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The Valuation of Inflation-Indexed and FX Convertible Bonds

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Abstract

Issuing convertible bonds has become a popular way of raising capital by corporations in the last few years. An important subgroup is convertibles linked to a price index or exchange rate. In this paper we extend the convertible pricing models of Tsiveriotis and Fernandes (1998) and McConnell and Schwartz (1986) to the case of indexation of the promised payments of the convertible to a general price index or to the price of foreign exchange. The theoretical framework derived in this paper considers two sources of uncertainty: both the underlying stock price and the consumer-price-index (or equivalently foreign-currency) are stochastic, and incorporate credit risk in the analysis. The extensions of two models enable to establish upper and lower bounds for the price of the indexed convertible.

We approximate the pricing equations by using Rubinstein (1994) three-dimensional binomial tree, and we describe the numerical solution. We investigate and compare the models with respect to the characteristics of the issuer, the economic environment and the security's characteristics. Moreover, we demonstrate the usefulness and the limitations of the pricing model by using convertible traded on the Tel- Aviv stock exchange.

1. Introduction

A convertible bond is a hybrid security, part debt and part equity, that while retaining most of the characteristics of straight debt, offers the right to forgo future coupon and principal payments, and instead, receive a pre specified number of the issuer's common stock. In recent years issuing convertible bonds has become a popular financial instrument. Between the years 1995 and 2000, based on dollar volume, the total market has grown at a 53.9% cumulative annual growth rate to $159 billion.[1]

In many financial markets convertible contracts as well as straight bonds link the promised payments to a general price index or the price of foreign exchange.[2] Japanese corporations have issued large amounts of convertible bonds with coupon and principal payments denominated in Euros or in U.S. dollars that can be converted to the issuer's stock traded in the domestic currency. In Israel, the coupon and the principal payments of most convertible bonds traded on the Tel-Aviv Stock Exchange (TASE) are linked to inflation as measured by the changes of the consumer-price-index (CPI) or to the Dollar/Shekel exchange rate.[3]

An important factor in the pricing of convertible bonds is credit risk. According to a recent Moody's sample between 1970 and 2000, default rates for rated convertible bond issuers are higher than those without convertible bonds in their capital structures.[4] Clearly credit risk has a crucial effect on convertible bond prices, and should not be ignored. In the last few years practitioners and academics have tried to incorporate credit risk in the pricing of nominal convertible bonds[5].

The main contribution of this paper is the derivation of a pricing model for inflation-indexed convertible bonds (hereafter IICB), where both the underlying stock and the CPI are stochastic and default risk is considered.[6] Our model can be easily applied to the valuation of foreign exchange convertibles where inflation uncertainty is replaced by foreign exchange one.

Since the work of Ingersoll (1977) and Brennan and Schwartz (1977 and 1980) the structural approach for the valuation of convertible bond, which was pioneered by Black Scholes (1973) and Merton (1974), is the ultimate choice. This method focuses on the capital structure of the firm where default may occur if the value of the firm's assets falls below the debt's face value.[7] Relying on this approach, Ingersoll (1977) and Brennan and Schwartz (1977, 1980) take the total value of the firm as a stochastic variable for pricing convertible bonds. Using the structural approach it is relatively easy to model the value of the convertible bond when the firm is in financial distress. The main drawback however of this approach is the need to estimate the total value and the volatility of the firm's assets, parameters that are not observable in the market.

To avoid this problem McConnell and Schwartz (1986, hereafter MS) present a valuation model for a zero coupon, convertible, callable, putable bond (LYON) based on the stock value as the stochastic variable. Since the stock price cannot become negative, it is impossible to simulate bankruptcy scenarios. To incorporate credit risk, they use an interest rate that is "grossed up" to capture the credit risk of the issuer, rather than the risk free rate.[8] However they treat credit spread as constant in their model meaning they do not take into account the fact that the credit risk of the convertible bond varies with respect to its moneyness. For this reason Bardhan et al. (1994) build the standard Cox, Ross and Rubinstein (1979, hereafter CRR) binomial tree for the underlying asset and consider the probability of conversion at every node. They choose the discount rate to be a weighted average of the risk free rate and the risky discount rate of an identical in quality straight corporate bond. The shortfall of this approach is its inability to take into account coupon payments or any contingent cash flow occurring due to call and put provisions.

To overcome these drawbacks Tsiveriotis and Fernandes (1998, hereafter TF) decompose the convertible bond into two components with different credit quality. The first is the debt only part of the convertible, which generates only cash payments and is exposed to default risk. The second component is the equity component, which is risk free, since the issuer can always deliver its own stock. They derive two joint PDE one for the "debt only component" and the other for the convertible bond price and approximate the solution by using the explicit finite difference method. Hull (2000) approximates these equations by using the more appealing CRR binomial tree.[9]

These single factor models can be adjusted to price IICB by using Fisher (1978) and Margrabe (1978) closed form solutions for an option to exchange one asset for another, where the underlying stock price and the CPI are both stochastic variables, and in the case of foreign currency convertible bond, the foreign currency replaces the CPI. However, since the conversion can take place anytime before maturity and the convertible bond usually has call and put provisions, the closed form solutions fail to price the convertible bond and a numerical method for the dynamics of the two correlated assets has to be applied, as suggested by Rubinstein (1994) and Boyle (1988) and others.[10]

In this paper we develop two valuation algorithms for the pricing of inflation-indexed convertible bonds (IICB) where both the underlying stock price and inflation are stochastic. Assuming a bivariate lognormal distribution for the underlying stock price and the CPI, we derive the governing PDE and the relevant boundary conditions for each of the two pricing models. Our theoretical framework can accommodate extensions of the MS and TF models. Following TF we incorporate credit risk by presenting two joint PDE, one for the convertible price and one for the artificial security - the "debt only component". Alternatively following MS we use the risky rate of the issuer as the discount rate of the convertible. By extending these two models for pricing IICB we poses upper and lower bounds for the IICB price. A numerical scheme for the dynamics of the two correlated assets is used, by applying Rubinstein (1994) three-dimensional binomial tree. This straightforward method is easy to implement compared to the finite different method.

We show how by applying the appropriate parameters MS, Margrabe (1978), CRR and Rubinstein (1994) models are nested as private cases within our extension of the MS model. We also show how Margrabe (1978), Rubinstein (1994), CRR and TF models are nested, as private cases, within our extension of the TF model for pricing the IICB.

By using a foreign exchange analogy our solution can be used with little modifications to price convertible bonds with coupon and principal payments denominated in a foreign currency, in the analogy the foreign currency corresponds to the consumer price index. [11]

We furthermore present numerical examples that demonstrate the usefulness of the model, the differences between the extensions of the MS and TF models, and illustrate how the models can be calibrated using market data. In a comparative statitcs analysis we study the sensitivity of the indexed convertible bond to credit spread, the correlation between the stock and the CPI returns, the CPI volatility and the real interest rates.

We examine empirically the different pricing models by using a sample that includes 291 daily observations from February 12, 2001, through April 13, 2003, of FX convertible bonds that are traded on the Tel-Aviv stocks exchange, which is characterized by the availability of high quality data. This is the first empirical study of the Tel-Aviv convertible-bond market. Each of the two extended pricing models is calibrated. The model-generated convertible bond prices are then compared to the market prices of the investigated convertible bonds. For the extended TF pricing models an overpricing of 1.58% is detected on average and for the extended MS model an underpricing of 2.31% is detected on average. For both models the average absolute prediction error is less than 5%, where the prices produced by the extended TF model are the closest to the market prices.

The rest of this paper is organized as follows. Section 2 describes the assumptions and derives the theoretical framework for pricing IICB. Section 3 presents the numerical binomial solution for the relevant pricing equations. Section 4 provides a sensitivity analysis of the convertible bond. Section 5 presents empirical application of the models for the pricing of indexed convertibles trade on the Tel Aviv Stock Exchange. Finally, concluding remarks are presented in section 6.

2. A Model for Pricing Inflation-Indexed Convertible Bonds with Credit Risk

In this section, we develop a valuation algorithm for the pricing of IICB. Unlike a nominal convertible bond that pays known coupons and principal payments the coupon and the principal payments of the IICB are linked to the changes of the consumer-price-index during the life of the convertible bond.

