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Persistence Characteristics of Latin American Financial Markets

Abstract

The financial rates of return from Latin American stock and currency markets are found to be non-normal, non-stationary, non-ergodic and long-term dependent, i.e., they have long memory. The degree of long-term dependence is measured by monofractal (global) Hurst exponents from wavelet multiresolution analysis (MRA). Scalograms and scalegrams provide the respective visualizations of these wavelet coefficients and the power spectrum of the rates of return. The slope of the power spectrum identifies the Hurst exponent and thereby the degree of scaling dependence that cannot be determined by Box-Jenkins type time series analysis. Our dependency and time and frequency scaling results are consistent with similar empirical findings from American, European, and Asian financial markets, extending the domain of the empirical investigation of the dynamics and risk characteristics of financial markets and refuting the hypothesis of perfectly efficient markets.

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1 INTRODUCTION

Financial researchers are continuously trying to improve the mathematical models for reliable identification of the price diffusion processes of financial markets. Currently, the most popular dynamic time series models are ARIMA and GARCH models. These models are based on the assumption of stationarity of the residual "innovations" in the time series, where market risk is measured by second-order moments only. This assumption is often combined with the assumption of normality (or Gaussian distribution) of these residual innovations.

Our critique of these "stationarity of residuals" models is twofold. First, the innovations of empirical financial time series are demonstrably not Gaussian and the moments of the distributions of these innovations tend to vary over time (Loretan and Phillips, 1994). Second, and more importantly, these "stationarity of residuals" models cannot properly model the observed long-term dependence, *i.e.*, long-term memory or LM processes, in particular time-frequency scaling processes, since they presume finite and integer lags. Cochrane (1988) has already shown that ARIMA models concentrate on the first few (constant) autocorrelations to capture the short-run features of the time series, and that they produce misleading results, when used to estimate long-run properties of time series. GARCH models exhibit similar time-frequency deficiencies.

We observe that empirical financial time series exhibit LM behavior, singularities, and discontinuities, which indicate that the occurrence of financial catastrophes in persistent markets and of turbulence in anti-persistent markets is more prevalent than predicted by analysis based on the Gaussian stationarity assumption. In addition, the higher order moments of the financial return series, such as skewness and kurtosis, prove to matter when measuring financial risk (Corazza and Malliaris, 2002). Finally, the complete distributions of empirical asset return innovations can be shown to be time-variant (Mandelbrot, Fisher and Calvet, 1997). Thus financial market return distributions are strictly non-stationary.

Such behavior of empirical financial time series challenges the Efficient Market Hypothesis

(EMH) of Fama (1970) based on martingale theory, which assumes stationarity and independence of the return innovations. An increasing number of studies have documented that empirical financial time series are not log-normal, but there have not been enough empirical studies focusing on the long-term dependence of time series produced by financial markets. Mandelbrot (1969, 1972) originally introduced the concept of the long-term persistence in economic series.

Substantial evidence is now emerging that theoretical long-memory processes do a better job in describing empirical financial data, such as forward premiums, interest rate differentials, and inflation rates. This evidence draws a renewed interest in Fractional Brownian Motion (FBM), fractionally integrated processes, and long-memory processes (Peters, 1994; Baillie, 1996; Robinson, 2003).

With such advanced modeling methodology, we can now more accurately characterize the behavior of financial time series. This is important for (1) risk measurements, (2) asset and options valuations, (3) portfolio selection and management, and (4) international capital allocation. For example, in the area of asset and options valuation, Black-Scholes' Geometric Brownian Motion (GBM) assumes Fickian neutral independence (i.i.d.) of the returns innovations. But many empirical researchers have observed non-Fickian degrees of persistence or long memory (LM) in the financial markets – a contradiction of Black-Scholes' GBM assumption.

The consequences of this new methodology could be substantial. Jamdee and Los (2004) graphically demonstrate the significant impact of the observed financial market persistence, *i.e.*, LM, on European option valuation. These findings impact the valuation of corporate remuneration packages, since warrants are written on a company's own (infrequently traded) shares for long expiration periods.

The accurate characterization of the financial time series data is also critical for risk management and international capital allocation. In the early 1990s, the vast majority of the Latin American countries launched market–oriented reforms. These initiatives attracted a large amount of foreign capital flowing into Latin American financial markets. In addition, as the US interest rates declined since 1990, US investors were induced to look for higher returns abroad, especially in the emerging markets (Edwards, 1998).

Investors seemed to believe that emerging markets always show higher rates of return on investments. However, Los (2000) documents that the emerging stock markets lost about 10% in their investment returns as measured by the IFC investable Index since 1994. Moreover, financial crises have been a more than normal occurrence in the 1990s, *e.g.*, the Mexican Financial Crisis of 1994, the Asian Financial Crisis of 1997, the Russian Debt Default Crisis of 1998, and the Brazilian Financial Crisis of 1999. The emerging markets appear to be highly volatile with extremely high levels of profits, but also of very substantial losses.

Extensive research is already done on Asian emerging financial market data; however, less research has been done on the LM processes of the stock prices and exchange rates of Latin American countries. Latin American markets form an important segment of emerging markets. They are small, speculative and exhibit high volatility in returns, and are vulnerable to crisis. The 1994 Mexican and 1999 Brazilian Financial Crises urged us to investigate the Latin American financial markets more carefully, and to focus on the areas that have been ignored, while expanding the empirical investigation of time and frequency dependence in financial market pricing.

This paper investigates the empirical characteristics of financial time series of six Latin American stock markets and five currency markets, focusing on measuring the degree of persistence of each financial market. It is the first paper that analyzes these empirical Latin American financial time series simultaneously in the time and frequency domains. Our results show that Latin American financial market returns are not stationary, and that they exhibit long-term dependence, and, thus, that they are not ergodic. This means that the use of statistical limit arguments to determine conventional financial statistics of these market time series, as used in CAPM and option valuation models, is scientifically erroneous.

In this paper, the degrees of the markets' persistence are measured by monofractal, (global, or homogeneous) Hurst exponents identified from the Wavelet Multiresolution Analysis (MRA) of

Mallat (1989a, b, and c). It enables a simultaneous time-frequency description of time series data in localized risk (volatility or energy) details. With the help of scalograms, which are visualizations of the wavelet resonance coefficients, shocks to the financial markets and the power of these shocks can be clearly identified and visualized. These wavelet resonance coefficients measure the degree of correlation of various segments of the time series with particularly shaped and sized wavelets (= "basis elements of complete analysis") according to their scale, which is proportional to the inversion of their frequency. Scalograms visualize the corresponding coefficients of determination and enable a more precise analysis of how market prices dynamically adjust to new information.

While a scalogram provides the time-and-frequency-localized analysis of the time series, a scalegram – which is the time-average of a scalogram – represents the classical autocorrelation function (ACF) of the time series in the scale domain. When applied to the innovations, it shows, especially, the “periodicities”, or, more precisely, aperiodic “cyclicities” of the financial markets, which cannot be easily identified by the static methodologies based on the "stationarity-of-innovations" assumption.

The paper is organized as follows. Section (1) provides a brief literature review of long-term dependence or long-memory. Section (2) describes the data and the methodology. It provides also a simple statistical distribution theory for the Hurst exponent based on wavelet MRA, showing that the accuracy of the MRA determination of this LM exponent is usually very high. Section (3) discusses the empirical results. Finally section (4) presents our main conclusions.

2 LONG TERM DEPENDENCE

Mandelbrot (1969, 1972) discusses the non-Gaussian distributions of financial prices and introduces the concept of the long-term persistence in economic series. Since his early papers, financial researchers have been searching for models that can identify such typical behavior of financial time series. Empirical econometric studies on long-term dependence often rely on the study by Geweke and Porter-Hudak (1983), who propose a method for the calculation of the fractional differencing parameter d of a Fractal Brownian Motion. In addition, Hosking (1981), and Granger and Joyeux (1980) propose Fractionally Integrated ARMA models to measure long-term dependence in combination with the conventional short-term serial correlations.

These models are extensively discussed in Beran (1992), Baillie (1996) and Robinson (1994, 2003). Among others, Baillie (1996) surveys the statistical and econometric work concerning long-memory and fractionally integrated processes that are associated with hyperbolically decaying autocorrelations and impulse response weights.

The topic of LM and persistence has recently attracted considerable additional attention in terms of a discussion of the behavior of the non-stationary second moments of log-normal pricing processes. Baillie, Bollerslev, and Mikkelsen (1996) apply the FIGARCH (Fractionally-Integrated-GARCH) process to exchange rates, Bollerslev and Mikkelsen (1996) apply the FIGARCH process to stock prices, and Breidt, Crato, and de Lima (1993), Crato and de Lima (1994), and Harvey (1993) find similar evidence of LM stochastic volatility in stock returns and exchange rates, respectively.

The R/S (Range-Scale) based exponent of Hurst (1951) is the most widely used statistic to measure global LM in time series. Greene and Fielitz (1977) and Aydogan and Booth (1988) used the original R/S analysis in common stock returns; while Lo (1991) uses the modified R/S statistic on returns from value and equal weighted CRSP indices from July 1962 to December 1987. Lo (1991) finds significant results from using the original R/S statistic, but, insignificant results from the application of his modified R/S statistic. He reports a lack of long-range persistence in annual returns from 1872 to 1986, but his non-corroborating results appear to be exceptional in the growing literature on long term memory or dependence in the financial markets.

Booth, Kaen, and Koveos (1982) apply the basic R/S statistic to exchange rates. Cheung (1993a), taking monthly data from January 1974 through December 1989, also finds some evidence of LM in the French Franc/US dollar rate, but no apparent departure from martingale behavior by the anchor currencies German Mark, Swiss Franc, or Japanese Yen.

Applying the very accurate wavelet MRA to high-frequency data for 1997, Karuppiah and Los (2000) find long-term dependencies in eight Asian currency markets (Japan, Hong Kong, Indonesia, Malaysia, Philippines, Singapore, Taiwan, Thailand) and in the German Mark (no longer existent). In particular, they were the first to find that the anchor currencies - the German Mark/Dollar and the Japanese Yen/Dollar rates are actually anti-persistent.[1] Corazza and Malliaris (2002) find long-term dependence in five currency markets: the British Pound, the Canadian Dollar, the German Mark, the Swiss Franc, and the Japanese Yen.

