The Equity Premium: Consistent with GDP Growth and Portfolio Insurance

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We find that the long-term equity premium is consistent with both GDP growth and portfolio insurance. We use a supply-side growth model and demonstrate that the arithmetic average stock market return and the returns on corporate assets and debt depend on GDP per capita growth. The implied equity premium matches the U.S. historical average over 1926–2001. Separately, we find that the equity premium tracks the value of a put option on the S&P 500. Our theory predicts a smaller equity premium in the future, assuming the recent regime shifts in dividend policies, interest rates, and tax rates are permanent.

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1. Introduction

The equity premium, typically defined as the difference between the S&P500 return and a risk-free return, is a critical input in portfolio allocation decisions as well as in capital budgeting decisions. It is also at the heart of the ongoing policy debate about whether a portion of the social security trust fund should be invested in the equity market or not. Consequently, it is crucial for finance professionals to be able to accurately gauge the size of the premium and to understand the factors that may change its value.

In their seminal paper, Prescott and Mehra (1985) show that the standard economic growth model is unable to explain the historical difference between the average stock return on a broad index versus risk-free bonds in the United States. This has become known as the equity premium puzzle. Although the fact that equity returns are higher than short-term bond yields makes intuitive sense, the puzzle is that standard risk measures cannot explain the size of the historical difference in returns. Many attempts at modifying the standard model fall short of fully explaining the size of the premium (Campbell, 2003; Kercholakota, 1996; Mehra, 2003). Although a few successful models have been proposed, the current asset pricing literature does not agree on a solution to the equity premium puzzle as well as other asset pricing anomalies.[1]

Empirical approaches to the study of the equity premium, although not articulating a full explanation of the puzzle, do offer alternative pathways in the quest for a solution to

[1] In two recent articles, McGrattan and Prescott (2000, 2001) claim to finally put the puzzle to rest. In their 2000 article, they show that the high ratio of market value of equity to gross national product (GNP) at the beginning of the year 2000 is rationally based when computing the value of corporate tangible and intangible capital assets. In the subsequent 2001 article, they show that during the postwar period, the large rise in equity values is as predicted by the theory, once the historical change in the taxation of dividends and the increase in holdings in tax-deferred accounts are accounted for. However, McGrattan and Prescott (2001) focuses on explaining recent equity returns and does not explain the historical difference between stock and risk-free returns.

this important puzzle. Asness (2000) draws a link between the size of the expected premium and differences in the volatility of stocks versus bonds for horizon periods of 20 years. However, his analysis does not account for the size of the average premium over the full historical record.[2] Ibbotson and Chen (2003) use a building-block approach to estimate the long-term stock market return, and relate some of the market returns components to gross domestic product (GDP) growth. However, they do not establish a firm theoretical basis for relating the *equity premium* directly either to GDP growth or to risk.

In this article, we show theoretically and empirically that in the long-run, the equity premium is directly related to GDP growth and is consistent with downside risk avoidance. Thus, we establish that the historical average value of the equity premium is fully accounted for by risk and growth-based explanations. These results in turn enable us to accurately gauge the size of the historical premium and to predict changes in the premium given extrapolated recent trends.[3]

First, we use a supply-side growth model and develop the macroeconomic equivalent of the standard sustainable growth formula found in corporate finance textbooks to determine the long-run average return on stocks. The average stock return depends on GDP per capita growth and the earnings retention ratio as well as a premium reflecting systematic risk, which is linked to de-trended earnings[4] and market-to-book volatilities and the response of dividend and share repurchase policies to these factors.

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[2] Asness (2000) uses the dividend and earnings yield as proxies for expected returns. Although the approximations may well be valid, there is evidence that bonds may be more risky than stocks for horizons of 20 years or longer when looking at ex-post return volatility, as shown by Siegel (2002).

[3] It is important to emphasize that we do not claim to resolve the equity premium puzzle here, because the puzzle is really about the failure of the standard asset pricing model. However, it is also a fact that *no other class of models* has been so far put forth to show the consistency of macroeconomic growth rates, risk, and other behavioral variables with the size of the premium, which is what we attempt to do here.

[4] Earnings are scaled (or de-trended) by book value of equity. In other words, we examine ROE volatility.

We then establish a relationship between GDP growth and the required returns on corporate assets and debt. Based on an empirical analysis at the aggregate corporate level, we obtain a value for the return on debt identical to the historical average of the 3-month T-bill over the period 1926–2001. Although this result is surprising, it is to some extent, an artifact of Flow of Funds data that exhibits a survivor bias, in the sense that it excludes defaulted or bankrupt companies over time. This result implies that in the long-run, survivor-biased corporate debt will be risk-free as the economy grows at a constant rate, where the risk-free rate is determined by the 3-month T-Bill, absent any term structure effects.