In order to price this type of convertible the following assumptions are made.[12]

(1) Investors can trade continuously in a complete, frictionless, arbitrage-free financial market. In particular it is assumed that there are no transaction costs, no restriction on short selling, and no differential taxes on coupons versus capital gains income.[13]

(2) The uncertainty in the economy is characterized by a probability space (Ω, F, P), where Ω is a state space, F is the set of possible events and P is the objective martingale probability measure on (Ω, F). The stock price S follows the stochastic differential equation

$$\frac{dS}{S} = (\mu_S - \delta)dt + \sigma_S dW_S \tag{1}$$

We also assume that the inflation process follows a geometric Brownian motion, with dynamics given by:

$$\frac{dI}{I} = \mu_I dt + \sigma_I dW_I\ , \tag{2}$$

where μ_s is the instantaneous expected return on the issuer's common stock, δ is the rate of dividend payout, μ_I is the instantaneous expected inflation rate. It is assumed that σ_s^2 and σ_I^2, which are respectively the instantaneous variances of the rate of return of the underlying stock, S, and the consumer-price-index, I, are constants. dW_S and dW_I are standard Wiener processes with correlation given by $dW_S dW_I = \rho_{SI} dt$.[14]

Using a foreign currency analogy, real prices correspond to foreign prices, nominal prices correspond to the domestic prices in local currency, and the CPI corresponds to the spot exchange rate. Garman and Kohlhagen (1983), assume that the process followed by a foreign currency is the same as that of stock providing a known dividend yield of δ, and therefore the drift under the risk neutral expectation of the foreign currency must be $(r - r_f)$, where r is the nominal domestic risk free rate and r_f is the foreign risk free rate.[15] By analogy, the CPI has a drift rate of $\mu_I = (r - r_r)$, where r_r is the real interest rate.[16]

Let $U(T, S, I)$ be the value at time t of an IICB with maturity at date T. The bond can be converted at any time to shares of the underlying stock S, and is paying a principal of F, that is linked to the changes in the CPI from the issuing date. The convertible pays fixed coupon payments, C, that are also linked to the CPI changes. To focus on the effects of inflation indexation on the convertible bond value we assume a generic IICB that is both non-callable and non-putable.

In the absence of risk of default by the issuer we can obtain its price dynamics by using Ito's formula for the dynamics of two correlated assets:

$$rU + f(t) = U_t + U_S(r-\delta)S + U_I(r-r_r)I + \frac{1}{2}(U_{ss}\sigma_S^2 S^2 + U_{II}\sigma_I^2 I^2 + 2U_{IS}\sigma_S\sigma_I\rho_{IS}IS) \tag{3}$$

where $U_S, U_I, U_{SS}, U_{II}, U_{SI}$ and U_t denote the first and second order partial derivatives of the value of the convertible bond with respect to S, I or t respectively and $f(t)$ represents the coupon payment function.[17]

Equation (3) does not account for default risk that is inherent in the convertible bond price. Our analytical framework can accommodate adjustments for credit risk in both ways suggested in the literature for a nominal bond. First, following MS model we use the risky rate of the issuer as the discount rate instead of the risk-free rate:

$$(r+cs)U + f(t) = U_t + U_S(r-\delta)S + U_I(r-r_r)I + \frac{1}{2}(U_{ss}\sigma_S^2 S^2 + U_{II}\sigma_I^2 I^2 + 2U_{IS}\sigma_S\sigma_I\rho_{IS}IS) \tag{4}$$

where cs is the credit spread implied by a similar in quality non-convertible bond of the same issuer.

We now cope with credit risk in a second way, by decomposing the convertible bond into two components with different credit quality, as is in TF. The underlying equity has no default risk since the issuer can always deliver its own stocks, on the other hand the issuer may fail to pay the coupon and principal payments, and thus introduce default risk. Following TF we define a hypothetical security, which is called the “debt only part of the convertible bond” that generates only cash payments, but no equity that an optimal holder of a convertible would receive. Since the convertible bond is a derivative security of the underlying stock, the "debt only" security can be considered a contingent claim with the same stock as its single underlying asset, thus the price of the debt only part, V, should follow the Black-Scholes (1973) equation. Since this security involves only cash payments by the convertible bond issuer, the relevant Black-Scholes equation should involve the credit spread of the issuer, i.e., the difference between the yield of a straight bond with the same credit quality as the convertible and a Treasury bond, identical in all respects except default risk. On the other hand, $(U-V)$ represents the value of the convertible related to payments in equity, and it should therefore be discounted using the risk free rate. The formulation of the convertible bond dynamics is obtained by the following system of two coupled equations, where Equation (5) refers to the convertible bond price, and equation (6) relates to the debt only component.[18]

$$r(U-V)+(r+cs)V-f(t)=U_t+\frac{1}{2}U_{SS}\sigma_S^2S^2+U_S(r-\delta)S \tag{5}$$

$$(r+cs)V-f(t)=V_t+\frac{1}{2}V_{SS}\sigma_S^2S^2+V_SrS \tag{6}$$

To extend the model to price IICB we add the terms which relate to the CPI from equation (3) to equations (5) and (6) and obtain the two PDE that evolve the IICB dynamics:

$$\begin{aligned} r(U-V)+(r+cs)V-f(t)=&\,U_t+U_S(r-\delta)S+U_I(r-r_r)I \\ &+\frac{1}{2}(U_{SS}\sigma_S^2S^2+U_{II}\sigma_I^2I^2+2U_{IS}\rho_{IS}\sigma_S\sigma_I SI) \end{aligned} \tag{7}$$

$$
\begin{aligned}
(r+cs)V - f(t) = & V_t + V_S(r-\delta)S + V_I(r-r_r)I \\
& + \frac{1}{2}(V_{SS}\sigma_S^2 S^2 + V_{II}\sigma_I^2 I^2 + 2V_{IS}\rho_{IS}\sigma_S\sigma_I SI)
\end{aligned}
\tag{8}
$$

Next, we characterize the boundary conditions according to the above defined terms of the IICB. The final conditions for the convertible bond price, U, and for the debt only component, V, can be written as:

$$
U(T,S,I) = \begin{cases} \lambda \mathrm{S} & \lambda \mathrm{S} \geq (F+C)\dfrac{I}{I_0} \\ (\mathrm{F}+\mathrm{C})\dfrac{I}{I_0} & \text{elsewhere} \end{cases}
\tag{9}
$$

$$
V(T,S,I) = \begin{cases} 0 & \lambda \mathrm{S} \geq (F+C)\dfrac{I}{I_0} \\ (\mathrm{F}+\mathrm{C})\dfrac{I}{I_0} & \text{elsewhere} \end{cases} ,
\tag{10}
$$

where λ is the conversion ratio, i.e., the number of shares of the underlying stock for which the convertible bond can be exchanged and I_0 is the value of the CPI on the issuing date of the convertible bond. [19] Since the bond can be converted at any time prior to maturity we are dealing with an American-type derivative, that has a free boundary conditions, where the upside constrains due to conversion are:

$$
U \geq \lambda S \qquad \forall\, t \in [t,T]
\tag{11}
$$

$$V = 0 \quad \text{if } U \leq \lambda S \qquad \forall\, t \in [t, T] \tag{12}$$

When we extend MS model according to equation (4) the boundary conditions are reduced to equations (9) and (11).

After having derived the models pricing equations and the relevant boundary conditions, we relate our results to previous contributions and show how these contributions are nested in our models.

