

# A Study of Neo-Austrian Economics using an Artificial Stock Market

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## Abstract

An agent-based artificial financial market (AFM) is used to study market efficiency and learning in the context of the Neo-Austrian economic paradigm. Efficiency is defined in terms of the “excess” profits associated with different trading strategies, where excess is defined relative to a *dynamic* buy and hold benchmark in order to make a clean separation between trading gains and market gains. We define an Inefficiency matrix that takes into account the difference in excess profits of one trading strategy versus another (“signal”) relative to the standard error of those profits

(“noise”) and use this statistical measure to gauge the degree of market efficiency. A one-parameter family of trading strategies is considered, the value of the parameter measuring the relative “informational” advantage of one strategy versus another. Efficiency is then investigated in terms of the composition of the market defined in terms of the relative proportions of traders using a particular strategy and the parameter values associated with the strategies. We show that markets are more efficient when informational advantages are small (small signal) and when there are many coexisting signals. Learning is introduced by considering “copy-cat” traders that learn the relative values of the different strategies in the market and copy the most successful one. We show how such learning leads to a more informationally efficient market but can also lead to a less efficient market as measured in terms of excess profits. It is also shown how the presence of exogenous information shocks that change trader expectations increases efficiency and complicates the inference problem of copycats.

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## 1 Introduction

In recent years it has become ever more popular to consider financial markets from other than a neoclassical rational expectations point of view. The latter considers financial markets to be in continuous equilibrium with informationally efficient prices. Empiricists have questioned the validity of this model, pointing to evidence of inefficiencies. Alternative views have been presented to better

match the empirical evidence. One with a distinguished history, that will be the focus of this paper, is the Neo-Austrian theory of financial markets.

Based on a recent rereading of the ideas of Friedrich Hayek and the Neo-Austrian theory of market processes (see, e.g., Hayek (1937), (1945), (1948) and (1978), Littlechild (1982), Rizzo (1990), Kirzner (1992) and (1997)), Benink and Bossaerts (2001) presented the first formal application of Neo-Austrian theory to financial markets. In the Neo-Austrian interpretation financial markets are continuously evolving from one inefficiency to another, never attaining the perfect, efficient equilibrium, yet strongly attracted towards it. Creative investors track and exploit profit opportunities generated by continuous shocks in a never-ending cycle. The result would be a stable process with pronounced regularities. According to Neo-Austrian theory, a competitive market provides a systematic set of forces, put in motion by entrepreneurial alertness (i.e. eagerness to make money), which tend to reduce the extent of ignorance among market participants.

The resulting knowledge is not perfect; neither is ignorance necessarily invincible. Equilibrium - read: market efficiency - is never attained, yet the market does exhibit powerful tendencies towards it. The fact that equilibrium is never attained is attributed to an erratically changing world where traders realize that their knowledge is imperfect. At the same time, the changes are never so extreme as to frustrate the emergence of powerful and pervasive economic regularities. Although traders can exhibit fully rational behaviour, in the sense that they try to optimize their financial position and wealth, the market process is not generating a rational expectations equilibrium (REE) and informationally efficient prices. Rational behaviour does not necessarily imply rational expectations.<sup>1</sup>

Imperfect knowledge is a key characteristic of Neo-Austrian thinking. According to Hayek, the problem of economic choice and ultimately the analysis of economic behavior in neoclassical theory is oversimplified, because it has been reduced to optimal behavior under constraints that agents are supposed to be very familiar with. These constraints concern: (1) preferences, (2) production and market technology, and (3) resources. In contrast, the Neo-Austrian view stresses that fundamental uncertainty and ignorance exist regarding these constraints. This uncertainty and ignorance is claimed to lead to disequilibrium, and disequilibrium itself generates further uncertainty and ignorance regarding the constraints. Disequilibrium thereby becomes self-enforcing and permanent.

However, alert participants in the market process, whom the Neo-Austrians define as entrepreneurs, try to get a - necessarily incomplete - picture of the nature of the disequilibrium in the marketplace, because disequilibrium generates profit opportunities. The actions of these entrepreneurs produce the very signals that are needed to reduce disequilibrium. What renders the market process a systematic process of coordination is the circumstance that each gap in market coordination expresses itself as a pure profit opportunity. The profit-grasping

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<sup>1</sup>Note that in the Neo-Austrian view the failure of markets to reach the informationally efficient equilibrium ought not to be attributed to costs of any nature (adjustment costs, information costs, trading costs, etc). As mentioned, the non-convergence has its origin in limitations of knowledge.

actions of successful entrepreneurs dispel the ignorance which was responsible for the profit opportunities, and thus generate a tendency towards coordination among market decisions. However, due to continuous change in the constraints, equilibrium is never reached.

In their paper Benink and Bossaerts (2001) construct an example of an economy with a continuously inefficient financial market. They adjust the memory of investors' trading rules in order to generate a market process that can be characterized as stable, cycling from one inefficiency to another. Despite the stability (stationarity), rational, risk-averse investors are unable to exploit all inefficiencies because they cannot make reliable inferences about them. This would be the case, for instance, if the memory of the return process is sufficiently long for statistics not to display their usual distributional properties needed to construct confidence intervals. Based on an analysis of average price changes, an investor will with high likelihood reject efficiency, yet the sign of the average is unreliable in predicting the sign in independent future replication. As a consequence, classical statistics cannot reliably assess the inefficiencies.

In contrast to Benink and Bossaerts, this paper places more emphasis on, and studies in detail, the learning processes and dynamics of a Neo-Austrian inefficient financial market. As mentioned above, the neoclassical rational expectations point of view considers financial markets to be in continuous equilibrium with informationally efficient prices. Pesaran (1989) notes that the idea of a REE involves much more than the familiar concept of the equilibrium of demand and supply. A REE can be characterized by three main features: (1) all markets clear at equilibrium prices, (2) every agent knows the relationship between equilibrium prices and private information of all other agents, and (3) the information contained in equilibrium prices is fully exploited by all agents in making inferences about the private information about others. Thus, in a REE prices perform a dual role - apart from clearing the markets they also reveal to every agent the private information of all the other agents. In effect, the concept of the REE requires that everybody knows (in a probabilistic sense) everything about the way the market economy functions. But as Hayek (1937) puts it:

“The statement that, if people know everything, they are in equilibrium is true simply because that is how we define equilibrium. The assumption of a perfect market in that sense is just another way of saying that equilibrium exists, but does not get us any nearer an explanation of when and how such a state will come about. It is clear that if we want to make the assertion that under certain conditions people will approach that state we must explain by what process they will acquire the necessary knowledge”.

The preceding implies that, for the REE to have any operational meaning, it is necessary that the processes by means of which people learn from experience and acquire the common knowledge necessary for the achievement of the REE, are specified fully and explicitly.

In this paper we use an agent-based artificial financial market (AFM) to generate simulations of inefficiencies and learning and investigate to what extent a Neo-Austrian interpretation of the resulting market dynamics is the most

natural<sup>2</sup>. Agent-based models are intermediate between empirical and analytic studies; the former offering grave problems in terms of inference, while the latter, perforce, come armed with a large number of model assumptions. Moreover, the complexity of the AFM can be tuned, so as to offer a more transparent model versus a more realistic one.

The most well known AFM is the Santa Fe model (see, e.g., Palmer, Arthur, Holland, LeBaron and Tayler (1994), Arthur, Holland, LeBaron, Palmer and P. Tayler (1997), LeBaron (1999, 2000 and 2001)). In this paper we use an alternative model - the so-called Neural Networks Chaos and Prediction Model (NNCP) (Gordillo and Stephens (2001a, 2001b and 2003)) - whose design was motivated by the desire to study relatively neglected elements, such as the effect of organizational structure on market dynamics and the role of market makers and information, all of which are important in the formation of market microstructure (see, for example O’Hara (1997)). Although capable of simulating more “realistic” dynamical scenarios, in this paper we use the NNCP in the context of a more transparent model, in which traders are associated with strategies chosen from a single one-parameter family, the parameter representing a trading bias linked to the informational advantage of the trader, zero bias representing noise traders. The resulting AFM, presented in section 2, can effectively be parametrized by three principle degrees of freedom: (1) the proportion of traders of a given type, (2) the number of different trader groups or strategies, and (3) the similarity between different trader groups - measured by distance in bias between two agents or groups. Learning is introduced in section 2.3 via the notion of “copycat” agents that observe the market, infer what is the most successful strategy and then copy it.

We use this AFM to investigate notions of efficiency and learning and examine to what extent the results are more naturally interpretable in a Neo-Austrian rather than a neo-classical framework. AFMs have been used, for example, by Chen and Yeh (2002), to consider efficiency as an emergent phenomenon. There however, efficiency was judged purely from the statistical properties of the returns series. However, as will be further discussed in section 3, predictability of the time series is not necessarily inconsistent with market efficiency. We therefore consider efficiency from the empirical point of view of whether or not traders can make excess returns systematically, defining a notion of excess profit that distinguishes between market gains and trading gains. To further distinguish between intelligent trading and “luck” we consider relative excess returns,  $I_{ij}$ , between trading strategies  $i$  and  $j$ , measured relative to the variance of these excess returns. In section 3, we introduce an *Inefficiency Matrix*, with matrix elements  $I_{ij}$ , which summarizes statistically the relevant relative inefficiencies in the market.

With these tools in hand, in sections 4.1 and 4.2, we investigate both efficient and inefficient markets in the absence of learning, showing in particular, in section 4.2.1, under what conditions a market may be inefficient, yet still

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<sup>2</sup>The results of AFMs in the past have mainly been analysed using an “evolutionary” as opposed to neoclassical view of markets (see, for instance, Farmer and Lo (1999) and Farmer (1998)).

be *observed* to be efficient. This possibility is due to the statistical inference problem that traders face in the light of noisy market data. In section 4.3 we show, paradoxically, that learning can lead to a more inefficient market in terms of excess profits, even though informationally the market was more efficient; and then, in section 4.4, we study how the arrival of new information affects inefficiency and learning. We then discuss the results in the framework of the Neo-Austrian paradigm and draw some final conclusions.

## 2 Description of the Model

For the purposes of the present investigation we consider a simplified form of the NNCP where a simulation is carried out for a prescribed number of *ticks* on a single risky asset. An agent can divide his/her wealth between this risky asset and a riskless asset (“cash”). At each *tick* an agent takes a position (buy/sell/neutral). Shares are bought in fixed size lots of one share.

Resources are finite and hence traders have portfolio limits associated with either zero cash or zero stock. Short selling is not permitted. Although the NNCP can consider different market clearing mechanisms, here we will consider only a double auction market. After each *tick* price is updated exogenously via a supply/demand type law as in (1)<sup>3</sup>.

$$p(t+1) = p(t)[1 + \eta(B(t) - O(t))] \quad (1)$$

In this equation, which is common to many AFMs,  $p(t)$  is price at *tick*  $t$  and  $B(t)$  and  $O(t)$  are the demand and supply at  $t$ , while  $\eta$  is a tuning parameter. Large values of  $\eta$  lead to large price oscillations while small values lead to slow price adjustments. Note that  $D(t) = (B(t) - O(t))$  depends not only on the positions taken by the agents but also on the mechanism used to match their trades, e.g. at what price two contrary trades will be matched.

