Seize the Moments: Approximating American Option Prices in the GARCH Framework

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Abstract

This paper proposes an efficient approach to compute the prices of American style options in the GARCH framework. Rubinstein's (1998) Edgeworth tree idea is combined with the analytical formulas for moments of the cumulative return under GARCH developed in Duan *et al.* (1999, 2002) to yield a simple recombining binomial tree for option valuation in the GARCH context. Since the resulting tree is univariate, the proposed approach represents a convenient approximation of the bivariate GARCH system. Numerical analyses are used to demonstrate the speed and accuracy of the proposed approximation.

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1 Introduction

Ever since its introduction by Engle (1982), ARCH (or the generalized version, GARCH) processes have been found to describe well time-varying volatilities of financial asset returns. In Duan (1995) and Kallsen and Taqqu (1998), theories have been developed for pricing derivative contracts using this class of models. Numerical methods have also been developed to enable the valuation of European and American style option prices. Heston and Nandi (2000) and Duan, Gauthier and Simonato (1999) have developed (quasi-)analytical approaches for computing European style option prices. For American options, Ritchken and Trevor (1999) and Duan and Simonato (2001) have come up with a modified lattice approach and a Markov chain method, respectively. The existing methods for pricing American options under GARCH are bivariate in nature. When compared to the binomial lattice method commonly used to price options in the one-dimensional diffusion framework, they are more intensive in both computing time and memory requirements. In many situations, computing option values quickly is of paramount importance. As such, one may be willing to compromise on the level of accuracy so as to gain computing speed and/or reduce memory needs. This paper is about designing a one-dimensional lattice to approximate American option prices under GARCH so as to take advantage of the computing speed and low memory requirements of such a lattice.

Rubinstein (1998) developed a one-dimensional recombining binomial tree to price European and American options under a general distribution function. His technique is based on the Edgeworth expansion to obtain discretized risk-neutral probabilities for a distribution with known first four moments. Extending the Edgeworth expansion idea of Jarrow and Rudd (1982), Rubinstein's Edgeworth tree can go beyond European options. For European options, the method's performance is solely determined by the quality of the Edgeworth expansion. There is an added complexity associated with American options, however. If a one-dimensional stochastic variable (i.e., asset price) is not sufficient to describe the stochastic evolution of the asset price system, there will be some loss of information by restricting the construction to the one-variable Edgeworth tree. The GARCH model is one such example. The asset price dynamic under GARCH is governed by the price-volatility pair, which can be viewed as a bivariate Markovian system. The lattice deduced from the terminal distribution of the asset price simply cannot replicate the GARCH system in its entirety. In other words, there will be some loss of accuracy if one proceeds to apply Rubinstein's Edgeworth tree idea to the American option valuation under GARCH. Despite this theoretical limitation, the Edgeworth tree method may still provide a reasonably accurate approximation for many practical applications. We find in this paper that this is indeed the case.

Rubinstein's Edgeworth tree technique can be operational as long as the first four moments of the cumulative return under the risk-neutral measure are known even if the true risk-neutral distribution is unknown. In the GARCH framework, the first four moments of the cumulative return under the risk-neutral measure can be obtained analytically by using the results in Duan *et al.* (1999, 2002). More specifically, analytical formulas to compute the first four moments of the cumulative return have been developed for the LGARCH, NGARCH, GJR-GARCH and EGARCH processes. In this paper, we combine these results with Rubinstein's Edgeworth tree to obtain an efficient method to approximate American option prices under GARCH. Although our approach does not have the convergence property shared by the methods of Ritchken and Trevor (1999) and Duan and Simonato (2001), it represents a practical alternative whenever computing time or memory requirement becomes an issue.

2 The Edgeworth Binomial Tree in the GARCH Context

2.1 Edgeworth binomial tree

As shown in Jarrow and Rudd (1982), an Edgeworth expansion using the first four moments of the risk-neutral asset distribution can be used to price European options because such options only depend on the asset price distribution at one time point. In order to price American options, one needs to describe the entire asset price path from the time of valuation to the maturity of the option contract. Rubinstein's (1998) method consists of using the Edgeworth expansion to approximate the risk-neutral asset price distribution at the maturity and then deducing from it an internally consistent binomial tree to describe the asset price evolution over the life of the option contract.