CASE 1. *When $\delta = 0$ and $cs = 0$, credit risk is zero (switched off) and in the absence of dividends it is well known that it is never optimal to exercise an American call option before the expiration date. Thus when $\delta = 0$ and $cs = 0$, the two extended models reduce to the well-known results* of *Margrabe (1978) of an option to exchange one asset for another.*

CASE 2. *When the $cs = 0$, the two models reduce to the well-known results* of *Rubinstein (1994) for pricing American options that depend on two correlated assets.*

CASE 3. *When $\sigma_I \rightarrow 0$, the CPI process as only a drift term $(r - r_r)$ and thus when $r = r_r$ the CPI process is switched off and the extended TF and MS two factors models are reverted to the TF one factor model and to the MS one factor model respectively.*

CASE 4. *When* $cs = 0$, $r = r_r$ *and the CPI process is switched off by setting* $\sigma_I \to 0$, *the two models revert to the classic Cox-Ross-Rubinstein (CRR) model [1979].*

3. Numerical Implementation

Since the closed form solutions fail to price the convertible bond a numerical method for the dynamics of the two correlated assets is applied. TF use the finite difference method to approximate the convertible bond price within a one-factor model. Based on this model, Hull (2000) approximates the convertible bond price by using the more appealing CRR binomial tree. In this section we extend this analysis to two factors model and demonstrate how to construct a recombining three-dimensional binomial tree that approximates the bivariate process of the stock price and the CPI, for pricing IICB.

At first, to approximate the dynamics of the diffusion processes we construct a Rubinstein (1994) three-dimensional binomial tree, where the underlying stock price and the CPI are the two stochastic variables, next we solve the convertible PDE with a recursive backward algorithm for each of the models while taking into account the boundary conditions that were derived in section 2.

Following equations (1) and (2) the stock price and the CPI dynamics follow a general geometric Brownian motion and the joined density of the two underlying assets has a bivariate lognormal distribution. The three-dimensional binomial tree is a discrete version of this process for time interval Δt. The time interval $[0,T]$ is divided into N equal intervals of length Δt, each of which will be denoted by i, where $i = 0,1,..., N$.

As in Jarrow and Rudd (1983) binomial tree, the initial stock price, S, can move up at any period by u or down by d with equal probability, where:

$$u = e^{\alpha_S \Delta t + \sigma_S^2 \sqrt{\Delta t}}, \ d = e^{\alpha_S \Delta t - \sigma_S^2 \sqrt{\Delta t}}.$$

The no arbitrage conditions are: $u > e^{r(i,i+1)\Delta t} > d$, where $r(i,i+1)$ is the future risk-free rate of interest between period i and period $i+1$. In general, the underlying stock price at each node is set equal to $Su^j d^{i-j}$, where $j = 0,1,...,i$ is the number of up movements of the stock price. Inflation uncertainty is introduced via four conditions. If the stock price moves by u, the value of the CPI, I, can move either by A or B with equal probability. If the stock price moves down by d, the CPI value can move by C or D with equal probability, where:

$$A = \exp[\alpha_I \Delta t + \sigma_I \sqrt{\Delta t}(\rho_{S,I} + \sqrt{1-\rho_{SI}^2})]$$

$$B = \exp[\alpha_I \Delta t + \sigma_I \sqrt{\Delta t}(\rho_{S,I} - \sqrt{1-\rho_{SI}^2})]$$

$$C = \exp[\alpha_I \Delta t - \sigma_I \sqrt{\Delta t}(\rho_{S,I} - \sqrt{1-\rho_{SI}^2})]$$

$$D = \exp[\alpha_I \Delta t - \sigma_I \sqrt{\Delta t}(\rho_{S,I} + \sqrt{1-\rho_{SI}^2})]$$

(13)

Where $\alpha_I = (r - r_r) - \frac{\sigma_I^2}{2}$.

To make the lattice for each state variable recombine the condition $AD = BC$ is imposed. By setting $A \neq C$ and $B \neq D$, it is possible to construct a nonzero correlation between the underlying stock price and the consumer-price-index. The three dimensional binomial process converges to the original continouous process as $\Delta t \to 0$.

From any node (i, S, I), the lattice evolves to four nodes, $(i+1, Su, IA)$, $(i+1, Su, IB)$, $(i+1, Sd, IC)$ and $(i+1, Sd, ID)$. Where IA, IB, IC, and ID are the values of the CPI in the different nodes. The CPI in each node, at any time period i and with j up movements of the stock price, is set equal to:

$$I(i, j, k) = I_0 e^{\alpha_I i \Delta t + \sigma_I \sqrt{\Delta t}\left[\rho_{SI}(2j-i) + (\sqrt{1-\rho_{SI}^2})(2k-i)\right]} \tag{14}$$

where $k = 0, 1, ..., i$.

The four nodes have associated risk-neutral probabilities of 0.25. [20] The tree consists of 2^{i+1} distinct nodes at each period and of total $(1+N)^2$ distinct nodes.

Given the value of the stock and the CPI at any node we can calculate the value of the convertible bond at each node by starting at maturity, where its value is known with certainty, according to the final conditions, and then moving backwards in time period by period to calculate the value at the earlier nodes while applying the free boundary conditions.

Applying the final condition (9) and the boundary condition (11), at any time the bondholder has two choices. She can hold (or redeem at maturity) the bond, which has the value at each node of $UH_{i,j,k}$ or she can convert the bond to stocks and receive $UC_{i,j,k}$. Summarizing: the value of the convertible bond, $U_{i,j,k}$, at each node, is worth the maximum of $UC_{i,j,k}$ and $UH_{i,j,k}$, which can be written as: $\mathrm{U}_{\mathrm{i,j,k}} = \max[UH_{i,j,k}, UC_{i,j,k}]$.

In order to incorporate credit risk into the pricing model according to TF model, the convertible bond value is decomposed into two components. The first is the debt only part of the convertible, $V_{i,j,k}$, which is discounted by the risk adjusted rate of the issuer, $r^*(i,i+1)$, while the equity component, $E_{i,j,k}$ is discounted by the risk free rate, $r(i,i+1)$. At each node the convertible bond value is equal to $\mathrm{U}_{i,j,k} = V_{i,j,k} + E_{i,j,k}$, that is the sum of the equity component and the bond component at the node.

In order to incorporate credit risk to the pricing model according to MS only a slight modification is needed in the numerical procedure - to discount the equity component by the risky rate of the issuer, as was done with the debt component.

At each final node (N, j, k) the holding value, $UH_{N,j,k}$, can be calculated by multiplying the promised final payment by the CPI yield, which is calculated in equation (14). The value received from immediate conversion, $UC_{N,j,k}$ is calculated at each final node, can be calculated as:

$$UC_{N,j,k} = \lambda S u^j d^{N-j},$$

$$UH_{N,j,k} = (F+C)e^{\alpha_I i\Delta t + \sigma_I \sqrt{\Delta t}\left[\rho_{SI}(2j-i) + (\sqrt{1-\rho_{SI}^2})(2k-i)\right]} \tag{15}$$

Given $UC_{N,j,k}$ and $UH_{N,j,k}$, we obtain the value of the equity and debt components at each final node:

$$E_{N,j,k} = \begin{cases} \mathrm{UC}_{\mathrm{N,j,k}} & \mathrm{UC}_{\mathrm{N,j,k}} \geq \mathrm{UH}_{\mathrm{N,j,k}} \\ 0 & \text{elsewhere} \end{cases} \tag{16}$$

$$V_{N,j,k} = \begin{cases} \text{UH}_{\text{N, j, k}} & \text{UH}_{\text{N, j, k}} > \text{UC}_{\text{N, j, k}} \\ 0 & \text{elsewhere} \end{cases} \tag{17}$$

At any time period prior to maturity, the holding value is calculated by adding the expected value of the debt component of the four leading nodes one time step later, multiplied by 0.25 and discounted at the appropriate risky rate, to the expected value of the equity component one time step later discounted at the risk free rate:

$$\begin{aligned} UH_{i,j,k} &= \frac{1}{4} e^{-r^*(i,i+1)\Delta t} (V_{i,j,k} + V_{i,j,k+1} + V_{i,j+1,k} + V_{i,j+1,k+1}) \\ &+ \frac{1}{4} e^{-r(i,i+1)\Delta t} (E_{i,j,k} + E_{i,j,k+1} + E_{i,j+1,k} + E_{i,j+1,k+1}) \end{aligned} \tag{18}$$