The wealth of an agent  $i$  at time  $t$  is given by  $W_i(t) = (E_i(t) + H_i(t)p(t))$ , where  $E_i(t)$  and  $H_i(t)$  are the amount of cash and number of shares that the agent possesses at time  $t$ .

### 2.1 Double Auction

The market clearing mechanism we use for the present simulations is a simple double auction, where at every *tick* each trader takes a position with an associated volume and at a given price, each trader being able to value the asset independently but with prices that are not too different. In this model price changes are induced only via the disequilibrium between supply and demand as measured by equation (1). Specifically:

1. At time  $t$  one lists all the positions taken by the agents and the associated volume and price. The agents' bids and offers are obtained at time  $t$  via a Gaussian distribution with mean  $\bar{p} = p(t-1)$ .

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<sup>3</sup>Price can also be updated endogenously as in the case of a market with *market makers* (see Gordillo and Stephens (2001a and 2001b) or Gordillo and Stephens (2003)).

2. A bid and an offer are matched only if they overlap, i.e.  $p_b(t) > p_o(t)$ . To realize a transaction we used: “*best bid/offer*”, where the highest bid and the lowest offer are matched at their midpoint successively until there are no overlapping bids and offers.
3. Price is updated via (1) using only those bids and offers that are not matched and that have overlap, i.e.  $p_b > p(t)$  and  $p_o < p(t)$ .

## 2.2 Trading Strategies

Although the NNCP market can accommodate many different strategy types we will illustrate our results in the context of a market with relatively few types of traders, as all that we wish to demonstrate can be observed in a simple setting. Essentially, we will consider a one-parameter family of traders described by a “bias”,  $d$  ( $d \in [0, 1]$ ), where the position probabilities are:

$$P(c) = \frac{2d}{3}, \quad P(n) = \frac{1}{3}, \quad P(v) = \frac{2(1-d)}{3} \quad (2)$$

where  $c$  represents Buy,  $v$  Sell and  $n$  Hold. For example, when  $d = 1/2$  then the corresponding probabilities are  $1/3, 1/3, 1/3$ . This limit corresponds to that of a liquidity or noise trader. An alternative, or complementary, interpretation is that it corresponds to a trader or investor that believes (correctly or incorrectly) that the market is efficient, having no statistical bias in favour of one position versus another. In contrast, a trader with  $d = 1$  has probabilities  $2/3, 1/3, 0$  and corresponds to a trader with a strong belief that the market will rise. Similarly, a trader with  $d = 0$  has a strong preponderance to sell, corresponding to a trader with a strong belief that the market will fall. We will denote a trading strategy from the above one-parameter family by the pair  $(100d, 100(1-d))$ . Thus, an uninformed, or liquidity, trader is denoted by  $(50, 50)$  and a maximally biased one, on the long side, by  $(100, 0)$ .

One could also imagine a biased trader to have a superior understanding of the underlying market dynamics - knowing that all else being equal a preponderance to buy/sell will lead to excess demand/supply, which in its turn will lead to a price increase/decrease, which will lead to a higher valued portfolio. In this sense we can think of these traders as being informed relative to their noise trading counterparts. It is important to realize that such considerations, such as the rationality of a trader, or what led a trader to adopt a particular strategy (e.g. risk preferences, utility function, information set etc.), are extraneous to our discussion in this paper, as our model is completely specified by the traders’ strategies and a market clearing mechanism. The presence of traders with a bias may create an excess demand (supply in the case of a sell bias). This excess demand thus drives the price via the price evolution equation (1). The actual excess demand depends on the actual composition of the population and the distribution of biases. Additionally, in the presence of learning it may also depend on the efficiency of the learning and how easily information may be inferred from the market.



### 2.3 Learning Mechanism

In order to introduce the concept of learning we consider “copycat” traders Gordillo and Stephens (2001a, 2001b). Copycats observe the success of different strategies in the market and copy the most successful one, updating their expectations periodically in the presence of new information. The copying process may be deterministic or probabilistic and the definition of success varied. For example, a copycat might copy that trader that has the portfolio with highest observed Sharp ratio, or the trader with highest daily returns over a certain period. Obviously, as we shall see, copycats face a difficult inference problem, having to distinguish between the best strategy and the best *observed* strategy. We will assume that there are no costs incurred in acquiring information for the copycats or, for that matter, any other trader.

We will consider a probabilistic copying mechanism whereby a copycat copies a strategy  $i$  with probability<sup>4</sup>

$$P_i(t) = \mu_i(t) / \sum_i \mu_i(t) \quad (3)$$

where  $\mu_i(t)$  is a measure of the success of strategy  $i$  at time  $t$ . Note that  $\mu$  may well depend on other parameters or timescales. An interesting one is the period of time over which success is measured. For instance, one copycat might look at the portfolio returns over the last 50 ticks, whereas another might consider returns over the last 50 days. Given that the copying process is stochastic it may be that the copycat does not copy the most successful strategy. The more successful a strategy is relative to others however, the more likely it is that this is the strategy copied. The stochastic nature of the copying process is used to reflect the inefficiencies inherent in the learning process. None of our results are qualitatively changed if a deterministic learning mechanism is used instead. This probabilistic selection process is carried out periodically, for example, every 100 ticks, thus permitting the copycats to incorporate new information into their analysis.

Copycats naturally try to copy informed traders to find the optimal strategy. This activates a learning process. However, complete learning is by no means guaranteed in the sense that they do not necessarily identify the best strategy. The quality of the learning depends on the signal to noise ratio, which in its turn depends on the trading parameters, such as trader biases, and the number of traders with a particular strategy.

Note that the learning might be incomplete even in the case where there is only one other strategy to learn if the learning is inefficient (if the update frequency for learning is high however the learning will tend to be more complete). We do not require the arrival of new exogeneous information - “external shocks” - to observe incomplete learning and the permanence of inefficiencies. As we will see, this will occur, for instance, when we have a wide variety of strategies that are quite similar with agents spread uniformly among them. It is important to

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<sup>4</sup>In the Evolutionary Computation literature this is known as “roulette wheel selection”.

emphasize here the diversity that these different types of informed trader bring to the market. Even though they all (potentially) receive the same information their response to it, as in a real market, can be markedly different.

### 3 Measures of Efficiency and Inefficiency

The efficient markets hypothesis is strongly linked to the idea that security prices fully reflect (publicly) available information. In this manifestation its confirmation or negation has been highly controversial due to the existence of the joint-hypothesis problem, formulated by Fama in his seminal 1970 and 1991 overview papers on efficient capital markets, wherein efficiency is determined only within the context of a particular asset-pricing model. A common corollary of the efficient markets hypothesis (that by some has been taken to be its definition) is that in an efficient market it is impossible to make excess profits in a systematic fashion, or that excess returns are unpredictable. However, Pesaran and Timmerman (1995) note that predictability of excess returns does not imply stock market inefficiency, and can be interpreted only in conjunction with, and in relation to, an intertemporal equilibrium model of the economy. Inevitably, all theoretical attempts at interpretation of excess return predictability will be model-dependent, and hence inconclusive. Fama (1991) states that it is only possible to test whether information is properly reflected in prices in the context of a pricing model that defines the meaning of “properly”. As a result, when anomalous evidence is found on the behaviour of returns, the way it should be split between market inefficiency or a bad model of market equilibrium is ambiguous. Furthermore, as Balvers, Cosimano and McDonald (1990) have pointed out it is possible to formulate an equilibrium model that leads to predictable returns.

A way to avoid the pitfalls of the joint-hypothesis problem is to take a completely empirical approach; defining inefficiency with respect to some measure that is not dependent on the existence of some underlying model, such as an asset-pricing model. Thus, one evaluates the economic significance of stock market predictability by seeing if the associated information could have been exploited successfully in investment strategies, thus leading to systematic excess returns. Of course, this begs the question of how do we define systematic and excess? - excess relative to what? In the literature it is common to measure excess relative to some ‘fixed’ benchmark such as the riskfree interest rate or to an index portfolio (the logical extreme of that being the market portfolio). One of the chief drawbacks of such measures is that they permit the possibility that uninformed traders acquire excess profits even when the market is efficient. For instance, a market composed of only uninformed (50, 50) traders is a purely random process. However, a given realization of this process over a fixed time interval can lead to a large net price difference relative to the initial price<sup>5</sup>. In this case noise traders who, on average, do not change their portfolio holdings,

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<sup>5</sup>Obviously, averaged over sufficient time, or over a large number of different realizations (difficult to do in a real market), the expected final price is the same as the initial price

may have excess profits relative to a fixed benchmark simply because, by chance, the price rose. Thus, as emphasized by Bagehot (1971), it is important to distinguish between trading gains and market gains. Additionally, in reference to fixed benchmarks the discussion of systematic does not naturally arise.

In order to eliminate these defects we choose as benchmark a “moving target” where excess profit during timestep  $t$  is related to the increase in the market value of an active trading portfolio in the timestep  $t$ , relative to the increase in the market value of a buy and hold portfolio in the same timestep. In this way an excess profit or loss for a given trader over the timestep  $t$  can only arise when there has been a net change in the trader’s portfolio holdings in the asset *and* a net change in the asset’s price. This choice of benchmark always refers the market dynamics to a zero sum game. More concretely, we define the “excess” profit of a trader  $j$  using a trading strategy  $i$  in the time interval  $t - 1$  to  $t$  to be

$$e_{ij}(t, t - 1) = (\delta V_{active}(i, j, t) - \delta V_{BH}(i, j, t)) \quad (4)$$

where  $\delta V_{active}(i, j, t)$  is the increase in portfolio value between  $t$  and  $t - 1$  for trader  $j$  by trading using a trading strategy  $i$ , while  $\delta V_{BH}(i, j, t)$  is the same quantity but using as trading strategy Buy and Hold.  $e_{ij}(t, t - 1)$  can also be written as  $e_{ij}(t, t - 1) = \delta n_{ij}(t) \delta p(t)$ , where  $\delta n_{ij}(t)$  is the change in portfolio holdings over the timestep  $\delta t = t - (t - 1)$  of the trader using strategy  $i$  and  $\delta p(t)$  is the change in asset price over this timestep.

The excess profit earned between times  $t'$  and  $t$  is

$$E_{ij}(t, t') = \sum_{n=t'+1}^{n=t} e_{ij}(n, n - 1) \quad (5)$$

One may also consider the excess profit per unit time over the interval  $t'$  to  $t$ , defined as  $(1/(t - t'))E_{ij}(t, t')$ . One may enquire as to the average excess profits associated with a particular trading strategy,  $E_i(t, t')$ , by summing over all those traders utilizing the strategy and dividing by the number of such traders

$$E_i(t, t') = \frac{1}{N_i} \sum_j E_{ij}(t, t') \quad (6)$$

In this case  $E_i(t, t')$  represents the excess profits earned between  $t$  and  $t'$  by a “representative” trader of the class  $i$ , i.e. it is the average excess profit per trader with the strategy  $i$ .

