Closed-End Fund Discounts in a Rational Agent Economy

by

Matthew Spiegel
University of California, Berkeley
Haas School of Business
S545 Student Services Building, #1900
Berkeley, CA 94720-1900
510-642-3421
spiegel@haas.berkeley.edu

November 24, 1997

Abstract

Nearly any standard financial model concludes that two assets with identical cash flows must sell for the same price. Alas, closed-end mutual fund company share prices seem to violate this fundamental tenant. Even when one considers several standard frictions, such as taxes and agency costs, classical financial models cannot explain the large persistent discounts found within the data. While the standard financial markets model may not explain the existence of large closed-end fund discounts, this paper shows that a rather close version of it does. In an otherwise frictionless market, if asset supplies vary randomly over time and agents posses finite lives a closed-end mutual fund's stock price may not track its net asset value. Furthermore, the analysis provides a number of conditions under which these discrepancies will lead to the existence of systematic discounts for the mutual fund's shares. In addition, the model provides predictions regarding the correlation between current closed-end fund discounts and current changes in stock prices and future changes in corporate productivity. As the analysis shows the same parameter values that lead to systematic discounts also lead to other fund price characteristics that resemble many of the results found within empirical studies.

Closed-end mutual funds tend to sell at a discount relative to their net asset value. How can this be? Any reasonable theory of capital markets should rule out arbitrage, and closed-end fund discounts appear to provide arbitrage opportunities. Malkiel (1977) put forth a number of frictions (such as taxes and agency costs) that might explain observed closed-end fund discounts. However, he shows that while these frictions can account for some of the observed discount they cannot account for all of it.[1] Lee, Shleifer and Thaler (1990) make the additional point that even if taxes and agency costs did explain the observed magnitude of closed-end fund discounts, there still remain a number of outstanding issues regarding time trends in the data that neither agency costs nor taxes seem well equipped to deal with.

In response to the inability of the standard financial model to fully explain the closed-end fund data, two alternative explanations have been proposed. In Lee, Shleifer, and Thaler (1991) small irrational agents create the discounts by trading on correlated sentiment. In Chordia and Swaminathan (1997) frictions including asymmetric information and market segmentation lead to closed-end fund discounts. While both of these models undoubtably help solve this puzzle, they do represent rather significant departures from the standard financial markets model in which rational agents trade in a frictionless environment. This paper argues that it may be too soon to give up on the standard capital markets model. True, unchanged, it obviously does not explain either observed closed-end fund discounts or their associated properties. But, as the analysis presented here shows, a rather close cousin of that model can.

By imposing two relatively minor modifications to the standard capital asset market's model this paper shows that it can both explain many of the empirical regularities and produce some, as yet, untested predictions. First, rather than living forever, agents in the model have finite lives. This rather simple, and realistic, change does a great deal to alter an investor's equilibrium behavior. When agents

---

[1]Additional work on the relationship between taxes and observed closed-end fund discounts has since been conducted by Brickley, Manaster, and Schallheim (1991), and Pontiff (1995). Both papers provide mixed evidence regarding the empirical validity of this hypothesis. With regard to agency costs, Barclay, Holderness, and Pontiff (1993) provide additional support regarding their influence on the discount.

only live for a finite amount of time they only care about price movements and dividend cash flows that will occur before their deaths. Of course this point is not new and can be found in articles such as Dow and Gorton (1994) and Holden and Subrahmanyam (1996). Second, the model assumes that asset supplies vary randomly through time. By adding both features to a frictionless multi-asset model a second equilibrium emerges from the standard capital markets model.[2]

Intuitively, the model shows that closed-end fund discounts can occur if people simply believe that they can occur. Arbitrage arguments, as to why discounts cannot exist, rest on the idea that if a discount does appear investors will try to take advantage of the situation by going infinitely long the fund and short the underlying securities. But imagine that you discover a closed-end fund that currently sells for less than its net asset value. Would you willingly take an infinitely long position in the fund and a short position in the underlying stocks? Not if you had to close out the position by some finite date, and you thought that the discount might widen prior to then. Instead you might take a small position depending upon your beliefs regarding the risk-return tradeoff. But this means that traders submit finite (rather than infinite) demands in response to discounts. If asset supplies vary then that variation will induce the discount to fluctuate over time, fulfilling people's beliefs.

The model's analysis not only shows that discounts can exist within a fairly classical model, but also that the existence of discounts can be linked to a number of other related phenomena. The same parameter values of the model that produce closed-end fund discounts also produce a number of other patterns, many of which have been documented in empirical studies. These include: (1) closed-end fund prices are more volatile than the underlying net asset value (Pontiff (1997)), (2) closed-end fund discounts are mean reverting (Thompson (1978), and Pontiff (1995)), (3) closed-end fund discounts widen when concurrent corporate productivity (perhaps proxied for by stock prices) goes up

---

[2]The fact that overlapping generations models can produce multiple equilibria goes back to Azariadis (1981).

unexpectedly (Malkiel (1977) and Pontiff (1995)), and (4) closed-end fund discounts are negatively associated with future aggregate economic activity (Swaminathan (1996)).

Like any other model, the theory presented here leaves a number of questions unanswered and others that are probably better answered by other theories. For example, Weiss-Hanley, Lee and Seguin (1996) find that investment firms work to support the price of a closed-end fund for about 29 days after the initial offering. This phenomena is completely outside the scope of the current model. Another phenomena related to the initial public offering price concerns the offering price itself. Peavy (1990) finds that at the initial public offering closed-end funds sell for about 7.41% above net asset value. While the model can in principle explain this, the explanation does not then conform to the finding in Hanley, Lee and Seguin (1996) that the issue price can only be supported through the intervention of the investment firms marketing the issue. Of course, the explanation my lie with systematic biases held by individuals as Lee, Shleifer and Thaler (1991) suggest. However, Chordia and Swaminathan (1997) note that underlying properties of the economy may also explain this phenomena. In the end, it will undoubtably turn out that a combination of several models will be needed to explain all of the data.

The paper's structure is as follows. Section 1 discusses the paper's relationship to the theoretical literature. Section 2 presents the model. Part 2.1 lays out the mathematical details describing the economy. Part 2.2 calculates the equilibrium conditions for the model's parameters. Next part 2.3 outlines the basic properties of the solution with regard to closed-end fund discounts, and provides some empirical predictions. Part 2.4 examines the relationship between closed-end fund discounts and future economic activity within the corporate sector. Section 3 concludes. Appendix A discusses the stability of the different equilibria, and Appendix B contains most of the proofs.

1 Relation to the Theoretical Literature

Lee, Shleifer and Thaler (1991) argue that investor sentiment drives the observed patterns associated with closed-end fund discounts. In their model the economy contains both rational investors

and those influenced by "sentiment." They argue that if investor sentiment is correlated across individuals then their demands can move prices if rational agents are unwilling to take fully offsetting positions.

Within the current model random supplies play a very important role and one that appears analogous to investor sentiment in Lee, Shleifer and Thaler (1991). Thus, the reader might legitimately ask if there are any equilibrium features that distinguish the two. In fact, there are at least four major differences. First, because all of the agents within the current model maximize utility functions there always exists an equilibrium in which closed-end fund discounts do not exist. Thus, even with random supplies closed-end fund discounts do not have to exist. Depending upon how one sets up a noise trader or sentiment model this may or may not be true.[3] Second, within equilibria that permit closed-end fund prices to deviate from their net asset value, a *decrease* in supply noise for the closed-end fund *increases* the discount and the volatility of the fund's per share price. Conversely, in a model of investor sentiment such as Lee, Shleifer, Thaler (1991), an increase in the demand volatility of such traders increases the fund's price volatility as well as the discount.

A third difference between an investor sentiment model and one using a mean reverting supply process lies in how they tie the empirical phenomena that closed-end fund discounts exhibit negative intertemporal serial correlation to other characteristics of the economy. In a random supply model changes in the discount are tied directly to changes in corporate assets and this ties the rate of mean reversion to concurrent changes in aggregate production. Fourth, tying the discount on closed-end funds to corporate productivity also induces a relationship between the current discount and future changes in corporate output. As the analysis of the model shows the parameter values that lead to the existence of closed-end fund discounts also lead to a positive correlation between the discount and concurrent

---

[3]As Bhushan, Brown and Mello (1997) demonstrate many results within the investor sentiment literature depend critically upon the assumed form of irrational behavior.

changes in value of the overall corporate productivity. Simultaneously, these same parameter values also

lead to a negative correlation between closed-end fund discounts and future corporate productivity. In

contrast, it is not clear why models of investor sentiment should produce non-zero correlations among

current discounts and future production levels.[4]

Another model that examines the closed-end discount can be found in Chordia and Swaminathan

(1997). In their view if the standard set of frictions cannot explain the closed-end fund puzzle, then they

suggest other frictions might.[5] They propose an explanation that includes both asymmetric information

and market segmentation. In their model market segmentation occurs because some of the investors are

restricted from trading in some markets. They then show that if there are two identical securities, and if

the informed investors cannot trade in one of them, then the other security (the "closed-end fund") may

sell at a discount. This type of segmentation, they argue, is consistent with evidence in Weiss (1989) that

institutional investors tend to shun closed-end funds.[6]

---

[4]This point can also be found in Swaminathan (1996), who notes that if investor sentiment forecasts future economic activity it is not clearly sentiment but perhaps a rational expectation.