Regarding the efficiency of emerging financial markets, Los (2000) uses nonparametric efficiency tests to ascertain the efficiency of Asian Stock markets (Hong Kong, Indonesia, Malaysia, Singapore, Taiwan, and Thailand) and finds that none of the markets is stationary or shows independent innovations. Nonparametric tests for the efficiency of Asian currency markets are also presented in Los (1999), where he finds that no Asian currency market exhibited complete efficiency in the Asian *annus horibilis* 1997.

Sadique and Silvapulle (2001) examine the presence of LM in weekly stock returns of seven countries by using R/S analysis, GPH procedures, and time and frequency domain versions of the score tests. They find that Korea, Malaysia, Singapore and New Zealand stock returns possess LM. The results of Sadique and Silvapulle (2001) contradict those of Cheung (1995), who uses Lo's modified R/S and fractional differencing to test for LM of stock returns in eighteen countries in Asia, Europe and North America and finds little support for LM in international stock returns.

Such contradictions and inconsistencies in the empirical finance literature regarding dependence in financial market pricing need to be resolved in an accurate fashion. These results affect the correct measurement of financial risk, and therefore the correct pricing of derivatives, the proper selection and management of investment portfolios, and, ultimately, the proper allocation

[1] This remarkable empirical obsevation by Karuppiah and Los (2000) was used to partially motivate the seminal paper by Elliott and Van Der Hoek (2003), which provides the mathematical underpinnings fo the current financial market persistence eseach.

of scarce international capital. Therefore, Gençay, Selçuk and Whicher (2002) and Los (2003) have collected the latest signal processing methodologies and technologies based on wavelet MRA to more carefully identify and measure the various degrees of long-term dependence, *i.e.*, the persistence and anti-persistence of financial time series. This paper applies some of their proposed advanced methodology and new technology.

3 DATA AND METHODOLOGY

3.1 Data

Our data set covers six daily stock market data series and five daily foreign exchange rate series. Since this paper focuses on measuring and identifying the overall persistence characteristics of financial markets, and how financial market prices adjust to shocks over long time periods, daily data may be more preferable than higher frequency data, since they are more manageable, computation-wise. On the other hand, traders and micro-market technicians would be interested in the intra-day, *i.e.*, very high frequency data, but such data are still extremely hard to obtain for the Latin American financial markets.

The six stock markets investigated in this paper include Argentina, Brazil, Chile, Colombia, Mexico, and Venezuela. Since Argentina dollarized its currency since the early 1980s, we do not include Argentina in our foreign exchange series. The data for daily stock indices are from the web site, "Yahoo, Finance!" and the data for foreign exchange rate series are from the site, "OANDA.com." It would be ideal to have a long and comparable data sets across countries, in both stock and currency markets. However, due to very restricted data availability, this paper deals with a somewhat unbalanced data set that (i) may not be very long, and (ii) has not exactly the same number of observations for each country. However, our admittedly imperfect data set is capable of showing the empirical characteristics of non-normality, non-ergodicity, and long-term dependence in an accurate fashion. Our conclusions would not change as more data are included in the sample, since we look at the data series in complete localized time-frequency domains.

Data descriptions, period coverage and numbers of observations are presented in Table 1. Note that Argentina has the same stock index as Uruguay, and Colombia shares the same stock index as Ecuador and Peru.

[TABLE 1, PANEL A, ABOUT HERE]

[TABLE 1, PANEL B, ABOUT HERE]

3.2 Methodology

The first part of our empirical analysis tests for the general time-frequency characteristics of the daily Latin American financial time series, such as the stationarity – both strict and wide sense - ergodicity, and long-term dependence. The second part of our analysis identifies the nature of the long-term dependence by measuring the Hurst exponent from the wavelet MRA. This wavelet MRA is visualized in the form of scalograms and scalegrams.

3.2.1 Measuring Stationarity, Ergodicity, and Independence

A stochastic process is said to be stationary in the strict sense, if the whole joint probability distribution remains invariant over time, *i.e.*, the joint distribution of any set of n observations $X_{t_1}, X_{t_2}, \ldots, X_{t_n}$ is the same as the joint distribution of $X_{t_{1+k}}$, $X_{t_{2+k}}, \ldots,$ $X_{t_{n+k}}$ for all n time points and for all lags k. The process is said to be stationary in the wide sense if the first two moments of the distribution remain invariant over time and the autocovariance function has only the finite integer lag k as argument, i.e., $E\{X_t\} = \mu$ and $Cov\{X_t, X_{t+k}\} = \gamma(k)$. In order to check for wide and strict sense stationarity, we calculate moving first, second, third, and fourth order moments with a fixed window size of 50 days to see if the series is stationary.

Ergodicity is defined by Mills (1999, p. 9) as follows: "... the process is ergodic, which roughly means that the sample moments for finite stretches of the realization approach their population counterparts as the length of the realization becomes infinite." In other words, if a process or time series is ergodic, its expected value or ensemble average, can be replaced by a sufficiently long time average. But when a time series is non-ergodic, this replacement is logically invalid. It is often said, as in Mills (1999, p.9) that " ... since it is impossible to test for ergodicity using just (part of) a single realization, it will be *assumed* from now on that all time series have this property" (of ergodicity).[2]

But such a maintained assumption is unnecessarily sweeping this important empirical issue under the rug, since we can check for ergodicity. To check for the ergodicity of an empirical series, we calculate the moving moments of time windows of increasing size for all series. If a time series is ergodic, the plotted moments of increasing windows should converge to constant values, *i.e.*, visualized as straight lines parallel to the abscissa of time, indicating that the values of moments do not change as the number of observations increases.

Independent random variables have no history. They are immeasurable using the past historical information, only measurable with respect to current information. To see if a data series exhibits independence, we compare the Autocorrelation Function (ACF) of the theoretical GBM and the ACFs of our empirical data series. Since the GBM assumes independence, and thus no serial correlation, its ACF drops immediately after the first lag. A time series is only long-term dependent, if its empirical ACF decays at a hyperbolic (or comparably slow) rate, which is much slower than the fast decay of the GBM.

3.2.2 Measuring the Degree of Long-Term Dependence

If the innovations of the rates of return are independent, the series can be represented by the GBM, whereas, if they are long-term dependent, they may be better modeled by a FBM. In both Brownian Motions, the rate of return is the first difference of the natural log of the price series:

$$x(t) = \ln X(t) - \ln X(t-1) \tag{1}$$

This rate of return is then fractionally differenced with white noise innovations

[2] Our emphasis!

$$(1-L)^d x(t) = e(t) \text{ where } -0.5 < d < 0.5 \text{ and } e(t) \sim i.i.d.(0, \sigma^2) \tag{2}$$

Here d is the fractional differencing exponent of Geweke and Porter-Hudak (1983). Only when $d = 0$, the FBM is a conventional GBM.

The time-domain ACF of FBM is proportional to (Los, 2003):

$$\gamma(\tau) = E\{x(t)x(t-\tau)\} \sim \sigma^2 \tau^{2d-1} = \sigma^2 \tau^{2H-2} \tag{3}$$

Equivalently, the frequency-domain power spectrum of an FBM $P(\omega)$ is proportional to:

$$P(\omega) \sim \sigma^2 \omega^{-(2H+1)} \tag{4}$$

and the wavelet scalegram or average wavelet spectrum of an FBM is proportional to:

$$P_W(a) \sim \sigma^2 a^{(2H+1)} \tag{5}$$

where, $H = d + 0.5$ is the monofractal Hurst exponent (Hurst, 1951), the scale $a = \frac{\omega_0}{\omega}$ with the fundamental angular frequency $\omega_0 \sim \frac{2\pi}{T}$, and $T =$ total number of observations.

The ACF, the power spectrum and the wavelet spectrum of the FBM clearly have scaling properties, with the ACF scaling according to time horizon τ, the power spectrum scaling according to frequency ω, and the wavelet spectrum according to the scaling level $a \sim \frac{1}{\omega}$. It is these time-and-frequency scaling properties of the FBM, which allows us to identify the Hurst exponent H as a measure of the degree of its long-term dependence using various methodologies. For example, the global Hurst exponent can be identified from the log plot of the power spectrum $P(\omega)$ of the FBM against the logarithm of frequency ω, since

$$\ln P(\omega) \quad = -(2H+1)\ln\omega + \ln\sigma^2 + \ln C \tag{6}$$

where C is a proportion constant. It can also be identified from the log plot of the wavelet scalegram of the FBM against the scaling level a, since

$$\log_2 P_W(a) = (2H+1)\log_2 a + \log_2 \sigma^2 + \log_2 D \tag{7}$$

where D is another proportion constant.[3] Thus, the current approach is to compute the Hurst exponent from the slope coefficient $-(2H+1)$ of the periodogram, *i.e.*, the spectral density of the empirical data set, or from the slope coefficient $(2H+1)$ of the scalegram, i.e. the scale density of the empirical data set.

This monofractal Hurst exponent is constrained in its value: $0 < H < 1$. The crucial LM test is the (potentially falsifying) test of the hypothesis of the neutral short memory of the GBM ($H_0 : H = 0.5$,or, equivalently, $d = 0$) against the two alternative hypotheses of a persistent FBM ($H_0 : 0.5 < H < 1$, or $0 < d < 0.5$) or anti-persistent FBM ($H_0 : 0 < H < 0.5$, or $-0.5 < d < 0$).[4]

Statisticians have tried to find an LM test statistic with a limiting probability distribution which can be easily computed and that has good power for hypothesis testing. Based on some admittedly "slightly defective heuristics" (Robinson, 2003, p.14), Geweke and Porter-Hudak (1983) argue that, asymptotically, the distribution of the log-periodogram regression estimate of d satisfies

$$\tau^{0.5}(\widehat{d} - d) \overset{d}{\to} N\left(0, \frac{\pi^2}{24}\right) = N(0, 0.41123) \tag{8}$$

giving rise to apparently simple inferential procedures. In addition, by employing a linear process for $x(t)$ based on martingale difference innovations, Robinson (1995a, b) used a Gaussian assumption and then rigorously established a more precise result, based on a slightly different estimator of the whole range $-0.5 < d < 0.5$:

[3] In wavelet analysis, usually dyadic scaling is used, so that $a = 2^j$, and thus also the dyadic logarithm $\log_2$ instead of the natural logarithm ln.