Applying the free boundary condition (11), the value received from immediate conversion at any time period prior to N is calculated as:

$$UC_{i,j,k} = \lambda S u^j d^{i-j} \ . \tag{19}$$

At periods where interest on the debt is paid the coupon value is multiplied by the CPI yield and added to the holding value of the convertible bond. Given $UC_{i,j,k}$ and $UH_{i,j,k}$, we obtain the value of the equity only component at any time period $i \in [0, N-1]$ as:

$$E_{i,j,k} = \begin{cases} \mathrm{UC}_{\mathrm{i,j,k}} & \mathrm{UC}_{\mathrm{i,j,k}} \geq \mathrm{UH}_{\mathrm{i,j,k}} \\ \frac{1}{4} e^{-r(i,i+1)\Delta t} (E_{i,j,k} + E_{i,j,k+1} + E_{i,j+1,k} + E_{i,j+1,k+1}) & \text{elsewhere} \end{cases}$$

(20)

Applying the free boundary condition from equation (12), the value of the debt component at any node at period $i \in [0, N-1]$ is worth zero in cases where the bond has been converted, in cases that the optimal policy is to hold the bond, the value of the debt component is the expected value of the debt component one time step later, discounted at the risky rate:

$$V_{i,j,k} = \begin{cases} 0 & \mathrm{UC}_{\mathrm{i,j,k}} \geq \mathrm{UH}_{\mathrm{i,j,k}} \\ \frac{1}{4} e^{-r^*(i,i+1)\Delta t} (V_{i,j,k} + V_{i,j,k+1} + V_{i,j+1,k} + V_{i,j+1,k+1}) & \text{elsewhere} \end{cases}$$

(21)

4. Comparative Statics Analysis

The comparative statics analysis of the IICB pricing models has three goals. First, numerical examples are provided to illustrate typical calibration results of the two versions of the pricing models. Second, we compare the performance of the extended TF model to the extended MS model in order to highlight the conditions in which the two pricing models are diverging or converging to the same values. Third, we study the effects of the unique market parameters that impact the IICB value according to each model. These parameters include the CPI's volatility, the real interest rate and the correlation between the returns. As a base case we assume an IICB with specifics and market data as described in Table 1. The IICB value for the extended TF model is equal 121.25, while the extended MS value is equal to 116.65.

To focus on the impact of the correlation between the CPI and the stock returns on the IICB theoretical value, according to the extended TF and MS models, Tables 2 and 3 and Figure 1 show the sensitivity of the IICB price with respect to the model's correlation parameter for different stock price and CPI's volatility. To emphasize the impact of the CPI stochastic behavior on the bond value we assume that the expected nominal interest rate is equal to the expected real interest rate and thus the drift term is equal to zero. Under this parameterization, when $\sigma_I \rightarrow 0$ equation (2), which described the stochastic behavior of the CPI, is redundant and equal to zero, and thus the IICB becomes a nominal convertible bond.[21] In Tables 2 and 3 the CPI's volatility receives values between 5% and 15%, the low level of volatility is similar to the actual volatility of the CPI returns in countries with low inflation rates, and the higher volatility is appropriate to the volatility level that exists in the currency markets.

When the correlation between the stock price returns and the CPI returns are high and positive (+0.5 in our example) and the equity value is equal to the bond face value (100 in our example) the IICB value is lower than the nominal convertible value by 1.18% and 2.17% according to the extended TF model for volatility levels of 5% and 15%, respectively. Similar results are obtained according to the extended MS method, for the same set of parameters, the IICB value is lower than the nominal convertible value by 1.23% and 2.27% for volatility levels of 5% and 15%, respectively. Since the two assets returns are positively correlated there is a good chance for the linked principal payment to be lower than the nominal principal payment and thus we observe a discount on the IICB.

When the correlation between the two assets returns is assumed negative (-0.5) we find the opposite phenomena. For CPI volatility of 5% the IICB value is higher than the nominal bond by 1.46% and 1.54% according to the extended TF and MS models respectively. For CPI volatility of 15% the IICB value is higher than the nominal bond by 5.12% and 5.37% for the two models respectively.

To analyze the price differences between the two pricing models we present in Figure 2 the price gap between the extended TF model and the extended MS model against the stock prices for positive, zero and negative correlation. At very low equity price conversion would probably not take place in any state, and thus the convertible bond synthesizes identical in all means straight corporate debt and the price difference between the models is negligible. As the stock price increases the probability of conversion and the price gap are increased, since the equity component is discounted in the TF model by the risk free rate, while at the MS model the bond is discounted with the risky rate of the issuer in each price state. For a given set of parameters as appears in Table 1, and for CPI's volatility of 15%, in case of negative correlation (-0.5) the price difference is higher than the one observed for positive correlation, since conversion will take place in larger number of price states. At very high equity price levels the impact of the discount factor on the IICB value becomes insignificant, since conversion would take place immediately and thus the price differences between the models is minor.

Table 4 and Figure 3 present IICB values for different CPI volatilities, correlations and levels of credit spread according to each extended model. As expected, for both models, the convertible value increases with CPI volatility and decreases with credit spread for negative correlation (-0.5). Interestingly, according to both models, when the correlation between the assets is positive (at the level of 0.5) the relationship between the IICB value and the CPI volatility is U-Shaped. Using a one period binomial tree and assuming a unit correlation between the two assets we can intuitively explain this result. At expiration there are only two possible states, up movement of the stock and the CPI, and down movement of these two assets. If the optimal policy in the up state is to convert the bond and the optimal policy in the down state is to redeem the bond, then the convertible bond value would decrease with the CPI volatility. On the other hand, when the optimal policy is to redeem the bond in the up state and to convert it in the down state, then the convertible value increases with the CPI volatility.[22]

Table 5 and Figure 4 provide the value of the IICB for combinations of stock price, real interest rate, and the initial level of the CPI (i.e., the cumulative change of the CPI yield from the issuing date till the current pricing date). Having in mind our foreign currency analogy we choose two levels of the CPI at the pricing date. In the first the CPI is equal to 1.2, and thus the accumulated inflation rate until the pricing date is equal 20% and in the second case the CPI level is 0.8 (decline of 20%) where in both cases the stock price is equal to 100 (at the money). In the first case, the conversion option is out-of-the-money and as a result the drift term of the CPI, $(r - r_r)dt$, has a relatively large effect on the convertible value. When the real interest rate is equal to the nominal interest rate (6%), and thus the drift term is equal to zero, the IICB is worth 124.7. A decrease of the real interest rate to zero would increase the convertible price to 129.9. In the second case, we assume a decrease in the CPI from the issuing date until the pricing date is –20%. In this case the conversion option is in the money, and thus the convertible bond value increases from only 105 to 106.5 according to the extended TF model.

5. Empirical Applications of the Inflation-indexed Convertible Bond Model

To demonstrate the extended models and to better understand it we present here an application of the model for the valuation of Machteshim-Agan Inc, FX linked convertible bond traded on the TASE (see Tables 6). On December 3, 2001, Machteshim-Agan convertible bond was traded at a price of 92.3 Agorot (Agorot100= 1 Israeli Shekel) per 1.00 Shekel par value of the bond. The market capitalization of the bonds was USD 67.4 million. The closing price of Machteshim-Agan common stock was 840.1 Agorot

According to the indenture agreement, each Machteshim-Agan FX-linked convertible bond has a face value of 1.00 Shekel and matures on November 20, 2007. If the security has not been converted prior to this date (and if the issuer does not default), the investor receives 1 Shekel that is linked to the Dollar/Shekel exchange rate during this period. The convertible bond pays a fixed annual coupon rate of 2.5% that is also linked to the exchange rate. At anytime before maturity the investor may elect to convert the bond into 0.0936 shares of Machteshim-Agan common stock (See Table 6).

To apply the IICB pricing model to Machteshim-Agan Inc it was necessary to calculate the local and foreign risk free interest rates, which were 6.66% and 4.04% respectively. [23] Besides these observable input parameters, the pricing model requires estimation of unobservable parameters inputs. These inputs include, the company's common stock volatility, the Dollar/Shekel foreign exchange volatility, the correlation between these two underlying assets and the appropriate credit spread and dividend yield. The common stock volatility and the exchange rate volatility used were the historic standard deviation of daily returns over the 365 trading days prior to the issue date of the FX convertible bond. The estimated stock and exchange rate annualized volatilities on the issue date were 28.03% and 4.5% respectively. The chosen credit spread is expressed in basis points over the government yield. Since the issuer had not issued any tradable straight bonds, the credit spread is estimated on the basis of its credit risk rating. Machteshim-Agan FX convertible bond was rated by the Israeli rating agency "Maalot" as AA, which parallels the Baa rating of Moody's international rating agency, so the credit spreads were calculated as the difference between Moody's Seasoned Baa index and the yield on US Treasury notes.[24] The dividend yield was estimated based on the stock's historical dividends during the last 12 months.