Rubinstein's method consists of first constructing a tree that recombines to yield $n+1$ nodes after n time steps. At the last step, the underlying asset value at the j^{th} node ($j = 0, 1, ..., n$), S_j, is set to be

$$S_j = s_0 \; e^{\mu\tau+\sigma\sqrt{\tau}x_j} \tag{1}$$

$$\mu = r - \frac{1}{\tau}\ln\sum_{j=0}^{n} P_j e^{\sigma\sqrt{\tau}x_j} \tag{2}$$

where s_0 is the initial asset price, r is the annual continuously compounded risk-free rate, τ is the time to expiration of the option (in years), $\sigma = \sqrt{Var(\rho_\tau)/\tau}$ is the annualized volatility rate for the cumulative asset return, $\rho_\tau \equiv \ln(s_\tau/s_0)$, and finally x_j is a mean 0 and variance 1 random variable with the corresponding probability distribution P_j. The probability distribution P_j is determined by modifying the binomial distribution using the Edgeworth expansion up to the fourth moment of ρ_τ. In contrast to the standard binomial lattice, the Edgeworth binomial tree need not have a constant move size or probability. In fact, all asset values and probabilities before the terminal time are deduced from the arbitrage-free principle. Finally, μ is used to ensure that the expected risk-neutral asset return equals r, a risk-neutrality condition. Once the Edgeworth binomial lattice is constructed, American option prices can be numerically computed by a backward recursion. Appendix A shows how x_j is created and how the skewness and kurtosis of ρ_τ are used to generate P_j.

2.2 Analytical moments under GARCH

Analytical expressions for the first four moments of the cumulative return are already available in the case of the LGARCH, NGARCH, GJR-GARCH and EGARCH models. If the asset's conditional expected return is specified to have a constant risk premium per unit of conditional standard deviation, denoted by λ, and the return innovation is normally distributed conditionally under the physical probability measure, the asset return dynamic with respect to the risk-neutral measure Q can be characterized. Duan (1995) showed that

$$\ln\left(\frac{s_{t+1}}{s_t}\right) = r_p - \frac{1}{2}h_{t+1} + \sqrt{h_{t+1}}\epsilon_{t+1}, \text{ for } t = 0, 1, 2, ... \tag{3}$$

where

$$\epsilon_{t+1}|\Phi_t \overset{Q}{\sim} N(0,1), \tag{4}$$

h_t is the conditional variance, Φ_t denotes the information set at time t, and r_p is the one-period risk-free rate (continuously compounded). If the length of one period is one calendar day, then $r_p = r/365$. Different versions of the GARCH model have their specific dynamic for h_t. In the case of the NGARCH model (Engle and Ng, 1993), the risk-neutral volatility dynamic becomes

$$h_{t+1} = \beta_0 + h_t[\beta_1 + \beta_2(\epsilon_t - \theta - \lambda)^2]. \tag{5}$$

For the GJR-GARCH model (Glosten *et al.*, 1993), it becomes

$$h_{t+1} = \beta_0 + h_t[\beta_1 + \beta_2(\epsilon_t - \lambda)^2 + \beta_3 \max(0, -\epsilon_t + \lambda)^2]. \tag{6}$$

Similarly, the risk-neutral volatility dynamic for the EGARCH model (Nelson, 1991) is

$$\ln(h_{t+1}) = \beta_0 + \beta_1 \ln(h_t) + \beta_4[|\epsilon_t - \lambda| + \gamma(\epsilon_t - \lambda)]. \tag{7}$$

Note that the LGARCH model (Bollerslev, 1986) can be viewed as a special case of the NGARCH model by setting $\theta = 0$ or a special case of the GJR-GARCH model by setting $\beta_3 = 0$. The parameters $\{\beta_i : i = 1, 2, 3, 4\}, \lambda, \theta$ and γ are parameters governing the volatility dynamic under different GARCH specifications. These parameters are subject to different restrictions to ensure that they are sensible specifications. For details, readers are referred to the respective papers that developed these models. The NGARCH, GJR-GARCH and EGARCH forms of the GARCH model are most popular because they permit the leverage effect, an important feature of financial asset returns.