[4] Physicists and astonomers use a color coding fo these various degees of persistence. When H approaches zero ($H \downarrow 0$), the time series is said to be blue noise. When $0 < H < 0.5$, the series is said to be anti-persistent or light blue. When $H = 0.5$, the series is neutral white noise. When $0.5 < H < 1$, the series' increments are said to be persistent, or pink noise. When H approaches 1 ($H \uparrow 1$), the series is said to be red noise. Once integrated white noise is brown noise (Cf. Los, 2003, p. 124, Table 4.3).

$$\tau^{0.5}(\widehat{d} - d) \xrightarrow{d} N(0, \frac{1}{4}) = N(0, 0.25) \tag{9}$$

This result provides also apparently simple asymptotic interval estimates, as well as a simple test of neutrality, $d = 0$, *i.e.* $H = 0.5$. Robinson's treatment, based on the theoretical *assumption* of Gaussianness, actually covers multiple time series, possibly differing memory parameters (= multifractality), and more efficient tests for equality of these parameters.

But both these limiting results are scientifically flawed, since they are based on the untenable presumption of independence, on inapplicable infinity (= limiting) arguments applied to the finite support for d or H, and prejudicial unidirectional projections (in case of Geweke and Porter-Hudak, 1983). The Hurst exponent $H = d + 0.5$ cannot be normally distributed, since it has known finite, support, $0 < H < 1$, implying that also the differencing parameter $d = H - 0.5$ must have finite support: $-0.5 < d < 0.5$. In contrast any normally distributed variable $z \sim N(0, \sigma^2)$ has infinite support: $-\infty < z < +\infty$.

The correct statistical theory for the H exponent, as identified from the wavelet MRA, is as follows. Flandrin (1992) and Flandrin and Gonçalvès (1996) proved that the detailed wavelet resonance coefficients, which correlate wavelets with particular segments of the time series $x(t)$,

$$W(\tau, a) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,a}(t)dt \tag{10}$$

- where $\psi^*_{\tau,a}(t)$ is a particularly localized wavelet - are Gaussian with mean zero and a variance that is their own value squared:[5]

$$W(\tau, a) \sim N(0, |W(\tau, a)|^2) \tag{11}$$

The elements of the wavelet scalogram consist, by definition, of these "coefficients of determination":

$$P_W(\tau, a) = |W(\tau, a)|^2 \tag{12}$$

[5] Despite the ∞ signs in the integral, the empirical support for these wavelet coefficients is finite, since the wavelet support is empirically finite: $1 < \tau < T$ and $1 < a < T$.

Each coefficient of the finitely tiled scalogram $P_W(\tau, a)$ is then chi-squared distributed with one degree of freedom and non-centrality parameter $P_W(a)$:

$$P_W(\tau, a) \sim \chi^2(1, P_W(a)) \tag{13}$$

where

$$P_W(a) = \int_{-\infty}^{+\infty} |W(\tau, a)|^2 \, d\tau \tag{14}$$

The properly sized chi-squared distributions for each of the scalogram coefficients $P_W(\tau, a)$ can be obtained by bootstrapping (*Cf.* Los, 2003, p. 252, Remark 338). The parameter $P_W(a)$ is then by itself chi-squared distributed with $\frac{T}{a}$ degrees of freedom and has itself as non-centrality parameter:

$$P_W(a) \sim \chi^2(\frac{T}{a}, P_W(a)) \tag{15}$$

Mathematical statistics teaches us that the mean/dispersion ratio of this chi-squared distribution is given by:

$$\frac{\mu\{P_W(a)\}}{\sigma\{P_W(a)\}} = \frac{[\frac{T}{a} + P_W(a)]}{\sqrt{2[\frac{T}{a} + 2P_W(a)]}} \approx \sqrt{\frac{T}{2a}} \tag{16}$$

which is larger when the number of observations T is larger, $T \uparrow \infty$ and the scale a is smaller, $a \downarrow 0$. Thus the scalegram $P_W(a)$ is better identified with more observations at finer scales of data resolution. For example, when we have $T = 10,000$ observations, for scale $a = 2$, we have $\frac{\mu}{\sigma} \approx 50$ or a 2% variation in the scalegram $P_W(a)$, but for scale $a = 8$, we have $\frac{\mu}{\sigma} \approx 25$ or a 4% variation. For one month of minute-by-minute observations $T \approx 40,000$, the variation is 1% and 2% respectively. As we saw earlier, the variation of the scalegram resides primarily in the identification uncertainty of its identifiable exponent $(2H + 1)$.

Therefore, together with all the engineers and scientists, who have confidently used wavelet MRA since Mallat (1989a, b and c) provided its rigorous theory, we are not too concerned about the potential lack of statistical determination or "insignificance" of this slope $(2H + 1)$ and thus of the Hurst exponent H, when the number of observations is large and the analysis is confined to the lower scales of resolution. Indeed, in all our empirical research on scaling in the financial markets we have observed that the actual statistical variation of slope $(2H+1)$ is very small, since most researchers use large numbers of observations.

Indeed, when Karuppiah and Los (2000) measured the Hurst exponent for nine Southeast Asian currency markets and checked how much it actually varied from month to month over a four month period, with ca. $40,000$ minute-by-minute observations/month, its variability was in the order of less than 2% either way over the first eight scales (Example: $H = 0.25+/-0.02$, which is how a physical scientist, who would not assume an underlying distribution, would accurately present this consistent slope measurement).

Classical statistical "significance testing" of unidirectional projection results based on inadmissible (and assumed) statistical distributions with infinite support cannot improve upon such precise range MRA measurement results, in particular at the finer scale resolutions. Moreover, this new wavelet analytic methodology is in agreement with the intuition that the smallest scale levels (= highest data resolution) should correspond with the highest degree of accuracy of exponent determination, while the largest scale levels (= lowest data resolution) should correspond with the lowest degree of accuracy.

3.2.3 Wavelet MRA and the Statistical Variance of Hurst Exponents

Our wavelet MRA is presented in the form of scalograms and corresponding scalegrams. In this paper, the wavelet resonance coefficients $W(\tau, a)$ are computed by Mallat's (1989a, b, and c) wavelet MRA with the Morlet-6 wavelet, using Kodak's online ION Script Research Systems Interactive Wavelet Program.[6] A wavelet scalogram $P_W(\tau, a)$, which is a visualization of the colorized squared wavelet resonance coefficients, identifies clearly the timing and power of all observed price innovations to the financial markets over all data frequencies. A wavelet scalegram

[6] A Morlet-6 wavelet can measure the precision of a non-smooth asymmetric distribution with 6 non-vanishing moments. In comparison, a smooth and symmetric normal distibution has only two unique non-vanishing moments. The Kodak's online ION Script Research Systems Interactive Wavelet Program can be found at: http://ion.researchsystems.com/IONScript/wavelet/

$P_W(a)$ is a time-averaged scalogram and is used to compute the monofractal Hurst exponents from the scaling slope of the resulting scale spectrum.

The monofractal Hurst exponent is constrained in its value: $0 < H < 1$. The crucial LM test is the (potentially falsifying) test of the hypothesis of the neutral short memory of the GBM ($H_0 : H = 0.5$,or, equivalently, $d = 0$) against the two alternative hypotheses of a persistent FBM ($H_0 : 0.5 < H < 1$, or $0 < d < 0.5$) or anti-persistent FBM ($H_0 : 0 < H < 0.5$, or $-0.5 < d < 0$).[7]

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$$\tau^{0.5}(\widehat{d} - d) \ \xrightarrow{d}\ N\left(0, \frac{\pi^2}{24}\right) = N(0, 0.41123) \tag{17}$$

giving rise to apparently simple inferential procedures. In addition, by employing a linear process for $x(t)$ based on martingale difference innovations, Robinson (1995a, b) used a Gaussian assumption and then rigorously established a more precise result, based on a slightly different estimator of the whole range $-0.5 < d < 0.5$:

$$\tau^{0.5}(\widehat{d} - d) \xrightarrow{d} N(0, \frac{1}{4}) = N(0, 0.25) \tag{18}$$

This result provides also apparently simple asymptotic interval estimates, as well as a simple test of neutrality, $d = 0$, *i.e.* $H = 0.5$. Robinson's treatment, based on the theoretical *assumption* of Gaussianness, actually covers multiple time series, possibly differing memory parameters (= multifractality), and more efficient tests for equality of these parameters.

[7] Physicists and astonomers use a color coding fo these various degees of persistence. When H approaches zero ($H \downarrow 0$), the time series is said to be blue noise. When $0 < H < 0.5$, the series is said to be anti-persistent or light blue. When $H = 0.5$, the series is neutral white noise. When $0.5 < H < 1$, the series' increments are said to be persistent, or pink noise. When H approaches 1 ($H \uparrow 1$), the series is said to be red noise. Once integrated white noise is brown noise (Cf. Los, 2003, p. 124, Table 4.3).

But both these limiting results are scientifically flawed, since they are based on the untenable presumption of independence, on inapplicable infinity (= limiting) arguments applied to the finite support for d or H, and prejudicial unidirectional projections (in case of Geweke and Porter-Hudak, 1983). The Hurst exponent $H = d + 0.5$ cannot be normally distributed, since it has known finite, support, $0 < H < 1$, implying that also the differencing parameter $d = H - 0.5$ must have finite support: $-0.5 < d < 0.5$. In contrast any normally distributed variable $z \sim N(0, \sigma^2)$ has infinite support: $-\infty < z < +\infty$.

The correct statistical theory for the H exponent, as identified from the wavelet MRA, is as follows. Flandrin (1992) and Flandrin and Gonçalvès (1996) proved that the detailed wavelet resonance coefficients, which correlate wavelets with particular segments of the time series $x(t)$,

$$W(\tau, a) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,a}(t)dt \tag{19}$$

- where $\psi^*_{\tau,a}(t)$ is a particularly localized wavelet - are Gaussian with mean zero and a variance that is their own value squared:[8]

$$W(\tau, a) \sim N(0, |W(\tau, a)|^2) \tag{20}$$

The elements of the wavelet scalogram consist, by definition, of these "coefficients of determination":

$$P_W(\tau, a) = |W(\tau, a)|^2 \tag{21}$$

Each coefficient of the finitely tiled scalogram $P_W(\tau, a)$ is then chi-squared distributed with one degree of freedom and non-centrality parameter $P_W(a)$:

$$P_W(\tau, a) \sim \chi^2(1, P_W(a)) \tag{22}$$

where

$$P_W(a) = \int_{-\infty}^{+\infty} |W(\tau, a)|^2 \, d\tau \tag{23}$$

[8] Despite the ∞ signs in the integral, the empirical support for these wavelet coefficients is finite, since the wavelet support is empirically finite: $1 < \tau < T$ and $1 < a < T$.

