stock SLAs. Similarly, the *Combined Financial Statements* (1980) published by the Federal Home Loan Bank Board indicate that expenses for "compensation and other benefits," when expressed as a percent of gross operating income, were 26 percent higher for mutuals in 1979 and about 22 percent higher in 1980 than those for stock SLAs.[5] Additional evidence of managerial profit diversion is provided by the growing literature on expense preferring behavior of financial institutions (see [2], [7], [8], and [29]).

Since an institution's capital position affects its deposit pricing policy, stock and mutual SLAs also can be expected to systematically differ in the interest rates they offer depositors. With incomplete deposit insurance coverage, as assumed in our model, the greater capital accumulation of stock SLAs should prompt them to offer lower deposit interest rates.

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[5] All other indicated categories of expenses--office occupancy, advertising, and all other--as well as total expenses were higher for mutual than for stock SLAs. Total expenses as a percent of gross operating income were in excess of 20 percent higher for mutuals than for stocks in both 1979 and 1980, despite the fact that stock SLAs grew faster than mutuals.

B. Deposit Insurance

In Section II, it was implicitly assumed that (a) deposit insurance is incomplete so that the deposit level depends on the firm's net worth as well as the interest rate, and (b) there is no regulatory capital requirement. Despite the prevailing nominally partial deposit insurance coverage, regulators often protect uninsured depositors by subsidizing mergers of distressed or insolvent institutions. Thus, the *de facto* extent of deposit insurance coverage is actually uncertain, but more nearly complete than the indicated coverage. This raises the question of how the results might be altered by assuming complete insurance coverage along with a concomitant regulatory capital (net worth) requirement, say, in the form of an upper bound on the deposits to capital ratio. Since the provision of complete deposit insurance would sever the relationship between the institution's net worth and the deposit supply function, the probability distribution $F(D_n | K_n, r_{D_n})$ would become independent of K_n, resulting in separability of the profit function equation (7) in K_n and r_{D_n}. However, this is only a special case of the model, involving weaker substitutability (condition (13)) between K and r_D in increasing profits. Consequently, qualitative results would remain unchanged. However, due to the weakened ability of net worth to attract depositors, the incentive to accumulate capital will diminish and profit diversions will increase.

If a capital requirement accompanies deposit insurance in the model, it becomes necessary to specify the penalty for violating the requirement. If the penalty is sufficiently large so that the manager finds it optimal to always satisfy the requirement, the qualitative properties of the manager's profit diversion policy should remain unaffected. However, the manager must now balance the benefits of current profit diversion against not only the future safety, but also the future ability to generate deposits in light of the binding capital constraint. Consequently, the profit diversions may diminish in order to satisfy the capital requirement. Thus, the regulatory capital requirement can be expected to offset the weakening of managerial incentives to accumulate capital resulting from the completion of deposit insurance coverage. In addition, an upper bound on deposit levels should induce a reduction in the deposit interest rates offered. Thus, it is anticipated that completion of deposit insurance will tend to increase profit diversions (reduce capital accumulation) while a regulatory capital requirement tends to offset this effect. However, predicting the net effect of these two changes is precluded by the model's level of generality and the qualitative nature of the results obtained.

C. Summary and Extensions

This paper has described optimal capital accumulation, managerial profit diversion, and deposit pricing in a rudimentary mutual deposit-type financial institution. The results have been used to examine differences in the behavior of stockholder-owned and mutual financial institutions, as well as aspects of deposit insurance. Thus, a special form of agency problem leads managers of mutuals to divert greater amounts of profit to personal emoluments than corresponding managers of stockholder-owned SLAs, and, as a consequence, the predicted rate of capital accumulation is lower among mutuals. These findings explain the widely observed operating cost and growth disparities between mutual and stockholder-owned SLAs.

With partial deposit insurance, the disparities in capital accumulation imply that mutuals will offer higher deposit interest rates than otherwise similar stockholder-owned SLAs. However, with complete deposit insurance augmented by a binding capital requirement, the deposit interest rate finding may be reversed. In light of the behavior of public regulators in dealing with distressed institutions, it is unclear which of the two assumptions regarding deposit insurance and capital requirements is more appropriate.