The timestep in the above may, of course, be chosen arbitrarily. In the simulations in this paper we will consider it to be the most fine-grained possible - a “tick”. In this case the maximum possible excess profit over a time interval,  $t'$  to  $t$ , for a fixed, constant transaction volume  $V$  per trade, is that associated with “perfect foresight”, given by

$$E_{ij}(t, t') = V \sum_{n=t'}^{n=t} |\delta p(n)| \quad (7)$$

The chief advantage of having excess profits measured at such high frequency is that, as we will see, statistical inference about excess profits is enhanced due to the greater sampling. For example, one would expect to be able to better judge the utility of a trader’s proprietary strategy if it has been used 1000 times as opposed to 10. Of course, this presumes that the traders trade frequently. For traders with longer time horizons more time is necessary in order to reach a given confidence level. In principle, our definition of excess profits could be implemented using real, high-frequency trading data where it could be used to evaluate the ability of traders to exploit short-term profit making opportunities. For example, to measure excess profits over the next 10 ticks to examine to what extent a trader was correctly predicting short-term market movements. Of course, it would not be sensible to use this criteria to judge between different fund managers. For example, a manager might increase his/her portfolio holding in a given stock to 2.6% from 2.1% in January as a result of expecting an appreciation in the price of the stock over a six month interval. It would be unfair to judge this investment over the first 10 ticks after the purchase! Similarly, it would be inappropriate to consider the performance of a day trader using a time horizon of a year, i.e. to consider the net change in the portfolio weighting over the year weighted by the net change in price over the year.

As excess profit is a stochastic variable there is always a non-zero probability that, over a given time interval, a trader makes a profit just by chance. Hence, it is natural to refer the magnitude of any excess profits (“signal”) to the degree of variance (“noise”) in the profits, measured, for example, in units of the standard deviation. Additionally, it is natural to compare the excess profits of one trader, or group of traders, to those of another group. In other words “relative” excess profits are the most important.<sup>6</sup> To take into account both these factors we introduce the following “signal-to-noise” measure

**Definition:** The *Relative Inefficiency*,  $I_{ij}$ , between two strategies or trader groups  $i$  and  $j$  evolving from time  $t$  to time  $t'$  is

$$I_{ij}(t, t') = \frac{(E_i(t, t') - E_j(t, t'))}{\left(\frac{\sigma_i^2(t, t')}{N_i} + \frac{\sigma_j^2(t, t')}{N_j}\right)^{\frac{1}{2}}} \quad (8)$$

where  $\sigma_i(t, t')$  is the variance in the excess profits of the representative agent of the strategy  $i$ . Given that we have defined an inefficient market to be one where traders can make excess profits we may define  $I_{ij}(t, t')$  as the “relative” inefficiency between the trading strategies, or trader groups,  $i$  and  $j$ .  $(E_i(t, t') - E_j(t, t'))$  measures the excess returns of trading strategy  $i$  relative to trading strategy  $j$ . The division into trader groups should be a partition, i.e. it should cover all traders and any trader should appear in one and only one trading group.

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<sup>6</sup>In the context of entire markets it has been suggested by Lo and McKinley (1999) that “relative” efficiency of markets is a more useful notion than that of absolute efficiency.

Dividing by the standard error,  $\sigma_{err}$ , means the resultant measure gives us a measure of the statistical confidence we can have in the relative excess returns of the two strategies. A reasonable criterion for concluding that two strategies are relatively inefficient over a given time interval is that  $I_{ij}(t, t') > n\sigma_{err}$  over that time interval, where  $n$  is a measure of the confidence interval we require. A reasonable value of  $n$  is  $n = 2$  though, of course, we may require a stricter criterion.

Finally, we may describe the inefficiency of the entire market by making the following definition:

**Definition:** The *Inefficiency Matrix*,  $\mathbf{I}^m$ , for a market  $m$  evolving from time  $t$  to time  $t'$  is the matrix with matrix elements  $I_{ij}(t, t')$  from equation (8).

We take the Inefficiency Matrix to give a complete description of the relative inefficiencies that exist in a market. In the case of multiple assets we can conceive of several possible Inefficiency Matrices, according to the way we group traders, strategies and the existing types of assets. For example, we could define an Inefficiency Matrix for each asset in the market which would indicate the relative inefficiency associated to the trading of specific stocks. However, we could also build the Inefficiency Matrix associated to groups of assets – clustered, for instance, through some hierarchical criterion – which would in turn indicate the relative inefficiency associated to the trading of particular types of stocks. Other arrangements are also possible (such as building Inefficiency Matrices through groups of agents characterized by a certain parameter), but they are, in any case, variations of the previous definition. In all these cases, the Inefficiency Matrices for different assets can differ in both their dimensionality (different numbers of trading groups in different assets) as well as in the explicit nature of their matrix elements (different types of trader groups). Due to the zero-sum nature of our measures this matrix is antisymmetric,  $I_{ij}(t, t') = -I_{ji}(t, t')$ , as, if trader group  $i$  is making profits at the expense of trader group  $j$ , then trader group  $j$  is making losses of exactly the same magnitude to trader group  $i$ .

We will define two trader groups or strategies,  $i$  and  $j$ , to be relatively inefficient with respect to each other over the time interval  $t \rightarrow t'$  if and only if  $I_{ij}(t, t') > n$ ,  $\forall \tau \in [t, t']$ , where the number  $n$  represents the degree of confidence we require in order to state that there is an inefficiency. As stated, a typical value would be  $n = 2$ .

An associated single inefficiency measure for the whole market is

$$\mathcal{I}^m(t, t') = \frac{1}{N'} \left( \frac{1}{2} \text{Tr}(-(\mathbf{I}^m)^2) \right)^{\frac{1}{2}} \quad (9)$$

where the trace is over all strategies or trader groups and the normalization factor  $N' = N(N - 1)/2$ , where  $N$  is the number of strategies or trader groups in the market. Note that this definition of inefficiency is totally endogenous, making no reference whatsoever to any external benchmark.<sup>7</sup> With this single

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<sup>7</sup>It is also possible to define inefficiency exogeneously by considering  $I_i^{ex}(t, t') =$

market measure we could in principle also consider the relative inefficiency of one market versus another.

It is important to emphasize that in a real financial market the question of whether a market is efficient or not is really an empirical one, as we do not have a valid underlying theory that can demonstrably prove a market to be efficient or not. Moreover, it is one that can only be answered statistically given that the evolution of a market is stochastic. In that sense the empirical question boils down to one of: can one *infer* that a financial market is efficient from a set of data. The luxury of an artificial market is that we can *create* an efficient or inefficient market and then vary the parameters of the market in order to study when, and under what conditions, it is possible or not to infer efficiency from observing the market. Additionally, in distinction to a real financial market we may obtain better statistics by “repeating history”, by rerunning the market over again.

## 4 Principal Results

### 4.1 What Does an Efficient Market Look Like?

Before presenting simulations of inefficiencies in the Neo-Austrian context we wish to first present some simulations of what represents an efficient market to use as a benchmark for comparing other results. Also, in the Neo-Austrian paradigm there are strong tendencies directing the market towards efficiency, even if it in reality never reaches such a state. We do not at this point consider a general form of efficiency but rather restrict ourselves to some simple, intuitive examples, in particular examining efficiency in the context of an homogeneous market.

In Figures 1 and 2 we see the distribution of excess profits for a group of 100 traders after 3001 auctions. In Figure 2 the traders use a (50, 50) strategy, i.e. they are liquidity traders, whereas in Figure 1 they are informed (90, 10) traders. In both cases the original distribution at  $t = 0$  was a single peak of 100 traders at the origin. The efficient market dynamics here is such that the initial peaked distribution spreads uniformly and symmetrically on average. Note that due to our choice of benchmark the losses of the traders to the left of the origin and the profits of the traders to the right sum to zero at all times. The variance of the distribution increases with time as  $\sigma^2(t) = A(n)t$ , where  $A(n)$  is a constant that measures the market liquidity - the less liquid the market the more volatile, and therefore the greater the dispersion. The dependence on the liquidity can be seen in Figures 1 and 2, where after 3001 auctions we see that the dispersion for 100 (90, 10) agents is substantially greater than that of 100 (50, 50) agents. Of

$$\frac{(E_i(t, t') - E_j^{ben}(t, t'))}{\left( \frac{\sigma_i^2(t, t')}{N_i} + \frac{\sigma_j^{ben2}(t, t')}{N_j^{ben}} \right)^{\frac{1}{2}}},$$

where  $E_j^{ben}(t, t')$  is the excess profit earned by a strategy  $j$  in some benchmark market. The most natural benchmark is an efficient market composed of purely uninformed liquidity traders, i.e. (50, 50) traders.

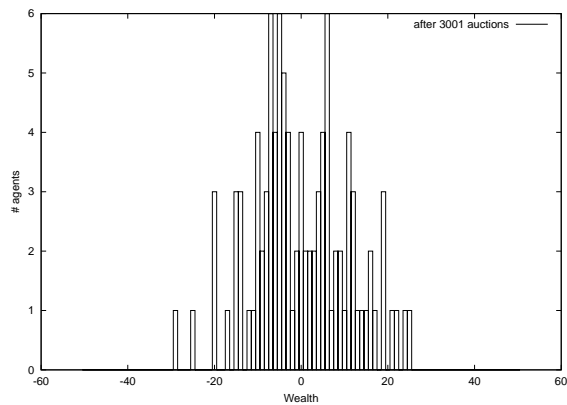


Figure 1: Histogram of excess profits for 100 (90, 10) traders

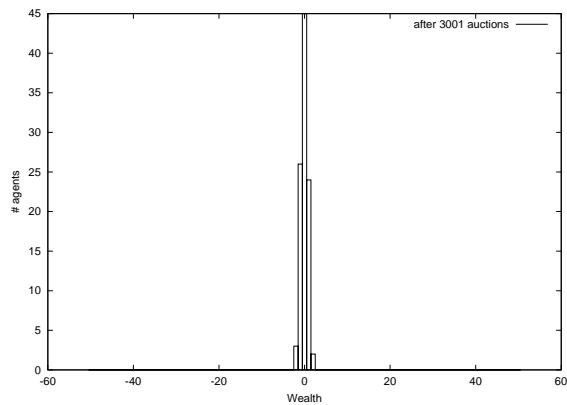


Figure 2: Histogram of excess profits for 100 (50, 50) traders

course, the price behaviour in both cases is very different. In the first case, due to the large excess demand, price increases very rapidly, whereas in the latter it is a random walk around the initial price. Both markets however are efficient according to our excess profit criterion, in that no trader or group of traders is making systematic excess profits at the expense of any other group. This, in fact, can be further confirmed by considering the behaviour of any particular trader or group of traders and observing that the evolution of their excess profits is a random walk of mean zero.