The properly sized chi-squared distributions for each of the scalogram coefficients $P_W(\tau, a)$ can be obtained by bootstrapping (*Cf.* Los, 2003, p. 252, Remark 338). The parameter $P_W(a)$ is then by itself chi-squared distributed with $\frac{T}{a}$ degrees of freedom and has itself as non-centrality parameter:

$$P_W(a) \sim \chi^2(\frac{T}{a}, P_W(a)) \tag{24}$$

Mathematical statistics teaches us that the mean/dispersion ratio of this chi-squared distribution is given by:

$$\frac{\mu\{P_W(a)\}}{\sigma\{P_W(a)\}} = \frac{[\frac{T}{a} + P_W(a)]}{\sqrt{2[\frac{T}{a} + 2P_W(a)]}} \approx \sqrt{\frac{T}{2a}} \tag{25}$$

which is larger when the number of observations T is larger, $T \uparrow \infty$ and the scale a is smaller, $a \downarrow 0$. Thus the scalegram $P_W(a)$ is better identified with more observations at finer scales of data resolution. For example, when we have $T = 10,000$ observations, for scale $a = 2$, we have $\frac{\mu}{\sigma} \approx 50$ or a 2% variation in the scalegram $P_W(a)$, but for scale $a = 8$, we have $\frac{\mu}{\sigma} \approx 25$ or a 4% variation. For one month of minute-by-minute observations $T = 40,320$, the variation is 1% and 2% respectively. As we saw earlier, the variation of the scalegram resides primarily in the identification uncertainty of its identifiable exponent $(2H + 1)$.

The skewness of the chi-squared distribution of the scalegram is very low:

$$skewness\{P_W(a)\} = \frac{2^{3/2}\left[\frac{T}{a} + 3P_W(a)\right]}{[\frac{T}{a} + 2P_W(a)]^{3/2}} \approx 2^{3/2}\left(\frac{a}{T}\right)^{1/2} \tag{26}$$

and vanishes, implying a symmetric distribution, when $T \uparrow \infty$. The kurtosis of this chi-squared distribution of the scalegram is:

$$kurtosis\{P_W(a)\} = 3 + \frac{12[\frac{T}{a} + 4P_W(a)]}{[\frac{T}{a} + 2P_W(a)]^2} \approx 3 + 12\left(\frac{a}{T}\right) \tag{27}$$

which approaches that of normality $(= 3)$, when $T \uparrow \infty$.

Therefore, together with all the engineers and scientists, who have confidently used wavelet MRA since Mallat (1989a, b and c) provided its rigorous theory, we are not too concerned about the potential lack of statistical determination or "insignificance" of this slope $(2H + 1)$ and thus of the Hurst exponent H. In particular, we are not concerned when the number of observations is large and the analysis is confined to the lower scales of resolution. In all empirical research on scaling in the financial markets reported thus far we have observed that the actual statistical variation of slope $(2H+1)$ is very small, since most researchers use large numbers of observations.

Indeed, when Karuppiah and Los (2000) measured the Hurst exponent for nine Southeast Asian currency markets and checked how much it actually varied from month to month over a four month period, with ca. $40,000$ minute-by-minute observations/month, its variability was in the order of less than 8% either way over the first eight scales (Example: $H = 0.25+/-0.02$, which is how a physical scientist would accurately present the measured range of the Hurst exponent without assuming an underlying distribution).

Classical statistical "significance testing" of unidirectional projection results based on inadmissible (and assumed) statistical distributions with infinite support cannot improve upon such precise range MRA measurement results, in particular at the finer scale resolutions. Moreover, this new wavelet analytic methodology is in agreement with the intuition that the smallest scale levels (= highest data resolution) should correspond with the highest degree of accuracy of exponent determination, while the largest scale levels (= lowest data resolution) should correspond with the lowest degree of accuracy.

4 EMPIRICAL ANALYSIS

Table 2 reports the descriptive statistics of the six stock indices and five exchange rates studied in this paper. We report the characteristics of both the daily price and return series. It is clear in the table that none of the series are Gaussian, since all data series have non-zero excess skewness and non-normal levels of kurtosis. The series exhibit consistent features across countries in terms of kurtosis, since all price series are platykurtic and all return series are leptokurtic.

[TABLE 2 ABOUT HERE]

Figure 1 contains the distribution plots of some stock index price series and exchange rate series. It reports the descriptive statistics of the price series, $X(t)$ and of their return series, $x(t)$ of stock indices and foreign exchange rates of Argentina, Brazil, Chile, Colombia, Mexico, and Venezuela.[9]

[FIGURE 1 ABOUT HERE]

Figure 2 reports the results of the visual check on ergodicity by way of windows of increasing length. The charts plot the four moments (on the Y-axis) against the number of observations (on the X-axis). The first four moments are computed with increasing window sizes of the data sets, and these windowed results are plotted. If the series are ergodic, the plotted moments should converge and then stay constant as the window sizes are increased. However, these plots clearly show that none of the series show ergodicity. There are sharp discontinuities and often divergences. Figure 2 only reports the plotted moments of stock indices and exchange rate of Mexico for both price and return series. The plotted increasing-window moments of the other countries behave very similarly and are available upon request.

[FIGURE 2 ABOUT HERE]

In addition to these increasing-window moments, the moving average moments of the first four orders, $E\{X^q(t)\}$ and $E\{x^q(t)\}$ for $q = 1, 2, 3$ and 4, are computed with a constant window size of fifty days for each series. The plotted moments fluctuate in value and do not converge, indicating that none of the financial series is stationary in either the strict or the wide sense, since their first four (moving average) moments are time-varying. These results are independent of the window size, since windows of different sizes show similar results. Figure 3 reports these checks on stationarity by way of moving windows. Again, we use Mexico as an example and the charts plot

[9] Since Argentina is dollarized, there is no floating FX rate for Argentina.

the four moments (the Y-axis) against the number of observations (the X-axis) and the similar charts of the other countries are available upon request from the authors.

[FIGURE 3 ABOUT HERE]

The first two moments of the exchange rate price and return series show some signs of apparent convergence, but the last two moments do not appear to converge. The conclusions from these ergodicity and stationarity checks is that none of the financial series in this study meet the assumptions of the conventional lognormal price diffusion models prevalent in the theoretical finance literature.

The usual stationarity models would assume that the empirical time series exhibit short-term (serial) dependence, which would be identified easily by their ACF. In Figure 4, we compare the ACF of Chilean and Venezuelan stock indices and exchange rates (both price and return) with the ACF of the simulated GBM, which declines immediately. It seems that the ACFs of stock price series die down faster than those of exchange rate series. However, they still exhibit long-term dependence, since their ACFs converge to zero at much slower rate than those of the GBM. The ACFs of the stock return series fluctuates.

[FIGURE 4 ABOUT HERE]

Having shown that the empirical time series are long-term dependent, and thus exhibit LM, we identify whether these series are persistent or anti-persistent, by calculating their monofractal Hurst exponents. The identified Hurst exponents are reported in Table 3. The results show that the stock markets of Argentina, Chile and Venezuela are persistent, as their Hurst exponents are significantly greater than 0.5. The Columbian stock market is surprisingly anti-persistent with $H = 0.42$ and is significantly different from 0.5, the Hurst exponent for the independent innovation of GBM. Only the Brazilian and Mexican stock indices confirm to the independence of the innovations of the GBM with Hurst exponents of 0.5. The exchange rates of all countries are persistent except for Mexico. The Hurst exponent of the Mexican exchange rate is 0.41 and is again significantly different from 0.5, showing Mexican foreign exchange market to be ultra-efficient (= fast mean-reverting) compared to those of other countries in our data set.

[TABLE 3 ABOUT HERE]

Figure 5 (Panel A through D) reports the results of the wavelet MRA for selected Latin American financial markets. There are three parts in each panel. Part a is the plot of the original time series and the type of wavelet used to analyze the time series (Morlet-6 wavelet). Part b is the localizing wavelet scalogram, MRA, or local power spectrum in a time-frequency frame. Finally, part c shows the wavelet scalegram, equivalent to the conventional average power spectrum turned 90 degrees.

[FIGURE 5 ABOUT HERE]

Panel A of Figure 5 shows the regime changes for Mexican peso. The Mexican peso experienced a change from pegged to float regime on December 20, 1994, from float to fixed on May 4, 1995, and from fixed to float again on October 16, 1995. The timing and power (= magnitude of local risk, or its local volatility) of these drastic currency regime changes are detected very sharply by the scalogram, where both floats are represented by substantial power at many frequencies. There is considerably less power during the pegged and fixed regimes. Starting from late 1998, a lot of noise trading is identified by the scalogram and occasional energy bursts, where they coincide with the Brazilian stock market crisis (September 10, 1998) and the Brazilian currency float (January 15, 1999).

The identified monofractal Hurst exponents for sub-periods show that the Mexican peso market has become more efficient over time. Before the first float, the identified Hurst exponent is 0.57, while it finally declined to 0.29 after the Brazilian stock market crisis, implying that the Mexican peso market went from mildly persistent to very anti-persistent or ultra-efficient, *e.g.* as efficient as a deep anchor-currency market. This market efficiency-enhancing trend is consistent with the global Hurst exponent for Mexican currency market reported in Table 3.

Panel B of Figure 5 shows the MRA of the Mexican peso return series. The Mexican peso crisis in December 1994 is represented by a turbulence vortex (= rapid frequency change from low to high frequency followed by an immediate reversal of that change), which in the scalogram is a singularity cone cutting through most of the frequencies with significant power. The second float is represented by a smaller vortex with much less power. The figure shows the powerful shock the currency crisis brought to the FX market trading, and how the market was trying to adjust to this shock with increased speed and efficiency of trading. The scalegram also illustrates that Mexican peso innovations do not consist of white noise: it shows clearly the institutional trading cyclicities at weekly, quarterly and yearly frequencies.

Panel C of Figure 5 presents Brazilian currency (real) regime change from fixed to float. During the fixed exchange rate period, the scalogram shows almost no power. By contrast, starting from the Brazilian real float in January 1999, the scalogram detects a lot of noise trading in Brazilian currency markets. These energy bursts happen at high frequencies over a lengthy time period, indicating how the market was trying to adjust itself to the new trading regime. The global Hurst exponent for the overall period is 0.66 apparently indicating an overall persistent Brazilian currency market. However, the high persistence of the overall period is due mainly to the pegged regime whose Hurst exponent is 0.67. The Hurst exponent after the float is only 0.46 showing the market becomes mildly anti-persistent and more efficient after the float, as clearly visualized by the scalogram. This contrast also suggests that a monofractal Hurst exponent of the complete time series does not describe the market innovations precisely. A Hurst exponent should be computed for each sub-period or a series of subperiods, implying that the financial maket is multifractal.