Table 6 presents all the necessary data for pricing the convertible bond, the observed price of the convertible bond and the models theoretical price. The extended TF theoretical price is larger than the market price by 2.5 Agorot, which is 2.7% of the bond price. According to the extended MS model the price is equal to 89.9 and thus the market price is greater than the model price by 2.4%. If we assume that the credit spread is equal to zero both models converge to Rubinstein (1994) model and the convertible price is equal to 107.2 Agorot, which is 16.6% above the market price. Figure 5 presents the theoretical price according to the extended TF and MS models and the market price of Machteshim-Agan convertible bond during the period between 3/12/01 to 13/4/03 (dd/mm/yy). The difference between the two models can be explained mainly by the behavior of the underlying stock price. Figure 6 presents the ratio between the two models against parity.[25] As stock price increases the probability of conversion increases and as a result the extended TF model yields values that are significantly larger than those of the extended MS model.

Using the methodology of Sterk (1982) and others to test options pricing formulas, Table 7 provides in the first three columns data about the maximum, minimum and mean percentage overpricing of each model. In the forth column we calculate the *model error ratio*, which is defined as the deviation of the theoretical value from the market price divided by the market price. Its average value can be written as:

$$AER = \frac{1}{M}\sum_{i=1}^{M}\frac{(U_i^{model} - U_i^{marketl})}{U_i^{marketl}}$$

where AER is the model average error ratio and M is the number of daily observations and $i = 1,...,M$. A negative value indicated an underpricing, i.e. the theoretical value is above the observed market price. The model's average error ratio is presented in the fifth column. The extended TF model average error ratio is equal to 0.93% indicating an overpricing, while the extended MS model indicates a negative average error ratio of 3.05% indicating an underpricing. The extended MS model prices may be biased downwards since the payoff in each state is discounted by the risky rate of the issuer. Amman, Kind and Wilde (2002) have investigated the price of French convertible bonds between 1999 and 2000 had found an average mispricing of 2.78% for the TF model.

The *absolute error ratio* is defined as the absolute value of the deviation of the theoretical from the observed market price divided by the observed market price of the observation. Its average value can be expressed mathematically by:

$$ABER = \frac{1}{M}\sum_{i=1}^{M}\frac{\left|U_i^{\mathrm{mod}\,el} - U_i^{marketl}\right|}{U_i^{marketl}}$$

The extended TF model average absolute error ratio is equal to 1.58% while the extended MS model average absolute error ratio is equal to 3.26%. These errors are relatively small compared to the 10% and 12.9% average overpricing that King (1986) and Carayannopoulos (1996) report respectively. The absolute average model error for Rubinstein (1994) model, which ignores credit risk, is equal to 18.9%, a similar in magnitude to the errors simple parity calculation (24.59%) and of the defaultable value (22.43%).

The last column shows the root mean squared error (RMSE) of the absolute mispricing. The RMSE shows the non-central standard deviation of the absolute deviations of model prices from market prices and it can be interpreted as a measure for the pricing fit of the model relative to market prices. This value ranges from 0.126 for the extended TF model to 0.181 and 0.436 for the extended MS and Rubinstein (1994) models respectively.

6. Summary of Main Findings and Concluding Remarks

Convertible bonds with coupon and principal payments that are linked to foreign currency or consumer-price-index are traded in numerous capital markets. In addition to uncertainty about the stock price and inflation (foreign exchange) these corporate securities are exposed to credit risk since the issuer can default on coupons or principal payments. Previous attempts to price these types of convertible bonds have not incorporated all these features and sources of risk.

In this paper we extend the previous literature on the valuation of convertibles bonds by providing two methods of adjusting to credit risk for inflation-indexed convertible bonds that allows for both, the underlying stock and the consumer-price-index, to be stochastic and incorporates exogenous credit spread.

We approximate the pricing equations by using a Rubinstein (1994) three-dimensional binomial tree. In one version of the model credit risk is introduced by extending Tsiveriotis and Fernandes (1998) convertible pricing method, while in the other version it is introduced by extending Brennan and Schwartz (1986) convertible pricing models. The extended two models provide upper and lower bounds for the indexed convertible price.

In our study of the convertible price sensitivity to the different risk factors we show first that positive correlation between the returns of the underlying stock and the CPI has a negative effect on the value of an inflation-indexed convertible bond. Second, when the correlation is negative the convertible bond price increases with CPI volatility, but when the correlation is positive the convertible bond price curve has a U-shape with respect to CPI volatility. Third, the higher the correlation, the faster will the convertible price converge to the conversion value as the credit spread increases. We also analyze the price differences between the two pricing models for different levels of stock price and correlation between the yields of the underlying stock and the CPI. We find that the price difference between the two models is relatively large when the conversion option is at the money, however, when the conversion option is deep in or out at the money the price difference is negligible

Empirical investigation of the pricing of Machteshim Agan’s convertible bond, which is traded on the TASE, produced absolute prediction errors of less than 1.58% and 3.26% for the extended TF model and the extended MS model respectively. These errors are a substantial improvement compared to the 10-12% biases reported by King (1986) and Carayannopoulos (1996). As expected, the extended TF model average error ratio that is equal to 0.93% indicates an overpricing, while the extended MS model indicates a negative average error ratio, i.e. underpricing of –3.05%.

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Table 1: Market and contract data for convertible bond pricing example.

Bond Maturity (T)	5 years	Equity Price (S)	100
Bond Coupon (C)	5%	Credit spread (cs)	2%
Coupon frequency	Every year	Equity Volatility (σ_S)	30%
Conversion ratio (λ)	1	Equity Dividend Rate (δ)	2%
Notional Amount (F)	100	Nominal Interest rate (r)	6%
Base CPI	100	Real Interest rate (r_r)	6%
Current CPI	100	Correlation (ρ)	0
CPI Volatility (σ_I)	10%		

	Extended TF	Extended MS
IICB Price:	121.25	116.65

Table 2: The theoretical value of the IICB according to the extended TF model for a combination of stock price, CPI volatility and asset's correlation.

Asset's correlation	CPI's Volatility	Stock price (S) 50	75	100	125
$\rho = -0.5$	$\sigma_I \to 0\%$	92.53	104.16	120.29	139.40
	$\sigma_I = 5\%$	93.58	105.73	122.05	140.97
	$\sigma_I = 10\%$	94.93	107.66	124.11	143.02
	$\sigma_I = 15\%$	96.41	109.74	126.45	145.32
$\rho = 0$	$\sigma_I = 5\%$	92.62	104.35	120.50	139.50
	$\sigma_I = 10\%$	93.12	105.04	121.25	140.21
	$\sigma_I = 15\%$	93.89	106.18	122.47	141.49
$\rho = 0.5$	$\sigma_I = 5\%$	91.63	102.78	118.87	138.00
	$\sigma_I = 10\%$	91.12	101.93	117.98	137.20
	$\sigma_I = 15\%$	90.93	101.65	117.68	136.95
Parity		50	75	100	125

The Parameters for the table are identical to the parameters of the base case, which appear at Table 1. The value of the base case appears in bold Italic. The percent of change between the inflation-index convertible bond value and the same but straight convertible bond value appear on barracks.

Table 3: The theoretical value of the IICB according to the extended MS model for a combination of stock price, CPI volatility and asset's correlation.