Our first key conclusion is that, in the long-run, the size of the premium (as expressed by the difference between the average stock return and the 3-month T-bill) is a function of GDP growth and other financial parameters such as marginal income tax rates. We empirically match the historical value of the S&P500 arithmetic average over 1926–2001, and the historical equity premium value of 8.1%.

Our second key result is that the equity premium is also consistent with a compensation for risk, when viewed in the context of a short-term portfolio insurance motive. We use an option-based approach and show that the equity premium is closely approximated by a put option premium on a $1 real investment in the market index when a long-term investor wishes to insure against year-to-year market volatility, by using the average yearly S&P500 volatility over 1926–2001.

Recent research argues that the actual equity premium is much smaller than indicated by historical long-term averages (Siegel, 1999; Jagannathan, McGrattan and Scherbina, 2000; Fama and French, 2002; and De Santis, 2004). In contrast, our result is that the *ob-*

served level of the long-run equity premium at 8.1% is fully consistent with the observed steady-state GDP growth and consistent with risk explanations as well, and thus it may constitute a legitimate input in capital budgeting and investment analyses.

Nonetheless, if one believes that the 1990's changes in dividend yields, income taxes, and interest rates represent permanent regime shifts, our portfolio insurance model does lead to a *lower* premium value consistent with the literature, when using parameter values from that period.

2. Long-term stock market return and GDP per-capita growth

Our first approach is to tackle the equity premium puzzle by using a supply-side growth model. Because the equity premium is typically measured by appealing to arithmetic historical average returns, which are sample estimates for unconditional expected returns, it is reasonable to focus on unconditional expectations (Fama and French, 2002; Ibbotson and Chen, 2003). We appeal to the natural properties of the economy's long-run steady-state growth path to derive our key relationship between the equity premium and GDP growth.

In the long run, Kaldor's (1961) stylized facts show that the U.S. economy has been characterized by a constant nominal (and real) GDP growth rate that equals the growth rate of capital, with a stable factor income distribution (labor vs. capital). In a stochastic framework, this amounts to saying that these year-to-year growth rates are stationary stochastic processes.

We establish a link between macroeconomic and finance variables by positing that in the long run, the unconditional expected growth of the economy's corporate capital stock must equals the unconditional expected growth in book value of a broad stock index.[5] The change in the index's book value of equity is driven by clean surplus accounting. We also postulate that the financial environment satisfies Miller and Modigliani's (1958) proposition and that the optimal capital structure is achieved for the whole economy.

Let us develop our notations. Let K_t denote the capital stock, B_t denote the book value of a broad equity index, V_t is the market value of the index, and e_t represents total earnings for the index; all at the *beginning of period t*. Let the variable $R_{t+1} = \frac{e_{t+1}}{B_t}$ denote the *ex-post* ROE at the end of period *t*. A portion b_{t+1} of earnings is paid out as dividends. Let the ex-post rate of net stock issues at the end of period *t* be denoted by $g_{s,t+1}$. Ex-post, the growth in *total* book value of equity is given by the following surplus equation:

$$\frac{B_{t+1}}{B_t} = 1 + (1 - b_{t+1}) \times R_{t+1} + \frac{g_{s,t+1}}{(1 + g_{s,t+1})} \times \frac{V_t}{B_t} \qquad (1)$$

Equation (1) states that the growth of book value for the equity index (S&P500) comes from two sources: the first term on the right-hand side represents internal reinvestment. The second term is the increment from issuing net new equity at a price equal to the current market price adjusted for the increase in supply of shares (price divided by the factor $(1+g_{s,t+1})$).[6] As mentioned, our key assumption is that the expected growth of equity

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[5] It is worth noting that our approach differs from the consumption-based capital asset pricing model (CCAPM). In our model, agents' optimizing behavior is in the background, and key behavioral parameters are considered exogenous. In addition, the typical solution of a CCAPM is a pricing kernel that involves agents' conditional expectations.