Panel D of Figure 5 presents Chilean stock index return plots. The scalogram shows that the shocks from Brazilian stock market crisis and Brazilian float affected also the Chilean stock market. Another striking vortex in this scalogram is corresponding to the widespread selling event of Chilean shares in January 2002 caused by the tender of Enerquinta shares. This singular event manifested itself as a large market price drawdown of around 30 percent of returns in the return plot, and in the scalogram the corresponding vortex cuts through most of the frequencies with large power. The scalogram thus identifies both the contagion from other markets and the shocks of single events in domestic markets. Again, the scalegram of Chilean stock index return indicates that this market is not truly identified by a GBM, since institutional cyclicities can be identified at weekly, biweekly, quarterly, and two-year frequencies. For a GBM, the scalegram of the innovations should show a flat straight line with a zero slope ($= 2H - 1$ with $H = 0.5$) across all trading frequencies, indicating white noise innovations.

5 CONCLUSIONS

Though the popular GBM price diffusion models claim to provide a good fit to financial time series data, their assumptions of normality, stationarity and independence of the residual innovations are falsified by the empirical data. In this study, we use accurate and complete signal processing methods to check such fundamental characteristics that are of importance for (1) risk measurements, (2) asset and options valuations, for (3) portfolio selection and management, and for (4) international capital allocation. The series included in this paper are the financial data from Latin American stock markets and currency markets. The stationarity, ergodicity and independence of the available financial time series of each Latin American country are checked and found to be non-existent. The series are also examined for their degrees of long-term time dependence. Their global degrees of long-term dependence are measured by the monofractal Hurst exponents identified from the respective wavelet MRAs.

The empirical evidence shows that the empirical financial data of Latin American financial markets possess characteristics that are different from what the conventional theoretic financial models assume. The first four moments of our empirical data series are non-stationary, non-ergodic and exhibit long-term dependence, and not independence of the innovations. When we compare the ACFs of the empirical data with those of simulated GBMs, we can easily detect the difference between the two, especially for Mexico for which we have a slightly larger data set.

Identified by their Hurst exponents, most Latin American financial markets are persistent, with Columbian stock market and Mexican exchange market are anti-persistent. Only the Brazilian and Mexican stock markets are, perhaps overall, identified by GBMs. The Hurst exponents only measures the general degree of dependence or persistence of the markets, but still fails to reflect how the market prices actually adjust to the shocks. Thus one needs to be very careful interpreting the values of reported Hurst exponents.

The wavelet MRAs reveals the Latin American financial time series to be highly non-stationary showing cyclicities and singularities. Scalograms facilitate detailed inspection of how the financial markets adjust to major interventions. Stock and foreign exchange market time and frequency adjustments phenomena are clearly observable after major interventions. Vortices can be found in the scalograms, corresponding to the major interventions such as the Mexican currency crisis, Brazilian stock market and currency crises, and a single sharp selling event in Chilean stock market. These findings are consistent with current studies of the observed contagion among Latin American countries.

The implication of this paper is that extremely caution needs to be exercised when applying conventional econometric residual stationarity dynamic process models to empirical financial series. Especially for the speculative and highly volatile emerging markets, like the Latin American stock and currency markets discussed in this paper, one needs to recall that the highly localized risks cannot be simply explained within the Markowitz mean-variance framework, since all distributions are non-Gaussian distributions. Moreover, the distributions of rates of returns do not scale gradually but sometimes change violently.

Investors and portfolio managers tend to believe that there are many opportunities of making profit in emerging markets before actually studying the empirical features of those markets. Indeed, it is impossible to predict the returns in those Latin American financial markets which are persistent, because they show sharp and unpredictable discontinuities, although there are long periods of sheer price inertia that may give the impression of predictability. Furthermore, being theoretically predictable does not mean that a time-invariant valuation model can be identified and that one can earn abnormal returns.

Persistent series often show rare unexpected, but sharp discontinuities, as clearly shown by the singular drawdown in the Chilean stock market. In contrast, anti-persistent markets show ultra-fast mean price reversions. They give the impression to be unpredictable, but are actually much more predictable than the persistent markets because their trading ranges remain very limited. In both cases, though, the current theoretical diffusion models are far from sufficient to completely identify the empirical financial markets for either prediction or invariant intrinsic valuation.

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7 TABLES

Table 1	**Data**	**Descriptions**	
Panel A:	**Stock**	**Market**	**Indices**
COUNTRY	INDEX NAME	PERIOD COVERED	NO. OF OBS.
Argentina	BUSE Merval Indx (MERV)	10/08/96 – 10/26/01	1253
Brazil	BRSP BOVESPA IND(BVSP)	04/27/93 – 10/26/01	2104
Chile	SASE Select Inx (IPSA)	06/07/97 – 10/26/01	1089
Colombia	Peru: Lima General Inx (IGRA)	04/28/98 – 10/26/01	871
Mexico	MXSE IPC GRAL IN (MXX)	11/08/91 – 10/26/01	2480
Venezuela	IBC INDEX (IBC)	04/28/98 – 10/26/01	852

Table 1	**Data**	**Descriptions**	
Panel B:	**Foreign**	**Exchange**	**Rates**
COUNTRY	INDEX NAME	PERIOD COVERED	NO. OF OBS.
Brazil	Brazilian Real (BRL)	10/22/95 – 12/18/01	2250
Chile	Chile	06/01/97 – 10/31/01	1614
Colombia	Colombia	04/01/98 – 10/31/01	1310
Mexico	Mexican Peso (MXP)	01/04/93 – 11/30/01	3253
Venezuela	Venezuelan Bolivar (VEB)	05/01/96 – 12/01/01	2041

Table 2	Descriptions:	Stock Indices		Foreign Exchange Rates	
		X(t)	x(t)	X(t)	x(t)
Argentina	Mean	546.92666	-0.0717289	NA	NA
	Variance	19665.256	5.309872	NA	NA
	Skewness	0.2846037	-0.5175271	NA	NA
	Kurtosis	-0.3714249	5.1442814	NA	NA
Brazil	Mean	8441.0358	0.2936553	1.521298356	0.0419508
	Variance	261.80521	9.1399316	0.267834563	0.6143049
	Skewness	0.098859	0.5147128	0.658967203	3.2949488
	Kurtosis	-1.0184553	7.9328273	-0.663923268	72.755232
Chile	Mean	113.63387	-0.03273	516.135	0.035
	Variance	314.29688	2.8530028	5332.400	0.175
	Skewness	0.5355462	-7.7918004	0.855	3.777
	Kurtosis	-0.4322389	164.71036	0.523	68.627
Columbia	Mean	1488.8049	-0.0545283	1917.441	0.043
	Variance	47782.299	1.3545021	109411.123	0.228
	Skewness	0.2620203	-0.0235501	-0.273	2.403
	Kurtosis	-1.3365513	4.64514	-1.320	24.313
Mexico	Mean	3748.3433	0.0560487	7.300334829	0.0336163
	Variance	3172037.5	3.3820698	5.920033696	0.8722933
	Skewness	0.4422013	-0.032757	-0.746398505	7.0552532
	Kurtosis	-1.0336986	4.8992809	-0.938361995	242.41246
Venezuela	Mean	5810.7764	0.0062416	591.0932425	0.0233582
	Variance	1759102.5	5.4498896	8466.752941	0.0177385
	Skewness	-0.2816602	1.2282356	0.128048915	-2.5937952
	Kurtosis	-0.8718848	13.333014	-1.432647717	56.709366

Table 3: Identified Hurst Exponents

This table reports the identified monofractal Hurst exponents of the stock indices and the foreign exchange rates. The Hurst exponents are identified from the logarithm of the scale density of the Fractional Brownian Motion (FBM) against the $\log_2(a)$ The slope coefficient of the resulting line is $(2H + 1)$, so that . A Hurst exponent of 0.5 indicates that the financial market follows a Geometric Brownian Motion (GBM), while a Hurst exponent between 0.5 and 1.0 means the market is persistent, and a Hurst exponent between 0 and 0.5 means the market is anti-persistent. The reported minimal accuracy range is based on the number of observations T daily observations reported in Tables 1A and 1B for each series and scale $a = 32$ days ($\approx$ one month of trading days).

	Stock Indices	**FX Rates**
Argentina	$0.79 +/- 0.18$	N/A
Brazil	$0.50 +/- 0.09$	$0.66 +/- 0.11$
Chile	$0.79 +/- 0.19$	$0.66 +/- 0.13$
Colombia	$0.42 +/- 0.11$	$0.61 +/- 0.09$
Mexico	$0.50 +/- 0.08$	$0.41 +/- 0.05$
Venezuela	$0.79 +/- 0.22$	$0.66 +/- 0.12$

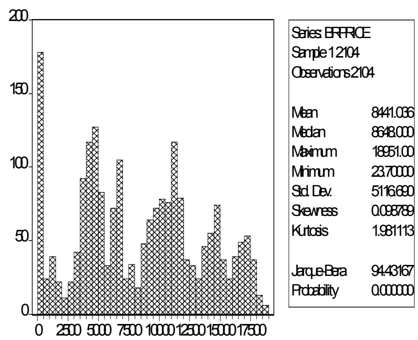
8 FIGURES

Figure 1: Probability Distribution of Latin American Financial Time Series

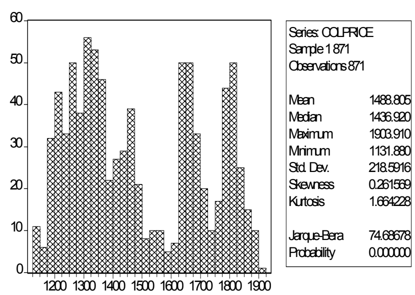
This figure reports the histograms of some selected stock indices (Panel A) and foreign exchange rates (Panel B). Notice their irregular non-Gaussian shapes.