Thus, we see that the hallmark of an efficient market is that no subgroup of traders make systematic excess profits at the expense of any other. This is manifest in the present graphs by the fact that the distribution is unimodal and symmetrical. In terms of the Inefficiency Matrix we can check that any chosen group of traders is not making systematic excess profits by considering the matrix element  $I_{ij}$ , where  $i$  refers to the group of traders of interest and  $j$

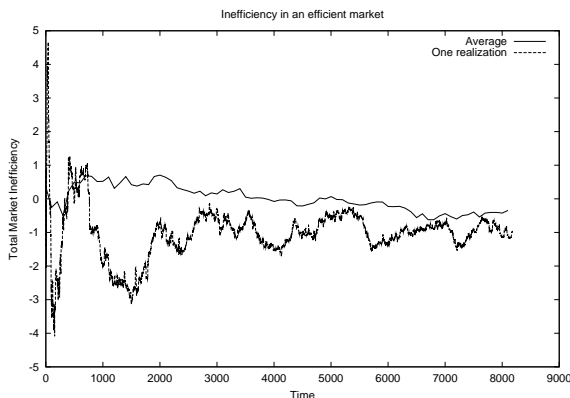


Figure 3: Graph of  $I_{ij}(t, 0)$  for a fixed subgroup of 50 (50,50) traders in an homogeneous market

refers to the rest.

In Figure 3 note that the scale of  $I_{ij}(t, 0)$  for a market of 100 (50, 50) traders is about the order of two or less. In fact, taking an average over 10 different experiments leads to a resultant curve that leads to consistency of the null hypothesis that the market is efficient. We can also see that the volatility of the curve diminishes as a function of time. This is a direct consequence of our definition of the Inefficiency Matrix. For an efficient market the numerator gives zero on average, while the denominator increases linearly with time. The interpretation of this fact is simply that as time passes  $I_{ij}$ , being a statistical measure, allows us to infer with a higher degree of confidence that the market is efficient. This also allows us to see that even if we decided that excess profits (or in this case losses) were being made, say over the first 1500 ticks, we may conclude that they are not systematic in that after this time there are no excess profits (or losses).

It is worth emphasizing again here why we are using a dynamic Buy and Hold benchmark. In a market of (90,10) traders all agents will make large profits relative to a static benchmark, such as a risk free interest rate. In this sense the market is very inefficient as in this scenario there is no mean reversion of the price so the market can continuously go up. However, no trader in the market is making systematic excess profits relative to any other. This aspect is made manifest by using a dynamic Buy and Hold benchmark and hence the market is seen to be efficient. However, in distinction to the case of a market with (50,50) traders the (90,10) market does not correspond to a rational expectations equilibrium in that there is a strong, continuous excess demand. In both cases however traders may be acting perfectly rationally with respect to optimizing their own utilities.



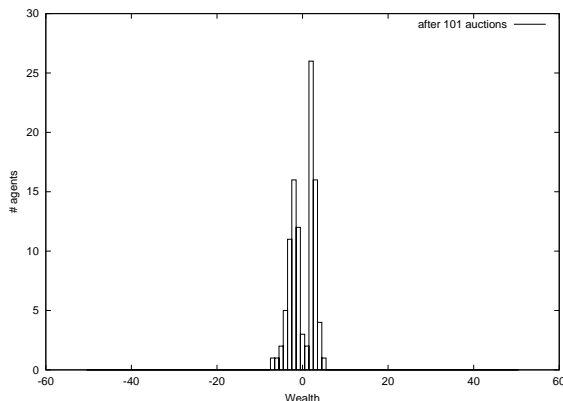


Figure 4: Histogram of excess profits for 50 (50, 50) and 50 (90, 10) traders after 101 ticks.

## 4.2 What Does an Inefficient Market Look Like?

Before discussing dynamical inefficiencies and, in particular, how inefficiencies begin and end, we consider the case where a time period exists such that the inefficiency persists across the entire time period. Specifically, we consider a market consisting of equal numbers of noise traders and informed traders with a (90, 10) bias.

In Figures 4 and 5 we see the histogram of excess profits for a group of 50 (50, 50) and a group of 50 (90, 10) traders. In Figure 4 we see the distribution after 101 ticks, and in Figure 5 after 3001 ticks. In the former we see how the distribution of excess profits begins to show a multi-modal structure. This is due to the fact that the (90, 10) traders are now making excess profits at the expense of the (50, 50) traders. The appearance of a multi-modal structure is symptomatic of a market inefficiency, the mean excess profit of the (90, 10) traders being positive while that of the (50, 50) traders is negative. This behaviour is fully confirmed in Figure 5, where we see that the distributions for the two types of traders are now fully decoupled.

In the context of the Inefficiency Matrix we show in Figure 6 the behaviour of  $I_{(100-x,x)(50,50)}$  for  $x = 5, 10, 15, 20, 25, 30, 35, 40$  and 45 for a market consisting of 50 traders of type  $(100 - x, x)$  and 50 (50, 50) liquidity traders. In this graph we can clearly see that the market is unambiguously inefficient, i.e.  $I_{(100-x,x)(50,50)} > 2$  for all markets that include informed traders, and that the inefficiency increases monotonically with time due to the monotonic increase in excess profits of the informed at the expense of the uninformed. Additionally, we see that the degree of inefficiency strongly depends on the degree of bias of the informed agents. The higher the bias the higher the excess demand and therefore the higher the average price increase between auctions. This in turn leads to higher excess profits for those traders that have a bias to buy. For larger biases the inefficiency increases approximately as  $t^{1/2}$ . This can simply be

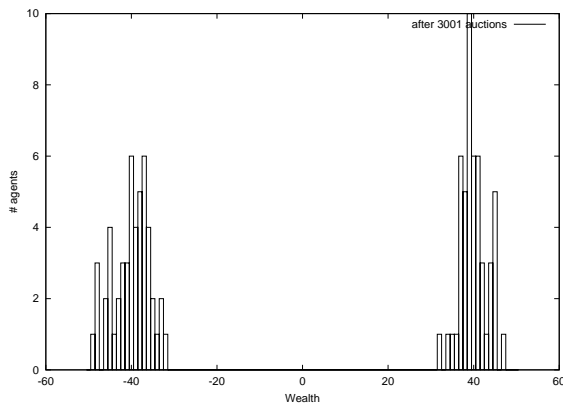


Figure 5: Histogram of excess profits for 50 (50, 50) and 50 (90, 10) traders after 3001 ticks

deduced from the fact that for a completely biased agent excess profit increases as  $t$  while the volatility in excess profits also increases as  $t$ .

Eventually the curves in Figure 6 begin to flatten out. This is a simple consequence of the existence of portfolio limits. As cash is used up exponentially by the informed as a function of time (due to the exponential increase in the price of the stock when both informed and uninformed agents are present), whereas the stock of the informed is used up only linearly (only one unit of stock can change hands every auction) the informed are the first ones to decouple from the market and for that reason the rate of increase of the inefficiency goes to zero and the inefficiency itself becomes constant. The stronger the bias of the informed the quicker they decouple.

#### 4.2.1 How Does Strategy Diversity Affect Market Efficiency?

Previously we introduced efficiency in the context of an homogeneous market while subsequently introducing the idea of inefficiency in the context of a market with only two types of strategy. In this sense one may think of the resultant market as being only minimally inhomogeneous (though, as we will see, this is erroneous) and naturally ask what happens in a more heterogeneous market. Hence, in this section we consider markets with more strategy diversity. Of course, there are different metrics for measuring diversity available to us. In the simplified one-parameter model under consideration, for  $N$  strategies, a detailed analysis of inefficiency naturally takes place in  $X_1 = [-1, 1]$  by considering the distribution of the  $N$  biases in this space, i.e. the distribution of points on this interval. Of course, simpler summary measures, such as just counting the number of strategies irrespective of their associated biases, can be useful. For instance, the average bias for a given set and the associated variance would be useful summary measures.

Intuitively, there are three basic degrees of freedom associated with the bias

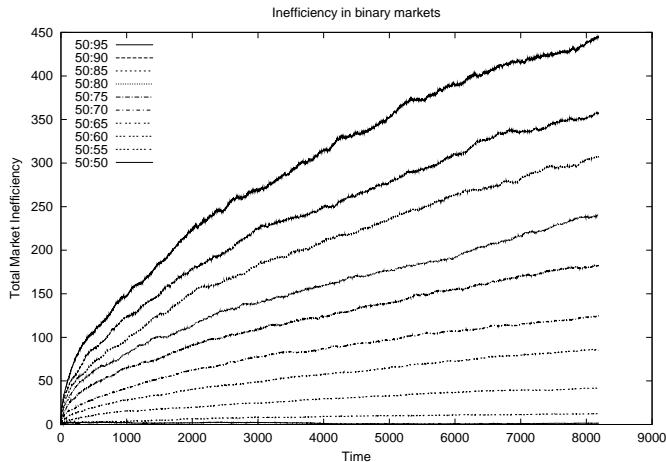


Figure 6: Graph of  $I_{ij}(t, 0)$  for 50  $j = (50, 50)$  and 50  $i = (100 - x, x)$  traders.

distributions: i) the proportion of traders associated with a given bias; ii) the number of different groups of traders; iii) the similarity between the strategies of different trader groups - measured in the present context by “distance” in bias between two agents or groups. It is thus of interest to ask how easy or difficult is it to infer inefficiency as a function of these components.

First, we examine inefficiency as a function of the proportion of traders. In Figures 7a and 7b we see the market inefficiency at a given fixed time as a function of the proportion of informed traders in a binary market consisting of 100 traders, where the curves represent averages over 10 experiments. The different curves correspond to different biases for the informed agents. The associated biases for the curves are 0 (i.e. (50, 50) traders) on the bottom curve to 49 (i.e. (99, 1) traders), on the top. Interestingly, we see that there always exists a maximum in the curve that corresponds to the proportion of informed to uninformed that yields maximum inefficiency for a given bias. We can see how the location of the maximum depends on the trader bias in Figure 7b, where the curve has the form  $Y = 59.7 + 20.17 \exp((55.0 - X)/6.07)$ .

There are two distinct effects at play in determining the maximum of the curve. One is that an inefficiency can arise only if an informed trader has an uninformed trader to profit from. This naively would lead one to believe that a market of equal numbers of informed and uninformed would be the most inefficient, as in this case for every informed there is an uninformed to exploit. However, one must also take into account that the inefficiency depends on the magnitude of the profits made by the informed. This in turn depends on how rapidly the asset price changes. The more informed there are, the more buy-sell disequilibrium there is, and the faster the price rises. So one effect favours equal proportions and the other an all informed market. The above graphs are a compromise between these two effects. In Figure 7b we see that the lower

the bias of the informed the larger the proportion of informed at maximum inefficiency.