Asset's correlation	**CPI's Volatility**	**Stock price (S)**			
		50	**75**	**100**	**125**
$\rho = -0.5$	$\sigma_I \to 0\%$	91.24	101.10	115.63	133.44
	$\sigma_I = 5\%$	92.21	102.67	117.41	135.18
	$\sigma_I = 10\%$	93.41	104.52	119.52	137.25
	$\sigma_I = 15\%$	94.77	106.55	121.84	139.59
$\rho = 0$	$\sigma_I = 5\%$	91.37	101.32	115.87	133.70
	$\sigma_I = 10\%$	91.80	102.01	***116.65***	134.44
	$\sigma_I = 15\%$	92.49	103.10	117.89	135.64
$\rho = 0.5$	$\sigma_I = 5\%$	90.50	99.85	114.21	132.14
	$\sigma_I = 10\%$	90.05	99.05	113.31	131.32
	$\sigma_I = 15\%$	89.90	98.78	113.00	131.04
Parity		50	75	100	125

The Parameters for the table are identical to the parameters of the base case, which appear at Table 1. The value of the base case appears in bold Italic. The percent of change between the inflation-index convertible bond value and the same but straight convertible bond value appear on barracks. The price of an identical in quality straight corporate bond is equal 86.82

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Table 4: The theoretical value of the IICB according to the extended TF model and the extended MS for a combination of CPI's volatility credit spread and correlation.

	Asset's correlation (ρ)	CPI's Volatility	Credit spread (cs) 0%	2%	4%
The Extended TF model	ρ = 0.5	σ_I –> 0%	124.99	120.29	116.03
		σ_I = 10%	122.48	117.98	113.92
		σ_I = 20%	122.47	117.99	113.93
	ρ = 0	σ_I = 10%	126.08	***121.25***	116.89
		σ_I = 20%	129.15	124.12	119.56
	ρ = – 0.5	σ_I = 10%	129.16	124.11	119.54
		σ_I = 20%	134.29	128.94	124.09
The Extended MS model	ρ = 0.5	σ_I –> 0%	124.99	115.63	108.18
		σ_I = 10%	122.48	113.31	106.16
		σ_I = 20%	122.47	113.30	106.16
	ρ = 0	σ_I = 10%	126.08	***116.65***	109.10
		σ_I = 20%	129.15	119.51	111.66
	ρ = – 0.5	σ_I = 10%	129.16	119.52	111.66
		σ_I = 20%	134.29	124.30	116.01
The value of an identical in quality straight corporate bond:			95.04	86.82	79.36

The Parameters for the table are identical to the parameters of the base case, which appear at Table 1. The value of the base case appears in bold Italic.

Table 5: The theoretical value of the IICB according to the extended TF model and the extended MS for a combination of stock price, real interest rate and CPI level.

		CPI yield = -20% (I=0.8)			CPI yield = 20% (I=1.2)		
	Real interest	0%	3%	6%	0%	3%	6%
	Stock price						
The Extended TF model	50	96.63	86.79	78.73	138.98	122.42	108.56
	75	107.53	99.69	93.54	144.95	130.19	118.10
	100	122.95	116.74	112.16	155.08	142.17	132.03
	125	141.19	136.38	132.97	168.49	157.48	149.10
	150	161.29	157.59	155.05	184.43	175.11	168.25
The Extended MS model	50	95.48	85.31	77.01	138.43	121.69	107.55
	75	104.74	96.49	90.13	143.22	127.96	115.51
	100	118.47	112.13	107.55	151.64	138.28	127.76
	125	135.50	130.87	127.80	163.33	151.93	143.31
	150	154.97	151.88	150.28	177.70	168.20	161.33

Table 6: Pricing Applications - Foreign-Currency linked Convertible bond

Machteshim-Agan on December 3 2001

Relevant Input:

Stock price	840.1	Local risk free yield	6.66%
Conversion ratio	100/1068	Foreign risk free rate	4.04%
Initial Exchange rate	4.22	Issuer's credit spread	3.83%
Current Exchange rate	4.24	Time to expiry	5.97
Stock volatility	28.3%	Exchange rate volatility	4.4%
Assets correlation	-0.34	Bond face value	100
Dividend yield	2.8%	Coupon rate	2.75
Num of principal payments	1	N	100

Convertible bond market price: 92.3

Pricing results of the extended TF model: 94.80

Equity component	40.13	Bond component	54.67
Straight bond price	74.36	Option price	20.44

Extended MS:	89.93	Rubinstein (1994):	107.58

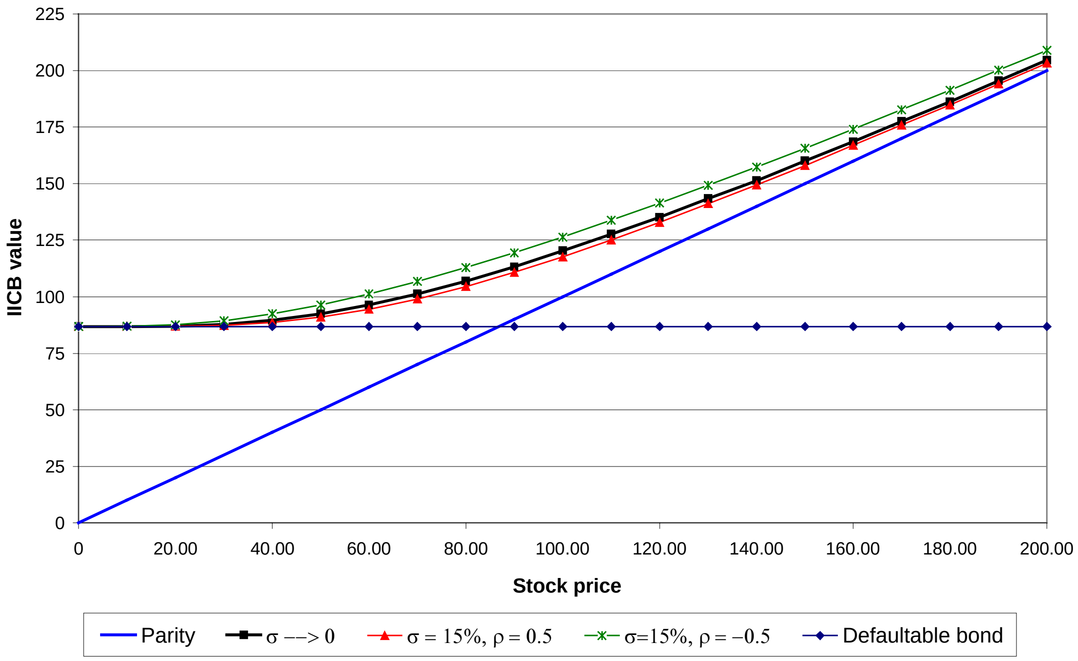
Table 7: Statistic of the input parameters and comprehensive pricing overview for Machteshim-Agan FX convertible bond

Data points	Mean of the input volatility	Mean of the dividend yield	Mean of the credit sprad (bp)	Correlation Stock/FX
291	27.01%	2.80%	4.35%	-0.24

Model type	Maximum percentage overpricing	Minimum percentage overpricing	Average model error ratio	Absolute model error ratio	RMSE
Extended TF	6.04%	-4.77%	0.93%	1.58%	0.126
Extended MS	3.34%	-8.78%	-3.05%	3.26%	0.181
Rubinstein (1994)	34.03%	6.89%	18.90%	18.90%	0.436
Parity	-11.62%	-38.68%	-24.59%	24.59%	0.497
Defaultable bond	-12.95%	-30.37%	-22.43%	22.43%	0.474