Panel A: Stock Market Indices

Brazil: Price Series

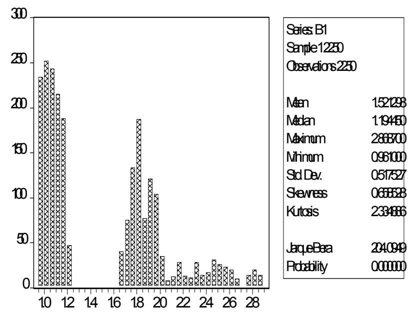


Colombia: Price Series

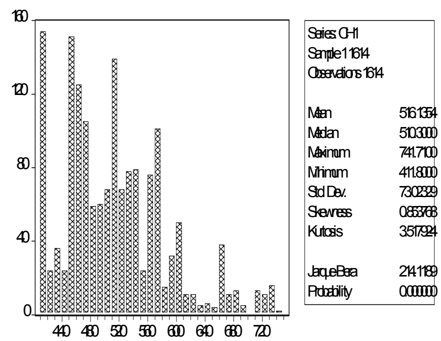


Panel B:Foreign Exchange Rates

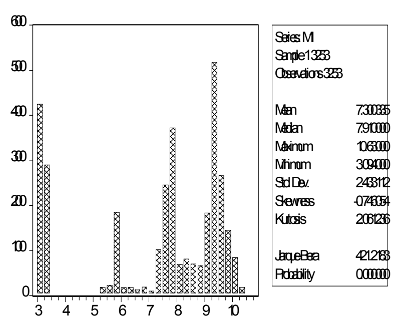
Brazil: Original Exchange Rate



Chile: Original Exchange Rate



Mexico: Original Exchange Rate



Venezuela: Original Exchange Rate

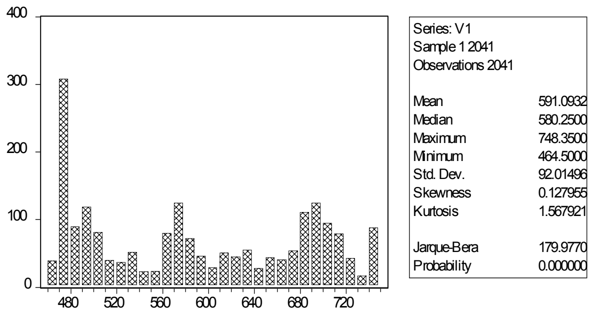
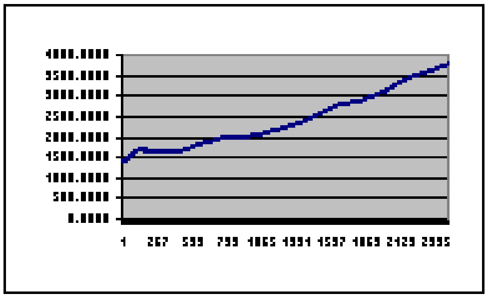


Figure 2: Four Moments of Stock Indices and Exchange Rate of Mexico with Increasing Windows

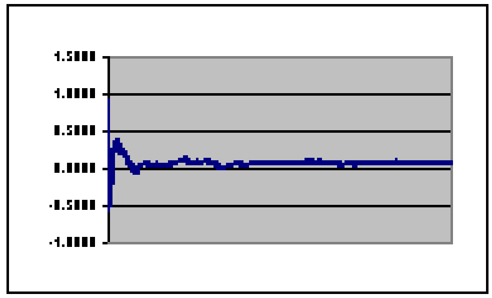
This set of figures plot the first four moments (on the Y-axis) against the number of observations (on the X-axis). The first four moments are computed with increasing window sizes of the data sets, and the windowed results are plotted.

Mean

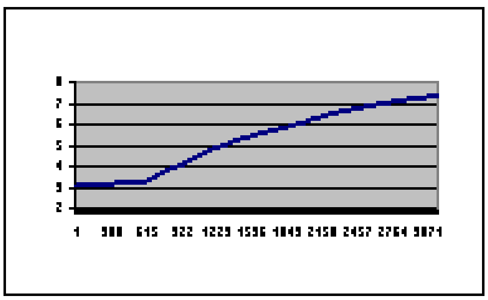
Stock Index Price



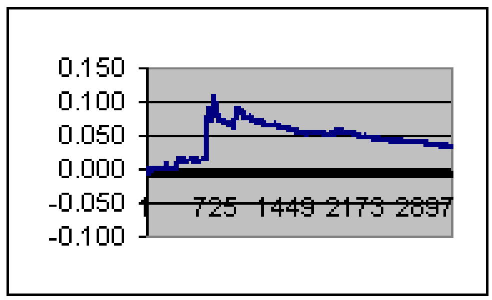
Stock Index Return



Exchange Rate Price

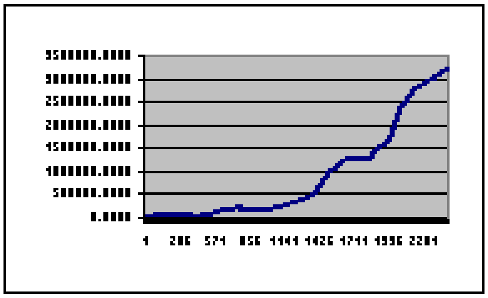


Exchange Return

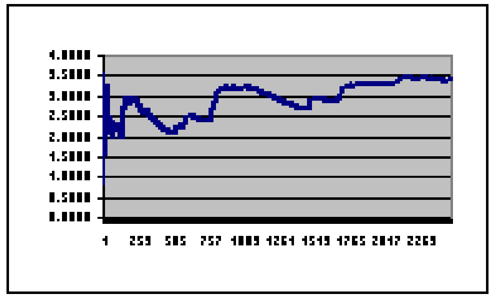


Variance

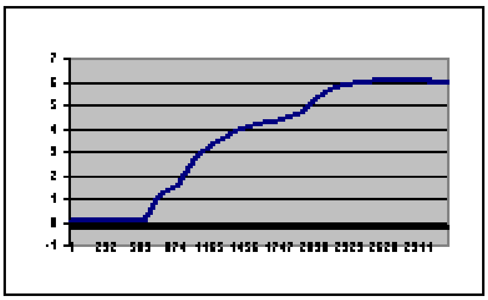
Stock Index Price



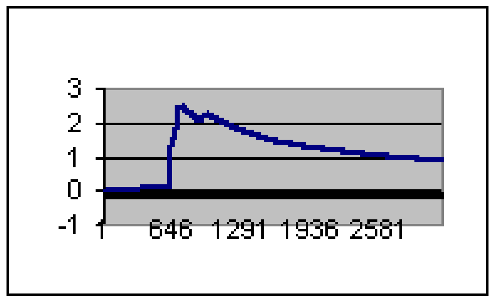
Stock Index Return



Exchange Rate Price

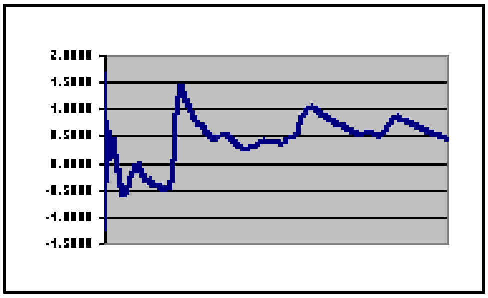


Exchange Return

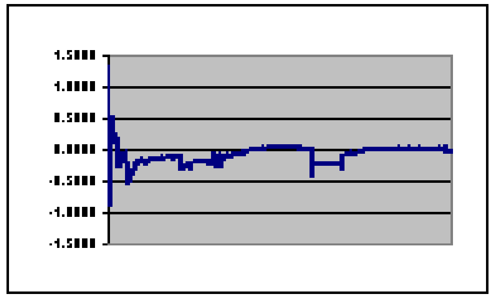


Skewness

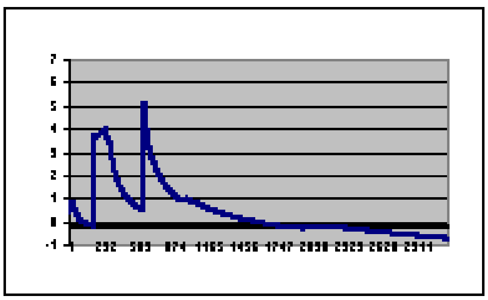
Stock Index Price



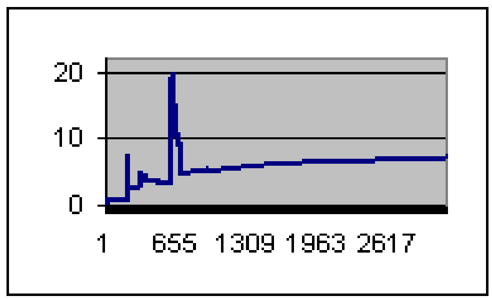
Stock Index Return



Exchange Rate Price

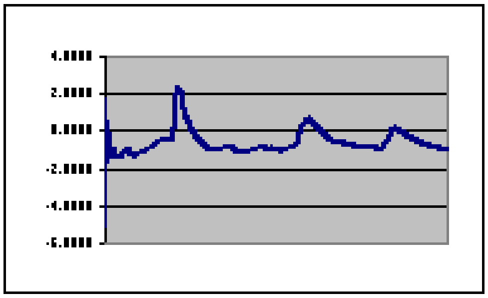


Exchange Return

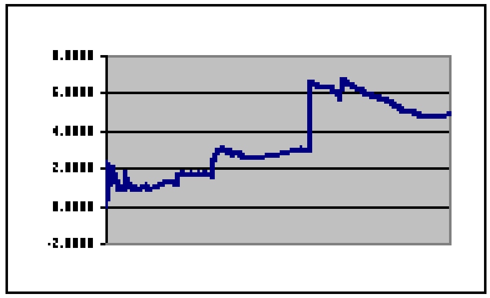


Kurtosis

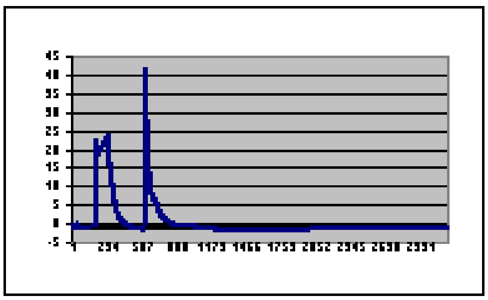
Stock Index Price



Stock Index Return



Exchange Rate Price



Exchange Return

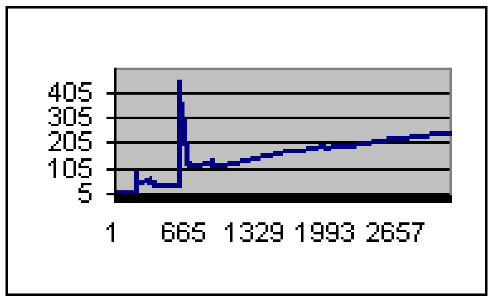
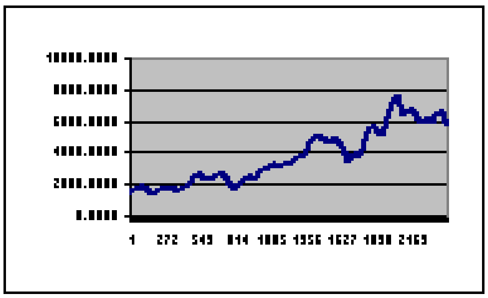