[5]In contrast, the current paper seeks to show that discounts can arise in the absence of transactions costs.

[6]The actual implications one should draw from Weiss' (1989) findings seem to be in dispute. Lee, Shleifer, and Thaler (1991) argue that small investors are more likely to herd via correlated investor sentiment than are large institutional money managers. Thus, they take the lack of institutional money in closed-end funds to imply that investor sentiment will play a larger role in the price process. However, there also seems to be some dispute as to which investors, if any, are more prone to herding. Froot, Scharfstein and Stein (1992) conjecture that professional money managers are the ones likely to herd. In terms of the direct empirical work on this issue, Grinblatt, Titman, and Wermers (1995) find some evidence of herding by mutual fund managers, but do not show if there is more or less relative to herding among small investors.

2   The Model

2.1   Description of the Economy

Readers familiar with the market micro-structure literature will recognize the model presented here as a

fairly standard "exponential-normal" economy with random supplies. Thus, there are no frictions in

either the form of transactions costs or taxes.[7] Everyone can trade every available security. In large part

these assumptions are employed in order to show that closed-end mutual fund discounts can occur within

a fairly classical setting and that these discounts will exhibit many of the empirical properties

documented in the literature.

The economy progresses through time via a sequence of overlapping generations. People live for

two periods, and consume only in their last period of life. Their preferences over consumption are

represented via a negative exponential utility function with risk aversion parameter θ. At the start of

period t a new generation of individuals are born. For simplicity, the model assumes that the

population's size does not vary over time, with each generation being of size "one."

Upon birth the period t generation is endowed with a $k\times1$ vector of risky assets (X_{t-1}) and a supply

of the riskless asset (β_{t-1}) that they can trade in the market with other individuals. As per standard

practice, the model treats the stock endowments of the new generation as shares of corporate equity that

are traded on the stock exchange. However, one can employ other interpretations without changing the

mathematical structure of the economy. For example, individuals may come endowed with real

productive assets (human capital). Upon birth individuals then sell their endowment capital directly to

the firms (which then turn the endowment capital into corporate capital) in exchange for corporate claims

---

(or a cash payment equal to the market value of the assets that they contributed to the firm). However, no matter how one interprets the situation, the only mathematically material aspect is that the model must somehow include random supplies of the risky assets traded within the economy.

While asset supplies may vary over time, it seems reasonable to suppose that on a per capita basis they do not tend to diverge towards plus or minus infinity. With a constant investor population, as in the current model, this restriction can be accommodated via the following mean reverting supply process:

$$N_{t+1} = A(\bar{N}-N_t) + N_t + \eta_{t+1} \tag{1}$$

where A equals a symmetric $k\times k$ matrix that determines the speed at which the $k\times 1$ vector of security supplies (N_t) revert to their normal level ($\bar{N}$), and η_t represents a $k\times 1$ vector of normally distributed shocks with mean zero and variance-covariance matrix Σ_η. Based upon equation (1) the stock endowment of each new generation equals N_{t+1}-N_t.

Throughout, the paper assumes that I-A is a positive definite matrix with eigenvalues between zero and one, where I is a $k\times k$ identity matrix. At the expense of algebraic complexity one can add another drift term, that does not mean revert, into the supply process without altering any of the paper's basic conclusions. To maintain the model's results one only needs that these drift processes tend to "stay together" over time.[8]

Intuitively, the assumption that asset supplies mean revert captures the idea that people will direct their endowment capital towards ventures where they can get the best return. Thus, people will tend to add capital to industries where it is in short supply (and thus the rewards higher) and away from

---

[8]Elimination of the mean reversion in the supply process can be accomplished by simply setting A to a matrix of zeros. Doing so will leave most of the model's conclusions unchanged, including many results on the price and volatility of closed-end funds relative to their underlying assets. However, without a mean reverting supply process the model will no longer produce a mean reverting discount, and the comparative statics concerning the impact of A on the equilibrium will obviously be lost.

industries where capital is in abundant supply (and thus the rewards lower). The matrix A captures this phenomena without imposing the considerable mathematical complexity that would be involved by allowing people to endogenously redirect their endowments at some exogenously specified cost. While such a generalization may yet provide insights into some interesting issues, the results that form the primary focus of this study seem unlikely to change since they hold for any matrix A that meets the restrictions given above. Endogenously calculating A from the forces directing the mean reversion of the security supplies would simply place some additional restrictions on A based upon some other exogenously given cost matrix. Thus, in an appeal to simplicity the current analysis simply takes A as given.

The k securities in the economy produce cash flows of $D_t' = (D_{t1}, D_{t2}, ..., D_{tk})$ in period t. The dividend flow follows a random walk with

$$D_{t+1} = D_t + \delta_{t+1} \tag{2}$$

where δ_t represents a $k{\times}1$ vector of normally distributed shocks with mean zero and variance-covariance matrix Σ_δ.[9] (For simplicity, the model assumes that the δ and η shocks are independent from each other.) Since some of the securities in question may include closed-end mutual funds, the matrix Σ_δ may be singular.

Let $P_t' = (P_{t1}, P_{t2}, ..., P_{tk})$ equal the equilibrium price vector for the securities. Then in the last period of a person's life their consumption equals

$$W_{t+1} = X_t'(P_{t+1} + D_{t+1}) + (1+r)\beta_t \tag{3}$$

where W_t equals period t wealth, X_t period t security demands, r the risk free interest rate, and β_t the

---

[9]Again, one can generalize this process to include terms that induce mean reversion and/or growth in the dividend process without altering the paper's basic conclusions.

investor's period t demand for the riskless asset.

Following standard practice in the literature, the riskless asset is in infinitely elastic supply and used by the population to either borrow from future consumption or store current consumption at no cost. Thus, in equilibrium the price of the risk free asset always equals one (a normalization), and stock prices adjust to set the current demand for risky assets equal to their current supply.

2.2   Equilibrium

Recall, that upon birth a trader enters the economy with an endowment of X_{t-1} shares, and β_{t-1} units of the risk free asset. He or she therefore faces a budget constraint that can be written as

$$(X_t - X_{t-1})'P_t + \beta_t - \beta_{t-1} = 0. \tag{4}$$

Use this to eliminate β_t from equation **(3)**, thereby producing

$$W_{t+1} = X_t'\left[P_{t+1} - P_t + D_{t+1} - rP_t\right] + (1+r)W_t. \tag{5}$$

Given the above wealth equation each trader will seek to maximize his expected utility from terminal consumption by selecting an appropriate demand vector X_t.

Notice that the investor does not need to know anything more about the model than the facts presented so far to make a demand decision. He only needs to hold conjectures about the mean and variance-covariance matrix for the vector containing next period's prices plus dividends. Knowledge concerning the actual process by which the economy forms prices is superfluous. Thus, it is possible that the investor does not know that supplies vary within the economy. Perhaps he even believes that prices should equal some constant (say $1/r$) times the current dividend. He would then be puzzled as to why prices violate this principle year after year. Nevertheless, he can still optimize his expected utility given equation **(5)** by selecting X_t to solve

$$max \quad X_t'\left[E_t(P_{t+1})-P_t+E_t(D_{t+1})-rP_t\right] + (1+r)W_t - \frac{\theta}{2}X_t'\Sigma_{PD}X_t, \tag{6}$$
$$X_t$$

where Σ_{PD} represents the investor's beliefs regarding the variance-covariance matrix of $P_{t+1} + D_{t+1}$.[10]

While traders do not need to posses conjectures concerning the price process, the modeler must. As in most exponential-normal settings a linear price process can be found that will satisfy the standard conditions for a rational expectations equilibrium. In this case the appropriate conjecture is that

$$P_t = \frac{1}{r}D_t + B_0 + BN_t \tag{7}$$

where B_0 is a $k\times 1$ vector and B a $k\times k$ matrix. Using the conjectured price process (7) in (6) implies that investors will implicitly optimize

$$max \quad X_t'\left[BA(\bar{N}-N_t)+D_t-rP_t\right] + (1+r)W_t - \frac{\theta}{2}X_t'\left[\left(\frac{1+r}{r}\right)^2\Sigma_\delta+B\Sigma_\eta B'\right]X_t, \tag{8}$$
$$X_t$$

which yields the equilibrium conditions listed in the next proposition.

