Stochastic Dominance Portfolio Analysis of Forestry Assets

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Abstract

We consider the forestry decision-making and harvesting problem from the perspective of financial portfolio management, where harvestable forest stands constitute one of the liquid assets of the portfolio. Using real data from Finnish mixed borealis forests and from the Helsinki stock exchange, we investigate the effect of trading the timber stock together with the forest land, or without the land (i.e., harvesting), on the portfolio efficiency. As our research methodology, we utilize the general Stochastic Dominance (SD) criteria, focusing on the recent theoretical advances in analyzing portfolio diversification within the SD framework. Our findings shed some further light on the question of how to model the forestry planning problem, and provide some comparative evidence of the applicability of the alternative SD test approaches.

Keywords: Forest Management, Portfolio Optimization, Stochastic Dominance, Diversification.

Introduction

During the last decades there has been increasing research interest in forestry decision-making under risk. A frequently followed approach is to consider forest as one investment alternative among other assets in a financial portfolio, the primary source of risk being the fluctuations in the stumpage prices (e.g. Thomson, 1991, and Reeves and Haight 2000). The conventional approach models the forest stands, including both the timber stock and the land, as a non-separable liquid asset (Mills and Hoover 1982).

The recent paper by Heikkinen (1999) considered the possibility to harvest and sell the timber stock, without trading the land, as a more realistic and relevant way of modeling the risk management problem of a forest estate in Finland, which has mixed Nordic borealis forests as the main assets. In principle, there is no reason why the timber-stock and the land could not be modeled as two separate assets. The operational forest management, for example, usually only concerns the harvesting decisions. For many land owners land sales are not an alternative, because land ownership has value as such or the land area may have other than economic value. These lands owners might, however, try to maximize their cutting incomes. While markets for both harvested timber and growing forest-land are established, trading growing timber stock without the land is quite unusual.

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In Finland, transactions concerning harvested timber are much more common than trading with forest land, while markets for growing stock are nonexistent. In early 1990's there were roughly 135.000 timber sales transactions per year in Finland, but just a one or two thousand transactions in the markets of forest land. Therefore, the existing timber stock can be considered a much more liquid asset than rural forest-land. However, increasing the timber stock, or modifying its species and assortments composition, is not feasible in the short run. This is an important consideration from the point of view of risk management. Modeling the timber harvesting decision problem as a part of the management of a mixed asset portfolio in a realistic way is not a trivially simple task. Seemingly innocent simplifications can make a big difference in the conclusions and recommendations.

If cuttings are implemented at the beginning of the planning period, some return for the next tree generations is generated during the planning period. If cuttings are not implemented, this return narurally does not exist. According to Heikkinen (1999) the return on next tree generations is so minimal that it can be ignored in a short run model and in the case of long rotation forests. This is also the case in our study. Reeves and Haight (2000) drew similar conclusions by assuming that the land is sold immediately after the cuttings. Although this question leaves room for remodeling, it falls beyond the scope of this study.

Focusing on timber-harvesting, Heikkinen demonstrated how the marker imperfections, which render purchases of new growing stock impossible, can be conveniently modeled by imposing a set of upper-bound restrictions (linear inequalities) on the portfolio weights of the feasible portfolio set. Somewhat surprisingly, the Mean-Variance (MV, Markowitz, 1952, 1959) analysis of Heikkinen did not reveal major differences in the MV efficient sets in the two cases of constrained timber harvesting and unconstrained land trade. Only for the lowest risk/return levels, the liquidity assumption for the land made a notable difference. Whether this result is due to the limitations of the MV model, remains an open question.

The MV model requires the asset returns to be normally distributed or the decision-maker's utility function to be of quadratic form. In many circumstances these assumptions appear questionable, not least in case of forestry and other natural resource assets. For example, there is empirical evidence that forestry returns (first differences of log prices) are not normally distributed (Toppinen and Toivonen, 1998; Heikkinen and Kanto, 2000). When the assumptions of MV do not hold, the Stochastic Dominance (SD) efficiency criteria offer the most immediate extension (see e.g. Bawa, 1982; or Levy, 1992, for a survey of SD). SD accounts for the entire probability distribution (not just the first two moments) and applies for the general classes of non-satiated and/or risk-aversive preference functions. In our SD approach the question is whether the current forest stands form an efficient portfolio strategy *as such*, or should some of them be harvested to generate cutting income which can be invested in the stock markets? It is well-known fact that it is not easy to specify decision-makers utility function. Decision-makers are often not willing or able to answer precise questions regarding their preferences. The SD approach involves only minimal assumptions concerning decision-makers preferences. The Second-order SD only assumes that decision-maker is risk averse and non-satiated. This assumption usually holds for majority of the decision-makers.

Unfortunately, the SD approach has had a serious shortcoming in dealing with portfolio *diversification*. While it is relatively simple to identify the MV efficient set of portfolios, until now there has not been any method of testing whether a given portfolio is SD efficient, let alone computing all SD efficient portfolios (i.e., the SD efficient set). This also explains why theoretically appealing SD criteria have attracted so little applications in finance and related fields.

The recent work of Kuosmanen (2001), and the subsequent developments by Post (2001), have to a great extent eliminated this handicap of SD: It turns out that the SD efficient set exhibits a relatively simple polyhedral structure, which can be analyzed by standard techniques and algorithms. In fact, one can test for SD efficiency by solving a simple Linear Programming problem, while MV analysis requires more complex Quadratic Programming.