The analytical formulas for the moments of the cumulative return under these GARCH models can be obtained by computing

$$E_0^Q\left[\rho_T^k\right] = E_0^Q\left[\left(Tr_p - \frac{1}{2}\sum_{i=1}^{T} h_i + \sum_{i=1}^{T}\sqrt{h_i}\epsilon_i\right)^k\right], \quad \text{for } T \in \{1, 2, ...\} \text{ and } k \in \{1, 2, 3, 4\} \tag{8}$$

where T is the maturity expressed in numbers of discrete periods. Expanding the expression inside the bracket and applying the expectation operator to the various terms obtains formulas with which the required moments can be computed. The final expressions are algebraically cumbersome but can be computed very quickly (in fractions of a second on a standard desktop computer). Interested readers are referred to Duan *et al.* (1999, 2002) for the analytical expressions. Matlab programs

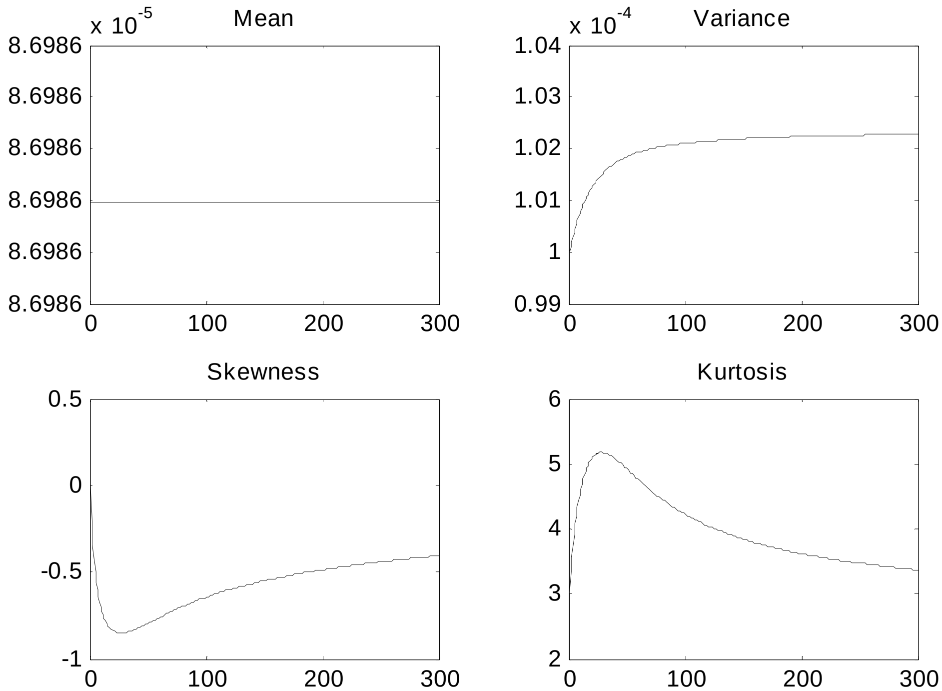

Figure 1: Term structure of moments for the cumulative return under NGARCH

implementing these formulas are available upon request. To better appreciate the nature of these moments, we have plotted them in Figure 1 using the NGARCH model with the parameter values: $r_p = 0.05/365$, $\beta_0 = 1e-5$, $\beta_1 = 0.7$, $\beta_2 = 0.1$, $\lambda + \theta = 1.0$, and h_1 is set to its stationary level. These moments are depicted as functions of maturity expressed in number of days[1]. As shown in these graphs, the distribution of the cumulative return quickly drift away from normality as maturity increases. This result is due to the stochastic mixture effect, a property possessed by the GARCH model. The distribution eventually reverts back towards normality as maturity is increasing. This result is tied to the central limit theorem. Slowness in the reversion to normality has a great deal to do with the high volatility persistence, which is a typical feature of financial data and is reflected in the chosen parameter values.