Future research may fruitfully incorporate additional institutional detail such as a capital requirement, a conversion option for the mutual manager, and an asset selection decision. If a conversion option is included, it may be possible to derive optimal stopping rules for the manager, where the stopping time corresponds to the time at which the manager decides to convert to the stock form and thereby modify the nature of his decision process. Further, given the results relating to the differences in capital accumulation and deposit pricing policies of the two organizational forms, it may be worthwhile to analyze the design of incentive compensation contracts for management. In practice, financial institutions often tie managerial compensation to some performance measure instead of paying a fixed salary as assumed here. For instance, some institutions offer bonuses to branch managers who increase their market shares. Viewed in the framework developed here, such bonus schemes can be interpreted as a mechanism to combat the inclination of managers to divert organizational resources and, therefore, represent an alternative to costly monitoring. In light of the disparate behavior of mutuals and stockholder-owned SLAs, an examination of alternative incentive compensation schemes may be especially useful. However, such an agency theoretic analysis can be expected to encounter formidable difficulties in the stochastic dynamic context of this paper.

APPENDIX

The proofs of Propositions 1, 2, and 3 require a sequence of lemmas which are first stated and proved below.

Lemma 1:

Let U and v be nondecreasing concave functions and define, for all (K,π),

$$V(K,\pi) = \underset{0 \leq C \leq \pi \vee 0}{\text{Max}} \{U(w+C) + \alpha\, v(K+\pi-C)\}. \tag{17}$$

Then $V(K,\pi)$ is nondecreasing and concave in (K,π).

Proof:

Since $v(K+\pi-C)$ and hence the maximand is nondecreasing in (K,π) for all C and since the feasible range $[0,\pi \vee 0]$ is nondecreasing in π, it follows that $V(K,\pi)$ is nondecreasing in (K,π). To show concavity, let $\lambda \varepsilon [0,1]$ and for any (K_1,π_1) and (K_2,π_2) define $(K,\pi) = \lambda(K_1,\pi_1) + (1-\lambda)(K_2,\pi_2)$. Denote the maximizing values of optimal C for (K_1,π_1), (K_2,π_2) and (K,π) by C_1^*, C_2^*, and C^*, respectively. Then

(18)

$$\begin{aligned} V(K,\pi) &= U(w+C^*) + \alpha\, v(K+\pi-C^*) \\ &\geq U(w+\lambda C_1^* + (1-\lambda)C_2^*) + \alpha\, v(\lambda(K_1+\pi_1-C_1^*) + (1-\lambda)(K_2+\pi_2-C_2^*)) \\ &\geq \lambda[U(w+C_1^*) + \alpha\, v(K_1+\pi_1-C_1^*)] + (1-\lambda)[U(w+C_2^*) + \alpha\, v(K_2+\pi_2-C_2^*)] \\ &= \lambda V(K_1,\pi_1) + (1-\lambda)\, V(K_2,\pi_2), \end{aligned}$$

proving concavity of V. The first and the last equalities in equation (18) follow by the definitions of C^*, C_1^*, and C_2^* as the maximizing values. The first inequality holds because $C_1^* \varepsilon [0, \pi_1 \vee 0]$ and $C_2^* \varepsilon [0,\pi_2 \vee 0]$ implies that $\lambda C_1^* + (1-\lambda)\, C_2^* \varepsilon [0, \pi \vee 0]$, so that $\lambda C_1^* + (1-\lambda)C_2^*$ is feasible with (K,π), while C^* attains the maximum in equation (17). Finally, the second inequality follows from concavity of U and v.

Lemma 2:

With U, v, and V as in Lemma 1, the optimal decision $C^*(K,\pi)$ yielding the maximum in equation (17) and $K + \pi - C^*(K,\pi)$ are both nondecreasing in (K,π).

Proof:

By concavity of v, $[v(K_2+\pi_2-C) - v(K_1+\pi_1-C)]$ is nondecreasing in C, whenever $K_2 \geq K_1$ and $\pi_2 \geq \pi_1$. Denoting $C_1^* = C^*(K_1,\pi_1)$ and $C_2^* = C^*(K_2,\pi_2)$, suppose $C_1^* > C_2^*$. Then, $v(K_2+\pi_2-C_1^*) - v(K_1+\pi_1-C_1^*) > v(K_2+\pi_2-C_2^*) - v(K_1+\pi_1-C_2^*)$ so that

$$[U(w+C_1^*) + \alpha v(K_2+\pi_2-C_1^*)] + [U(w+C_2^*) + \alpha v(K_1+\pi_1-C_2^*)] >$$

$$[U(w+C_1^*) + \alpha v(K_2+\pi_1-C_1^*)] + [U(w+C_2^*) + \alpha v(K_2+\pi_2-C_2^*)],$$

which contradicts optimality of C_1^* in (K_1, π_1) and C_2^* in (K_2,π_2). Hence, it follows that $C_2^* \geq C_1^*$.