In this paper we revisit the forest risk management problem of Heikkinen (1999), utilizing the latest SD tools. Our main objective is to investigate the influence of the constraints on purchasing growing timber stock, as modeled by Heikkinen, to the portfolio efficiency in terms of the more general SD criteria. We find it interesting to investigate the robustness of the earlier MV results and conclusions regarding the MV assumptions. As a valuable by-product, this also enables us to compare the approaches of Kuosmanen (2001) and Post (2001), and highlight their relative merits in the context of the present application. By sharing our findings and experiences from this application, we also hope to provide valuable guidelines for further development of the SD approach.

The rest of the paper unfolds as follows. The next section introduces the basic SD notions and outlines the diversification analysis proposed by Kuosmanen (2001). We then review the two alternative test procedures proposed by Kuosmanen (2001) and Post (2001), respectively. This is followed by a description of the empirical forest management problem and the data set. We then apply and adapt the SD method to the present data, and discuss the results. The concluding section puts forth some interesting routes for future research.

Diversification and Stochastic Dominance

This section introduces the basic terminology and outlines some recent advances in the SD methodology. First of all, it is worth noting there are an infinite number of different SD criteria (i.e., SD of order n, $n = 1,2,\ldots$), so we have to be more specific about the meaning of SD in the present context. For simplicity, we focus exclusively on the Second-order SD criterion (henforth SSD). Consider two arbitrary risky portfolios j and k with the return distributed according to the cumulative distribution functions (CDFs) G_j and G_k, respectively.

Definition: Portfolio j <u>dominates</u> portfolio k by Second-order Stochastic Dominance, denoted by $j\mathrm{D}_2 k$ if and only if

$$\int_{-\infty}^{z} \left[G_k(t) - G_j(t) \right] dt \geq 0 \ \ \forall z \in \mathbb{R}, \text{ and } \int_{-\infty}^{z} \left[G_k(t) - G_j(t) \right] dt > 0 \text{ for some } z \in \mathbb{R}.$$

The SSD criterion has the following well-known economic interpretation in terms of the Expected Utility Theory. Consider the von Neumann - Morgenstern utility function $U : \mathbb{R} \rightarrow \mathbb{R}$. SSD dominance jD_2k is equivalent to the condition that *all* non-satiated and risk-aversive investors (with $U'(z) \geq 0, U''(z) \leq 0 \ \forall z \in \mathbb{R}$) prefer portfolio j to k (Fishburn, 1964).

In empirical portfolio analysis, the underlying probability distributions G are not known. The analysis is geared at estimating the distribution functions from the data. We hence assume a finite (and therefore discrete) sample of return observations of the n assets from m time periods indexed as $T \equiv \{1, 2, ..., m\}$ and $N \equiv \{1, 2, ..., n\}$. This gives a panel data representable in the form of matrix $Y \equiv (Y_1 ... Y_n)^T$ with $Y_j \equiv (Y_{j1} ... Y_{jm})$. Assuming away shortsales,[1] we denote the portfolio weights by $\lambda \in \Lambda$, where $\Lambda \equiv \left\{ \lambda \in \mathbb{R}^n_+ \left| \sum_{i=1}^n \lambda_i = 1 \right. \right\}$ denotes the feasible domain. The set of feasible portfolios (characterized here as return time series) is hence

(1) $\qquad \Psi \equiv \left\{ y \in \mathbb{R}^m \left| y = \lambda Y; \lambda \in \Lambda \right. \right\}.$

Much of the appeal of the SD approach lies in its avoidance of arbitrary assumptions regarding the function form of the underlying distributions. Rather, the SD approach ' lets the data speak for themselves' . To derive the empirical distribution function (EDF) for an arbitrary portfolio *I*, the standard approach is to rearrange elements of return vector $y_i \in \Psi$ in non-decreasing order, and denote the resulting ranked return vector by x_i, i.e. $x_{i1} \leq x_{i2} \leq ... \leq x_{im}$. Observe that this operation involves a loss of potentially valuable information on the time-series structure of the observations. We will henceforth reserve y, Y for the time series, and x for the ranked data. Based on x_i, we further construct the cumulative sum vector $x'_i = (x'_{i1} ... x'_{im})$ where $x'_{ik} = \sum_{j=1}^k x_{ij}$. Utilizing the function $O_i(z) : \mathbb{R} \rightarrow \mathbb{N}_+$:

(2) $\qquad O_i(z) \equiv Max\{t \in T | z \geq x_{it}\},$

we can construct the empirical distribution function (EDF) for asset *i* simply as

(3) $\qquad H_i(z) = O_i(z) / m .$

The EDF defined by (3) provides a non-parametric minimum variance unbiased estimate of the underlying unobservable CDF. Therefore, a vast number of empirical studies apply SD criteria directly on EDF H_i when the underlying CDF G_i is not known (see e.g. Bawa, 1982, or

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[1] Observe that if our data set contains at least $m + 1$ linearly independent return vectors Y_i and shortsales are not limited, then the portfolio set Ψ spans the entire m dimensional Euclidean space (i.e. infinite returns are possible). Therefore, it would be necessary to impose some constraints on the shortsale possibilities. However, the shape of the portfolio set Ψ is likely to be sensitive to the particular specification of such constraints. For simplicity, we completely exclude the possibility of shortsales in the following.