[1]In these graphs, the mean is defined as $E_0^Q\left[\rho_T\right]/T$, the variance as $Var(\rho_T)/T$, the skewness as $E_0^Q\left[z_T^3\right]$ and the kurtosis as $E_0^Q\left[z_T^4\right]$ with $z_T = \left(\rho_T - E_0^Q\left[\rho_T\right]\right) / \left(\sigma\sqrt{T}\right)$.

For options with short maturities, the prices obtained by the GARCH model will quickly deviate from those obtained by the Black-Scholes model for which skewness and kurtosis equal 0 and 3, respectively. In the case of long-term options, the GARCH model will yield values different from than those of the Black-Scholes model even though the standardized cumulative return tends to be normally distributed. These differences in option values are due to the fact that, in the GARCH framework, the appropriate variances for long-term options are different from the variances for short-term options. In Figure 1, variance is increasing with maturity because we set the initial conditional variance to the long-run average of one-period returns. If the initial variance is substantially lower than the average value, the relationship can be reversed. A clear message emerges from these graphs; that is, if GARCH models are appropriate descriptions of the dynamic of the stock price, the Black-Scholes formula will work better for long-term options. However, there will likely be an under or overvaluation if one simply plugs historical volatility into the Black-Scholes formula for long-term options.

3 A simulation analysis

In this section we present the results of a simulation study examining the precision of the proposed method. Since our analysis indicates that the Edgeworth binomial tree has a similar performance for the LGARCH, NGARCH, GJR-GARCH and EGARCH models, we only present the results for the NGARCH option pricing model. The simulation study adopts an approach similar to that of Broadie and Detemple (1996). A test pool of 500 American put options is simulated. For all options, we assume that the initial underlying asset value, s_0, is 100. Each option faces a different parameter set, consisting of parameter values randomly selected from predetermined distributions, independently from one another. The parameters are chosen from the following distributions: the number of days to maturity is uniformly distributed between 30 and 270; the strike price, K, is uniformly distributed between 70 and 130; r is uniformly distributed between 0 and 0.1 with a probability of 0.8, and equals 0 with a probability of 0.2. The NGARCH parameter values are drawn from the following distributions: β_0 is uniformly distributed between 0 and 10^{-4}; β_1 is uniformly distributed between 0 and 1; β_2 is uniformly distributed between 0 and 1; $\lambda + \theta$ is uniformly distributed between 0 and 1; the initial conditional variance, h_1, is uniformly distributed between 0.5 and 1.5 times the stationary variance under the risk-neutral probability measure Q. The simulated GARCH parameter value sets that violate the normality reversion conditions given in Duan *et al.* (1999) are discarded. Also imposed are ranges for skewness and kurtosis due to the limitations of the Edgeworth expansion discussed in Rubinstein (1998). Specifically, we limit the parameter sets that imply a skewness between -0.8 and 0.8 and a kurtosis between 3 and 5.5. The Edgeworth binomial tree prices are computed using the number of steps equal to the maturity (in number of days) of the option. This choice ensures that the number of early exercise points permitted by the Edgeworth binomial tree coincides with the number of allowable exercise time points under the daily GARCH model.

The benchmark prices in this study are obtained by the Markov chain method of Duan and Simonato (2001) with 301 states for the underlying asset price and 101 states for the conditional variance. We choose to use the Markov chain method to obtain the benchmark prices because (1) it yields option prices that theoretically converge to the right values, and (2) the existing practical Monte Carlo methods for American options are known to be biased downward with an unknown magnitude. To gain some idea about the precision of the Markov chain method, we present in Table 1 the comparison of the European option prices obtained by the Markov chain with those obtained using the $200,000$-path empirical martingale simulation method of Duan and Simonato (1998). In almost all cases, the Markov chain prices are accurate within a penny relative to the corresponding Monte Carlo prices.