We now consider how inefficiency depends on the number of and distance between trader groups, in the present circumstances measured by the difference in bias between the two groups. We consider a market of 50 liquidity traders and 50 informed traders of type  $(50+x, 50-x)$ , where  $x = 0, 1, 2, 3, 4$  and  $5$ . In other words, we are considering the case of relatively weak biases, which can be interpreted as meaning that trader expectations are similar. In Figure 8 we see a comparison of the inefficiency in these 6 markets as a function of time. Note that using our efficiency criterion of  $I_{ij} > 2$  the markets with  $x = 1, 2$  and  $3$  are indistinguishable from an efficient market over the timescale considered of 8000 ticks. For  $x = 4$  the market would not be considered inefficient until after about 4000 ticks and the  $x = 5$  market until after 2000 ticks. The important point to emphasize here is that all these markets are theoretically inefficient in the sense that they are composed of inhomogeneous trader groups one having a more advantageous strategy than the other. This advantage is an intrinsic element of the market. However, as we have remarked, observationally, inefficiency is an inference problem. Here we see that these markets are observationally indistinguishable from efficient markets over certain time scales due to the fact that the informed strategies are not sufficiently superior to the uninformed strategy to lead to any observable inefficiency over the relevant timescale.

In Figure 9 we see a graph of inefficiency as a function of liquidity (average number of operations per tick) and average distance between strategies. The graph is an average over the results of ten experiments, where each experiment consisted of 100 traders divided into ten groups of ten, where their biases were selected at random from the interval  $d \in [0.5, 1]$ . In this case, liquidity is a proxy for the average bias, a higher average bias being associated with a higher probability to buy, and therefore less liquidity due to the low probability to find sellers. Similarly, the average distance is a measure of the variance and hence acts as a proxy for the number of different strategies as the strategies are chosen probabilistically, i.e. the more strategies we have the smaller the average distance between them is likely to be. Clearly, we can see that inefficiency decreases as the average distance between strategies decreases. In other words the more distinguishable the strategies are the more inefficient is the market. We also see that inefficiency decreases as a function of liquidity - the more liquidity the more efficient is the market for a given informational "imbalance".

## 4.3 Inefficiency and Learning: A Neo-austrian Interpretation

### 4.3.1 The Neo-Austrian Paradigm

At the beginning of this paper we discussed the Neo-Austrian interpretation of financial markets which implies that financial markets are continuously evolving from one inefficiency to another, never attaining the perfect, efficient equilibrium, yet strongly attracted towards it. Creative investors track and ex-

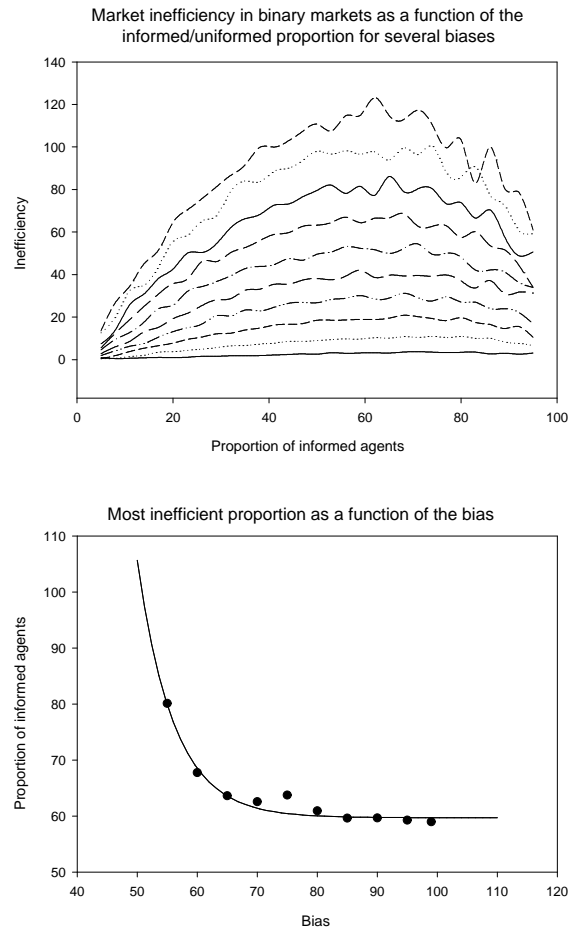


Figure 7: Graph of  $(t, 0)$  as a function of the proportion of informed to uninformed (Figure 7a) above) and of the informed bias (Figure 7b) below).

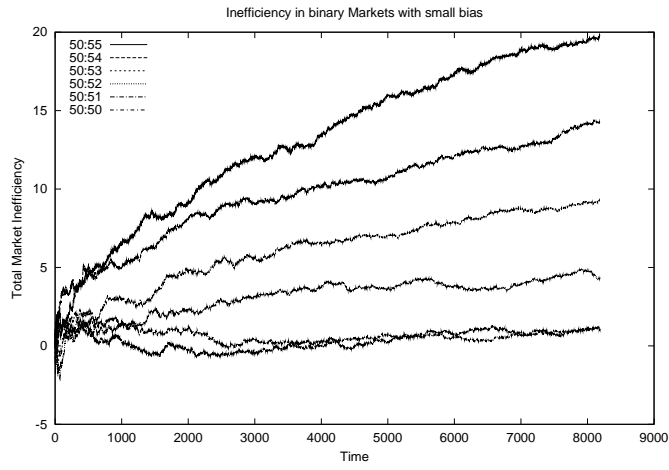


Figure 8: Graph of the total inefficiency for six binary markets with small bias as a function of time.

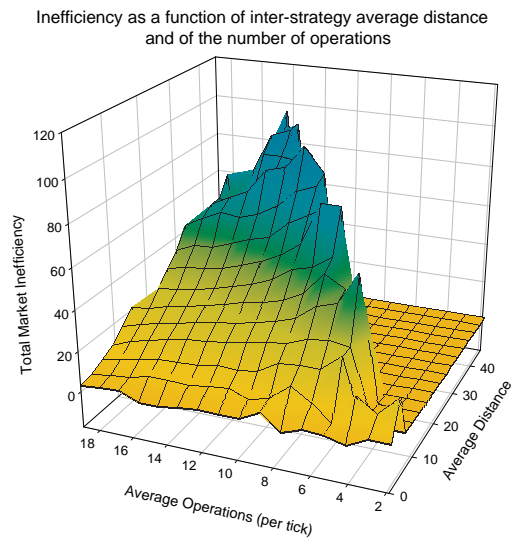


Figure 9: Graph of inefficiency as a function of liquidity and average distance between strategies

exploit profit opportunities generated by continuous shocks in a never-ending cycle. These alert participants in the market process, who are defined as “entrepreneurs” by the Neo-Austrians and as “informed traders” and “copycats” in our AFM, try to get a - necessarily incomplete - picture of the nature of the disequilibrium in the marketplace, because disequilibrium generates profit opportunities. The actions of these entrepreneurs produce the very signals that are needed to reduce disequilibrium. What renders the market process a systematic process of coordination is the circumstance that each gap in market coordination expresses itself as a pure profit opportunity. The profit-grasping actions of successful entrepreneurs dispel the ignorance which was responsible for the profit opportunities, and thus generate a tendency towards coordination among market decisions. However, due to continuous change in the constraints of the underlying economy, equilibrium is never reached.

In the following section we will study learning processes in an inefficient financial market from a Neo-Austrian point of view. At this point, we do not yet include changes in the constraints so that we can analyze learning processes that are uninterrupted. However, in section 4.4 we will study learning processes that are interrupted by exogenous information shocks.

### 4.3.2 Distribution of Excess Profits in the Presence of Learning

If we introduce learning via copycat agents then we may investigate how learning affects the existence and evolution of inefficiencies. In Figures 10-12 we see the evolution of the excess profits associated with a group of 35 (50, 50), 60 (90, 10) agents and 5 copycat agents. The initial strategy of the 5 copycats is (50, 50), so they begin as liquidity traders. They observe the market for 700 ticks then copy a strategy via roulette wheel selection. After 101 ticks, in Figure 10, we already see a decoupling between the (90, 10) agents and the rest. At this moment the copycats have not updated their expectations and hence make excess losses just the same as the (50, 50) traders. In Figure 11 we see the situation after 801 ticks, seeing the 5 copycat traders begin to decouple from the group of liquidity traders. The alert copycats have detected the existence of traders with systematic excess profits and are now copying them, whereupon they begin to reduce their excess losses at the expense of the liquidity traders. In Figure 12, after 1101 ticks we see that the copycats are about to start making excess profits.

The natural Neo-Austrian interpretation here is that the informed are entrepreneurs that are exploiting a profit making opportunity. The copycats are other alert entrepreneurs that respond to the signal of the original informed traders. This signal identification is prone to error due to the uncertainty associated with statistically inferring what is the optimal strategy to use, i.e. entrepreneurs can make mistakes. The alert copycat entrepreneurs through their profit seeking behaviour increase market coordination by increasing the number of informed traders. If the market consisted only of informed and copycats (who were initially uninformed) then, depending on the completeness of the learning process, in principle, complete market coordination could take place, though

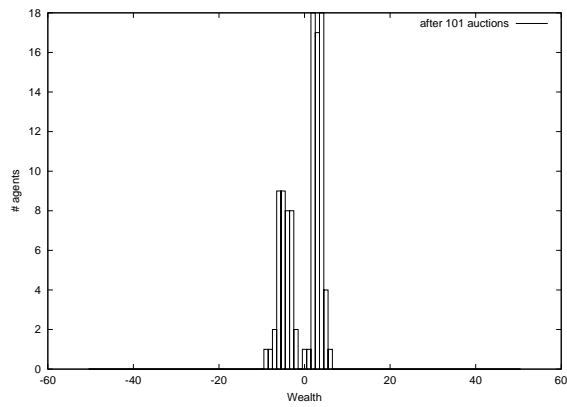


Figure 10: Histogram of excess profits for 35 (50, 50), 60 (90, 10) agents and 5 copycat agents after 101 ticks

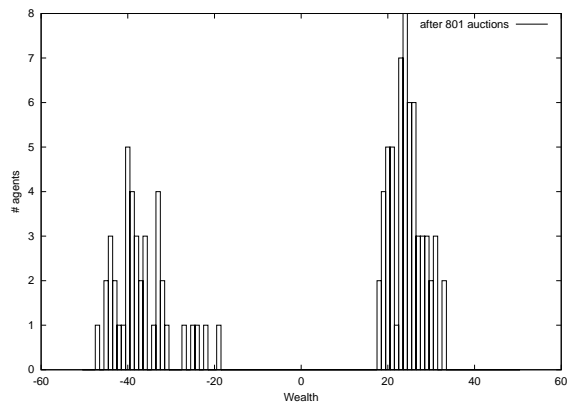


Figure 11: Histogram of excess profits for 35 (50, 50), 60 (90, 10) agents and 5 copycat agents after 801 ticks



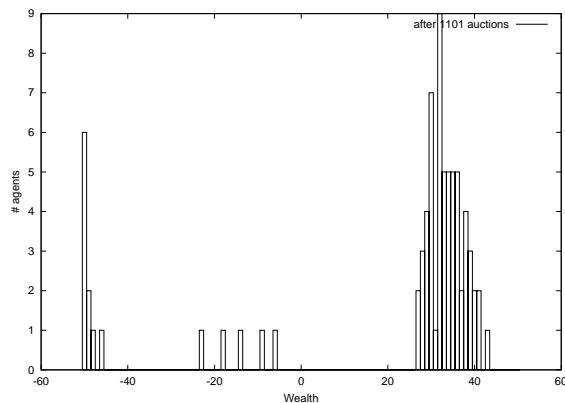


Figure 12: Histogram of excess profits for 35 (50, 50), 60 (90, 10) agents and 5 copycat agents after 1101 ticks

the resulting “equilibrium” does not necessarily have to be associated with an equilibrium price, i.e. the market does not necessarily have to clear.