To prove that $K_2+\pi_2-C_2^* \geq K_1+\pi_1-C_1^*$, note that U and v are concave and monotone, so their derivatives exist almost everywhere and for an interior maximum in equation (17),

$$U'(w+C^*(K,\pi)) = \alpha\ v'(K+\pi-C^*(K,\pi)). \tag{19}$$

Since $C^*(K,\pi)$ is nondecreasing and U is concave, the left-hand side in equation (19) is decreasing in (K,π). This, together with the concavity of v, implies that $[K + \pi-C^*(K,\pi)]$ on the right-hand side of equation (19) must be increasing in (K,π). If $C_1^* = \pi_1$, then by the constraint in equation (17), $C_2^* \leq \pi_2 = C_1^* + (\pi_2 - \pi_1)$ and $K_2 + \pi_2-C_2^* \geq K_1 + \pi_1-C_1^*$. If $C_2^* = 0$, then $C_1^* = 0$ and $K_2 + \pi_2-C_2^* \geq K_1 + \pi_1 - C_1^*$.

Lemma 3:

Under the assumptions of Lemma 1, $V(K,\pi)$ has the substitutability property (i.e., $\frac{\partial^2 V}{\partial K \partial \pi} \leq 0$) and $\frac{\partial V}{\partial \pi} \geq \frac{\partial V}{\partial K}$.

Proof:

Suppose $C^*(K,\pi)$ is in the interior of $[0,\pi \vee 0]$. Then differentiating equation (18) and using equation (19) yields

$$\frac{\partial V}{\partial K} = \alpha\ v'(K + \pi-C^*(K,\pi)).$$

By Lemma 2, $K + \pi-C^*(K,\pi)$ is nondecreasing in π, which together with the concavity of v implies that $\frac{\partial V}{\partial K}$ is nonincreasing in π. If $C^*(K,\pi) = 0$, then $V(K,\pi) = U(w)+\alpha v(K + \pi)$ and the concavity of v implies that $\frac{\partial V}{\partial K} = \alpha\ v'(K + \pi)$ is nonincreasing in π. Similarly, if $C^*(K,\pi) = \pi$, then $V(K,\pi) = U(w+\pi) + \alpha v(K)$, and the first result follows.

To prove the second result, note that $\frac{\partial V}{\partial K} = \frac{\partial V}{\partial \pi}$ if $C^*(K,\pi) \in (0,\pi)$. If $C^*(K,\pi) = \pi$, then $[U'(w + \pi) - \alpha v'(K)] \geqslant 0$ and $\frac{\partial C^*}{\partial \pi} \geqslant 0 = \frac{\partial C^*}{\partial K}$, so that $\frac{\partial V}{\partial \pi} = [U'(w+\pi) - \alpha v'(K)] \frac{\partial C^*}{\partial \pi} + \alpha v'(K) \geqslant \alpha v'(K) = \frac{\partial V}{\partial K}$.

Lemma 4:

Let $V(K,\pi)$ be nondecreasing, substitutable, and concave in (K,π). Suppose that the complementary distribution function $\bar{H}(\cdot|K, r_D)$ of π is stochastically increasing, substitutable, and concave in (K,r_D) in the sense of equations (11), (12), and (13). Then

$$W(K, r_D) = E[V(K,\pi)|K, r_D] \tag{20}$$

is nondecreasing, substitutable, and concave in (K,r_D).