We measure the aggregate relative pricing error for the test pool of American put options by a root mean square error (rmse), defined as:

$$\text{rmse} = \sqrt{\frac{1}{m}\sum_{i=1}^{m}\left(\frac{C_i(b) - C_i}{C_i}\right)^2}, \qquad (9)$$

where C_i is the i^{th} Markov chain price, $C_i(b)$ is the i^{th} Edgeworth binomial tree price, and m is the number of option prices in the test pool. Although our sampling procedure results in the pricing of 500 options, we restrict the analysis to a subset of 333 options whose Markov chain prices are greater than or equal to 0.50. This restriction avoids having a large pricing error due to a small divider.

3.1 Results

The test pool yields a rmse measure of 0.01816. It should be pointed out that an option price obtained using an Edgeworth tree in the GARCH framework has an inherent approximation error related to the use of a single stochastic variable to proxy a two-variable environment. This error is fundamental and cannot be mitigated by increasing the number of steps in the lattice. The rmse measure thus gives the expected loss in accuracy. The loss in accuracy is, however, compensated by a substantial increase in computation speed. The gain is often by a factor greater than 100 when compared to the time required by the Markov chain method. We will document the gain in computation time at the end of this section.

To better understand the source of pricing errors, we divide the test pools into options with a high volatility persistence ($\beta_2\left(1+(\lambda+\theta)^2\right)+\beta_1 > 0.85$) and a low volatility persistence ($\beta_2\left(1+(\lambda+\theta)^2\right)+\beta_1 \leq 0.85$). The rmse measures for these two cases become 0.0334 and 0.0161, respectively. This suggests that the Edgeworth binomial tree is more accurate when the volatility persistence is low. We also divide the test pool into in-the-money and out-of-the-money options to obtain rmse measures of 0.0048 and 0.0489, respectively. A poorer performance for out-of-the-money options was, to a large extent, expected because of the small divider effect. Finally, we classify options according to the magnitude of the relative pricing error and present the corresponding frequency of occurrence in Figure 2. The result indicates that 33% of the sample has a negligible relative pricing error. There are only 3.5% of the sample has a relative pricing error greater than 5%. Large relative pricing errors are, in all cases, associated with deep out-of-the-money options, which are options with a strike-to-asset-price ratio below 0.9.

In Tables 2 and 3, more detailed results with regards to the performance of the Edgeworth binomial tree are presented. The Markov chain prices, with 301 states for the underlying asset price and 101 states for the conditional variance, are again used as benchmarks. The results are for two sets of parameters: $\{\beta_0 = 0.00001,\ \beta_1 = 0.7,\ \beta_2 = 0.1,\ \theta+\lambda = 0.2\}$ and $\{\beta_0 = 0.00001,\ \beta_1 = 0.8,\ \beta_2 = 0.1,\ \theta+\lambda = 0.2\}$. They respectively represent low and high levels of volatility persistence. In each panel, the first set of numbers corresponds to the case where h_1 is set equal to the stationary variance, h^*. The second and third sets respectively correspond to the cases where h_1 is fixed at 20% above and below h^*. The risk-free interest rate is 5% per annum and the initial