Figure 1

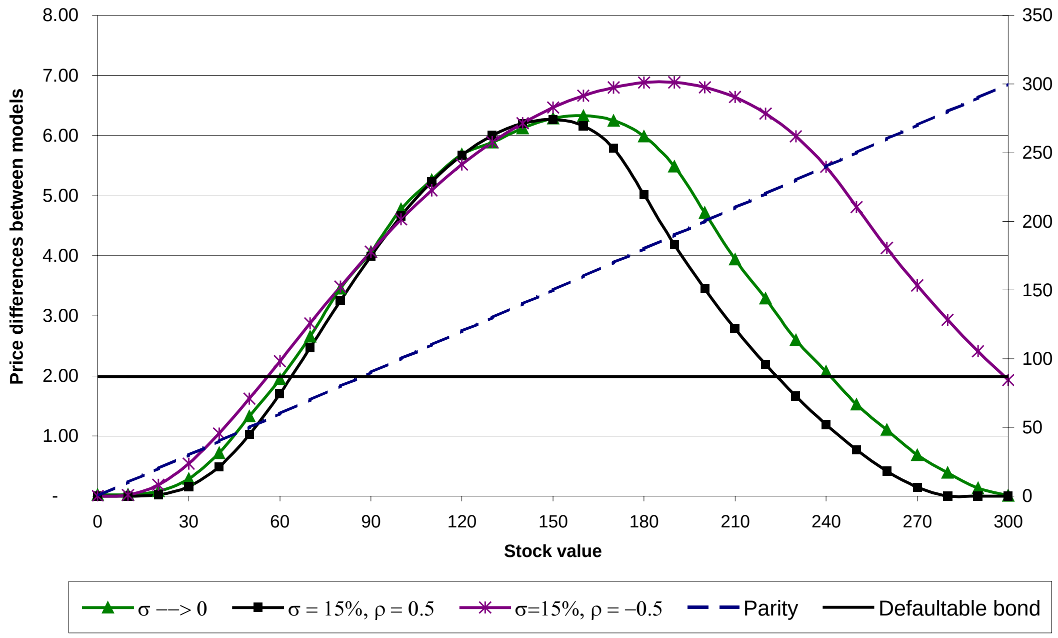
The values of the IICB for a combination of stock price, CPI volatility and correlation according to the extended TF model.



The parameters for the figure are these of the base case as presented at Table 1. The price of an identical in quality straight corporate bond is equal 86.82

Figure 2

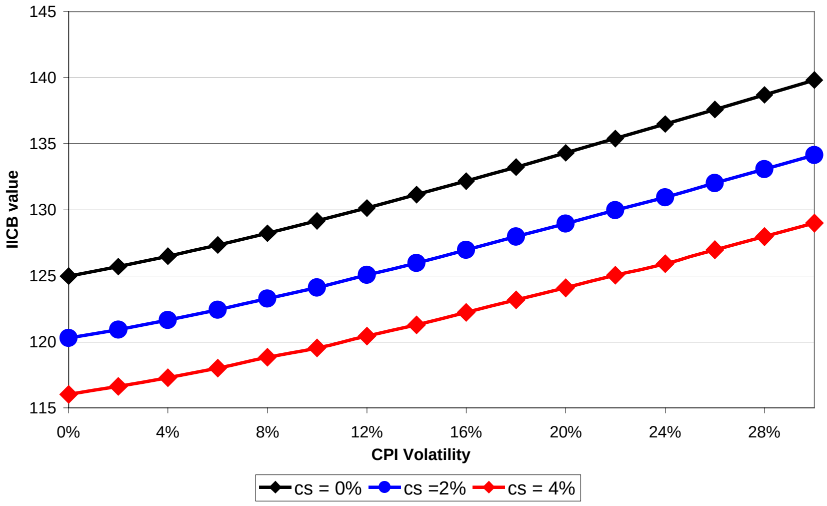
The price differences between the extended TF model and the extended MS model against stock price for various level of correlation.



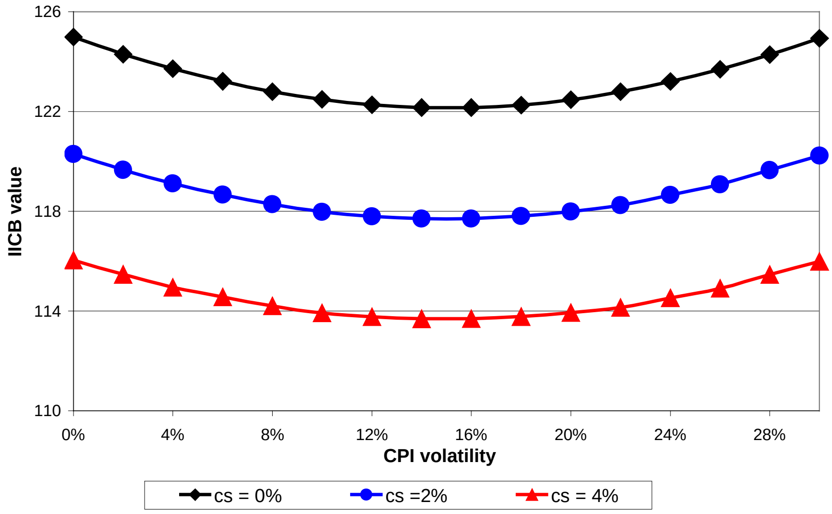
The parameters for the figure are these of the base case as presented at Table 1. The price of an identical in quality straight corporate bond is equal 86.82

Figure 3

The value of the IICB for combination of stock price, CPI's volatility and correlation according to the extended TF model.



$\rho = -0.5$

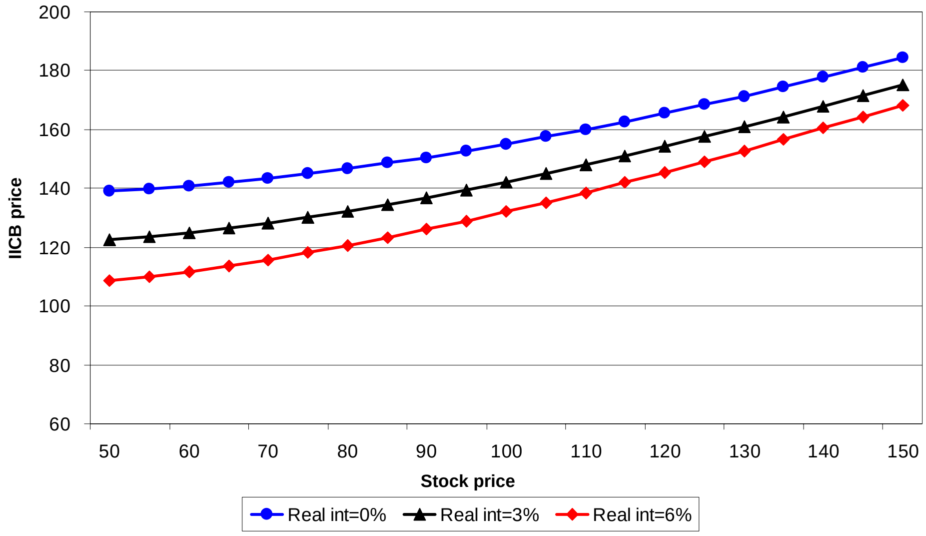


$\rho = 0.5$

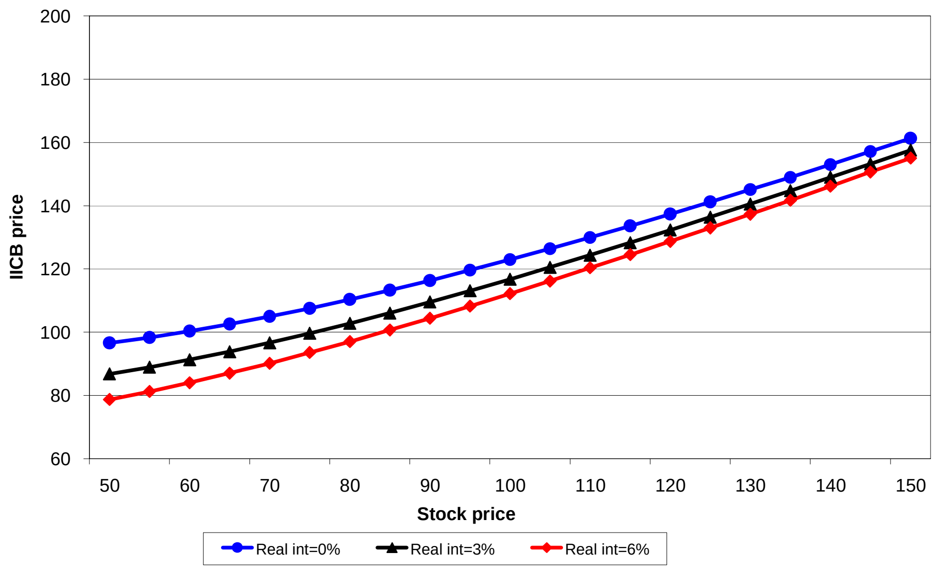
Parameters: See Table 1.

Figure 4

The values of the convertible bond for a combination of stock price, real interest rate and CPI level according to the extended TF model.