### 4.3.3 Inefficiency in the Presence of Learning

We now consider in Figures 13 and 14 the Inefficiency Matrix for informed agents with bias  $d$ , uninformed agents and copycats. There are 3 matrix elements and therefore 3 curves. In these experiments learning events take place every 1600 ticks. We show first in Figure 13 the results for  $d = 0.9$ . From this graph we can see that the copycats after the first learning event also make excess profits at the expense of the uninformed. We may ask why they do not earn as much as the informed given that they learn so quickly? The answer to this question is that they do, but the reason why the relative inefficiency between informed and copycats decreases after learning is that the variance associated with the excess profits of the copycats is much higher.

It is interesting to note the reason for this: learning is imperfect, hence after the learning event some copycats have determined correctly the optimal strategy while others have not and remain uninformed. This result is consistent with the Neo-Austrian insight of imperfect knowledge where entrepreneurs (“copycats”) try to get a “necessarily incomplete” picture of the nature of the disequilibrium in the marketplace.

Thus, within one group of traders we have a situation, such as seen in Figures ??, where the distribution is multimodal. The variance associated with such a distribution is obviously very large compared to a unimodal distribution and will keep increasing until learning is sufficiently complete. Intuitively, the decrease in inefficiency is due to the fact that it is now harder to infer if or not the copycats have a strategy which leads to excess profits, as at any given time some have learned the optimal strategy and others not.

In Figure 14 we see the same 3 matrix elements as in the previous figure but

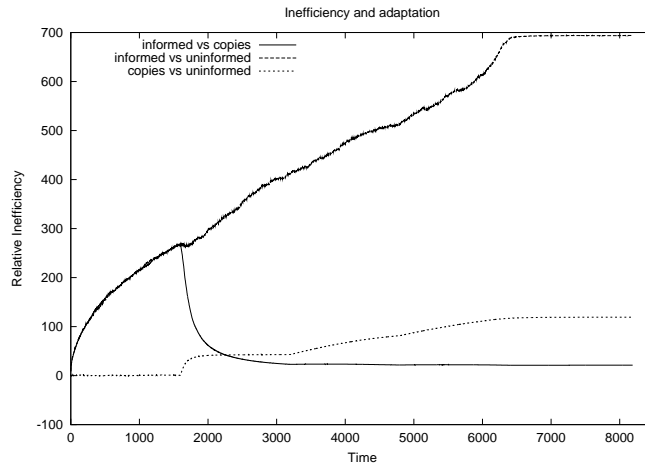


Figure 13: Relative inefficiency of biased agents of type (90,10), uninformed agents and copycats

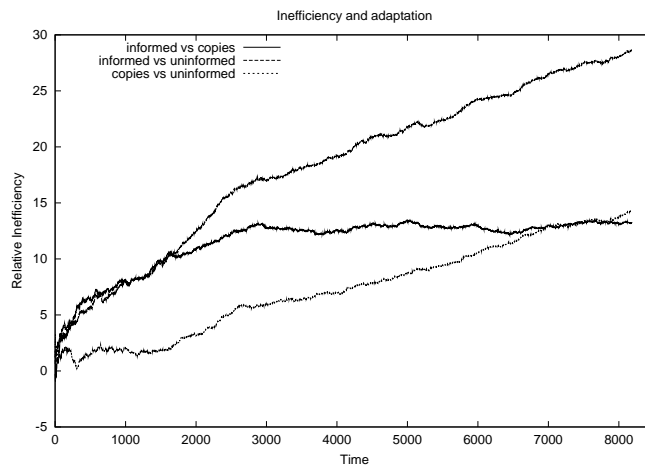


Figure 14: Relative inefficiency of informed agents of type (55,45), uninformed and copycats

	low-low	low-medium	low-high
uninformed	150	70	30
informed	30	30	30
copycats	20	100	140

Table 1: Initial numbers of different trader types for different experiments.

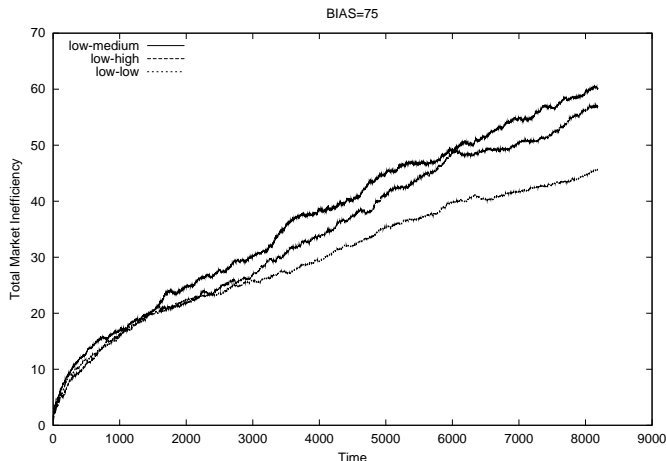


Figure 15: Total market inefficiency in experiments where the proportion of informed agents changes due to copycat learning. Bias of informed agents is 75.

now for a much smaller bias,  $d = 0.55$ . Notice now that the inefficiency of the informed relative to the copycats is less than that between the informed and uninformed after the first learning event, showing that some of the copycats have learned the optimal distribution. However, unlike the case with bias  $d = 0.9$  the curve does not drop suddenly. The reason for this is that: as the bias is so weak, even though some copycats learn the optimal strategy they do not have that much more excess profit. Hence, the distribution associated with the copycats, although multimodal, remains much more compact than in the case of a large bias. Hence, the variance is much smaller.

We may also investigate how the addition of copycats leads to more or less efficiency for the entire market. We have seen in Figure 7, in the case of different proportions of informed and uninformed agents, that increasing the number of informed agents can lead to a more *or* less efficient market depending on the net number of informed after the increase. With this in mind we consider three different regimes involving copycat learning in markets with 200 agents. The proportions of informed, uninformed and copycat traders in each regime are shown in Table 1. The proportions have been chosen so that for the “low-low” regime, even in the presence of perfect learning the proportion of informed

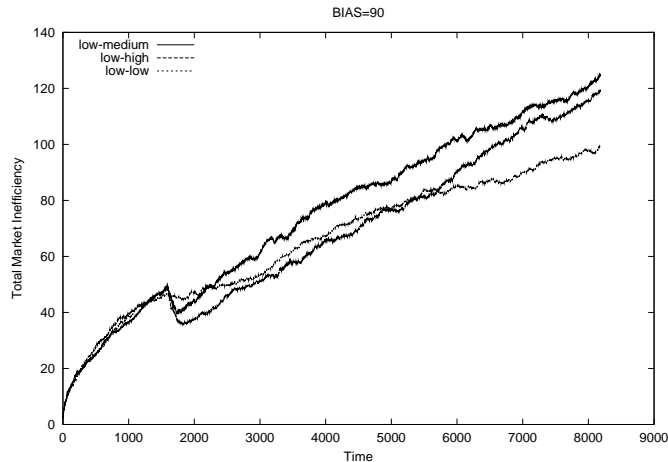


Figure 16: Total market inefficiency in experiments where the proportion of informed agents changes due to copycat learning. Bias of informed agents is 90.

would be to the left of the maximum in Figure 7a. Similarly, once again for perfect learning, the “low-high” regime is such that the proportion of informed after learning is to the right of the maximum in Figure 7a. Finally, the regime “low-medium” is such that after perfect learning the proportion of informed corresponds to the regime of maximum inefficiency in Figure 7a. In Figures 15 and 16 we see the total market inefficiency for these markets. For bias  $d = 0.9$  we see the characteristic decrease in inefficiency after the first learning event seen in Figure 13 due to the high variance of the associated multimodal distribution. This decrease is much less notable for the “low-low” market as the rate at which the two peaks of the distribution separate is much smaller in this case. Note that, as expected, asymptotically at least, the “low-medium” market is the most inefficient, followed by the “low-high” with the “low-low” market being the most efficient.

#### 4.3.4 Inference and Copycats

In this section we will further explore the difficulties that copycats face in trying to infer from the market which are the most useful strategies to copy. However, before presenting the experiments with their results, we will discuss some aspects of the copycat’s adaptation process. As it has been mentioned previously, in order to adapt their strategies to the market’s conditions, copycats must “play” a roulette formed from, say, the profits of each strategy in the market (Eq. 3). It is useful to recognize the stochastic effects of this game, in particular those produced by the composition of the market. We can illustrate this by thinking of a market where the copycats copy via roulette wheel selection the most popular

strategy. In this case the probability to copy a strategy  $i$  is

$$P_i(t) = s_i(t) / \sum_{j=1}^m s_j(t); \quad (10)$$

where  $s_i$  is the number of agents with strategy  $i$ . One question we can answer is the expected number of copycats that will adopt strategy  $i$  at time  $t$ . Let  $C$  be the number of copycat agents,  $I$  the initial number of agents with strategy  $i$  and  $N$  the total number of agents (i.e.  $N = \sum_{j=1}^m s_j(t)$ ). When  $t = 1$  (the first adaptation) it follows that

$$X_i(1) = CP_i(1) = IC/N; \quad (11)$$

where  $X_i(t)$  is the average number of copycats that adopt strategy  $i$  at time  $t$ . Now, when  $t = 2$ , the number of agents with strategy  $i$  is  $I + X_i(1)$ , and in general, after  $K$  learning events, one has

$$X_i(K) = CP_i(K-1) = IC/N + IC^2/N^2 + \dots + IC^K/N^K = I \sum_{j=1}^K C^j/N^j \quad (12)$$

Therefore

$$X_i(K)_{K \rightarrow \infty} = IC/(N - C). \quad (13)$$

which is the expected maximum number of copycats that will copy the most popular strategy. In Figures 17 and 18 we show the result of the first 20 adaptations in markets with different values of  $I$ ,  $C$  and  $N$ , where copycats adapt every tick and the most popular strategy is associated with informed (51, 49) agents. The graphs are a result of averaging over 10 different runs.

In Figures 17 and 18 we see how the number of correct copycats asymptotes to a value close to that given by (13). There is a slight difference in that the graphs are for copycats that copy the most profitable strategy. However, for weak bias we see that (13) gives a good approximation. More generally, it gives a lower bound for the number of correct copycats.