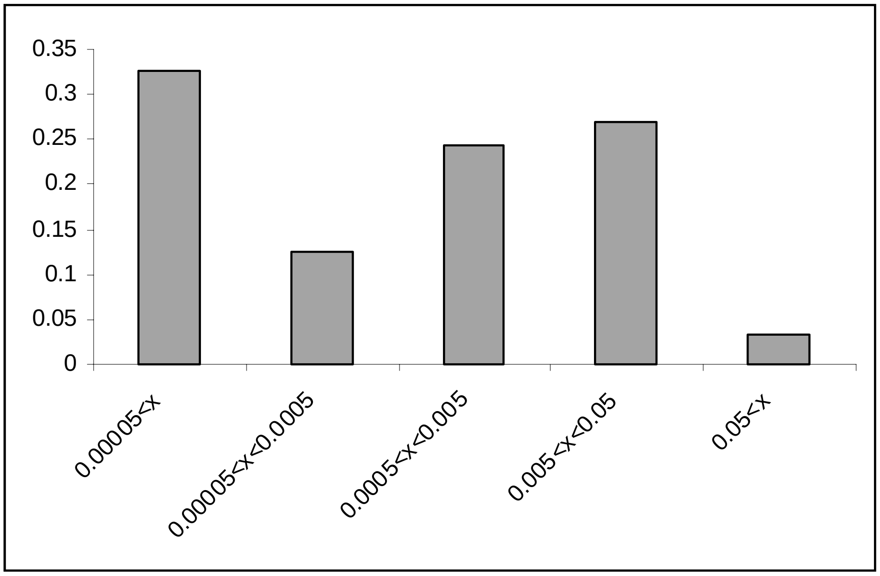

Figure 2: Figure 2. Frequency distribution of the relative pricing error

asset price is at \$50. The results are presented for four different maturities: 10, 30, 90 and 270 days. Finally, we consider three moneyness ratios (1.1, 1.0 and 0.9), defined as the strike-to-asset-price ratio. For the first set of parameters, we obtain penny accuracy in almost all cases. For the second set of parameters which correspond to a higher volatility persistence, the Edgeworth tree method is less accurate. This is especially true for longer maturity options. Nevertheless, the pricing errors in all cases are reasonably accurate relative to the magnitudes of the corresponding option prices.

To shed some light on computation times, we report the time taken by the Markov chain and the Edgeworth tree methods (both coded in C) in computing an American option with 90 days to maturity. The Edgeworth binomial tree takes approximately 0.06 seconds on a standard desktop computer whereas the Markov chain with 201 states for the underlying asset price and 75 states for the conditional variance takes about 8 seconds. For an option with 200 days to maturity, a Markov chain of a larger dimension is required to achieve penny accuracy. In this case, the Edgeworth tree takes about 0.12 seconds but the Markov chain with 301 states for the underlying asset price and 101 states for the conditional variance takes about 33 seconds. It is obvious that the Edgeworth binomial tree should also be faster than the method of Ritchken and Trevor (1999), which is a trinomial lattice accompanied by vectors of option values, corresponding to different volatilities, at all nodes.

4 Concluding remarks

The Edgeworth binomial tree is a fast method for computing American option prices when the cumulative return is not normally distributed. In the case of the GARCH model, we have shown that the Edgeworth binomial tree may be more desirable when computing speed is an important consideration. As expected, the Edgeworth binomial tree contains an inherent approximation error in the case of the GARCH model, which is simply due to the use of unique stochastic variable to proxy two stochastic variables. The magnitude of the pricing errors, however, appear to be well within the tolerance level for many applications. In short, the Edgeworth binomial tree adds to our set of practical tools for valuing American options under GARCH.

A Edgeworth tree construction

The tree construction starts first by considering an n-step binomial distribution with $n+1$ possible values denoted by $y_j = \frac{[(2j)-n]}{\sqrt{n}}$ for $j = 0$ to n and the associated probability $b_j = [n!/j!(n-j)!](1/2)^n$. Given a pre-specified skewness and kurtosis, the binomial distribution is modified by the Edgeworth expansion up to the fourth moment to yield:

$$f_j = \left[1 + (1/6)\xi(y_j^3 - 3y_j) + (1/24)(\kappa - 3)(y_j^4 - 6y_j^2 + 3)\right] b_j$$

where $\xi = E^Q\left[z_\tau^3\right]$ is the skewness and $\kappa = E^Q\left[z_\tau^4\right]$ the kurtosis of the cumulative return for the option's maturity under the risk-neutral measure with $z_\tau = \frac{\rho_\tau - E^Q[\rho_\tau]}{\sigma\sqrt{\tau}}$. Scaling is needed to ensure that the probabilities sum up to one because the Edgeworth expansion only apporximates a probability distribution. The scaling operation is

$$P_j = \frac{f_j}{\sum_j f_j}.$$

The variable y_j based on the probability P_j is no longer a binomial random variable and can be standardized to have mean 0 and variance 1 as follows:

$$x_j = \frac{y_j - M}{V}$$

with $M = \sum_j P_j y_j$ and $V^2 = \sum_j P_j (y_j - M)^2$. The variable x_j is then used in equation (1) to create the terminal asset price and the corresponding risk-neutral probability of a single path to node j:

$$p_j = P_j / [n!/j!(n-j)!].$$

Working backwards, the rest of the tree can be deduced easily. Denote by (p_j, S_j) and (p_{j+1}, S_{j+1}) the probabilities and asset prices at two adjacent nodes where the subscript j and $j+1$ indicate the upper and lower branch. The backward recursion based on the arbitrage-free principle can be used to find the price and probability pair, i.e., p and S, for the preceding node; that is,

$$\begin{aligned} p &= p_j + p_{j+1} \\ S &= \left[\frac{p_{j+1}}{p} S_{j+1} + \frac{p_j}{p} S_j\right] \exp(-r \times \tau / n) \end{aligned}$$

where τ is the maturity of the option in years. It is clear that the induced asset price at the origin equals the initial asset price, s_0, because of equations (1) and (2).

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Table 1: The performance of the Markov chain method in pricing European options under NGARCH

$\beta_0 = 0.00001, \beta_1 = 0.70, \beta_2 = 0.10$ and $\lambda + \theta = 0.50$

	Maturity = 10 days			Maturity = 30 days			Maturity = 90 days			Maturity = 270 days		
K/S_0	1.10	1.00	0.90	1.10	1.00	0.90	1.10	1.00	0.90	1.10	1.00	0.90
$h_1 = h^* \times 1.00$												
Monte Carlo	4.92	0.44	0.00	4.78	0.72	0.01	4.53	1.13	0.09	4.31	1.64	0.38
Markov Chain	4.92	0.44	0.00	4.78	0.72	0.01	4.53	1.13	0.09	4.31	1.64	0.38
$h_1 = h^* \times 1.20$												
Monte Carlo	4.92	0.46	0.00	4.78	0.73	0.01	4.54	1.14	0.09	4.31	1.64	0.39
Markov Chain	4.92	0.46	0.00	4.78	0.73	0.01	4.54	1.14	0.09	4.32	1.65	0.39
$h_1 = h^* \times 0.80$												
Monte Carlo	4.92	0.41	0.00	4.78	0.70	0.01	4.53	1.13	0.09	4.30	1.63	0.38
Markov Chain	4.92	0.41	0.00	4.78	0.70	0.01	4.53	1.12	0.08	4.30	1.63	0.38

Monte Carlo is the European option price by the Monte Carlo simulation with 200,000 sample paths. Markov Chain is the European option price by the Markov Chain method with 301 states for the underlying asset and 101 states for the volatility.

Table 2: The performance of the Edgeworth binomial tree in pricing European and American options under NGARCH (low persistence)

$\beta_0 = 0.00001, \beta_1 = 0.70, \beta_2 = 0.10$ and $\lambda + \theta = 0.50$