Proposition 1: In equilibrium

$$B_0 = \frac{1}{r}BA\bar{N}, \tag{9}$$

_____

[10]The mean-variance optimization problem follows from the assumptions that traders have exponential utility functions, and that for a given position in stocks and bonds that their future wealth is normally distributed.

$$B = \frac{1}{2\theta}[rI+A]\Sigma_\eta^{-1/2}\left\{-I + \left[I-4\theta^2\left(\frac{1+r}{r}\right)^2\Sigma_\eta^{1/2}[rI+A]^{-1}\Sigma_\delta[rI+A]^{-1}\Sigma_\eta^{1/2}\right]^{1/2}\right\}\Sigma_\eta^{-1/2},$$

(10)

and thus

$$P = \frac{1}{r}\left[D_t + B[rN_t+A\bar{N}]\right].$$

(11)

Proof: See Appendix B for the proof of this and all other propositions.

Notice that the equilibrium equation describing B can always accommodate a singular dividend variance-covariance matrix (Σ_δ). In contrast, without a redefinition of the security set, the variance-covariance matrix of the supply process (Σ_η) must be of full rank.

While the B matrix appears rather complex one can simplify it to some degree. Define Γ as a matrix of orthonormal eigenvectors and Ω as a diagonal matrix of eigenvalues and write

$$\Gamma\Omega\Gamma^{-1} = 4\theta^2\left(\frac{1+r}{r}\right)^2\Sigma_\eta^{1/2}[rI+A]^{-1}\Sigma_\delta[rI+A]^{-1}\Sigma_\eta^{1/2},$$

(12)

which allows one to simplify B to

$$B = \frac{1}{2\theta}[rI+A]\Sigma_\eta^{-1/2}\Gamma[-I+(I-\Omega)^{1/2}]\Gamma^{-1}\Sigma_\eta^{-1/2}.$$

(13)

This brings out the importance of the square root term in B. The square root term produces two distinct equilibria for each eigenvalue since the ith element in $(I-\Omega)^{1/2}$ has two roots $+\sqrt{1-\omega_i}$ and $-\sqrt{1-\omega_i}$, where

ω_i represents the ith diagonal element of Ω. Thus, if one can deal with portfolios that are proportional to the eigenvectors in Γ (eigen-portfolios) one can say a great deal about their behavior if one knows something about the associated eigenvalue. Note that within equation **(13)** the second term in square brackets contains what looks like a diagonal matrix of eigenvalues for B. While this is not exactly true, one can define a pseudo-eigenvalue matrix $\Lambda = .5[-I + (I - \Omega)^{1/2}]$ and its ith diagonal element as λ_i. The next couple of propositions demonstrate that zero dividend portfolios are particularly easy to work with since they are eigen-portfolios, with associated eigenvalues in Ω equal to zero, and equal to either zero or minus one in Λ.

Empirical studies that examine closed-end mutual fund discounts are actually talking about the properties of a zero dividend portfolio that goes long the mutual fund and short the underlying assets. To single out such portfolios, let Z designate an arbitrary portfolio (with at least some non-zero security holdings) such that the holder of portfolio Z never expects to receive any net dividend payments. Thus, a portfolio that holds a short position in a closed-end fund and a long position in the underlying assets forms a special case within the set of all possible zero dividend portfolios. The key to valuing such portfolios lies within the following proposition.

Proposition 2: Let Z represent an arbitrary k×1 vector, while E and F represent arbitrary k×k matrices. Now assume that Z′E=0. Then $Z'F[I - F^{-1}EF'^{-1}]^{1/2}F' = \pm Z'FF'$.

This proposition makes the analysis of a zero dividend portfolio tractable. In general, the matrix B in the equilibrium pricing equation **(10)** is very complicated. However, when pre-multiplied by a zero dividend portfolio and post-multiplied by $rI+A$ it collapses to the relatively simple $-(r\theta)^{-1}Z'[rI+A]\Sigma_\eta^{-1}[rI+A]$. This relationship allows one to say quite a bit about both the price and variance of the zero dividend portfolio.

Proposition 3: Assume Σ_δ is a singular matrix and that Z represents a portfolio such that $Z'D_t = 0$ in all states of nature, and that $Z'\Sigma_\delta = 0$. Then the value of the portfolio Z equals

12

$$Z'P_t \;=\; \begin{cases} -\dfrac{1}{r\theta}Z'[rI+A]\Sigma_\eta^{-1}[rN_t+A\bar{N}] & \text{if } \lambda_Z=-1 \\[2em] 0 & \text{if } \lambda_Z=0, \end{cases} \qquad\qquad \textbf{(14)}$$

where λ_Z is the pseudo-eigenvalue of B associated with Z and

$$Z'\Sigma_P Z \;=\; \begin{cases} -\dfrac{1}{\theta^2}Z'[rI+A]\Sigma_\eta^{-1}[rI+A]Z & \text{if } \lambda_Z=-1 \\[2em] 0 & \text{if } \lambda_Z=0 \end{cases} \qquad\qquad \textbf{(15)}$$

equals its variance.

For a zero dividend portfolio there exist two possible equilibria. When the associated eigenvalue equals 0 the standard set of results hold and the portfolio's value and variance both equal zero. However, there also exists another equilibrium (the negative root equilibrium) where the portfolio, despite paying zero in all states of nature exhibits a non-zero price and a positive variance. How can this be? The problem faced by potential arbitragers is that they may need to unwind their portfolio in a time period where the price has moved *even further away* from zero, thereby leading them to incur a loss. As in Dow and Gorton (1994), so long as agents have finite lives *they* do not view the "arbitrage" portfolio as providing *them* with an arbitrage since *they* may incur a loss if they try to take advantage of the situation. The pricing equation **(14)** tells us the amount by which the arbitrage portfolio's price can deviate from zero. Notice that it is not some arbitrary number. While agents are risk averse they are happy to take positions in portfolios that have sufficiently high returns given the risk faced by the individual purchasing the portfolio.

The economy's infinite horizon provides a key element within the model towards the creation of multiple equilibria. Without it, agents can backwards program the set of realized prices and the negative root equilibria will collapse. Thus, according to the model, a closed-end fund that everyone expects to

"open up" should lose its discount.[11] Brickley and Schallheim (1985) find something similar. Fund discounts do disappear when funds open up. However, the discounts do not disappear all at once. Nevertheless one can reconcile this result with the current model. Suppose that until the actual date of opening there remains some uncertainty as to whether or not the fund will in fact open up. Then the discount will narrow gradually as one approaches the day of reckoning at which point it will disappear. One can test this hypothesis by seeing if the discount narrows faster when funds provide greater assurances that they will in fact open up.

Equations **(14)** and **(15)** also show that within the negative root equilibrium relatively small supply shocks can lead to economically significant results. To see this, note that within both equations the supply shock's variance-covariance matrix enters only through its inverse. Thus, even if supply shocks tend to be fairly small they can still account for both the size and volatility of the observed closed-end fund discounts.[12]

In order for the model to produce closed-end fund prices that differ from their underlying net asset values, the economy must "select" the negative root equilibrium. This begs the question as to why most, if not all, closed-end funds trade in this equilibrium as opposed to the positive root equilibrium. The answer may lie in the sequencing of events. Imagine that initially one fund begins trading and its price deviates from its net asset value, that is, it trades in the negative root equilibrium. Initially investors may be surprised but they see this occur and accept it as part of the market's behavior. Now another fund starts up. What should the investor believe? It certainly seems reasonable for an individual investor to expect that this fund will also sell for a price other than its net asset value. In which case trading will take place under the negative root equilibrium. Thus, due to the sequential nature of fund

---

[11]A closed-end fund is said to "open up" if at some date it becomes an open ended fund.

[12]For a detailed discussion of the general relationship between supply and price volatilities see Spiegel (1997). That paper shows that relatively modest supply shock volatilities can produce price volatilities commensurate with those generally observed in the overall stock market.

openings, if the initial fund trades at a price other than its net asset value, all future funds may also

exhibit the same behavior.[13]

2.3   Closed-end Mutual Funds and Their Properties

Closed-end fund values do not deviate from the value of their underlying assets by a mean zero

amount. Rather closed-end funds are generally priced at a discount, with occasional periods of time

during which they sell at a premium. A model that tries to explain this phenomena should show more

than the possibility that the closed-end fund will sell for a price that differs from the underlying

securities. It should also explain why closed-end funds generally, but not always, sell for less. The

current model can do so if one believes that the supply shocks associated with the mutual fund are small

relative to those for the underlying stocks. This seems plausible if one returns to the assumption that

changes in the supply of human capital drive the supply shocks. The companies representing the

underlying stocks presumably came into being in response to real production opportunities and thus their

production may be greatly influenced by the current supply of talent. Conversely, the mutual fund

represents a financial institution that simply repackages claims and thus one might expect its production

to be relatively immune to the current talent pool, which translates into small supply shocks within the

model. If the variance of a mutual fund's supply is in fact relatively small, then the economy should

respond by producing a relatively low value for the closed-end mutual fund and a relatively high price

volatility.