[6] Using the current price adjusted ex-post is more accurate than using *next* period's price because our adjustment precisely corrects for the price pressure due to share growth, without adding the noise of new market information incorporated in next period's price. This particular adjustment works when stock demand elasticities are close to one. Shleifer (1986) provides evidence of unitary stock demand elasticity for the S&P500.

book value of a broad equity index (S&P500) equals the expected growth in total capital assets at the economy level:

$$E\left(\frac{K_{t+1}}{K_t}\right)=E\left(\frac{B_{t+1}}{B_t}\right) \tag{2}$$

where $E(\cdot)$ denotes the unconditional expectation operator. We assume unconditional expectations satisfy the following:[7]

i) The expected market to book ratio $E\left(\frac{V_t}{B_t}\right)=1$.

ii) The expected ROE equals the long-run average return on stocks μ.

iii) The expected long-run growth rate of population n and net new shares g_s are both constant.

iv) The expected growth rate of capital equals the long run average growth rate of GDP denoted by g.

v) The expected long-run payout ratio is the long run average ratio b.

Using equation (2) and condition iv) we trivially get:

$$E\left(\frac{B_{t+1}}{B_t}\right)=1+g \tag{3}$$

Taking unconditional expectations of both sides of equation (1), then substituting equation (1) into (3), by using the rest of the conditions above, then equation (3) becomes:

[7] These conditions are minimal for the stock market to be in a stationary equilibrium. Condition i) comes from the residual accounting literature (e.g.; Ohlson, 1995) since companies that grow at the same rate as the economy have zero residual earnings growth and thus market value must converge to book value on average. Condition ii) in fact follows from condition i). Philips (1999) documents that condition ii) seems to hold since the average S&P500 ROE has been very close to the average stock return over the last thirty years. Conditions iii), iv) and v) state that these variables follow stationary stochastic processes.

$$1 + \mathrm{g} = 1 + (1-\mathrm{b})\mu + E\left(\frac{\mathrm{g}_{s,t+1}}{(1+\mathrm{g}_{s,t+1})}\right) - COV(b_{t+1}, R_{t+1}) + COV\left(\frac{\mathrm{g}_{s,t+1}}{(1+\mathrm{g}_{s,t+1})}, \frac{V_t}{B_t}\right) \quad (3')$$

Strengthening assumption iii) we posit that in the stationary equilibrium net new share growth g_s equates population growth *n*.[8] Let $\mathrm{g}_\mathrm{y} = \mathrm{g} - n$ denote the GDP per capita growth rate. Rearranging (3'), and using the approximation $E\left(\frac{\mathrm{g}_{s,t+1}}{(1+\mathrm{g}_{s,t+1})}\right) \approx n$, we get:

$$\mu = \frac{\mathrm{g}_\mathrm{y} + COV(b_{t+1}, R_{t+1}) - COV\left(\frac{\mathrm{g}_{s,t+1}}{(1+\mathrm{g}_{s,t+1})}, \frac{V_t}{B_t}\right)}{1-\mathrm{b}} \quad (4)$$

Equation (4) is the central result of this section. It shows that the long-run nominal stock return is a direct function of the GDP per capita growth rate. This return also depends on the retention ratio $(1-\mathrm{b})$ and the difference between two covariances: 1) the covariance between dividend payout and the index ROE and 2) the covariance between market to book ratio and the normalized growth rate of shares next period.[9] Thus, the long-run nominal stock return equals the long-run expected growth rate of GDP per cap-

[8] In the long run, aggregate stock wealth cannot grow faster than GDP, to rule out permanent bubbles. The same must be true on a per capita basis, given that the distribution of wealth is stable in the steady state. On one hand, the supply of net new shares has to match at least the growth of new investors. On the other hand, a growth of shares in excess of population growth would depress earnings-per-share and thus dividends and capital gains. This would depress stock prices permanently. Corporations would then have an incentive to repurchase shares. Using the Federal Flow of Funds, over the period 1946–2002, the growth in total stock market value was 8.40%, whereas it was 7.20% for the S&P500 over the same period. Since the S&P500 was a relatively constant fraction of the overall market value (about 60%), and the index is on a per-share basis, it is evident that the difference of 1.2% represents net share growth, about equal to long-term population growth. Loderer, Cooney, and Van Dunen (1991) show that stock prices are reduced on average by 1% around announcements of secondary equity offerings over a period covering 1969–1982. This effect is consistent again with the fact that in the long-run net new share growth happens at the rate of average population growth.

[9] These two expressions play a similar role as that played by the covariance between consumption growth and equity returns in the standard consumption-based capital asset pricing, which reflects both the riskiness of growth as well as the intertemporal optimal consumption choices in light of expected returns (e.g.; Hansen and Singleton, 1983). It is also worth noting that in these models the covariance expression treats consumption growth essentially as an *exogenous* variable.

ita plus a risk premium term (the difference of covariances), divided by the percentage of new earnings retained.