Levy, 1992, for surveys).[2] In this respect, a well-known result from the majorization theory provides a useful starting point:

Theorem 1: *The following equivalence results hold for empirical distributions of all portfolios j and k:*

$$j\mathrm{D}_2 k \Leftrightarrow x'_{jt} \geq x'_{kt} \; \forall t \in T, \text{ and } x'_{jt} > x'_{kt} \text{ for some } t \in T.$$

Proof. Follows directly from the result of Karlin and Novikoff (1963), see e.g. Aboudi and Thon (1994, p. 509-510) for discussion.

The inequalities of Theorem 1 can be easily checked by enumeration. Hence, Theorem 1 forms a basis for a simple but effective ' crossing algorithm' for testing SD relationships by a pairwise comparison of asset returns.

The notion of SD *efficiency* assumes some scarcity for the investment opportunities. Focusing on the portfolio set Ψ, SD efficiency is characterized by the following definitions:

Definition 2: Portfolio k: $y_k \in \Psi$ is FSD (SSD) <u>efficient</u> in set Ψ, if and only if, $j\mathrm{D}_1 k$ ($j\mathrm{D}_2 k$) $\Rightarrow y_j \notin \Psi$. Otherwise k is FSD (SSD) inefficient.

Definition 3: The set $\Delta(y_0) \equiv \left\{ y \in \mathbb{R}^m \,\middle|\, y\mathrm{D}_2 y_0 \right\}$, $l = 1,2$, is called the *dominating set of* the evaluated portfolio y_0.

The dominating set relates to SD efficiency in the following sense:

Lemma 1: *Portfolio y_0 is SSD efficient if and only if the dominating set of y_0 does not include any feasible portfolio, i.e. $\Psi \cap \Delta(y_0) = \varnothing$.*

Proof. Follows directly from Definitions 2 and 3.

As an immediate corollary, if we can identify the dominating set, we can test SD efficiency by checking whether the intersection of the dominating set and the portfolio set is empty. Unfortunately, characterization of the dominating and the efficient set is a highly complicated problem. Only very recently, Kuosmanen (2001a) derived the explicit characterizations of the SSD dominating sets for an arbitrary evaluated portfolio y_0. We next briefly review these results.

Matrix $W = \left[W_{ij} \right]_{m \times m}$ is called *doubly stochastic* if its elements are non-negative real numbers and all its rows and columns sum up to unity. Formally, the set of doubly stochastic matrices is henceforth denoted by

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(4) $$\Xi \equiv \left\{ \left[W_{ij} \right]_{mxm} \Big| 0 \leq W_{ij} \leq 1; \; \vec{1}W = \left(W\vec{1}^{T} \right)^{T} = \vec{1} \right\}.$$

Note that the set of *permutation matrices* is a subset of Ξ, but the converse is not true. Utilizing the famous theorem of doubly stochastic matrices by Hardy, Littlewood, and Polya (1934), Kuosmanen (2001b) derived the following analytical characterization of the dominating set:

Theorem 2: $\Delta(y_0) = \left\{ y \in \mathbb{R}^m \Big| \exists W \in \Xi : y \geq y_0 W; \; y \neq y_0 P \; \forall P \in \Pi \right\}$

As a direct corollary, the set $\Delta(y_0)$ is closed, monotonous, and symmetric with respect to the diagonal ray of risk-free assets. In addition, it is a convex set, which is very convenient from the operational point of view.
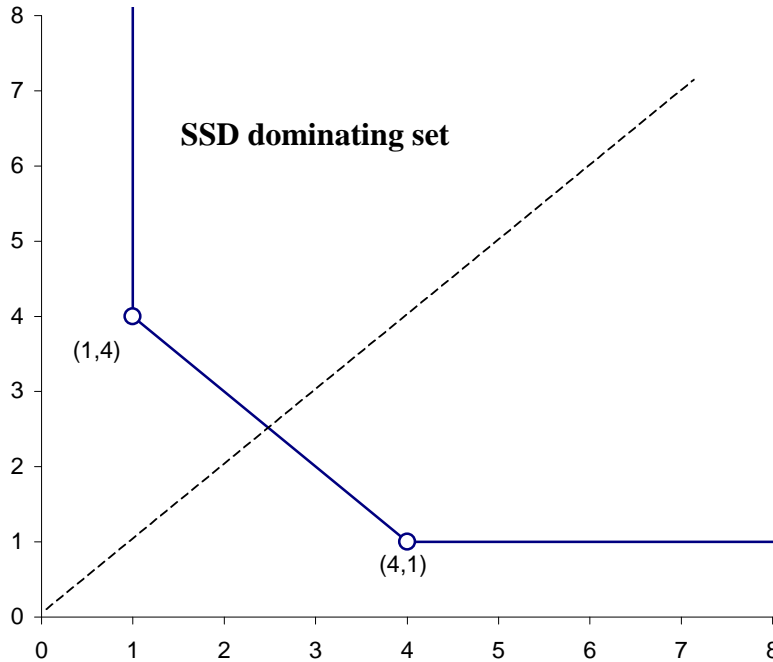

Figure 1: The SSD dominating set of portfolio (1,4).

Figure 1 graphically illustrates the SSD dominating set of an arbitrary vector $y_0 = (1,4)$. Note that the smallest risk-free return that dominates portfolio y_0 by SSD equals 2.5, i.e. 2.5 > 1 and 2.5 + 2.5 = 1 + 4, which equals the mean return of the portfolio y_0. This confirms the well-know fact that an option with the smaller mean cannot dominate by SSD (Hadar and Russel, 1969).