To prove the following propositions it is useful to first define Y as the zero dividend portfolio

that is short the mutual fund and long its underlying stocks, Y_u as the underlying stock portfolio, and y_m as

---

[13]For a further discussion of the economic conditions under which the model's results will hold
the reader should consult Appendix A. That appendix considers the economic forces that might lead the
economy to choose one equilibrium over another, and whether or not the negative root equilibrium can be
eliminated via the creation of additional long lived corporations.

a portfolio that only holds the mutual fund.[14] Thus,

$$Y = \begin{pmatrix} Y_u \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ y_m \end{pmatrix} \tag{16}$$

and where the 0 vectors in **(16)** are included to make the terms in parenthesis conformable with each other.

Proposition 4: Assume that the variance of the mutual fund's supply shock ($\sigma^2_{\eta m}$) is independent of all other supply shocks, and that A is a diagonal matrix. Further assume that the equilibrium in the economy sets pseudo-eigenvalue (λ_y) associated with Y equals -1. Then, for $\sigma^2_{\eta m}$ sufficiently small the expected value of the zero dividend portfolio Y is positive, and the variance of the closed-end fund's price will exceed the variance of its net asset value.

The results in Proposition 4 seem to conform with the two primary findings in the empirical closed-end fund literature. To a large degree closed end funds exhibit both relatively low prices and higher volatilities than a portfolio comprised of the underlying securities.

Notice that non-dividend shocks driving the results in Proposition 4 push the model in exactly the opposite direction from the analogous forces in the investor sentiment model of Lee, Shleifer, and Thaler (1991). In their model investor sentiment introduces a non-dividend based shock to the system, and they find that a higher variance on the investor sentiment parameter leads to a higher variance in the security. But in the current model, at least within the equilibrium of interest, a lower variance on the non-dividend based shock (i.e. the supply shock) leads to a higher variance and discount on the zero dividend portfolio containing the closed-end mutual fund. This difference my prove useful when attempting to reconcile the data. In general, theories have had been unable to fully account for the rather large discounts observed in

---

[14]Note that the portfolio that contains only the mutual fund is represented by a "y" in keeping with the convention of using lower case letters for scalar variables.

the data without the imposition of empirically unreasonable parameter values.[15] However, in the current

model a *small* variance on the mutual fund's supply shocks lead to large discounts. Thus, modest

parameter values appear to have some chance of reconciling the data.

Another aspect of the equilibrium is the influence the elements of A have on both the existence of

closed-end mutual fund discounts, and the volatility of the closed-end fund's per share price relative to

that of the underlying assets. Once again consider the idea that human capital shifts influence the asset

supplies. As argued earlier assets outside the closed-end fund represent companies that produce real

goods and services and it seems likely that their production will depend heavily upon the current talent

pool. In contrast, the talent pool for the production of closed-end fund services seems likely to be

relatively stable. Within the model these relative influences are represented in part by the matrix A, with

larger diagonal elements of A and smaller off diagonal elements of A representing a supply process that

exhibits lower volatility over time as the talent pool changes.[16] Thus, according to the arguments given

above the diagonal value of A associated with the closed-end fund should be relatively large, and the off

diagonal elements relatively small. As the next proposition shows, if these conclusions accurately

represent the economic forces at work, then the closed-end fund will sell at a discount and exhibit excess

price volatility relative to its net asset value.

---

[15]As discussed earlier Malkeil's (1977) paper came to this conclusion with regard to taxes and
agency costs. More recently, a paper by Brauer (1993) comes to the same conclusion with regard to the
investor sentiment hypothesis.

[16]The off diagonal elements have the opposite influence as the diagonal elements because they
represent the influence of the supply of security i on the supply of security j. A more detailed
explanation can be found in the discussion following Proposition **8**.

Proposition 5: Assume that the variance of the mutual fund's supply shock ($\sigma^2_{\eta m}$) is independent of all other supply shocks, and that A is a diagonal matrix. Let a_{mm} represent the element of A associated with the mutual fund. Then an increase in a_{mm} increases the closed-end fund's discount's expected value and variance.

The relationship documented in Proposition 5 provides a way to test the model against data outside that provided by the stock market. Actual aggregate corporate output equals the supply of outstanding shares times the current dividend produced by each share. Proposition 5 states that there should exist a relationship between the aggregate output statistics for the industries in a closed-end fund and that fund's discount. If the fund holds industries whose asset supplies mean revert relatively slowly then the discount on the fund will be relatively large.

Propositions 2 and 3 show that the zero dividend portfolio forms an eigenvector of both the dividend shocks' variance-covariance matrix and the square root term in *B*. Because this greatly simplifies the pricing equations one can say a great deal about the covariance of its price with any other portfolio.

Proposition 6: The covariance between a zero dividend portfolio and any other portfolio equals

$$Cov(Z'P, \ X'P) \ = \ \frac{1}{4\theta^2}Z'[rI+A]\Sigma_\eta^{-1}[rI+A]X. \tag{17}$$

Notice that even though the pricing equation is rather complex, the factors that influence the covariance between the zero dividend portfolio and any other portfolio are rather simple.

Some empirical studies have found that closed-end mutual fund discounts appear to decline when small firms have unusually large returns, and increase in periods where small firms exhibit below normal returns. This is really a statement about the correlation between closed-end fund discounts and small

stock returns, which are apparently correlated.[17] Equation **(17)** shows how this can happen. Define

$\Psi\Pi\Psi^{1} = [rI+A]\Sigma_n^{-1}[rI+A]$, so that Ψ equals a matrix of orthonormal eigenvectors, Π a diagonal matrix of

eigenvalues with π_i as its ith diagonal element. With this definition one can decompose any arbitrary

vector X into a weighted sum of the eigenvectors in Ψ. Thus, if there are k securities one can rewrite **(17)**

as

$$Cov(Z'P,\ X'P)\ =\ \frac{1}{4\theta^2}\sum_{i=1}^{k}\pi_i w_i(Z)w_i(X) \tag{18}$$

where $w_i(Z)$, and $w_i(X)$ satisfy

$$Z\ =\ \sum_{i=1}^{k}w_i(Z)\psi_i,\ \text{and}\ X\ =\ \sum_{i=1}^{k}w_i(X)\psi_i, \tag{19}$$

if ψ_i equals the ith eigenvector in Ψ. Because, $[rI+A]\Sigma_n^{-1}[rI+A]$ is a positive definite matrix its

eigenvalues must be strictly positive. Equation **(18)** basically describes a principal components analysis

of the correlation between the zero dividend portfolio and any other portfolio. The empirical statement

that closed-end fund discounts appear correlated with the returns to small company stocks is equivalent

(within the model) to a statement that they share similar weights (w) in **(19)**.[18] Notice, that the equation

_____

[17]There is quite a bit of disagreement about this conclusion. Lee, Shleifer, and Thaler (1991) presented the initial evidence regarding the correlation between closed-end fund discounts and small stock returns. Chen, Kan and Miller (1993a, and 1993b) then presented new evidence that this correlation either does not exist or it is very weak. They also reinterpret the findings in Lee, Shleifer and Thaler (1991) to imply that the evidence does not support the existence of a correlation between closed-end fund discounts and small stock returns. Chopra, Lee, Shleifer, and Thaler (1993a, 1993b) dispute their conclusions. Whatever the case may be, the predictions presented in this paper are about the potential correlation between closed-end fund discounts and any other arbitrary portfolio. While one can apply the model's findings to small firm portfolios, it is also possible to apply them to portfolios with other characteristics.

[18]This type of analysis resembles some of the statistical work found in Lee, Shleifer and Thaler (1991) and Chen, Kan and Miller (1993a). These papers regress closed-end fund discounts on the returns

underlying the principal components decomposition (equation **(17)**) does not include the variance-covariance matrix governing the dividend process. This is not too surprising since the variance of a zero dividend portfolio obviously equals zero. However, it does mean that the covariance between the zero dividend portfolio and the underlying securities depends on variance-covariance matrix of the supply shocks. The rather extensive literature on small stock returns indicates that small stocks incorporate a return "factor" that does not appear to influence large company stocks. Thus, there is every reason to believe that one of the eigenvectors in Ψ may represent a "small stock" portfolio. Under this scenario the closed-end fund discount will be correlated with small stock returns if the appropriate value of $w(Z)$ is positive.