Because the retention ratio $\left(1-\mathrm{b}\right)$ and steady-state growth rate $\mathrm{g_y}$ are determined in the background by optimal consumption-investment decisions, equation (4) simply establishes that the long-run equity return is a function of these choices. It is important to note that expression (4) is the macroeconomic equivalent to the long-term sustainable growth formula found in standard corporate finance textbooks (for example, Brealey, Myers and Marcus, 1999).[10] The difference with the standard formula is that ours applies to the corporate sector as a whole.

Furthermore, the sustainable growth rate is determined by the long-run GDP per capita growth rate and what essentially constitutes an added systematic risk premium. When the first covariance $COV(b_{t+1}, R_{t+1})$ is large, this means that companies pay out a greater fraction of earnings when their ROE is high (procyclical), which exacerbates the volatility of cash flows and thus price volatility. On the other hand, when the second covariance $COV(\frac{\mathrm{g}_{s,t+1}}{(1+\mathrm{g}_{s,t+1})}, \frac{V_t}{B_t})$ is large, greater stock issuance is associated with periods of high market-to-book ratios (procyclical). In that case, greater stock issuance will bring the market-to-book ratio back down, and vice versa in periods of low valuation. Thus, price volatility is dampened.

Our model essentially predicts that stock markets with *countercyclical* aggregate dividend payouts and *procyclical* new shares growth overall will experience a *reduction* in

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[10] Our result agrees with Arnott and Bernstein (2002), who find that the stock market return is directly related to per capita GDP growth.

the impact of market risk and therefore a lower long-term stock market return. For example, during periods of large *downside* earnings volatility and low valuation, firms can reduce their investors' exposure to systematic risk in two ways. One method is to smooth out the dividend stream by temporarily paying *higher* dividends. Another possibility is to boost earnings *per share* by issuing fewer shares or even by buying back existing shares. Any such outcome is the result of optimizing behavior, which depends on the aggregated preference parameters of the particular economy.[11]

The arithmetic average yearly population growth rate is n = 1.19%[12] and is assumed equal to the growth rate of shares g_S. Using yearly data, the arithmetic average nominal growth rate of GDP per capita over the 1926–2001 period is $g_y \cong g - n$ = 6.65% – 1.19% = 5.46%, and the average S&P500 dividend payout is 55.5%. Our estimate for the covariance between dividend payout and ROE is –0.51%. Strikingly, this value is identical to the value of the sample covariance between the market to book ratio and the subsequent period (normalized) shares growth rate, over 1925–2001.[13] This means that over the pe-

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[11] The result in expression (4) implies that, even though dividend payout policy is constant, the stock market return will be different for two economies with widely different business cycles and growth risks. The second covariance term will raise the stock market return for the higher risk economy, when that covariance is *negative*. That is, when the growth of shares is principally driven by steady financing needs of the corporations making up the index. In that instance, a low price-to-book environment necessitates higher growth of shares to meet a given financing need, although the need may be smaller during a contraction than during an expansion phase.

[12] This rate corresponds to the growth rate of the total U.S. population (source: Bureau of the Census) over the period.

[13] We construct a measure of book value for the S&P500 by using data from Robert Shiller's Web site www.irrationalexuberance.com. We assume that the book value of the index in 1871 was equal to its market value. Year-to-year, we add retained earnings to the previous year's book value. Our computation of the ROE is then done from year-end 1926 until 2001. The market-book ratio for the S&P500 is assumed similar to that of the aggregate corporate sector. The data is from the Federal Flow of Funds over the period 1952–2001. The missing market-to-book values for 1926-1951 are reconstructed by back-trending corporate market and book values, using regressions of the log of these variables on linear time trends over 1952-2001. Again, because the S&P500 was a relatively constant fraction of the overall market value (about 60%), an index for the number of stock shares in the S&P500 is constructed by dividing the market value of corporate equity by the value of S&P500 index.

riod, both dividend payouts and net new share issuance have been *countercyclical* in the United States, thereby creating coupling effects that offset the risk premium. Hence, we get:

$$\mu = \frac{5.46\%}{(1-55.5\%)} = 12.27\%$$

This final value is nearly identical to the arithmetic average nominal stock return value of 12.2% estimated for example by Siegel (2002) for the period 1926–2001. Hence, we have derived an exact analytical relationship linking the average real stock return and long-term GDP per capita growth. Examining equation (5), we observe that the smaller the retention ratio is, the greater the stock return is for a *given GDP per capita growth rate.*[14]