Efficiency Tests

By Lemma 1, we may test SD efficiency of any given portfolio y_0 by simply checking whether the dominating set Δ and the portfolio set Ψ share common portfolios, in other words, whether

any dominating portfolio is feasible. Thus, consider the test statistic $\theta_2^N(y_0)$ obtained as the optimal solution to the following Linear Programming (LP) problem:

(5)

$$\theta_2^N(y_0) = \max_{\lambda, W} \frac{1}{m}(Y^+\lambda - y_0 W)\vec{1}^T$$

$$s.t.$$

$$Y^+\lambda \geq y_0 W$$

$$\sum_{i=1}^{m} W_{ij} = \sum_{j=1}^{m} W_{ij} = 1 \; \forall i, j \in T$$

$$\sum_{k=1}^{n+1} \lambda_k = 1$$

$$W_{ij}, \lambda_k \geq 0 \; \forall i, j \in T, k \in N$$

where $Y^+ = \begin{pmatrix} y_0 & Y \end{pmatrix}^T$. Assuming all elements of y_0 and Y are finite, the optimal solution to (5) always exist.

The test statistic θ_2^N has a natural interpretation as the "inefficiency premium" of the evaluated portfolio: $\theta_2^N(y_0)$ indicates the difference of the mean return between the evaluated portfolio and the dominating portfolio with the highest mean return. In other words, even a most risk-aversive investor could gain at least $\theta_2^N(y_0)$ by following the portfolio strategy $\lambda^* \equiv \arg\max \theta_2^N(y_0)$, which does not involve more risk compared to the present situation. Intuitively, the objective function maximizes the inefficiency premium, while the first constraint guarantees SSD dominance (by Theorem 2). The remaining constraints simply secure that W is a dounbly stochastic matrix and λ is a vector of portfolio weights of the benchmark portfolio.

Following Kuosmanen (2001), θ_2^N is a necessary test statistic.

Theorem 3: $\theta_2^N(y_0) = 0$ *is a necessary condition for SSD efficiency of portfolio y_0.*
Proof. Kuosmanen (2001).

However, $\theta_2^N(y_0) = 0$ is not yet a sufficient condition. In particular, it is easy to verify that any equal-mean portfolio $y_0 W$, $W \in \Xi - \Pi$ dominates the original portfolio y_0 by SSD. To test whether any of those dominating portfolios might be feasible, we may calculate the following test statistic:

$$\theta_2^S(y_0) = \min_{W,\lambda,s^+,s^-} \vec{1}(s^+ + s^-)\vec{1}^T$$

$$s.t.$$

$$y_0 W = Y^+ \lambda$$

(6)

$$\sum_{i=1}^m W_{ij} = \sum_{j=1}^m W_{ij} = 1 \; \forall i, j \in T$$

$$W_{ij} = \tfrac{1}{2} + s_{ij}^+ - s_{ij}^-$$

$$\sum_{k=1}^{n+1} \lambda_k = 1$$

$$W_{ij}, s_{ij}^+, s_{ij}^- \lambda_k \geq 0 \; \forall i, j \in T, k \in N$$

Theorem 4: $\theta_2^N(y_0) = 0$ *and* $\theta_2^S(y_0) = \dfrac{m^2}{2}$ *is a necessary and sufficient condition for SSD efficiency of portfolio y_0.*

Proof. Kuosmanen (2001)

We find these quite remarkable results in the sense that we have shown that the unsolved problem of testing SD efficiency actually boils down to a very standard class of LP problems. Very effective simplex and interior point methods are generally available for solving large-scale LP problems. Consequently, the SSD test statistics can be computed by a usual desktop PC. Moreover, the computational cost should not prevent complementing the test by other (computationally intensive) numerical techniques like the bootstrapping approach. See Post (2001) for some encouraging simulation results.

In fact, there also exists a more straightforward way of testing SSD efficiency, as suggested by Post (2001). Consider the following alternative test statistic:

$$\hat{\theta}_2(y_0) = \min_w y_0 w$$

$$s.t.$$

(7)

$$Y_j w \leq 1 \; \forall j \in N$$

$$w_l < w_k \; \forall l, k \in T : y_{0l} > y_{0k}$$

$$w > \vec{0}^T$$

Theorem 5: *Portfolio y_0 is SSD efficient in Ψ if and only if $\hat{\theta}_2(y_0) \geq 1$.*

Proof. See Post (2001).

The test statistic $\hat{\theta}_2$ is by all essential parts identical to that proposed by Post (2001), derived from the Expected Utility interpretation of SSD. We may view this result as a separating hyperplane theorem: By convexity of the dominating set Δ and the portfolio set Ψ, portfolio y_0 is efficient if there is a linear hyperplane which (strictly) separates the dominating set from the

portfolio set. The weights w can be interpreted as the slope coefficients of this separating hyperplane. The purpose of the test is to check whether appropriate slope coefficients exist or not.