Another empirical phenomena revolves around the finding by Thompson (1978) and Pontiff (1995) that closed-end fund discounts tend to be mean reverting. As the next proposition shows this empirical pattern can be guaranteed within the model by simply adding the assumption that A is a diagonal matrix equal to $a\mathrm{I}$, where $a>0$ is a scalar.

Proposition 7: If A can be written as aI then under the negative root equilibrium the closed-end fund discount will exhibit mean reversion. Mathematically, let $Y'P(\bar{N})$ represent the expected value of the closed-end fund discount. If $Y'P_t>Y'P(\bar{N})$ then $E(Y'P_{t+1})<Y'P_t$ and if $Y'P_t<Y'P(\bar{N})$ then $E(Y'P_{t+1})>Y'P_t$. Clearly one can greatly relax the proposition's restrictions on A and still retain the basic result in a number of cases. Intuitively, if the underlying production process in the economy exhibits mean reversion, then so will various statistics produced by the stock market.

The above propositions outline the parameter values that lead to expected closed-end fund discounts. Nevertheless, with some probability the discount will temporarily turn into a premium if the realized supply shocks take on the appropriate values. Thus, the model offers a potential explanation for

---

for 10 different stock size deciles. One can think of this as a search for the eigen-portfolio with the largest eigenvalue.

the creation of closed-end funds, and their subsequent returns. In those periods of time when there exists a premium investment firms can profitably create closed-end funds.[19] Then, due to the mean reversion in the discount (Proposition 7), the premium should then proceed to disappear and eventually become a discount.

At first blush, the above results seem to conform with Peavy's (1990) empirical findings on the creation of new closed-end funds, their initial prices and subsequent returns. Also, one should note, that the very same patterns arise in the investor sentiment model of Lee, Shleifer, and Thaler (1991). Alas, neither the current explanation nor that provided by the investment sentiment model is perfect. While it is true that closed-end funds are initially issued at a premium, Weiss-Hanley, Lee and Seguin (1996) show that the price would immediately collapse in the aftermarket without the very strong support provided by the investment firms. If in fact closed-end funds are only issued when they should sell at a premium then this type of support should not be needed. Thus, while both this model and the investor sentiment model can account for the creation of closed-end funds, they cannot reconcile all of the data. It thus seems more likely that the economic forces leading to the creation and sale of new closed-end funds differ from those that determine prices in the secondary market.

2.4   The Relationship Between Closed-End Fund Discounts and Aggregate Production

Intuitively, the most direct relationship between closed-end fund discounts and corporate productivity should involve changes in the contemporaneous values. Indeed there seems to be some indirect evidence that this relationship is a positive one. That is, when the economy does well closed-end fund discounts seem to increase. The evidence is only indirect since most studies have only examined the relationship between closed-end fund discounts and stock market returns, and this is not the same

---

[19]Obviously, within the current model investment houses cannot expend resources to create new funds. While ideally one would like to add this feature explicitly, the mathematics become considerably more complicated and would take the general discussion far afield. Nevertheless, one suspects that most of the primary results would not change.

thing as current corporate productivity. However, with that caveat in mind, both Malkiel (1977) and

Pontiff (1995) find that when the stock market does well, closed-end fund discounts increase. Thus, one

might expect a study that focused on corporate output to find a similar relationship, as would be

predicted by the next proposition.

Proposition 8: The covariance between changes in the discount, and changes in total current corporate

production equals

$$\text{Cov}(Y'(P_{t+1}-P_t),\ N'_{t+1}D_{t+1}-N'_tD_t)\ =\ -\frac{1}{\theta}Y'AD_t. \tag{20}$$

Let the subscript m designate the location in the vector Y that holds the short position in the closed-end

mutual fund. Then the covariance increases as a_{mm} increases, and if all of the values of Y_u, and D_t are

non-negative, then the covariance increases as a_{im} decreases.

The proposition shows two things. First, decreasing the values of the off diagonal elements of *A*

associated with the closed-end fund increases the covariance. Intuitively, one can think of these elements

as determining the influence that asset supplies in one security have on the asset supplies of another.

When the a_{ij} ($j{\ne}i$) are negative then large supplies in one asset tend to drive down supplies in other assets.

Second, for the diagonal element of *A* associated with the closed-end fund, increasing its value increases

the covariance. As discussed earlier increasing this parameter increases the speed of mean reversion in

the asset's supply. Together these two points lead to a simple inference. An increase in the speed at

which supplies mean revert leads to an increase in the covariance between changes in the discount and

total current corporate production.[20]

Proposition 8 deals with contemporaneous changes in the closed-end fund discount and corporate

---

[20]One can increase the speed of mean reversion either directly, via an increase in the a_{ii}, or
indirectly via a decrease in the a_{ij} for $j{\ne}i$.

production. However, based on Swaminathan's (1996) study, it also appears that there exists a relationship between closed-end funds discounts and *future* corporate output. Unfortunately, fitting the model to Swaminathan's (1996) results runs into problems associated with the "economy's initial conditions." The mean reverting supply process prevents one from simply calculating a meaningful relationship between closed-end fund discounts, and aggregate production. Not only will the results depend upon the initial starting conditions, but also the one, two, three and other future correlations will differ from each other. Another problem lies with the fact that empirical studies do not provide the necessary information to calculate a set of reasonable starting conditions for the security supplies. All of this suggests that it may prove more fruitful to look for correlation statistics that do not depend upon the model's initial conditions, at least with respect to the supply process.[21]

In order to calculate unconditional statistics, one needs to examine the model's "steady state." This can be achieved by looking at the period t value of the statistic based upon the available information at time 0, and then letting t go to infinity. In this way the influence of the initial conditions can be purged from the results. For the problem at hand, the unconditional covariance between the closed-end fund's discount and future corporate productivity equals:

$$\lim_{t \to \infty} \text{cov}(Y'P_t, \ [N_{t+1}'D_{t+1} - N_t'D_t] | N_0) \equiv \lim_{t \to \infty} E_0 \Big\{ Y'[P_t - P(\bar{N})][N_{t+1}'D_{t+1} - N_t'D_t] \Big\}. \tag{21}$$

where $P(\bar{N})$ represents the price vector given N equals $\bar{N}$, and N_0 represents the value of N in period 0. The first term in the covariance calculation equals the period t discount on the closed-end mutual fund

---

[21]As will become apparent, to a limited degree, the results in this section do depend upon the initial starting conditions for the dividend process. There is nothing that can be done about this since the model assumes that the dividend process does not mean revert. Thus, the expected value of the dividend vector in any future period is simply its current value. One can eliminate this problem by simply imposing a mean reverting process on the dividends. That will only change the results in the subsequent sections in that a vector representing the long run average dividend (a term analogous to $\bar{N}$) would replace D_0 in the equations that follow.

relative to its unconditional mean. The second term equals the difference between aggregate corporate

production in periods $t+1$ and t.

Proposition 9: The unconditional covariance between closed-end fund discounts and future aggregate

production equals

$$\lim_{t \to \infty} \operatorname{cov}(Y'P_t, \ [N_{t+1}'D_{t+1} - N_t'D_t]|N_0) \ = \ \frac{1}{\theta}Y'[rI+A]\Sigma_\eta^{-1}\left[\sum_{\tau=1}^{\infty}(I-A)^{\tau-1}\Sigma_\eta(I-A)^{\tau-1}\right]AD_0. \tag{22}$$

Proposition 9 provides an empirical prediction regarding current discounts and future aggregate

corporate production; a result that cannot be readily produced by models that rely on either investor

sentiment or transactions costs. While investor sentiment may be correlated with current market

conditions, why should it forecast future aggregate production? Transactions costs models suffer from

similar problems. While they may be associated with current economic predictions it is not clear why

they should forecast future conditions.

To obtain additional insights into Proposition 9's results consider the case where Σ_η and A are

both diagonal matrices. In this case the right hand side of equation **(22)** simplifies to

$$\operatorname{cov}(Y'[P_t-P(\bar{N})], \ [N_{t+1}'D_{t+1} - N_t'D_t]) \ = \ \sum_{i=1}^{k}\frac{1}{\theta}y_id_{0i}\frac{r+a_{ii}}{2-a_{ii}}. \tag{23}$$

Again, in line with the notation developed so far, small letters stand for scalar elements within their

corresponding matrices. Thus, y_i equals the ith element of Y, while d_{0i} represents the period 0 dividend

provided by security i, and a_{ii} equals the ith diagonal element of A. Recall, that Y represents a zero

dividend portfolio that holds a long position in the closed-end fund's underlying assets and a negative

position in the closed-end fund itself. Consider the case where all of the initial dividends are positive and

the closed-end mutual fund only holds long positions in its underlying assets. Then the ith element in the

24

summation within equation **(23)** will take on non-negative values for securities held by the closed end fund (since the corresponding y_i will be positive in such cases) and a non-positive value for the element of y_i representing the short position in the closed-end fund itself.