Proof:

For expositional simplicity, assume that all functions are continuously differentiable in their arguments. Now, integrating by parts, rewrite equation (20) as

$$W(K,r_D) = \bar{V}(K) - \int_{-\infty}^{\infty} H(\pi|K,r_D) \frac{\partial V(K,\pi)}{\partial \pi} d\pi$$

where $\bar{V}(K)$ is the (possibly infinite) limit of $V(K,\pi)$ as $\pi \to \infty$. Then the monotonicity of W follows from

$$\frac{\partial W}{\partial K} = \frac{\partial \bar{V}}{\partial K} - \int_{-\infty}^{\infty} [H\frac{\partial^2 V}{\partial \pi \partial K} + \frac{\partial H}{\partial K}\frac{\partial V}{\partial \pi}] d\pi$$

which is nonnegative since V is increasing and substitutable in (K,π) and $\bar{H}$ is stochastically increasing in K. Similarly,

$$\frac{\partial W}{\partial r_D} = -\int_{-\infty}^{\infty} \frac{\partial H}{\partial r_D} \frac{\partial V}{\partial \pi} d\pi$$

is nonnegative since r_D is chosen from the range $[0,r(K)]$ of values over which $\bar{H}(\cdot|K,r_D)$ is stochastically increasing. Next, to show subadditivity of W, note that

$$\frac{\partial^2 W}{\partial r_D \partial K} = -\int_{-\infty}^{\infty} [\frac{\partial H}{\partial r_D}\frac{\partial^2 V}{\partial \pi \partial K} + \frac{\partial^2 H}{\partial r_D \partial K}\frac{\partial V}{\partial \pi}] d\pi$$

which is nonpositive by (stochastic) monoticity and subadditivity of V and $\bar{H}$.

Similar routine but cumbersome computations yield concavity of W based on monotonicity and concavity of V and $\bar{H}$ and substitutibility of V.

Proof of Proposition 1:

One proceeds by backward induction on n. Since $V_N(k,\pi) = R(K + \pi)$ and since R is monotone and concave in its argument (by equation 4)), it is also monotone, concave, and substitutable in (K,π), so that the result holds for n=N. Suppose that for some (n+1) we have $V_{n+1}(K,\pi)$ nondecreasing, concave, and substitutable in (K,π). By equations (11), (12), and (13), $\bar{H}(\cdot|K,r_D)$ is nondecreasing, concave, and substitutable in (K,r_D). Therefore, by Lemma 4,

$$W_{n+1}(K',r_D) = E[V_{n+1}(k',\pi')|K',r_D]$$

is nondecreasing, and concave (and substitutable) in (K',r_D). Hence,

$$v_{n+1}(K') = \underset{r_D \,\varepsilon\, [0,r(K')]}{\text{Max}} \{W_{n+1}(K',r_D)\}$$

is nondecreasing and concave in $K' \geqslant 0$. This, together with the concavity of U, as in equation (1), implies by Lemmas 1 and 3 that, with $K' = K + \pi - C$,

$$V_n(K,\pi) = \underset{C \,\varepsilon\, [0,\ \pi \vee 0)}{\text{Max}} \{U(w+C) + \alpha v_{n+1}(K + \pi - C)$$

is nondecreasing, concave, and substitutable in (K,π), thus completing the induction argument.

Proof of Proposition 2:

As in the proof of Proposition 1, the functions U and v_{n+1} are nondecreasing and concave and $C_n^*(K,\pi)$ is that value of C which attains the maximum in equation (14). The result now follows from Lemma 2.

Proof of Proposition 3:

As in the proof of Proposition 1, $W_{n+1}(K',r_D)$ is substitutable in (K',r_D), i.e., $[W_{n+1}(K_2', r_D) - W_{n+1}(K_1',r_D)]$ is nonincreasing in r_D whenever $K_2' \geqslant K_1'$. Suppose that $r_{D_2}^* \geqslant r_{D_1}^*$, contrary to the required result, where $r_{D_2}^* = r_D^*(K_2')$ and $r_{D_1}^* = r_D^*(K_1')$. Then,

$$W_{n+1}(K_2',r_{D_2}^*) - W_{n+1}(K'_1,r_{D_2}^*) \leqslant W_{n+1}(K_2',r_{D_1}^*) - W_{n+1}(K_1',r_{D_1}^*)$$

i.e.,

$$W_{n+1}(K_2', r^*_{D_2}) + W_{n+1}(K_1', r^*_{D_1}) \leq W_{n+1}(K_1', r^*_{D_2}) + W_{n+1}(K_2', r^*_{D_1}).$$

But this contradicts $r^*_{D_2}$ and $r^*_{D_1}$ yielding the maxima of $W_{n+1}(K_2', r_D)$ and $W_{n+1}(K_1', r_D)$, respectively.

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