The only difference of problem (7) to Post's original formulation is that (7) imposes two strict inequality constraints, while Post uses weak inequalities. In our framework, the strict inequalities are required for a genuine necessary and sufficient test, to take into account the fact that portfolio $y_0 W$, $W \in \Xi - \Pi$, is generally "less risky" than the original portfolio y_0. Post artificially avoids the inconvenience of strict inequalities by imposing a more stringent, non-standard definition of SSD, which, among other things, breaks down the intuitive link between the SSD and the Mean-Variance criteria. In particular, $y_0 W$, $W \in \Xi - \Pi$ (i.e., portfolios with the equal mean return but smaller variance) do <u>not</u> dominate y_0 according to Post's definition of SSD.

The basic LP algorithms do not allow for strict inequalities. Still, we may use the standard trick and use an arbitrarily small sensitivity parameter ε to convert the strict inequalities into the standard LP form

(8)
$$w_l \leq w_k + \varepsilon \ \forall l, k \in T : y_{0l} > y_{0k}$$
$$w \geq \varepsilon \cdot \vec{1}^T$$

Replacing the strict inequality constraints of (4.3) by (4.4) allows us to derive a necessary and sufficient test as the limiting case $\varepsilon \to 0$. For any real-valued $\varepsilon > 0$, we have a *sufficient* test: $\widehat{\theta}_2(y_0) \geq 1$ implies efficiency, but $\widehat{\theta}_2(y_0) < 1$ does not imply inefficiency. For $\varepsilon = 0$, however, we have a *necessary* test: $\widehat{\theta}_2(y_0) \geq 1$ does not verify efficiency, but $\widehat{\theta}_2(y_0) < 1$ does confirm inefficiency. To obtain a necessary and sufficient efficiency diagnosis, we can solve a pair of test statistics with $\varepsilon = 0$ and some small $\varepsilon > 0$, and check whether the conclusions are mutually consistent. If not, we can decrease the value of the sensitivity parameter ε for the sufficiency test until diagnoses become unisonous, or we have to declare the test inconclusive.

The infinitesimally thin line between the efficient and the inefficient sets highlights a possible disadvantage of the test statistic (7): If y_0 is efficient, one may need a long (potentially infinite) series of iterations to find sufficiently small $\varepsilon > 0$ that confirms efficiency. On the other hand, the advantage of the latter test is its simplicity: Problem (5) involves $3m^2 + n$ variables in contrast to the m variables of Problem (5), and hence the latter can be solved much faster. While the computation time of (5) is measured in minutes or hours depending on the data size, the time of solving problem (7) is measured in seconds. (See Table 4 below for some evidence.) These hard data led Post (2001) to describe his test procedure as the first "tractable" test of SSD for large applications.

However, there are some attractive features in the computationally more demanding test statistic θ_2^N which deserve consideration. First, the test statistic itself has an intuitive interpretation as a cardinal "inefficiency" measure, which allows for comparing the inefficiency premiums of various inefficient portfolios, for example. By contrast, Post's test statistic is merely an arbitrary dummy. Second, in case of inefficiency diagnosis, the optimal solution to (5) always

identifies a dominating portfolio as a benchmark, while (7) does not. This feature may have significance in the portfolio management applications. Third, perhaps most importantly, in (5) we can directly impose additional constraints on the feasible set of portfolio weights λ. This is not possible in formulation (7). This proves a pivotal advantage for the Kuosmanen approach in the forest management application to follow.

Application

To assess the usefulness of the SD developments outlined above, we revisit the harvesting problem of a real forest holding in Eastern Finland, which has been earlier studied in the MV framework by Heikkinen (1999), and in terms of a dynamic stochastic optimization model in Heikkinen (2001). These studies operate with explicitly highly-specified utility functions, even though there was no information about decision makers preferences. As a difference to these studies we apply now SD approach that is more general, i.e., fewer assumptions are needed. The land area of the forest holding is 22.6 hectares, which is partitioned in the official forestry plan into 14 stands according to the geographic location. Each stand has a unique mixture of timber species and assortments as well as physical growth, and hence stands are managed independently. In practice, all harvestable stands are mixed stands, which include at least saw-logs and pulpwood.

Cutting is usually implemented by forest stand, not by tree assortment or species. This is because it is unprofitable to select only certain species from the stand and because in practise all the mature trees include both sawlogs and pulpwood. Finnish forestry legislation forbids final felling of young stands. Regional Forestry Centers determine which stands are harvestable. In the present case, final felling is allowed only in four forest stands aging between 70 and 90 years. Therefore, only the timber stock on those specific four stands can be considered as a liquid asset.

Table 1. Characteristics of the four harvestable stands

	Stand #162	Stand #163	Stand #165	Stand #173	Total	Prices €/m^3 1996:12
Pine sawlogs		16 m^3	17 m^3			41.73
Spruce sawlogs	67 m^3	1 m^3		267 m^3		34.66
Birch sawlogs						16.50
Pine pulpwood		9 m^3	16 m^3			20.82
Spruce pulpwood	25 m^3	2 m^3		191 m^3		41.53
Birch pulpwood		3 m^3	14 m^3			15.84
Total	92 m^3	31 m^3	47 m^3	485 m^3	628 m^3	
Growth (%)	3.2	4.1	4.1	3.7		
Value (€)	2 843	940	1 195	13 232	18 210	
Portfolio weight	0.16	0.05	0.07	0.72		
Area (ha)	0.7	0.5	0.6	2.2	4	

Table 1 presents descriptive statistics about timber species and assortments, growth, monetary value, weight in the original portfolio, and land area of the four stands considered. Two stands consisted mainly primarily of spruce sawlogs and pulpwood, the other two mainly of pine sawlogs and pulpwood. The annual growth of stands varied between 3.2 and 4.1 percents. The total timber volume of these four stands was 628 m^3 and the value 18,210 Euro using the prices of 12/1996.