The actual magnitude of **(23)** clearly depends upon a number of elements. However, the influence of each individual a_{ii}, the rate at which a particular supply process mean reverts, can be broken out to produce some empirically interesting properties.

Proposition 10: Assume that Σ_η and A are diagonal matrices. Then an increase in a_{mm} or a decrease in a_{ii} for $i \neq m$ decreases the unconditional covariance between the current closed-end fund discount and future corporate production.

Once again, a relatively small valued mean reversion parameter for the closed-end fund's supply leads to results that are roughly consistent with the data.

By now it should be clear that the model can reconcile a number of empirical patterns via the same restrictions on the relative speed with which asset supplies mean revert. Proposition 5 shows that an increase in a_{mm} (the element of A associated with the closed-end fund) leads to a larger discount and higher volatility on closed-end fund's price. Proposition 8 shows that increasing the value of a_{mm} increases the covariance between the discount and changes in current corporate productivity. Finally, Proposition 10 shows that increasing a_{mm} induces a negative covariance between current closed-end fund discounts and future production. Thus, the model provides a fairly parsimonious explanation for a number of observed phenomena.

Together Propositions 5, 8, and 10 link the same set of parameter values to multiple results, and thus they provide a set of associated testable hypotheses. If the underlying industries held by a closed-end fund show relatively slow rates of mean reverting production then one expects this to imply four things about the closed-end fund's price:

1. it will display a relatively large discount,

2. it will be relatively volatile,

3. it will have a particularly strong covariance with unexpected contemporaneous changes in corporate valuations, and

4. it will display a negative covariance with future production.

Furthermore, these hypotheses are easily tested against the empirical data. Simply look at the industry holdings for each fund, and then compare the rate at which productivity within each industry mean reverts relative to its trend. Funds that hold industries with relatively slow levels of mean reversion should be responsible for a disproportionate fraction of the statistical patterns that have been observed in the data between close-end fund discounts and the other economic variables that researchers have examined.

3   Conclusion

Financial theory rests to a large degree upon a paradigm in which stocks trade within a frictionless market, populated by competitive agents possessing homogenous information sets. Even absent this, at the very least most theories rule out arbitrage possibilities. Alas, this world view cannot explain why the stock market frequently prices closed-end mutual funds at a substantial discount from their net asset values. In a frictionless market arbitrage, if nothing else, should preclude any discrepancies from occurring let alone long term discrepancies with non-zero means.

As Malkiel (1977) points out, the standard financial paradigm (even with the addition of some standard frictions such as taxes and agency costs) simply cannot explain the large persistent closed-end fund discounts observed in the data. However, the model presented here shows that one can reconcile large parts of the data by altering the standard financial model in other modest ways. In particular if asset supplies vary over time, and if rational investors have finite lives then closed-end fund discounts can appear in what is otherwise a very traditional model. These discounts appear and persist despite a lack of frictions from either transactions costs, or informational asymmetries. For better or worse it

appears that death is a rather large friction. Individuals with finite lives cannot arbitrage between closed-end funds and the underlying securities simply because they risk having the price move against them during their lifetime. If these investors believe that closed-end fund prices can deviate from the fund's net asset value, then they will act accordingly by submitting demands that exhibit some inelasticity. Such behavior will of course verify their beliefs since closed-end fund prices will then deviate from underlying net asset values.

The model not only provides a setting in which closed-end fund discounts can exist, but it also produces a number of empirical predictions that also seem to concur well with patterns in the data. In particular, a number of propositions show that the same parameter values that lead to the existence of a closed-end fund discount will also lead to a fund price that is more volatile than the underlying net asset value, to a discount that exhibits a positive covariance with current changes in corporate production, and whose magnitude displays a negative covariance with future economic productivity. Since all four of these regularities appear in the data, the model may provide a simple explanation capable of connecting them to one source.

4   Bibliography

Azariadis, C., 1981, "Self-Fulfilling Prophecies." *Journal of Economic Theory*, 25, 380-396.

Barclay, M., C. Holderness, and J. Pontiff, 1993, "Private Benefits from Block Ownership and Discounts on Closed-End Funds," *Journal of Financial Economics*, 33, 263-291.

Bhushan, R, D. Brown, and A. Mello, 1997, "Do Noise Traders 'Create Their Own Space'?," *Journal of Financial and Quantitative Analysis*, 32, 25-45.

Brickley, J., S. Manaster, and J. Schallheim, 1991, "The Tax-Timing Option and the Discounts on Closed-End Investment Companies," *Journal of Business*, 64, 287-312.

Brickley, J. and J. Schallheim, 1985, "Lifting the Lid on Closed-End Investment Companies," *Journal of Financial and Quantitative Analysis*, 20, 107-117.

Brauer, G., 1993, "'Investor Sentiment' and the Closed-End Fund Puzzle: A 7 Percent Solution," *Journal of Financial Services Research*, 7, 199-216.

Chen, N., R. Kan, and M. Miller, 1993a, "Are the Discounts on Closed-End Funds a Sentiment Index?," *Journal of Finance*, 48, 795-800.

Chen, N., R. Kan, and M. Miller, 1993b, "Are the Discounts on Closed-End Funds a Sentiment Index?: A Rejoinder," *Journal of Finance*, 48, 809-810.

Chopra, N., C. Lee, A. Shleifer, and R. Thaler, 1993a, "Yes, Discounts on Closed-End Funds are a Sentiment Index," *Journal of Finance*, 48, 801-808.

Chopra, N., C. Lee, A. Shleifer, and R. Thaler, 1993a, "Yes, Discounts on Closed-End Funds are a Sentiment Index: Summing Up," *Journal of Finance*, 48, 811-812.

Chordia, T. and B. Swaminathan, 1996, "Market Segmentation, Imperfect Information, and Closed-End Fund Discounts," working paper Owen Graduate School of Management, Vanderbilt University.

Dow, J. and G. Gorton, 1994, "Arbitrage Chains," *Journal of Finance*, 49, 819-849.

Froot, K., D. Scharftstein, and J. Stein, 1992, "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation," 47, 1461-1484.

Grinblatt, M., S. Titman, and R. Wermers, 1995, "Momentum Investment Strategies, Portfolio Performance, and Herding: A Study of Mutual Fund Behavior," *American Economic Review*, 85, 1088-1105.

Hanley, K. Weiss-, C. Lee, and P. Seguin, 1996, "The Marketing of Closed-End Fund IPOs: Evidence from Transactions Data," *Journal of Financial Intermediation*, 5, 127-159.

Holden, C. and A. Subrahmanyam, 1996, "Risk Aversion, Liquidity, and Endogenous Short Horizons," *Review of Financial Studies*, 9, 691-722.

Lee, C., A. Shleifer, and R. Thaler, 1990, "Closed-End Mutual Funds," *Journal of Economic Perspectives*, 4, 153-164.

Lee, C., A. Shleifer, and R. Thaler, 1991, "Investor Sentiment and the Closed-End Fund Puzzle," *Journal of Finance*, 46, 75-109.

Malkiel, B., 1977, "The Valuation of Closed-End Investment Company Shares," *Journal of Finance*, 32, 847-859.

Peavy, J., 1990, "Returns on Initial Public Offerings of Closed-End Funds," *Review of Financial Studies*, 3, 695-708.

Pontiff, J., 1995, "Closed-End Fund Premia and Returns Implications for Financial Market Equilibrium," *Journal of Financial Economics*, 37, 341-370.

Spiegel, M., 1997, "Stock Price Volatility in a Multiple Security Overlapping Generations Model," forthcoming *Review of Financial Studies*.

Strang, G., 1976, *Linear Algebra and its Applications*, Harcourt Brace Jovanovich, Inc., New York.

Swaminathan, B., 1996, "Time-Varying Expected Small Firm Returns and Closed-End Fund Discounts," *Review of Financial Studies*, 9, 845-887.

Thompson, R., 1978, "The Information Content of Discounts and Premiums on Closed-End Fund Shares," *Journal of Financial Economics*, 6, 151-186.

Vayanos, D., 1996, "Transaction Costs and Asset Prices: A Dynamic Equilibrium Model," working paper Massachusetts Institute of Technology, Sloan School of Management.

Weiss, K., 1989, "The Post-Offering Price Performance of Closed-End Funds," *Financial Management*, 18, 57-67.

5   Appendix A - Stability of the Equilibria

Both roots of the eigenvalue matrix yield equilibria that are fully consistent with the generally accepted principles governing Nash equilibria. In all cases each trader's actions maximize his or her utility given the actions of all other traders, while the equilibrium prices equilibrate supply and demand in each asset. Thus, issues of stability (to the extent they exist at all) must be addressed outside the usual game theoretic arguments.