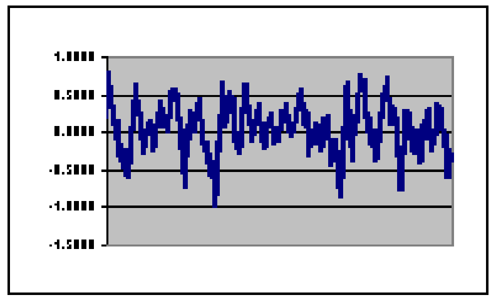
Figure 3: Four Moving Moments of Stock Indices and Exchange Rate of Mexico with Fixed Window Size

Mean

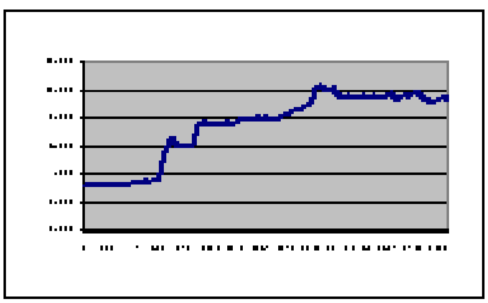
Stock Index Price



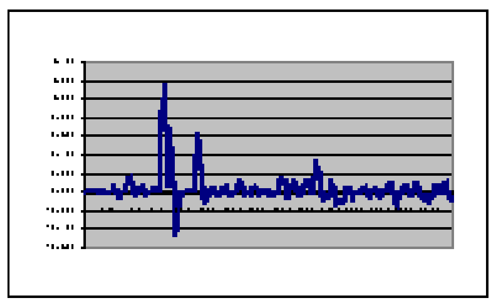
Stock Index Return



Exchange Rate Price

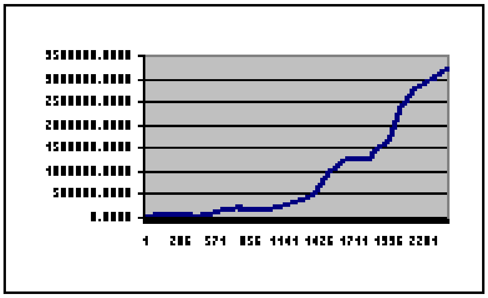


Exchange Return

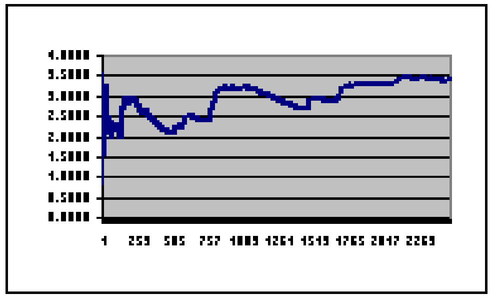


Variance

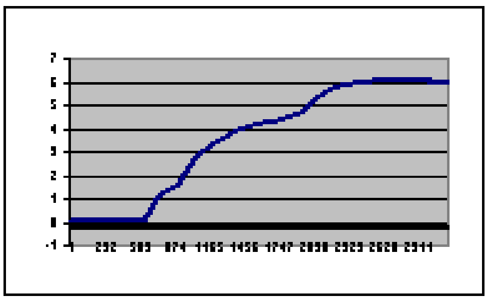
Stock Index Price



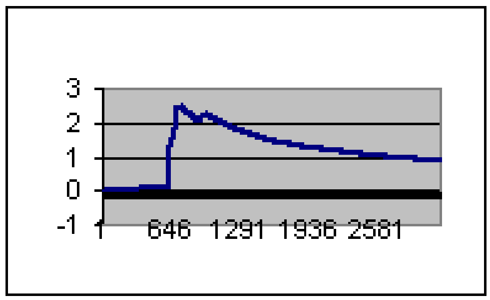
Stock Index Return



Exchange Rate Price

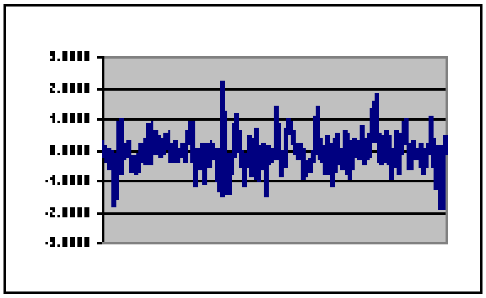


Exchange Return

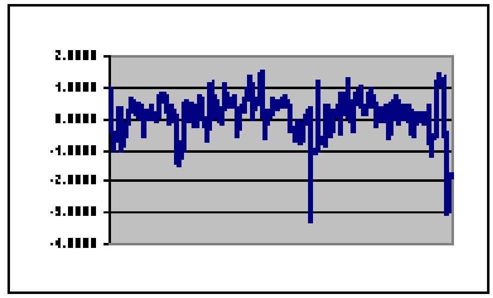


Skewness

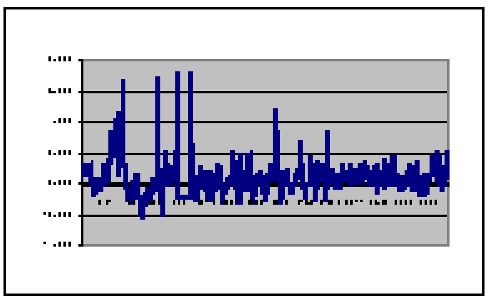
Stock Index Price



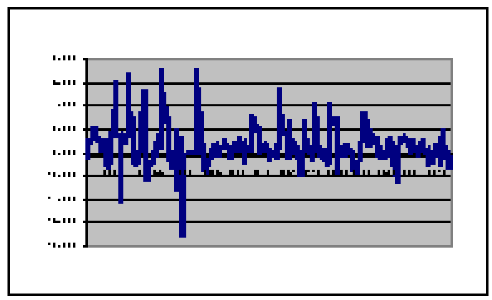
Stock Index Return



Exchange Rate Price

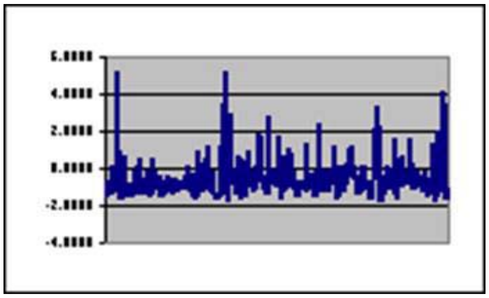


Exchange Return

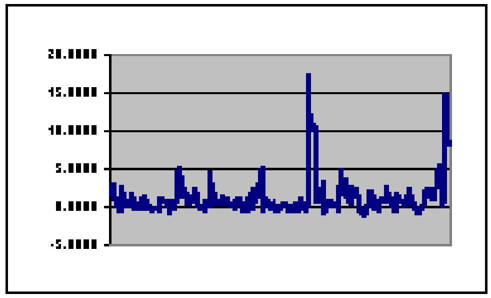


Kurtosis

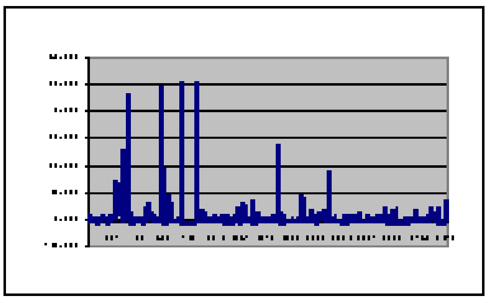
Stock Index Price



Stock Index Return



Exchange Rate Price



Exchange Return

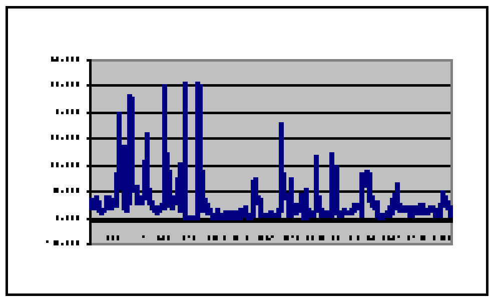
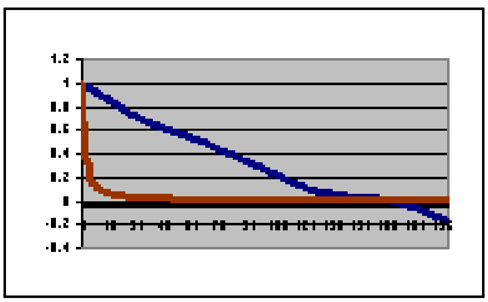


Figure 4: Comparison of Empirical ACFs and Geometric Brownian Motion ACF

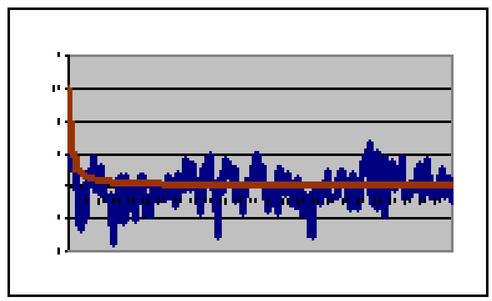
This set of figures compares the rapidly declining ACF of the theoretical Geometric Brownian Motion (GBM) with the slowly declining ACFs of the empirical stock series and exchange rates of selected Latin American countries. The price and return series (on the Y axis) are plotted against the number of observations (on the X axis).