For simplicity, we use the general stock market index of Helsinki Stock Exchange (HEX) to represent equities as an alternative investment possibility. The monthly arithmetic averages of the daily closing values of HEX were calculated to make data comparable to stumpage prices, which are reported on monthly basis. Our historical data ranges from October 1985 to December 1996. During this period, the stock marker of Helsinki offered the mean return of 16 per cent, with a standard deviation of 0.21. The forest stands offered a much more modest growth with mean return of roughly 6 per cent. On the other hand, the growth was also more stable: The standard deviation of the returns varied between 0.07 and 0.10 for all four forest stands. (For more detailed statistics and discussion, see Heikkinen, 1999).

The investment problem of the landowner is to reallocate his liquid timber assets worth 18,210 Euro in a productive way. We consider two basic short-term strategies. The first is to harvest the timber immediately, and invest (for a moment) to equities traded in the stock market. The second alternative is to postpone harvesting to the future, in hope of a price increase for timber as well as the yielding additional physical growth. Of course, both the stock market and the future timber prices are risky. For the present purposes we consider the physical growth to be deterministic. This is a valid assumption in our application, because even though there is small variation in physical growth in practice, the variation of physical growth is insignificant if it is compared with price variation.

Besides the two pure strategies discussed above, we consider a full continuum of ' mixed' diversified strategies. Since we do not consider land sales but merely timber harvesting, the liquid timber asset can be considered perfectly divisible. For example, it is possible to harvest b percent ($0 \leq b \leq 1$) of the liquid timber and invest it in the stock market, and hedge the risk by leaving the remaining $1 - b$ percent of timber to be harvested in the future. The risk profile of the total portfolio can be influenced by targeting fellings in specific stands, i.e., it is not necessary to harvest all stands to the uniform degree. However, we restrict attention to final fellings in the sense that it is not possible (or cost effective) to target harvest to specific species within any given stand to change the composition (and hence the risk profile) of the stand. By this simplification, the historical returns of each forest stand can be computed as the average of the increases in stumpage prices weighted by the composition (species/thickness) of the timber plus the physical growth.

In the spirit of Heikkinen (1999), our main objective is to investigate the influence of the market imperfections, which present themselves in the form of constraints on acquire additional timber stock of similar quality, to the portfolio efficiency. For example, Stand #163 constitutes only 5% of the total timber portfolio. It might be in the interest of the forest owner to increase the share of this type of forestland with valuable pine sawlogs, to achieve a more balanced risk exposure. However, the differences in species and assortment decomposition and the physical

growth rate make each stand unique in terms of the risk profile. Therefore, it is not self-evident that forest-land with similar characteristics is available on the local/regional markets, at the prevailing market-prices.

Most earlier studies applying portfolio optimization techniques have simply assumed that forest stands can be traded like financial assets. The paper by Heikkinen (1999) was the first study to explicitly model the constraints for buying additional forest land with similar forest land. Specifically, given the original portfolio weights $\lambda_{162} = 0.16$, $\lambda_{163} = 0.05$, $\lambda_{165} = 0.07$, $\lambda_{173} = 0.73$, and $\lambda_{HEX} = 0.00$. Heikkinen imposed the linear constraints

(9)
$$\lambda_{162} \leq 0.16,$$
$$\lambda_{163} \leq 0.05,$$
$$\lambda_{165} \leq 0.07,$$
$$\lambda_{173} \leq 0.73,$$

in the standard Mean-Variance quadratic optimization problem to rule out purchases of additional forest land. This is especially relevant in the present Finnish case, where the timber is a reasonably liquid asset, whereas the forest land is not.

Empirical results

To check whether the MV approach would suffice or whether there is indeed case for SD analysis, we tested the normality assumption of returns required by MV. There are several statistical tests for testing normality hypotheses, including corrected Kolmogorov-Smirnov tests, Anderson-Darling test, and the chi-squared test. Since we have no a priori hypotheses of the mean and the variance of the theoretical distribution, the most basic test procedures do not apply. We therefore resorted to the Shapiro-Wilks W-test, a standard approach which has performed well in comparisons (see e.g. Royston, 1992, for further details). Table 2 reports the results.

Table 2: Shapiro-Wilks normality test statistics and the associated significance levels (n=135)

Asset	Test statistic	p-value
HEX	0.792	<0.0001
Stand #162	0.862	<0.0001
Stand #163	0.820	<0.0001
Stand #165	0.902	<0.0001
Stand #173	0.793	<0.0001