One way to examine the stability of each equilibrium is to see whether the system tends to "drift towards or away" from the equilibrium point if it starts a small distance away from the equilibrium. Such arguments are necessarily heuristic since game theoretic arguments do not include an explanation as to how the economy selects one equilibrium or another. However, with this caveat in mind the forces that drive the two equilibria can be seen within the following graph.
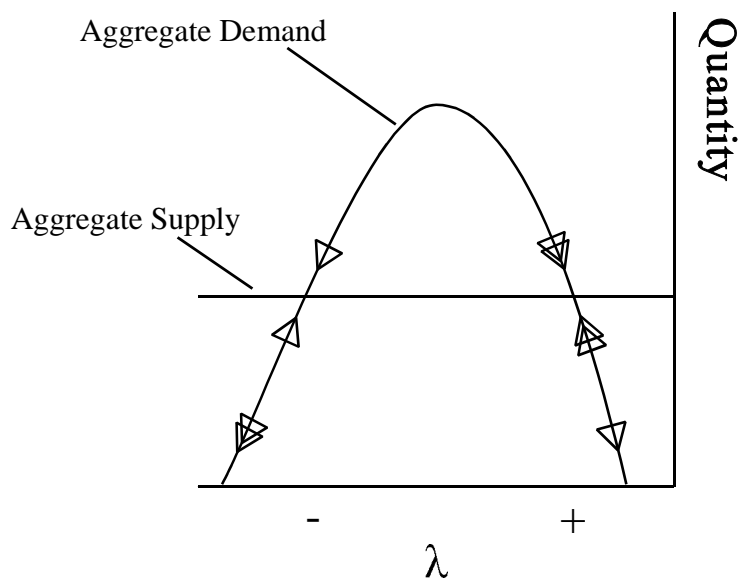

Figure 1: "-" and "+" = value of some eigenvalue λ under the negative and positive root equilibria respectively. Single triangles represent the "out of equilibrium" forces on λ from the stock's variance, while the double triangles do the same for the stock's price.

30

The arc in **Figure 1** represents the aggregate demand of the population for the eigen-portfolio associated with the pseudo-eigenvalue λ, while the straight line represents the eigen-portfolio's aggregate supply. In the scalar case the value of B is simply proportional to λ, thus in the vector case one can think of an increase in λ as equaling an increase in B. In fact, this interpretation will make the following discussion somewhat easier to understand in terms of the influence the pseudo-eigenvalues have on stock price volatility via equation **(11)**.

The plus and minus signs within **Figure 1** designate the value of λ under the positive and negative root equilibria. Notice, that there are two market clearing values for λ because an increase in λ does not uniformly increase or decrease the aggregate demand. With a positive supply of the asset, an increase in λ has two opposing influences on demand. First, it increases the stock price thereby discouraging investors from purchasing the asset. Second, it reduces the variance of the stock's price (since an increase in λ moves its value towards zero) which encourages investors to purchase the asset. Depending on the current value of λ one or the other influence will dominate.

The arrowheads in **Figure 1** indicate the direction of each force's influence on an auctioneer that uses a tatonnement process to set λ by considering λ's influence on the price and variance separately. First, consider the area to the right of the plus sign. In this region, the auctioneer notices that there exists insufficient demand for the stock. The auctioneer can increase demand by lowering the price variance, and he can do this by increasing λ. Thus, the single arrowhead points to the right. Alternatively, the auctioneer can increase demand by decreasing the price of the stock, which he can accomplish by decreasing λ. Thus, the double arrowheads point to the left and into the plus sign. These same arguments can also be applied to the other regions in the graph thereby yielding the forces indicated by the remaining arrowheads.

Notice that in terms of the stability arguments given above neither the positive nor the negative

root equilibrium has an advantage. The impact of λ on the security's price tends to favor the positive root equilibrium, while its impact on the variance tends to favor the negative root equilibrium.

One might suspect that even if an equilibrium is stable for an economy with finitely lived investors, it may not be stable if these same investors can set up infinitely long lived mutual funds. In order to make the argument concrete, consider a period t generation that attempts to profit by setting up such a mutual fund when the price of the zero dividend portfolio has a positive price. Under this scenario the fund sells the zero dividend portfolio and distributes the cash to its investors. In period $t+1$ the investors then offer to sell their shares in the fund to next generation at a price of zero by pointing out that there are no cash flows associated with the fund. As part of the old generation's sales pitch they must convince the young generation that too will find ready buyers at a price of zero one period later. In fact, the period $t+1$ old generation must convince the young generation that every subsequent generations will successfully convince its own descendants to accept the fund shares at a price of zero.

Now, suppose that after setting up the fund the price of the zero dividend portfolio increases even further, and consider the position of the generation that is "supposed" to purchase the shares at a price of zero. If these investors want to hold this same mutual fund they can go out and sell the same zero dividend portfolio themselves, and then *keep* the revenues themselves. Clearly, this strategy at least weakly dominates that of purchasing the fund shares from the previous generation. The young generation winds up holding the same assets plus the cash from shorting the underlying securities, rather than with a position in just the underlying securities. Even so, might the young generation still agree to make the purchase? Not if they think there exists an arbitrarily small possibility that the next generation will not agree to transfer the shares at a level below the fund's net asset value. Consider such "trembles" and let the trembles go to zero. For any positive tremble each member of the young generation strictly prefers to create his or her own position by trading in the underlying assets rather than accept the mutual fund shares from the old generation. Thus, under what would seem to be a fairly reasonable application

of "trembling hand" perfection, the creation of an infinitely lived mutual fund cannot eliminate the

negative root equilibrium. The mutual fund is simply a corporate shell and cannot alter the fundamental

tradeoffs available to investors.[22] Unfortunately, no corporation or mutual fund can change the fact that

the underlying claimants have finite lives.

---

[22]As with any other model in this genre, all of the shocks are normally distributed in order to make the model tractable. Thus, prices can go negative and investors do not enjoy limited liability protections. However, the dominance argument does not appear sensitive in this regard and if anything limited liability should strengthen them. With limited liability a mutual fund with a negative net asset value will simply declare bankruptcy. This possibility may severely limit its ability to short the "expensive" security.

6   Appendix B

Proof of Proposition 1**:** Given the wealth equation **(5)** and the conjectured pricing equation **(7)** the

trader will seek to solve the problem given by equation **(8)**. Differentiating **(8)** therefore produces the

first order conditions that must be satisfied by the investor's demands. Next, match terms to solve for B_0,

and B. Since aggregate demand cannot depend upon the current dividend, one must have that

$$B_0 = \frac{1}{r} BA\bar{N}. \tag{24}$$

Now set X_t equal to N_t in order to produce an equation in terms of the matrix B,

$$B\Sigma_\eta B' + \frac{1}{\theta} B[rI+A] + \left(\frac{1+r}{r}\right)^2 \Sigma_\delta = 0. \tag{25}$$

This is a quadratic matrix equation in B. To solve it let $B_1 = B[rI + A]$, and use this to replace B in the

above equation. After making this substitution pre and post multiply the equation by $\Sigma_\eta^{1/2}[rI+A]^{-1}$ and its

transpose respectively. Now let

$$Y = \Sigma_\eta^{1/2}[rI+A]^{-1}B_1[rI+A]^{-1}\Sigma_\eta^{1/2} \tag{26}$$

and note that Y is a symmetric matrix, implying that $Y = Y'$. With all of these substitutions in place the

quadratic equation reduces to

$$Y^2 + \frac{1}{\theta}Y + \left(\frac{1+r}{r}\right)^2 \Sigma_\eta^{1/2}[rI+A]^{-1}\Sigma_\delta[rI+A]^{-1}\Sigma_\eta^{1/2} = 0. \tag{27}$$

In order to solve **(27)** for Y add and subtract $.25\theta^2 I$, which will complete the square. This yields a solution for Y equal to

$$Y = -\frac{1}{2\theta}I + \left[\frac{1}{4\theta^2}I - \left(\frac{1+r}{r}\right)^2 \Sigma_\eta^{1/2}[rI+A]^{-1}\Sigma_\delta[rI+A]^{-1}\Sigma_\eta^{1/2}\right]^{1/2}. \tag{28}$$

Reversing through all of the substitutions produces equations **(9)**, **(10)**, and **(11)**. Q.E.D.