3. The return on corporate assets and return on debt

Whereas the first piece of the equity premium puzzle was the return on the stock market, the second piece is the return on debt. In this section, we focus on deriving the average return on corporate debt for the economy. As an intermediate step, we examine the economy's required return on corporate assets (RRCA) from the investors' standpoint. The required return on corporate assets is the discount rate that makes the present value of

[14] If all earnings are reinvested (b = 0), so that no dividends or share repurchases occur and growth is financed internally, the best return that investors could expect to earn is GDP per capita growth. This suggests that leverage may enhance stock returns by allowing firms to maintain asset and earnings growth and still boost stock returns through dividends and repurchases.

expected future after-tax cash flows accruing to creditors and equity holders equal to the market value of the corporate asset base in the economy.[15]

Again, we predict that in the long run, the expected growth of the capital stock at the economy level equals the growth in book value for the S&P500. Furthermore, the corporate debt-equity ratio also must be constant in the steady state given an environment a la Miller and Modigliani (1958). Future cash flows are given by after personal income tax $\mathrm{T_t}$, expected corporate dividends $\mathrm{D_t}$, and net interest payments $\mathrm{IP_t}$ on outstanding corporate debt.[16] Assuming market efficiency, the market value of all corporate assets must equal the present value of all expected future cash flows net of taxes that accrue to creditors and shareholders at the economy level. Formally, this is expressed as:

$$\text{MV Corporate Assets}_0 = E\sum_{t=1}^{\infty}\frac{(1-\mathrm{T_t})\times\left(\mathrm{D_t}+\mathrm{IP_t}\right)}{\Pi_{j=1}^{j=t}\left(1+\mathrm{RRCA_j}\right)} \tag{5}$$

where $E(\cdot)$ again stands for the unconditional expectation operator. We assume that in every period *j*, the corresponding discount rate $\mathrm{RRCA_j}$ is constant and equal to the long-term average RRCA given by:

$$\mathrm{RRCA} = \mu\times(1-\mathrm{L})+\mathrm{r_D}\times\mathrm{L} \tag{6}$$

The long-term nominal stock return is again denoted by μ. The variable $\mathrm{r_D}$ stands for the nominal return on corporate debt/bonds, and L stands for the fraction of the investor's portfolio invested in debt or alternatively corporate leverage. We hypothesize that in the

[15] Fama and French (1999) compute an IRR based on operating and investment cash flows. Our approach is grounded in the steady-state analysis of the economy. In that context, the fraction of dividend plus net interest payments over the market value of corporate assets should be constant, whereas its components may not be constant. In fact, this fraction of total payments has been relatively constant over the period examined, whereas the relative size of interest payments compared with dividends has increased.

[16] Cash flows are in nominal terms. This current model does not incorporate capital gains taxes because dividend cash flows and interest are assumed paid forever. The tax rate, $\mathrm{T_t}$, is an average marginal tax rate that blends dividend income and interest income tax rates.

long run, the sum of expected dividends plus interest payments to investors is a constant fraction λ of GDP, as they cannot *together* grow faster than GDP. Again, let g stand for the nominal long-term average GDP growth rate. Given an average marginal tax rate of T, the above-mentioned expression (5) becomes:

$$\text{MV Corporate Assets}_0 = \sum_{t=1}^{\infty} \frac{\lambda(1-T)\times \text{GDP}_0 \times (1+g)^{\text{t}}}{\left(1+\text{RRCA}\right)^{\text{t}}} = \frac{\lambda(1-T)\times \text{GDP}_0}{\left(\text{RRCA}-g\right)} \tag{7}$$

Equation (7) is the steady-state version of equation (5). The result in equation (7) indeed is the standard growing perpetuity formula applied to the entire asset base of the corporate sector. Thus, the RRCA is also given by:

$$\text{RRCA} = g + \lambda(1-T)\times \frac{\text{GDP}_0}{\text{MV Corp Assets}_0} \tag{8}$$

Finally, combining expressions (6) and (8), we get an expression for the debt return as:

$$\text{r}_\text{D} = \frac{g + \lambda(1-T)\times \dfrac{\text{GDP}_0}{\text{MV Corp Assets}_0} - \mu \times (1-\text{L})}{\text{L}} \tag{9}$$

We use Flow of Funds data and National Income and Product Accounts data from the Bureau of Economic Analysis, for the nonfinancial corporate sector over the period 1954–2001. The variables used are net interest payments, dividend payments, total debt, market value of equity and book value of equity. Average marginal tax rates on dividend and interest income are obtained from Estrella and Fuhrer for the period 1954–1979 and from the National Bureau of Economic Research TAXSIM model for the period 1980–

1999.[17] Because our marginal income tax data is limited to 1954–1999, we extrapolate the 1999 taxes rates for the years 2000 and 2001.