The interpretation of the W-tests is clear: In all case of all assets the null hypothesis of normality is rejected at 99% confidence level or higher. By visual inspection, all 5 assets appear to have a bell-shaped distribution, symmetric around the mean. Normality is violated mainly because of the high kurtosis, not because of any notable skewness. Figure 2 illustrates visually

how the cumulative return distribution of Stand #165 deviates from the CDF of a normal distribution with the equal mean and variance levels.
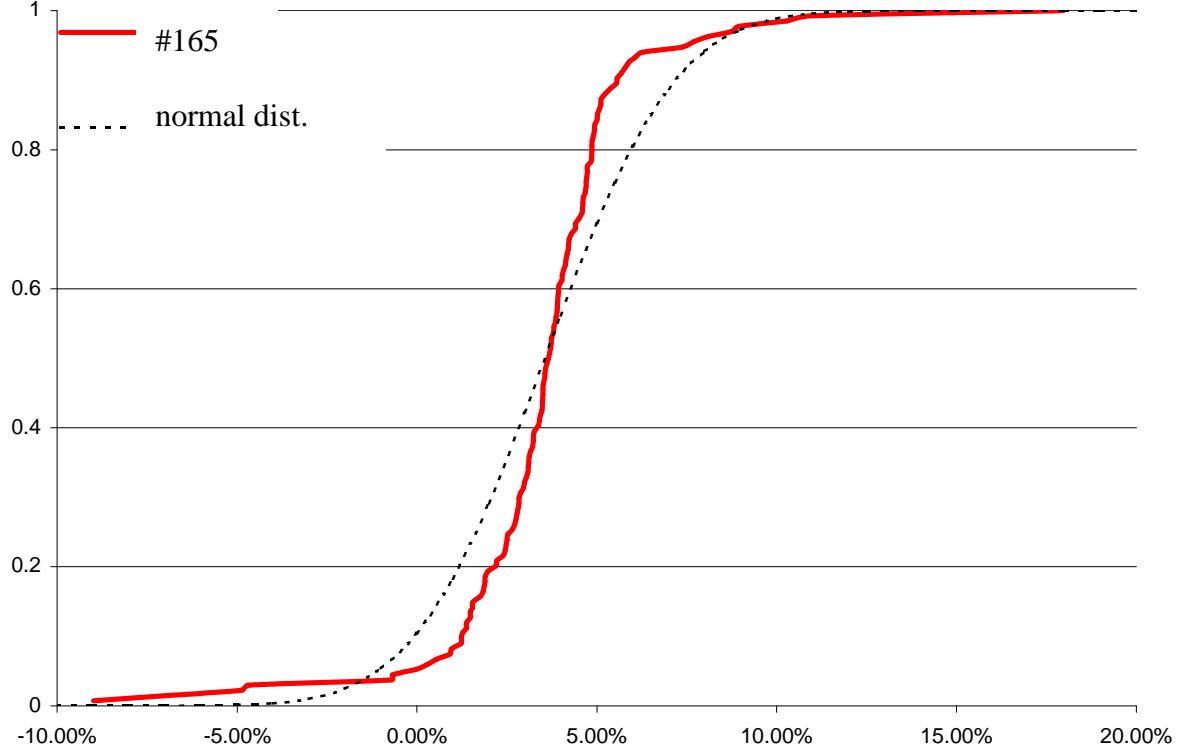

Figure 2: Cumulative return distribution of Stand #165 (solid curve), compared to the normal distribution with the equal mean and variance (dashed curve).

We next applied the SSD tests to above described data to investigate the influence of the purchasing constraints (9) on SSD efficiency. We used the GAMS.IDE software powered by the CONOPT2 solver, and a 1.7 GHz processor with 256 MB of physical memory, for solving the test statistics θ_2^N, θ_2^S, and $\widehat{\theta}_2$ for the original portfolio $\lambda_{162} = 0.16$, $\lambda_{163} = 0.05$, $\lambda_{165} = 0.07$, $\lambda_{173} = 0.73$, and $\lambda_{HEX} = 0.00$. Table 3 reports the values of the test statistics.

Table 3: Influence of purchasing constraint on SSD efficiency

	Kuosmanen		Post
	θ_2^N	θ_2^S	$\widehat{\theta}_2$
No constraint	0.0008	-	0.4920
Constrained case	0.0000	8978.00	n.a.

In the unconstrained case, both the Kuosmanen and the Post approaches diagnose the present forest portfolio as inefficient. The Kuosmanen statistic reveals that the mean return could be increased only by the minimal 0.08 percentage points without altering the risk profile. The Post statistic confirms the inefficiency diagnosis, but does not provide any additional information. (As the necessary condition was already violated, the sufficient θ_2^S statistic was omitted as meaningless.)

Furthermore, from the optimal solution of (5), we learn that $\lambda_{162}^* = 0.906$ and $\lambda_{HEX}^* = 0.094$. That is, the maximum return with the current risk profile would be obtained by harvesting all other stands, but acquiring almost equal amounts of additional forest land similar to Stand #162. That is, mature spruce forest with over 70 per cent of the timber suitable for sawlogs. Of course, one could favor more homogenous spruce stands in the long term forest planning. In this respect, the result is well in line with the recent trends in forestry practices in Finland. In the short run, however, this would require trading the forest land. Moreover, it is highly unlikely that the present land ownership could be instantaneously traded for large areas of old spruce forests. Therefore, the unconstrained case seems not very realistic for the short run planning. It should be also noted that the expected gains of 0.08 per cent points are very minimal. This should be balanced against the transaction costs of making radical changes of the investment strategy, the environmental and the aesthetic losses due to increased homogeneity of the tree species, as well as the option value of postponing the semi-irreversible harvesting decision, which have not been duly accounted for in the present analysis.