Proof of Proposition 2: Proof: Let S and V represent k×k matrices, with V diagonal, such that $SVS^{-1} = F^{-1}EF'^{-1}$. Thus, S represents a matrix of eigenvectors and V a diagonal matrix of eigenvalues for $F^{-1}EF'^{-1}$. Using these relationships the following equalities must hold,

$$\begin{aligned} Z'F[I-F^{-1}EF'^{-1}]^{1/2}F' &= Z'F[I-SVS^{-1}]^{1/2}F' = \\ Z'F\left[S(I-V)S^{-1}\right]^{1/2}F' &= Z'FS(I-V)^{1/2}S^{-1}F. \end{aligned} \tag{29}$$

From the definitions of F, S, and V the following equalities must also hold

$$Z'FSVS^{-1}F' = Z'FF^{-1}EF'^{-1}F' = Z'E = 0, \tag{30}$$

where the last equality follows from the proposition's assumption that $Z'E=0$. Thus, Z is an eigenvector of $F^{-1}EF'^{-1}$ with an associated eigenvalue 0. Define $Z'F = Y'$ and select S orthonormal with the exception of the eigenvector that is proportional to Z. Call the eigenvector associated with Z, s_z and normalize it so that $Z's_z = 1$. Eigenvectors with these properties must exist since $F^{-1}EF'^{-1}$ is symmetric, see Strang (1976). Now the ith diagonal element of $(I-V)^{1/2}$ equals ± 1 since the ith diagonal element of V equals 0. So,

$$Z'FS(I-V)^{1/2}S^{-1}F' = \pm Z'FF' \tag{31}$$

since the square root of +1 equals ± 1. Q.E.D.

35

Proof of Proposition 3: To derive equation **(14)** pre-multiply equation **(11)** by Z' after using **(10)** to substitute out for B. Then use Proposition 2 to produce **(14)**.

To derive **(15)** note that

$$\Sigma_P = \frac{1}{r^2}\left[\Sigma_\delta + B\Sigma_\eta B'\right]. \tag{32}$$

Now pre and post multiply by Z and then use Proposition 2 to derive **(15)**. Q.E.D.

Proof of Proposition 4: Under the negative root equilibrium, the expected value of the zero dividend portfolio Y can be written as

$$E(Y'P) = -\frac{1}{r\theta}Y'[rI+A]\begin{bmatrix}\Sigma_{\eta 1}^{-1} & 0 \\ 0 & \sigma_{\eta m}^{-2}\end{bmatrix}[rI+A]\bar{N}. \tag{33}$$

Here $\Sigma_{\eta 1}$ represents the supply shock variance-covariance matrix for the securities in Y_1. Expanding equation **(33)** produces

$$E(Y'P) = -\frac{1}{r\theta}\left[Y_1'(rI_1+A_1)\Sigma_{\eta 1}^{-1}(rI_1+A_1)\bar{N}_1 - y_m(r+a_m)^2\sigma_{\eta m}^{-2}\bar{n}_m\right], \tag{34}$$

where variables with the subscript "1" represent the appropriate submatrices and subvectors that correspond to the elements in Y_1. The variables with a subscript "m" represent the scalar elements of the appropriate matrices and vectors associated with y_m. Clearly for $\sigma_{\eta m}^2$ small enough $E(Y'P)$ must be strictly positive, proving the first claim in the proposition.

Under the negative root equilibrium the payoff variance from holding Y equals

$$Y'\Sigma_P Y = \frac{1}{\theta^2} Y'[rI+A] \begin{bmatrix} \Sigma_{\eta 1}^{-1} & 0 \\ 0 & \sigma_{\eta m}^{-2} \end{bmatrix} [rI+A]Y. \tag{35}$$

Expanding this equation into the components associated with Y_1 and y_m produces an equation similar to (34) with the $\bar{N}$ and $\bar{n}$ terms replaced by Y_1 and y_m respectively. Taking the limit as $\sigma_{\eta m}^2$ goes to zero proves the second claim in the proposition. Q.E.D.

Proof of Proposition 5: The proof of the first statement in the proposition follows immediately from equation (33). The proof of the second statement uses (35) and follows along the same lines as the arguments given in Proposition 4 following (35). Q.E.D.

Proof of Proposition 6: The derivation is similar to that of equation (15). Use the same arguments with Z as the zero dividend portfolio, and X as an arbitrary portfolio. Q.E.D.

Proof of Proposition 7: Using (14) write the realized change in the closed end fund discount as,

$$Y'(P_{t+1} - P_t) = -\frac{1}{\theta}(r+a)Y'\Sigma_\eta^{-1}(N_{t+1} - N_t). \tag{36}$$

now use (1) to find the expectation of (36) as

$$E[Y'(P_{t+1} - P_t)] = -\frac{1}{\theta}a(r+a)Y'\Sigma_\eta^{-1}(\bar{N} - N_t). \tag{37}$$

The difference between the current closed end fund discount and its unconditional expected value equals

$$Y'(P_t - P(\bar{N})) = -\frac{1}{\theta}(r+a)Y'\Sigma_\eta^{-1}(N_t - \bar{N}), \tag{38}$$

which clearly has the opposite sign as (37) since $0 \le a \le 1$. Q.E.D.

Proof of Proposition 8: From equation **(14)** $Y'(P_t\text{-}P_{t+1})$ can be written as

$$Y'(P_t - P_{t+1}) = -\frac{1}{\theta} Y'[rI + A]\Sigma_\eta^{-1}(A(\bar{N} - N_t) + \eta_{t+1}). \tag{39}$$

From the equations specifying the dividend and supply processes (equations **(2)** and **(1)**) one finds that

$$N'_{t+1}D_{t+1} - N'_t D_t = (A(\bar{N} - N_t) + N_t + \eta_{t+1})'(D_t + \delta_{t+1}) - N'_t D_t. \tag{40}$$

Plugging the above two equations into the formula for the covariance between two variables produces

$$E\left\{-\frac{1}{\theta}Y'[rI + A]\Sigma_\eta^{-1}\eta_{t+1}\eta'_{t+1}D_t\right\} = -\frac{1}{\theta}Y'[rI + A]D_t = -\frac{1}{\theta}Y'AD_t, \tag{41}$$

where the second equality follows from the assumption that $Y'D_t = 0$. Q.E.D.

Proof of Proposition 9: To prove the proposition use equation **(14)** to write

$$Y'(P_t - P(\bar{N})) = -\frac{1}{\theta}Y'[rI + A]\Sigma_\eta^{-1}(N_t - \bar{N}). \tag{42}$$

Next, simplify the second term in the covariance calculation by writing the period $t+1$ values for the dividend and security supply in terms of period t values and shocks. Thus,

$$N_{t+1}'D_{t+1} - N_t'D_t = [A(\bar{N} - N_t) + \eta_{t+1}]'D_t + [A(\bar{N} - N_t) + N_t + \eta_{t+1}]'\delta_{t+1}. \tag{43}$$

Now plug equations **(42)**, and **(43)** into the covariance formula in **(22)**, and then eliminate the D_t by using

$$D_t = D_0 + \sum_{\tau=1}^{t}\delta_\tau. \tag{44}$$

At this point, note that the N_t terms do not depend upon the realization of any of the δ_t terms and recall that the δ_t have both mean zero expectations and zero correlations with any other non-dividend shocks within the system. Thus, the expectation of δ_t with either N_τ or η_τ for any t or τ equals zero. This eliminates all of the δ_t terms from the calculation.

To eliminate the N_t terms one needs to calculate the value of N_t in terms of N_0. To do this repeatedly use **(1)** to move forward in time from period 0 to period t and thereby produce

$$N_t \;=\; (I-A)^t N_0 \;+\; \sum_{\tau=1}^{t} (I-A)^{\tau-1}[A\bar{N}+\eta_{t+1-\tau}]. \tag{45}$$

The at t goes to infinity $(I-A)^t$ goes to zero and $\sum_{\tau=1}^{t}(I-A)^{\tau-1}A\bar{N}$ goes to $\bar{N}$ since (by assumption) the eigenvalues of I-A lie between zero and one. Thus, both the N_0 and $\bar{N}$ terms disappear from the calculation of **(22)**. This leaves the following elements to account for in the limit

$$\lim_{t\to\infty}\ \text{cov}(Y'[P_t-P(\bar{N})],\ [N_{t+1}'D_{t+1}-N_t'D_t]) \;=\;$$

$$\lim_{t\to\infty}\ E\left\{\frac{1}{\theta}Y'[rI+A]\Sigma_\eta^{-1}\left(\sum_{\tau=1}^{t}(I-A)^{\tau-1}\eta_{t+1-\tau}\right)\left(\sum_{\tau=1}^{t}(I-A)^{\tau-1}\eta_{t+1-\tau}\right)'A-\eta'_{t+1}\right\}D_0 \tag{46}$$

the right hand side of which simplifies to **(22)**. Q.E.D.

Proof of Proposition 10: Simply differentiate equation **(23)** to prove the proposition. Q.E.D.