I = 1.2



I = 0.8

Parameters: See Table 1.

Figure 5: Machteshim-Agan Inc convertible bond market price, the extended TF model price and the extended MS model price for the period 12/02/01-04/13/03 (mm/dd/yy)

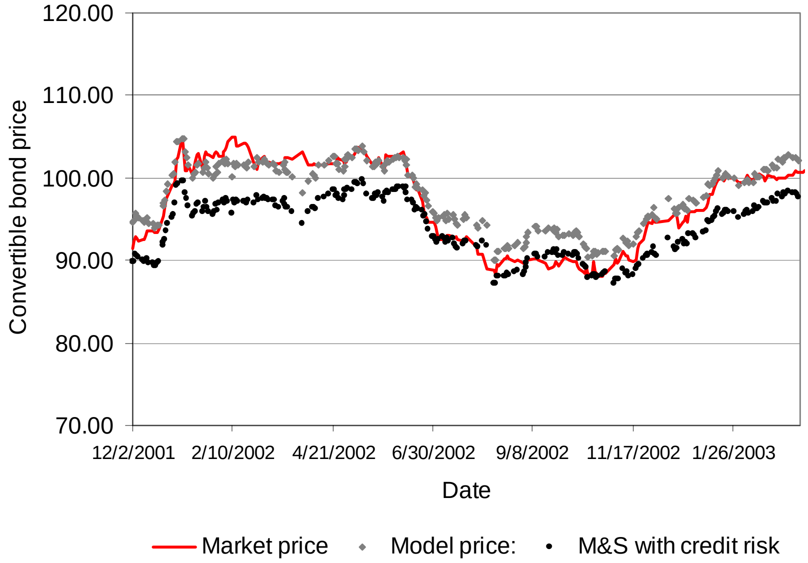
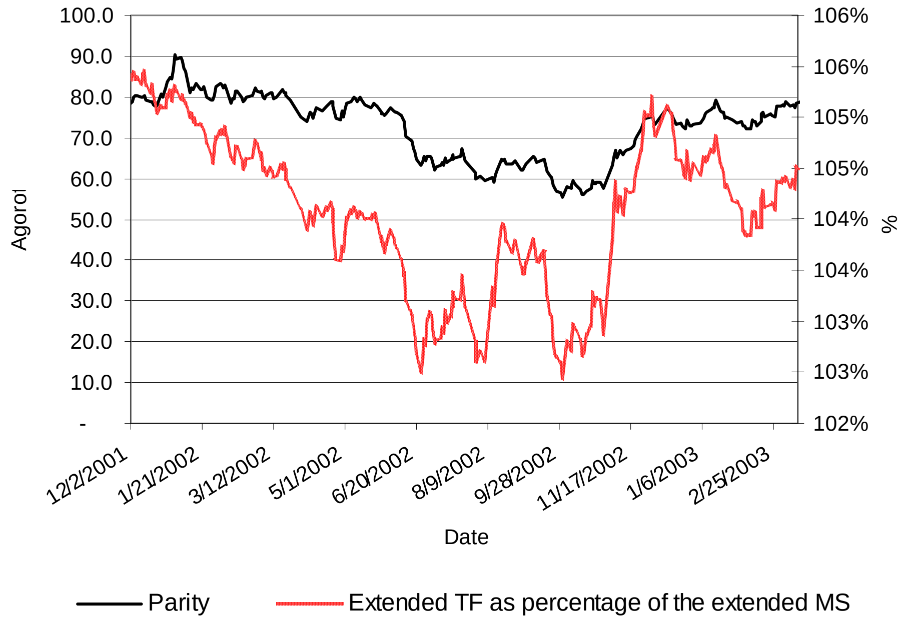


Figure 6: Machteshim-Agan Inc convertible bond value according to the extended TF model as percentage of the extended MS against parity (the stock price multiplied by the conversion ratio) for the period 2/12/01-13/4/03 (mm/dd/yy)



Endnotes

[1] See Stumpp (2001).

[2] In Israel virtually all intermediate and long term government bonds are linked to inflation (or the exchange rate); In Great Britain about 20% of government bonds issued in the last decade have been inflation linked; In 1997 the U.S. Treasury started issuing such bonds, called Treasury Inflation Protected Securities (TIPS).

[3] According to the Bank Of Israel, the amount of new issues of CPI-Indexed and FX linked convertible bonds during the years 1997-2002 was three billion shekels (about 650 million dollar).

[4] This high level of default is partly explained by the fact that generally convertibles are issued in the form of junior subordinated debt, which places them low in the priority of payment. Furthermore, the indentures covering convertibles often contain just few of the covenants that afford protection to traditional bondholders.

[5] The meaning of "nominal convertible bond " in this paper is a convertible bond that promised a nominal principal and coupon payments that are in the same currency as the underlying stock of the issuer.

[6] There are extra two sources of randomness- the stochastic behavior of interest rates and the stochastic behavior of the real interest rates or the foreign interest rates, depends on the bond feature. But we prefer to assume that those factors are constant since we want to focus on the influence of indexation on the convertible price. Brennan and Schwartz (1980) find that the effect of stochastic term structure on convertible prices is insignificant.

[7] Merton (1974) shows that company's equity can be viewed as a European call option on the total value of the firm assets, with a strike price equal to the face value of debt, where default can only occur at debt maturity.

[8] A discussion on MS model can be found at Ammann, Kind and Wilde (2002).

[9] Takahashi, Kobayashi and Nakagawa (2001) test the model empirically by using Japanese convertible bonds prices; Ammann, Kind and Wilde (2002) use a broader sample of French convertible bonds to test the pricing model.

[10] A Quanto option is an example of an option on two different assets (foreign currency and equity), however it is a European type option with terms that differ from an inflation indexed convertible and can be priced using a closed form solution, see Derman, Karasinsky, and Wecker (1990)

[11] Recently, Yigitbasioglu (2002) introduces a pricing model for FX convertible bonds by relying efficiently on the change of numeraire technique and solving the pricing equation by using the Crank-Nicholson scheme.

[12] Some of the assumptions could be relaxed. In particular, it would be possible to let the nominal and the real interest rates change over time as in Merton (1973), or to let the covariance and the volatilities to change over time as in Ho, Stapleton and Subrahmanyam (1995). The added complexity would not add significant insights to the present paper.

[13] However, there are recent evidence that differential state taxes on corporate versus government bonds may be important for the determination of corporate bond yields, see Elton, Gruber, Agrawal, and Mann (2001).

[14] The above stochastic process for the dynamics of the CPI can be found in Friend, Landskroner and Losq (1976) and in Benninga, Bjork and Wiener (2001).

[15] Discussion on this relation can be found at Garman and Kohlhagen (1983).

[16] By definition, a real bond provides complete indexation against future movement in price T periods ahead. Although inflation-indexed bonds provide incomplete indexation for the coupon and principal payments, because of reporting lags, Kandel, Ofer and Sarig (1993), show empirically, using Israeli bond data, that differences between expectations of past inflation embedded in bond prices and actual inflation rates are small in magnitude.

[17] See Black and Scholes (1973)

[18] Although Tsiveriotis and Fernandes (1998) assume in their paper that the credit spread is constant it can easily be relaxed and modeled as a time-dependent parameter.

[19] Discussion on the boundary conditions of a convertible bond can be found at Brennan and Schwartz (1977,1980).

[20] A similar expression for the correlated asset price for Rubinstein three-dimensional binomial tree can be found in Haug (1997).

[21] In most real world cases the inflation expectation is positive and thus the drift is positive and does not equal zero as in the chosen example.

[22] A discussion of the optimal exercise regions of American options on multiple assets can be found in Broadie and Detemple (1997).

[23] The local yields were calculated as the average of the intermediate Israeli government bonds yields at each pricing date (named “Shahar”). The foreign yields are the average

yields on the 5 and 7 years constant maturity treasury bonds indexes, as is daily published by the U.S. Treasury.

[24] We assume that all of the corporate- Treasury yield spread is due to credit risk; however taking a smaller credit spread would not affect our results significantly.

[25] Parity is equal to the current stock p rice multiplied by the conversion ratio.