To determine leverage, we use book value of equity. This approach is supported by the evidence of stability of book leverage over the period 1951–1996 documented by Fama and French (1999). The average value for the book leverage ratio over our period is 38.05%; the average value for the ratio of the net total payments to investors over total market value of assets (before taxes) $\lambda \frac{\text{GDP}_0}{\text{MV Corp Assets}_0}$ equals 3.67%. The average blended marginal (dividend plus interest) income tax rate is 35%. Table 1 below summarizes the main parameters for the model.

Table 1: Estimated parameter values for return on corporate assets and debt formulas

g	g_y	b	T	L	λ	$\frac{\text{GDP}_0}{\text{MV Corp Assets}_0}$	μ
6.65%	5.46%	55.5%	35%	38.05%	3.27%	112%	12.27%

From these parameters and equation (9), we obtain a value for the *real* RRCA of 5.89%, based on a long-term GDP nominal growth rate of 6.65% and inflation rate of 3.14%. This result is nearly identical to Fama and French's (1999) estimate of 5.95% over the period 1950–1996. Finally, using formula (10) our estimate for the nominal re-

[17] Even though there is a difference in methods between the two approaches, we do not believe that these differences affect our conclusions in any significant way.

turn on debt is 3.74%, which is close to the real T-bill arithmetic historical average of 3.93% for 1926–2001.[18]

This result is surprising. It suggests that the overall corporate debt may be considered risk-free in the long run! However, we must recognize that the Flow of Funds corporate debt data has a survivor bias. The corporate debt data do not retroactively correct for defaulted debt, and thus the Flow of Funds reports interest payments that typically begin to shrink earlier than the reported principal does, which biases computed rates downward.[19]

4. Growth and the long-run equity premium

Finally, we derive the difference between the return on corporate equity versus corporate bonds. Let us define $\Delta COV = COV(b_{t+1}, R_{t+1}) - COV(\frac{\mathrm{g}_{s,t+1}}{(1+\mathrm{g}_{s,t+1})}, \frac{V_t}{B_t})$. Based on expressions (4) and (9), the difference in returns is given by the following formula:

$$\mu - \mathrm{r_D} = \frac{1}{\mathrm{L}}\left[\frac{g\mathrm{b} - n + \Delta COV}{(1-\mathrm{b})} - \lambda(1-T)\times\frac{\mathrm{GDP}_0}{\mathrm{MV\ Corp\ Assets}_0}\right] \qquad (10)$$

The difference depends positively on the rate of growth of GDP, on the blended marginal income tax rate T and on systematic risk, and negatively on leverage and the net growth of shares. The fact that the difference between stocks and corporate bonds seems

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[18]Fama and French (1999) discuss the use of simple versus compounded returns as discount rates. They argue that under certain conditions the expected one period simple return is the appropriate discount rate. Otherwise, a more appropriate method is to use a weighted average of simple and compound returns.

[19]This leads to an understatement of the actual long-term return on debt that should probably include a default premium. Thus, the growth of actual issued volume of debt should exceed GDP growth by an amount equal to the average corporate default rate. Note also that any inflation risk and interest rate risk are already embedded in the T-Bill rate. Furthermore, because we are examining *arithmetic* average returns, capturing 1-year investment horizons, the inflation and interest rate risks may bear a smaller effect than for debt yields representing multiyear horizons. It turns out that using market-value based average historical leverage instead of book-value leverage ratio lowers our estimate of the return on debt even more because the value of the leverage ratio is 34.23% over 1954–2001.

related to GDP growth is sensible if we note that a bond is a claim to a fixed income stream, whereas a stock is a claim to both growing dividends and earnings streams.

In section 3, we show that the return on corporate debt r_D nearly equals the risk-free rate in the long run. This result implies that formula (10) characterizes the equity premium as well.[20] Over the period of 1926–2001, and after combining our prior estimates for the long-term return on debt and the return on stocks, we obtain a value for the premium of equity over the 3-month T-Bill equal to 8.3%, which closely matches the historical estimate of 8.1% over the examined period.

5. Portfolio insurance and the equity premium

The literature on the equity premium has so far shown scanty evidence of the link between the premium and risk. Asness (2000) attempts to empirically reconcile the premium with measures of the difference in risk between bonds and stocks and achieves some relative success for rolling market periods of 20 years, but falls short of explaining the premium over the full historical record.