Chile

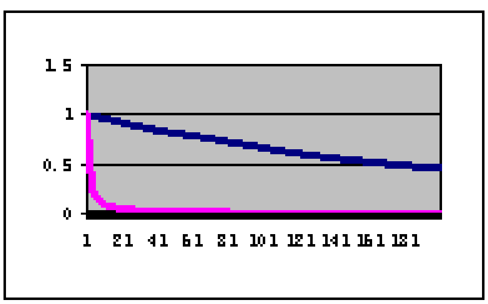
Stock Index Price



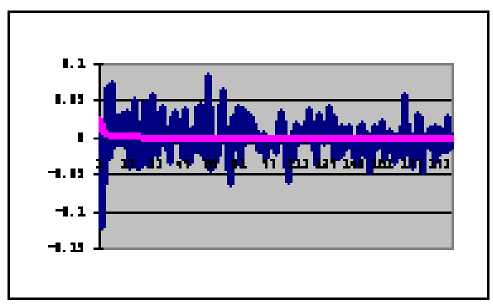
Stock Index Return



Exchange Rate Price

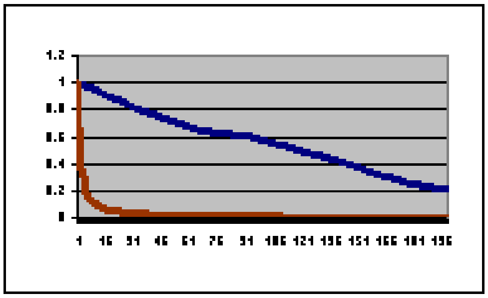


Exchange Return

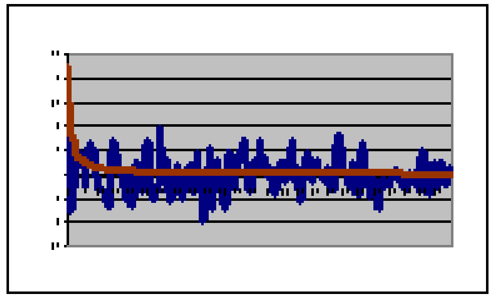


Venezuela

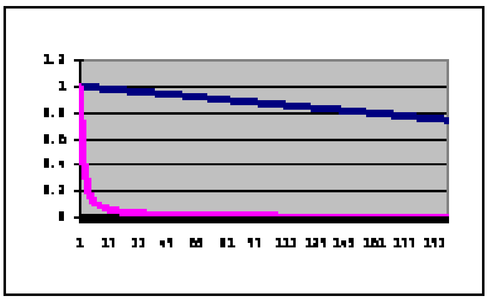
Stock Index Price



Stock Index Return



Exchange Rate Price



Exchange Return

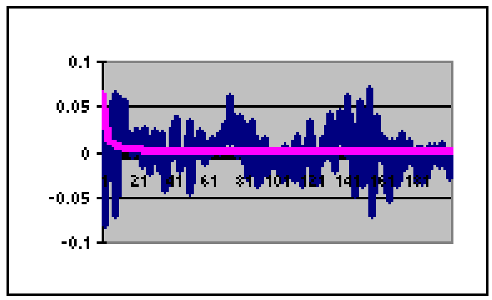
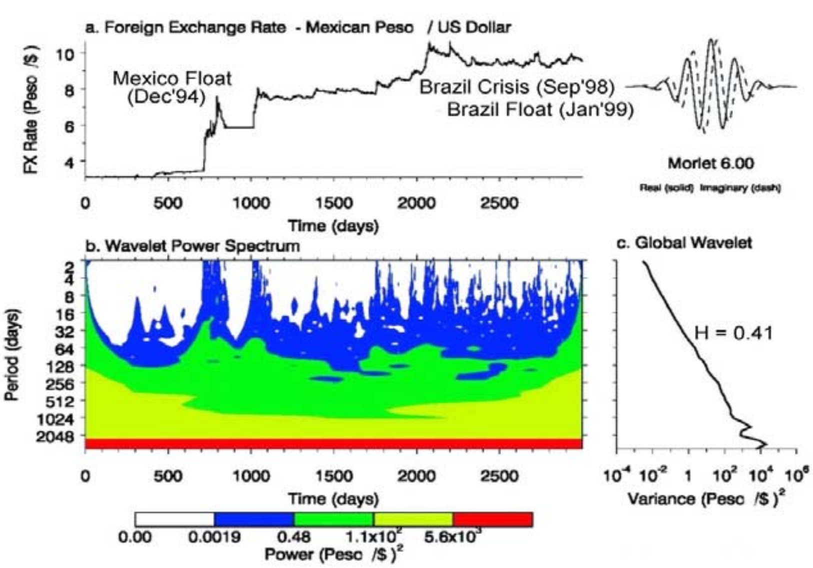


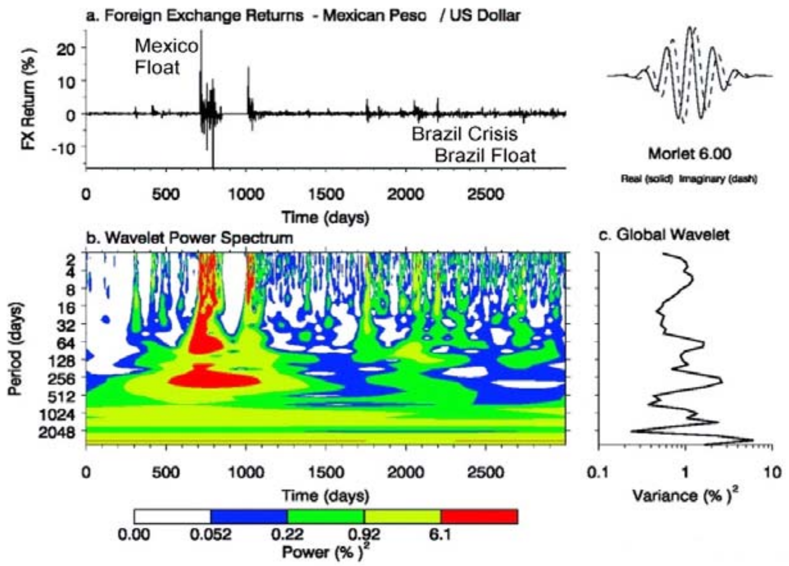
Figure 5: Wavelet Scalograms and Scalegrams

This portfolio of figures contains four panels consisting of time series, scalograms and scalegrams based on wavelet multiresolution analysis. Panel A analyzes the exchange rate series of Mexican peso, Panel B analyzes the FX rate of return series of Mexican peso, Panel C analyzes the exchange rate series of Brazilian real, and Panel D analyzes the rates of return of the Chilean stock indices. There are three parts in each plot. Part a is the plot of original time series and the type of wavelet used to analyze the time series (*i.e.*, the Morlet−6.0 wavelet). Part b is the scalogram, which is a colorized plot of the squared value of the wavelet resonance coefficients, i.e. the time-fequency-localized "coefficients of determination.". Part c is the scalegram, which plots the variances of the wavelets against the scales and can be viewed as the statistical time average of the scalogram. It demonstrates the monofractal scaling of the time series. When $H = 0.5$ we look at a GBM, when $0 < H < 0.5$, we look at an anti-persistent FBM (*e.g.,* Mexico)and when $0.5 < H < 1$, we look at a persistent FBM (*e.g.,* Brazil).

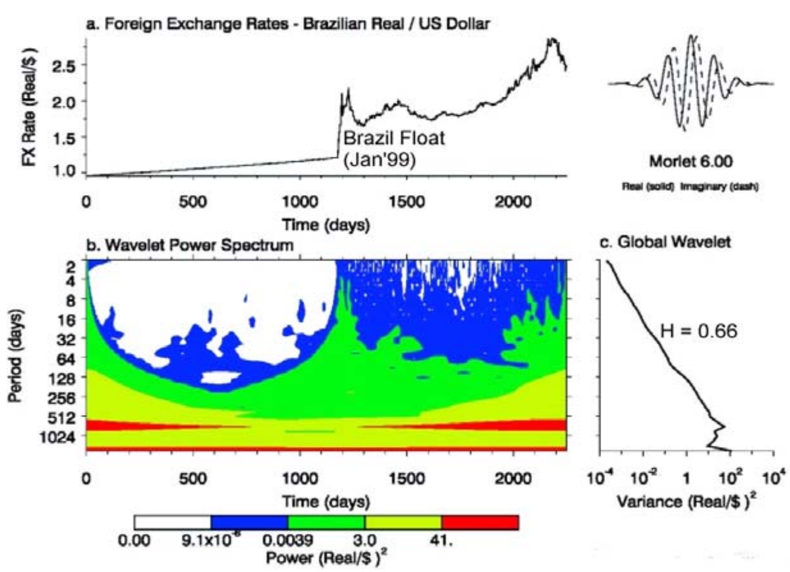
Panel A



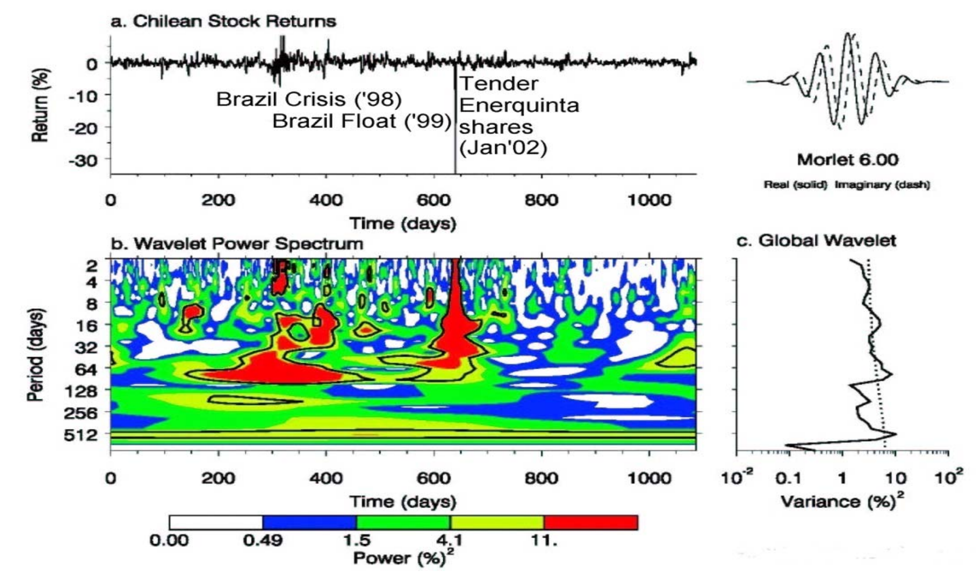
Panel B



Panel C



Panel D


a. Chilean Stock Returns
Return (%)
0
-10
-20
-30
Brazil Crisis ('98)
Brazil Float ('99)
Tender
Enerquinta
shares
(Jan'02)
0 200 400 600 800 1000
Time (days)
Morlet 6.00
Real (solid) Imaginary (dash)
b. Wavelet Power Spectrum
Period (days)
2
4
8
16
32
64
128
256
512
0 200 400 600 800 1000
Time (days)
0.00 0.49 1.5 4.1 11.
Power (%)^2
c. Global Wavelet
10^{-2} 0.1 1 10 10^2
Variance (%)^2