Consider next the constrained case where increasing the timber stock is not an option in the short run, that is, the pure harvesting problem. As discussed above, we cannot incorporate additional constraints on the portfolio weights in Post's approach, so the Kuosmanen tests are the only alternative. Interestingly, the original portfolio turns out to be efficient if the buying constraint is imposed. That is, the current forest portfolio is the optimal, expected utility maximizing choice for at least some non-satiated risk-aversive investors. In other words, we can rationalize the observed emphasis on the under the land-trading constraint, applying the SSD criteria. Of course, this does not imply the risk profile of the current portfolio matces optimally with the preferences of the forest owner. It merely underlines that it is not possible to find a preferable alternative portfolio without learning more of the forest owners preferences.

Now, let us contrast these results to those of the MV analysis of Heikkinen. The MV approach minimizes the variance of the portfolio, given an exogenously specified target mean return. From the scenarios considered by Heikkinen, the case of 6 per cent mean return target is closest to the original mean return of 5.9 per cent. In the unconstrained case, also the MV approach finds the original portfolio inefficient. Interestingly, the efficient portfolio also involved a similar buying and harvesting strategy, concentrating as much as 43.5 per cent of the total wealth in the forest similar to Stand #162. The MV analysis indicated that the optimal diversification strategy would decrease the portfolio variance by roughly 5.9 per cent.

In case of land-trading constraint, the SD and MV approaches yielded different qualitative results. While under SD criteria the original portfolio was diagnosed efficient, the MV criteria suggest inefficiency. However, the MV efficient benchmark portfolio at the target mean return of

6 per cent does not deviate substantially from the original forest portfolio. In fact, only 2 per cent of the total wealth is reallocated from the timber assets to the stock market. This would yield a 1.8 percent decrease in the portfolio variance.

As a conclusion, most risk-aversive forest managers would probably postpone harvesting and preserve the present composition of the portfolio in light of these results. Indeed, both SD and MV fail to demonstrate any significant gains by further diversification of the portfolio to the stocks, either in terms of higher mean return or lower variance. Less risk-aversive forest owners could scan the SD or the MV efficient subsets of the portfolio set Ψ to identify preferable diversification strategies. Since this would involve more active interaction with the forest managers, and effective methodologies for identifying and representing the efficient subsets remain to be developed, we refrain from further operational recommendations.

We also obtained evidence of the relative computational burden of the different tests. Table 4 reports the computation times of the different test statistics (in seconds). As expected, Post's test statistic proved the quickest to solve, requiring only two seconds for complete execution, whereas the Kuosmanen version took over 10 minutes in the unconstrained case, and almost 8 minutes in the constrained one. Furthermore, confirming the efficiency status took over 2 hours of computation time in the constrained case. Still, we think in many situations this may be an affordable price to pay for a meaningful efficiency score, and for the identity of the dominating portfolio yielding the highest mean return. For sure, in the constrained case the Kuosmanen approach was the only option.

Table 4: Total computation times of different SD statistics (seconds)

	Kuosmanen		Post
	θ_2^N	θ_2^S	$\widehat{\theta}_2$
No constraint	628.4	-	2.3
Constrained case	461.9	7290.9	n.a.

Conclusions and future research

We think the SD approach could provide useful tools for portfolio analysis when forest and/or other natural resource assets with non-normal return distributions are involved. Portfolio analysis can be made using minimum number of assumption if compared with M-V or some other portfolio optimisation tools that require highly spesific information about decision-maker preferences. We have reviewed the theoretical framework and the operational tests recently outlined in Kuosmanen (2001), and integrated to it some useful extensions from the subsequent work of Post (2001). We revisited the forestry portfolio management case of Heikkinen (1999) to shed further light on some important practical aspects of the SD tests. Our main focus was on studying the impact of the possibility to purchase additional quantities of the timber-land with the desired species and quality characteristics. This is an important question in the Finnish forestry environment where the liquidity of the timber harvest is much higher than the liquidity of the land.

Interestingly, we found that the purchasing constraint made a big difference in the SD efficiency of the portfolio. If desired type of forest land would be for sale, it would pay to trade the present forest holdings to homogenous spruce forest in the short run. This strategy would yield 11 percent points higher mean return without changing the present risk exposure or profile in any way. In the short run, however, it is not so simple to trade with the forest land. Therefore, in the short run planning it is relevant to impose the constraint that the shares of the timber assets cannot be increased in the portfolio.

From methodological perspective, our application proved an interesting case for comparing the two alternative test approaches proposed by Kuosmanen and Post, respectively. Our application confirmed the theoretical fact that Post's test requires less time and computational resources. However, Post's test cannot accommodate additional restrictions for the portfolio weights, so the Kuosmanen approach was the only option in the more short term planning problem.

In the present application we focused on the new SD tools, ignoring the possible intertemporal dynamics in the time series, as well as the possible sampling errors. The future applications could pay further attention on integrating these aspects in the SD framework. The SD approach could prove a useful analytical framework for dealing with "catastrophic" events. For example, forestry risks also include such threats as fire, tornados, heavy snow, flood and pests, which shape the left tail of the return distribution. However, we see there at least two major difficulties in application of SD to assets exposed to catastrophic events. First, since the catastrophic events are relatively rare, the observed time series data may fail to adequately account for these risks. Second, the SD criteria are sensitive to sampling error, especially in the left tail of the distribution. To circumvent the problems with the left tail, one might enrich the empirical distribution by a combination of a theoretical catastrophe model and/or the bootstrapping technique. Although this paper did not extend the analysis towards these directions, it will to serve as a useful starting point for future investigations and extensions which address specifically these two difficulties.

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