In this section, we show that option pricing can help us derive a measure for the equity premium that is directly related to observed historical stock return volatility. In fact, we show that the long-run equity premium is closely related to an investor's objective of averting downside risk.

Consider an investor adopting the following long-term strategy: every year, invest $1 in a stock index and buy a put option on the index (with a real $1 strike), sell the stock at the end of the year. This is an instance of seeking portfolio insurance by using a protec-

[20] Again, if it weren't for the survivor bias in the Flow of Funds corporate debt data, we should expect that corporate debt returns would incorporate a default premium.

tive put (Merton, Scholes, and Gladstein, 1982). The yearly maximum loss is limited to the loss of the put premium. In the long run, the expected stock return must be the arithmetic average of the index return. We postulate that yearly market volatility is expected to equal the annual volatility given by a long-term horizon estimate. In that case, the put option price remains constant. We also posit that the index pays a known dividend.

Formally, let $\tilde{\mu}$ be the arithmetic average *real* stock return, let $\tilde{r}$ be the arithmetic average *real* risk-free rate, q is the real after-tax dividend yield, and c and p are the respective option prices for the European call and put. Our goal is to find the value of this put option. Assume that the current index price is \$1 and the strike price is \$1 (real), and using compounded rates of returns, the put-call parity formula for a dividend paying stock (Hull, 2003) leads to express the put price as:

$$p = c + e^{-\tilde{r}} - e^{-q} \tag{11}$$

Following Rubinstein (1984), we define the *real* risk-neutral expected *future* value of the put-call relationship as follows:[21]

$$\mathrm{E}(p) = \mathrm{E}(c) + 1 - e^{\tilde{\mu}-q} \tag{12}$$

Given that the expected capital gains rate is greater than zero, the expected value for the put option must be zero, because it is expected to be out-of-the-money. Therefore, we get:

$$\mathrm{E}(c) = e^{\tilde{\mu}-q} - 1 \tag{13}$$

[21] We are applying Rubinstein's (1984) approach to real (deflated) values of expected call and put prices. Rubinstein also defines these expectations for a horizon h as a fraction of one year and for volatility estimates that may differ for individuals compared with the overall market. Here we posit that h is arbitrarily close to one and that individual estimates are arbitrarily close to the market volatility.

Thus, the real expected future value of the call option equals the real expected capital gains rate.[22] We postulate that the present value of the expected future call value discounted at the risk-free rate is a good approximation for the current call value, that is:[23]

$$c \cong e^{-\tilde{r}} \mathrm{E}(c) \tag{14}$$

Using equations (13) and (14), and plugging the resulting call price into (11), we obtain:

$$e^{q} p \cong e^{\tilde{\mu}-\tilde{r}} - 1 \tag{15}$$

Hence, the equity premium can be expressed as:

$$\tilde{\mu} - \tilde{r} \cong \mathrm{Ln}(1 + e^{q} p) \tag{16}$$

Thus, the risk premium approximately equals the value of a European put option on a stock index compounded at the dividend yield rate (using the average annual standard deviation of prices). Applying Black and Scholes' (1973) approach, we can evaluate the price of such a put option. It is well known that the resulting value is independent of the expected stock return and investor preferences.[24] Assume that the strike price and the current stock price are both equal to \$1 and that the option's maturity is 1 year. The standard formula for an option on a dividend paying stock index (with taxes) is given by Scholes (1976).[25] According to our strategy, an investor sells her stock at the end of each year.

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[22] It is interesting to note that this same result can be derived using Black and Scholes' (1973) call option approach to corporate equity. In that case, we would have to assume that stocks are initially purchased using zero-coupon debt and that the minimum *real* required return on debt is zero.

[23] Using parameter values introduced later, we find that the Black–Scholes current call price equals 6.56%, whereas our estimate is 6.50%.

[24] This approach is subject to the standard criticism of the normality assumption of the stock market return distribution (e.g., Fama 1965).

[25] Scholes (1976) prices a European call option. A put option can be priced using the put-call parity formula.