Returning to the problem of learning: The objective of a copycat is to acquire the optimal strategy (i.e. the strategy that maximizes profits constrained to existing market conditions); conversely, the objective of the biased traders is to create an excess demand. This excess demand thus drives the price via the price evolution equation (1) along with the profits of informed agents. Additionally, both the excess demand and the profits of the informed traders depend on the composition of the entire population as well as on the distribution of biases. In this scenario, copycats try to copy informed traders to find the optimal strategy. This activates the learning process. However, complete learning is by no means guaranteed in the sense that they do not necessarily identify the best strategy. The quality of the learning depends on the signal to noise ratio (i.e. the size of the different regions in the roulette), which in its turn depends on the agent biases and the market composition. Note that learning might be incomplete even in the case where there is only one other strategy to learn.

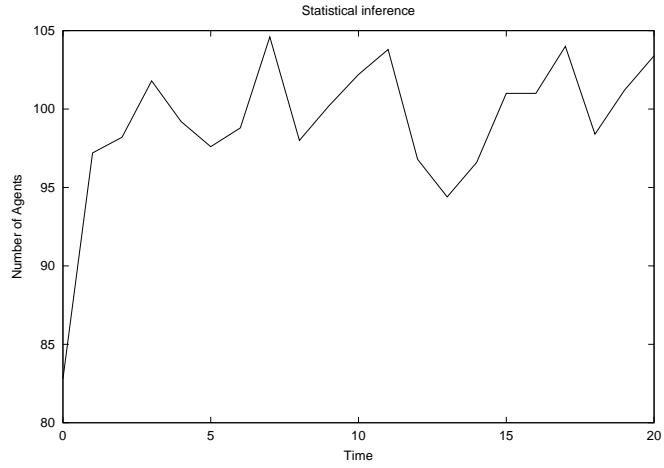


Figure 17: Number of copycats that learn strategy  $i$  in simulations with  $I = 300$ ,  $C = 150$  and  $N = 600$ .

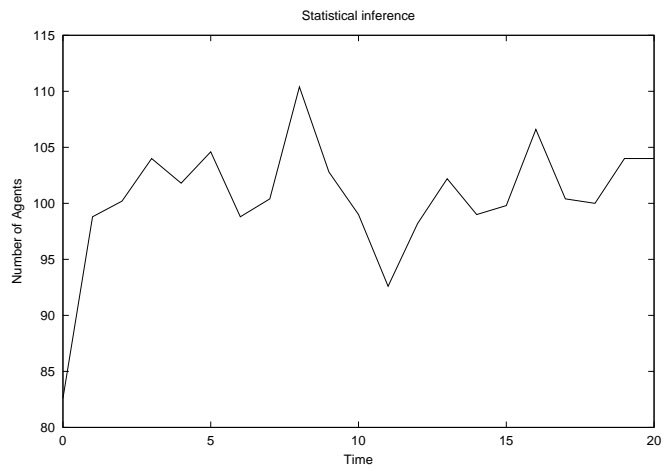


Figure 18: Number of copycats that learn strategy  $i$  in simulations with  $I = 200$ ,  $C = 200$  and  $N = 600$ .

Informed Bias=75					Informed bias=90						
exp	1	2	3	4	5	exp	1	2	3	4	5
low-low	50	49	62	55	55	low-low	50	50	55	80	75
low-medium	40	70	82	84	85	low-medium	59	83	83	90	93
low-high	39	61	75	76	75	low-high	44	74	78	77	78

Table 2: Percentage of copycats that have learned the optimal strategy after each learning event (“exp”).

Informed Bias=75					
exp	1	2	3	4	5
low-low	40/160	40/160	42/158	41/159	41/159
low-medium	70/130	100/100	111/89	112/88	116/84
low-high	84/116	116/84	135/65	137/63	135/65
Informed Bias=90					
exp	1	2	3	4	5
low-low	40/160	40/160	41/159	46/154	45/155
low-medium	89/111	113/87	114/88	121/79	123/77
low-high	92/108	134/66	140/60	137/63	138/62

Table 3: Ratio of informed to uninformed after each learning event (“exp”).

This can be amply illustrated by returning to the experiments associated with Table 1. Table 2 gives results for the quality of learning by showing after each learning event (denoted by “exp”) the percentage of copycats that have correctly learned the optimal (i.e. informed) strategy. Finally, Table 3 shows after each learning event the relative numbers of informed to uninformed traders.

Table 2 clearly show that, for a given regime type, learning is more efficient the stronger the informed bias. For example, for “low-low” markets after 5 learning events the percentage of informed copycats is 75% for bias  $d = 0.9$  and 55% for bias  $d = 0.75$ . This is, of course, intuitively interpretable in that in the former there is a stronger information signal than in the latter. Interpreting the results from the point of view of the different regimes, we see that for “low-low” markets, for a bias of 0.75 the number of copycats that learn the informed strategy is barely more than it would be had they picked randomly between the two strategies. This plainly shows the inference problems the copycats face - and this is in the case of a large bias and where 15% of the market started off informed! Interestingly, the highest learning percentages are associated with the “low-medium” regime as in this case the large number of copycats is such that even if the initial learning is random, 50% of the copycats will learn the optimal strategy and they in turn will provide a good information signal for the other copycats to follow in subsequent learning events. Notice that both the “low-medium” and “low-high” regimes lead to very large increases in the percentage

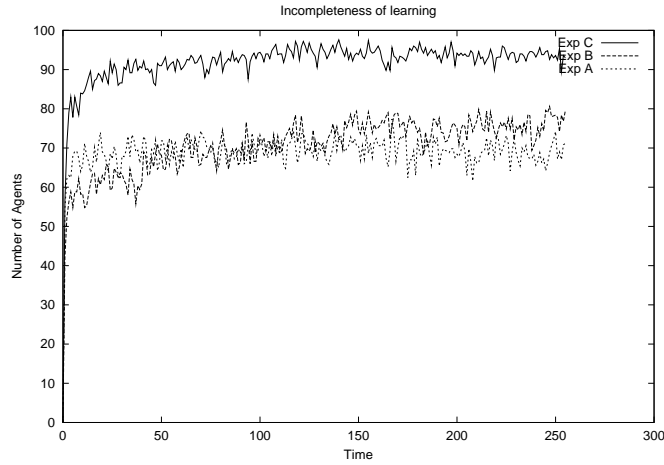


Figure 19: Incompleteness of learning: number of copycats that learn the optimal strategy in experiments with different market compositions and daily learning.

of informed copycats after the second learning event due to the aforementioned effect of copycats who have chosen the optimal informed strategy, by chance or by learning, providing a further signal for other copycats to detect. However, in the case of “low-high”, the asymptotic percentage is smaller than for “low-medium”, due to the fact that the smaller number of uninformed means that the excess profits of the informed are less, as there is less opportunity to exploit the uninformed.

As a further illustration of the incompleteness of learning consider Figures 19 and 20, where we show the number of copycats that learn the correct strategy in three different experiments. In the first case (Experiment A), the market is composed of 20 agents of each of the following strategies: (50, 50), (60, 40), (70, 30), (80, 20), (90, 10), 100 (99, 01) agents, and 100 copycats. In Experiment B the market is formed by 100 uninformed agents (i.e. with a (50, 50) strategy), 100 (60, 40) agents and 100 copycats. Finally, Experiment C was composed of 100 (50, 50) agents, 100 (99, 01) agents and 100 copycats. The roulette at time  $t$  was built using  $E_i(t, 0)$ , that is, the profits calculated since the beginning of the experiment. We show two sets of results - one where the copycats update their learning every day, and another where they update every 50 days.

In Experiment A, the optimal strategy is (99, 01). However, the presence of other strategies with lesser yields confuses the copycats in such a way that only about 70% of them present successful learning, i.e. that identify the optimal strategy, (the average due purely to the composition of the market is 50% in all cases; this can be derived through simple probabilistic arguments with the use of the roulette). In Experiment B we can observe that the interaction of the



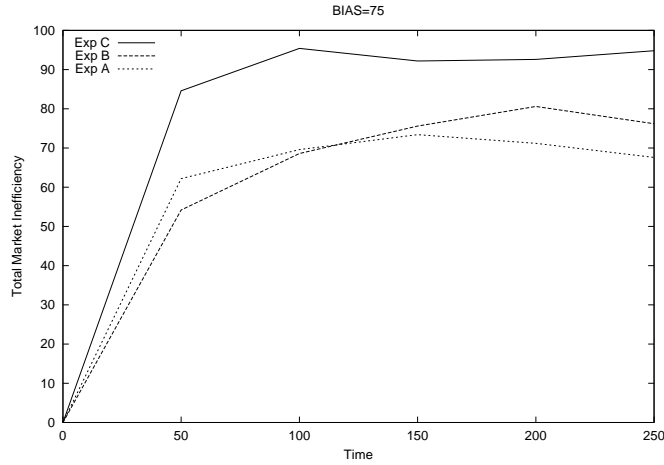


Figure 20: Incompleteness of learning: number of copycats that learn the optimal strategy in experiments with different market compositions and learning every 50 days.

(50, 50) and (60, 40) strategies generates only a relatively small signal, hence explaining why the number of copycats that learn the best strategy is only slightly bigger than the average that would arise purely due to the market’s composition. Experiment C shows the imperfection of the learning process, even in a market with a very large difference in biases, i.e. that due to the stochastic nature of the roulette wheel complete learning cannot take place. We see in general then that the efficiency of learning depends on the market biases and the diversity of strategies in the market, as well as the stochastic nature of the roulette wheel selection.

#### 4.4 Efficiency and Learning in the Presence of Exogeneous Information shocks

An important element in both neoclassical and Neo-Austrian thinking is the arrival of new information. In the neoclassical paradigm this new information is random and alters trader expectations accordingly. Here we model the arrival of new information via an information “shock” wherein the perceptions and expectations of some, or all, of the market participants are changed. Specifically, we consider markets with uninformed traders, two types of informed trader with strategies (90, 10) or (55, 45) and copycats. In the following experiments the traders change their perception of the market after an information shock. The uninformed remain uninformed. However, the informed, due to the shock, change their perceptions by, after a shock, choosing randomly with probability 1/2 one of the two informed strategies. Despite the shocks the optimal strat-

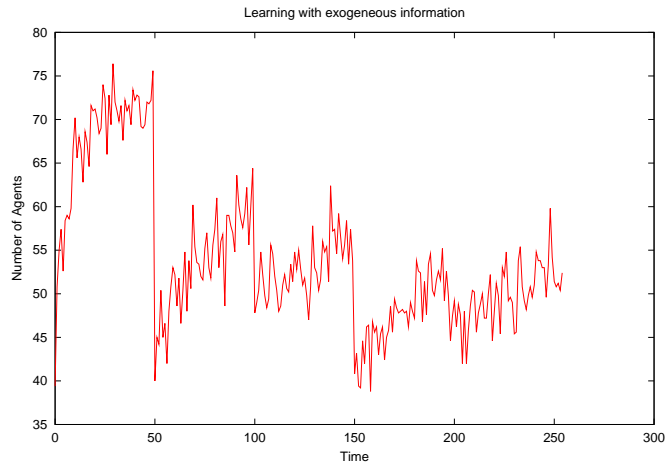


Figure 21: Learning in markets with exogeneous information shocks: copycats with long-term memory.

egy throughout is to choose the informed strategy with maximum bias. What changes are the perceptions of the informed as to what trading bias they should implement. However, given that it is probability  $1/2$  for the informed to change bias the market statistically remains homogeneous in that, on average, after any given shock, there are the same number of informed with bias  $(90, 10)$  as with  $(55, 45)$ .