	Maturity = 10 days			Maturity = 30 days			Maturity = 90 days			Maturity = 270 days		
K/S_0	1.10	1.00	0.90	1.10	1.00	0.90	1.10	1.00	0.90	1.10	1.00	0.90
$h_1 = h^* \times 1.00$												
Markov Chain Euro.	4.92	0.44	0.00	4.78	0.72	0.01	4.53	1.13	0.09	4.31	1.64	0.38
Edgeworth Euro.	4.92	0.43	0.00	4.78	0.72	0.01	4.53	1.14	0.09	4.31	1.64	0.38
Markov Chain Amer.	5.00	0.44	0.00	5.00	0.73	0.01	5.00	1.19	0.09	5.10	1.84	0.42
Edgeworth Amer.	5.00	0.43	0.00	5.00	0.73	0.01	5.00	1.19	0.09	5.08	1.82	0.41
$h_1 = h^* \times 1.20$												
Markov Chain Euro.	4.92	0.46	0.00	4.78	0.73	0.01	4.54	1.14	0.09	4.32	1.65	0.39
Edgeworth Euro.	4.92	0.44	0.00	4.78	0.73	0.01	4.54	1.14	0.09	4.31	1.64	0.39
Markov Chain Amer.	5.00	0.46	0.00	5.00	0.75	0.01	5.00	1.20	0.09	5.10	1.85	0.42
Edgeworth Amer.	5.00	0.45	0.00	5.00	0.75	0.01	5.00	1.20	0.10	5.08	1.83	0.42
$h_1 = h^* \times 0.80$												
Markov Chain Euro.	4.92	0.41	0.00	4.78	0.70	0.00	4.52	1.12	0.08	4.30	1.63	0.38
Edgeworth Euro.	4.92	0.41	0.00	4.78	0.71	0.01	4.53	1.13	0.09	4.30	1.63	0.38
Markov Chain Amer.	5.00	0.42	0.00	5.00	0.72	0.01	5.00	1.18	0.08	5.09	1.83	0.41
Edgeworth Amer.	5.00	0.41	0.00	5.00	0.72	0.01	5.00	1.18	0.09	5.08	1.82	0.41

Markov chain Euro. (Amer.) is the European (American) option price by the Markov chain method with 301 states for the underlying asset and 101 states for the volatility. Edgeworth Euro. (Amer.) is the option price by the Edgeworth binomial tree approach.

Table 3: The performance of the Edgeworth binomial tree in pricing European and American options under NGARCH (high persistence).

$\beta_0 = 0.00001, \beta_1 = 0.80, \beta_2 = 0.10$ and $\lambda + \theta = 0.50$

	Maturity = 10 days			Maturity = 30 days			Maturity = 90 days			Maturity = 270 days		
K/S_0	1.10	1.00	0.90	1.10	1.00	0.90	1.10	1.00	0.90	1.10	1.00	0.90
$h_1 = h^* \times 1.00$												
Markov Chain Euro.	4.93	0.68	0.00	4.85	1.13	0.08	4.98	1.85	0.42	5.51	2.87	1.20
Edgeworth Euro.	4.93	0.66	0.00	4.85	1.11	0.10	4.97	1.85	0.46	5.52	2.90	1.24
Markov Chain Amer.	5.00	0.68	0.01	5.00	1.14	0.09	5.21	1.90	0.43	6.01	3.07	1.27
Edgeworth Amer.	5.00	0.66	0.00	5.00	1.13	0.11	5.19	1.90	0.46	5.96	3.07	1.30
$h_1 = h^* \times 1.20$												
Markov Chain Euro.	4.93	0.73	0.01	4.86	1.18	0.10	5.00	1.88	0.44	5.54	2.89	1.22
Edgeworth Euro.	4.93	0.70	0.00	4.87	1.16	0.12	4.99	1.88	0.47	5.54	2.91	1.25
Markov Chain Amer.	5.00	0.73	0.01	5.00	1.19	0.10	5.23	1.94	0.45	6.03	3.10	1.29
Edgeworth Amer.	5.00	0.71	0.00	5.00	1.17	0.12	5.21	1.93	0.48	5.98	3.09	1.31
$h_1 = h^* \times 0.80$												
Markov Chain Euro.	4.92	0.63	0.00	4.84	1.08	0.07	4.95	1.82	0.40	5.50	2.85	1.19
Edgeworth Euro.	4.93	0.61	0.00	4.84	1.07	0.09	4.95	1.82	0.44	5.51	2.88	1.23
Markov Chain Amer.	5.00	0.63	0.00	5.00	1.09	0.07	5.18	1.87	0.41	5.98	3.05	1.25
Edgeworth Amer.	5.00	0.62	0.00	5.00	1.09	0.09	5.17	1.87	0.45	5.95	3.05	1.29

Markov chain Euro. (Amer.) is the European (American) option price by the Markov chain method with 301 states for the underlying asset and 101 states for the volatility. Edgeworth Euro. (Amer.) is the option price by the Edgeworth binomial tree approach.