Taking this into account, after a shock, a copycat must re-learn the correct strategy under the new market conditions. An interesting problem is determining how much endogeneous information a copycat needs to learn the best strategy, given the dynamic conditions of the market. In the examples presented earlier on learning, each copycat used all the history of the traders' profits to make a decision, i.e. each agent had "long-term" memory. We may pose the question: Will the same information be as useful in a system with changing perceptions? In Figures 21 and 22 we show the results of two experiments that shed some light on this question. In Experiment D the copycats try to copy a strategy using only the information generated by the market after each shock. Thus, they have only "short-term" memory as they do not keep in their memory any information prior to the shock. In contrast, Experiment E depicts the case in which copycats have long-term memory, preserving the entire information of the market's history without distinguishing data obtained before and after shocks. In Figures 21 and 22 are the results associated with 100 uninformed, 100  $(90, 10)$ , 100  $(55, 45)$  and 100 copycats.

We can see in this figure that the learning process where traders only use as their learning information set market information from the last shock until the present moment is more efficient, as after each shock the number of copycats

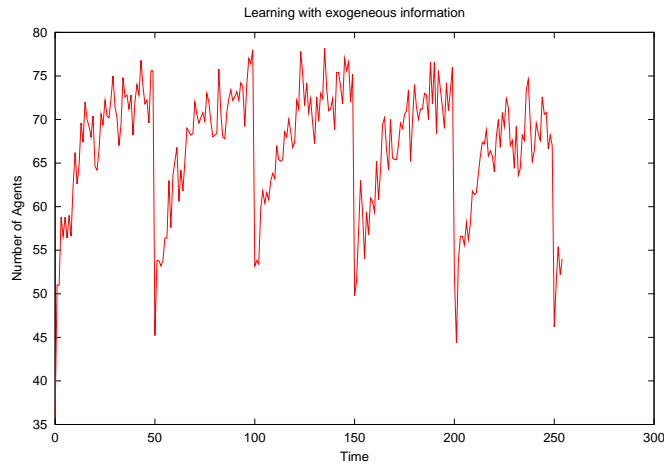


Figure 22: Learning in markets with exogeneous information shocks: copycats with short-term memory.

that have learned correctly increases to the same value. In contrast for copycats with long term memory the learning process deteriorates due to the fact that after a shock the copycats may keep copying those informed who were most successful before the shock, i.e. those with bias  $(90, 10)$ , but who after the shock are sub-optimal, with bias  $(55, 45)$ .

## 5 A Neo-Austrian Analysis of the Results

Although the results of our simulations stand on their own, independent of interpretational frameworks, as our chief goal was to examine the Neo-Austrian paradigm using an AFM, and as all the relevant elements are now in place, it behooves us to re-examine what we have found in that light.

In the simulations we have bit by bit built up the key ingredients of Neo-Austrian theory. Of course, they are also key elements in real markets. In section 4.4, we see, in a simplified setting, all the key elements of the Neo-Austrian point of view in play. The market contains entrepreneurs (informed traders) who, in the absence of uncertainties associated with exogeneous information, are making excess returns by exploiting their uninformed counterparts. The resulting market is inefficient. The market also contains other entrepreneurs (copycats), alert to the existence of any possible profit making opportunity (inefficiency). The key problem for these entrepreneurs is to identify the right opportunity by being able to infer correctly the right strategy to copy, i.e. to avoid mistakes and identify the correct “signal”. This in turn depends on the “signal to noise ratio” characteristic of the inefficiency which, in its turn, depends, in this model, on the trader biases and the composition of the market, as well as the adaptation

frequency of the copycats, i.e. how frequently they revise their expectations. Copycat learning leads to more coordination between copycat entrepreneurs and informed entrepreneurs. However, as we showed previously, in distinction to a strict Neo-Austrian interpretation, this coordination may lead to a more inefficient market, depending on the relative proportions of uninformed, informed and copycats. In this sense our simulations, although generally in accord with the Neo-Austrian point of view, offers a richer interpretation of the learning process experienced by entrepreneurs. If all market participants, besides the informed, are potential entrepreneurs then learning can lead inexorably towards “equilibrium” - meaning an efficient (in our simple model - homogeneous, in that all traders have the same (informed) expectations) market but not necessarily with an equilibrium price. How close this state can be approached depends on the quality and completeness of the learning process. Generically, complete learning will not be achieved.

So, alert entrepreneurs (informed) can exploit, or even create, inefficiencies (“disequilibrium”), which creates signals (profit making opportunities) that other entrepreneurs (copycats) identify and try to exploit, potentially leading to a situation of more market coordination and less disequilibrium. Arrival of new (unanticipated) information leads to a change in expectations and this change has a degree of uncertainty associated with it. The relative success of different informed entrepreneurs can change due to changed expectations even though the optimal strategy (the strategy with maximum bias available) does not. In this new environment other alert (copycat) entrepreneurs are faced with the task of identifying anew the optimal strategy to copy. Now, the potential to make mistakes depends crucially on the information set that the copycats use for their decision making. This is an important new source of potential error.

In between information shocks the market is inefficient, as there is always a set of informed traders exploiting the uninformed. However, averaging over shocks the market becomes more and more efficient, as traders that were making excess profits during one period can be making excess losses in another. Copycats who base their decisions on time periods that include shocks face the prospect of misidentifying the optimal strategy by copying the strategy of an informed trader who was optimal before the shock but suboptimal afterwards. On the contrary, copycats who base their decisions on short timescales run the risk of not having sufficient data to statistically identify with sufficient confidence the optimal strategy. The optimal dataset for a copycat to use is that from the last information shock to the present time. If shocks arrive too frequently however, there is not enough time for a copycat entrepreneur to gather enough information to reliably infer what is the optimal strategy in that period or, even if a correct identification is made, there is not enough time to exploit the information. Thus, tendencies towards efficiency can be quite complex, depending on many factors, even in our simple model.

## 6 Conclusions

The main goal of this paper was to study Neo-Austrian ideas about market efficiency and learning, usually expressed in qualitative language, in the formal setting of an agent-based AFM market - the NNCP. Unlike a real market, the luxury of an AFM is that we can *create* an efficient or inefficient market and then vary the parameters of the market in order to study when, and under what conditions, it is possible to infer efficiency from observing the market.

To avoid the joint-hypothesis problem we defined a purely empirical quantitative measure of efficiency, defining an inefficient market in terms of whether or not there exist traders making systematic, excess profits. To distinguish between trading gains and market gains excess was defined relative to a dynamic buy and hold benchmark. As excess profit is a stochastic variable it is most naturally measured in units of the standard deviation of the excess profits. In this way one can not only distinguish between traders profiting from an active trading strategy as opposed to those profiting purely from market gains, but one can also distinguish between those traders who have a “lucky” trading strategy versus an “intelligent one. We introduced the concept of an Inefficiency Matrix which summarizes the relative inefficiencies between the different trading strategies in the market.

Using the Inefficiency Matrix as the principle measure, we performed a series of simulations in the context of a transparent model where the traders used trading strategies taken from a one-parameter family associated with a trading “bias”, zero bias corresponding to noise or liquidity traders. Learning was introduced using the concept of a copycat trader who observes the market, statistically estimates which is the most successful strategy and then copies it. It is important to emphasize that all these elements have been used in previous studies, in Gordillo and Stephens (2001a and 2001b) and (2003), without any reference to the Neo-Austrian paradigm. In other words, our model was not designed with the Neo-Austrian viewpoint in mind. Rather, our motivation was to use a previously designed model to see to what extent its results were most naturally interpreted in the Neo-Austrian as opposed to the neoclassical framework.

The results of this paper are in general consistent with the Neo-Austrian interpretation of markets as opposed to the neoclassical rational expectations point of view that considers financial markets to be in continuous equilibrium with informationally efficient prices. The results, in fact, lend substantial insight into and enrich many of the key elements of Neo-Austrian theory which, as mentioned, is a predominantly qualitative theory. For instance, we saw that the existence of alert entrepreneurs (copycats) could even lead to an increase in inefficiency rather than a decrease, depending on the proportion of informed to uninformed agents in the market.

In the Neo-Austrian interpretation of financial markets, financial markets are continuously evolving from one inefficiency to another, never attaining the perfect, efficient equilibrium, yet strongly attracted towards it. Creative investors track and exploit profit opportunities generated by continuous shocks

in a never-ending cycle. The result would be a stable process with pronounced regularities. According to Neo-Austrian theory, a competitive market provides a systematic set of forces, put in motion by entrepreneurial alertness (i.e. eagerness to make money), which tend to reduce the extent of ignorance among market participants. Interestingly, and contrary to other alternative theories explaining inefficient financial markets, in the Neo-Austrian view the failure of markets to reach the informationally efficient equilibrium ought not to be attributed to costs of any nature (adjustment costs, information costs, trading costs, etc). This description fits very well the results of the simulations of section 4.4, where “creative investors” (informed traders and copycats) track and exploit profit opportunities generated by exogeneous information shocks.

With their emphasis on imperfect knowledge the Neo-Austrians put themselves at the heart of the famous debate on risk and uncertainty (see, e.g., Knight (1921)). Neoclassical financial economists believe that uncertainty can be reduced to “objective” risk, depending on knowledge of the objective probability distribution implied by the true model of the economy that economic agents know or are capable of learning. However, Neo-Austrians tend to emphasize that economic agents have to cope with imperfect knowledge and fundamental uncertainty. Just as post-keynesian economists, they claim that neoclassical theory fails to specify how agents will be able to overcome fully this uncertainty, i.e. that it can be reduced to the “objective” probability distribution implying rational expectations and efficient markets. Contrary to post-keynesians, however, Neo-Austrians claim that alert market participants have powerful incentives to learn about the true nature of uncertainty and related disequilibrium, because disequilibrium generates profit opportunities. Thus, the market process is viewed as a stabilizing process with a powerful tendency towards equilibrium and efficiency. With their trust in the market process Neo-Austrian economists are intellectually close to their neoclassical colleagues, although they arrive at this result from a rather different perspective on the underlying economic reality. We believe that our results on learning and inference for copycat entrepreneurs perfectly illustrate this point of view.

Due to their flexibility and adjustable complexity, we believe that AFMs are an ideal vehicle for addressing some of the deepest and most difficult questions about efficiency and rational expectations. Further, by using a purely empirical measure of inefficiency, as we have done here, complications due to the joint-hypothesis problem can be avoided. We believe that combining the two gives a powerful framework within which other fruitful studies may be carried out.

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