We posit that he or she is taxed at the marginal ordinary income tax rate and that this tax rate equals the dividend income marginal tax rate *T*. Hence, the formula is:

$$
\begin{aligned}
& p = e^{-\tilde{r}} N(-d_2) - e^{-q} N(-d_1) \\
\text{with} \quad & d_1 = \frac{\tilde{r} - q + \sigma^2 / 2}{\sigma} \\
& d_2 = d_1 - \sigma \\
& q = (1-T) \times \text{dividend yield}
\end{aligned}
\tag{17}
$$

Thus, using the relationship between the put option and the equity premium expressed in (16) and the put pricing formula (17), we obtain an independent estimate of the premium. We apply formula (17) with the following parameters: the dividend tax rate corresponds to the average marginal rate over 1954–1999 (with 1999 values used for the years 2000 and 2001). The value for the tax rate is 40%.[26] The standard deviation of stock index returns is a historical estimate using continuously compounded annual *real* total S&P500 returns over 1926–2001. We adjust total returns according to Hull (2003), by removing the effect of dividends on stock volatility to account for the risky portion of stock prices. The value of the standard deviation σ is estimated at 18.87%.

Our estimate of the inflation-adjusted pretax average S&P500 dividend yield is 4.20% over the same period. The value for the real risk-free rate corresponds to the T-Bill arithmetic average real rate of 0.76% over that period as well. We then arrive at a value for the put option premium *p* of 8.29%, and a value for the risk premium of 8.16%, which is nearly identical to our prior estimate for the risk premium.

Note that the insurance strategy presented above does not fully guarantee a risk-free return on a *yearly* basis. To achieve that goal, the investor could for example sell a call

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[26] The estimates for 1954–1979 are from Estrella and Fuhrer (1983) and for 1980–1999 from the NBER TAXSIM model.

option in addition to owning a protective put. However, over a long-term investment horizon, stocks are on average as "riskless" as bonds, in the sense that they deliver an average return that is more and more certain with longer horizons, based on real long-term GDP per capita growth. The difference between riskless securities and stocks is that riskless securities are a perfect hedge against short-term market downside risk, whereas stocks obviously are not. Our result shows that the long-run equity premium reflects a portfolio insurance motive since there would otherwise be an opportunity for a riskless *long-term* arbitrage, in the sense that put-call parity would be violated on average.

Our result demonstrates that the risk-free rate and the long-term stock return are jointly defined in relation to the premium on a put option. Our analysis of the return to corporate assets above, when combined with the result of this section, leads us to offer a new view of the debt versus stock trade-off, in which bondholders and stockholders mutually benefit from co-financing corporate assets at the macroeconomic level. Stockholders will benefit from leveraging assets since they can boost equity returns via receiving higher dividends all the while maintaining constant asset and earnings growth rates.

In this new view, it is useful to consider that an investor following our portfolio insurance strategy is a debt holder. Debt holders by choosing to hold debt instead of equity settle to earn a lower long-term return (than equity holders), because they choose to *fully* insure against the loss of their principal in the short run. Thus, in equilibrium, agents will be indifferent between holding debt or equity, because insuring one's principal is costly. Debt holders are in effect "paying" a portfolio insurance premium to stockholders, and thus end-up with a return equal to the risk-free rate on average.

This argument has an interesting implication regarding the traditional view of compensation for risk. The standard CAPM uses a "bottom-up" approach to the equity premium: the equity premium is *added* to the risk-free rate to compensate stockholders for the extra risk borne by them. Our logic of portfolio insurance presents a top-down view of the premium: the premium is *subtracted* from the long-term equity return to obtain the risk-free rate, because insurance is a cost.[27] This logic obviously does not violate the spirit of CAPM.

6. A shrinking equity premium?

Jagannathan, McGrattan and Scherbina (2000) and Fama and French (2002) argue that ex-post returns are a distorted view of expected returns and that a lower equity premium should be used compared with the historical average.

Table 2: Estimates of the equity premium using the portfolio insurance model. 1970–2004.

	Beg. Year	Div. Yield	Div. Tax Rate	SD Mkt. Return	Real T-Bill	Est. Premium
Period	1970	3.62%	36.48%	12.61%	1.38%	5.38%
Ending	1982	3.21%	29.08%	11.68%	3.26%	4.08%
December	1990	2.46%	27.84%	10.51%	2.25%	3.87%
1999	1995	1.78%	29.06%	10.90%	2.89%	3.49%
Period	1970	3.32%	35.23%	12.69%	1.17%	5.44%
Ending	1982	2.83%	29.11%	12.31%	2.63%	4.49%
December	1990	2.15%	28.13%	12.03%	1.62%	4.65%
2004	1995	1.68%	29.18%	13.25%	1.67%	4.91%

[27] In the limit case of 100% debt-financing, corporate debt holders would require the same return as "equity" holders (or T-Bill return plus risk premium). However, to avoid short-term principal losses, debtholders would have to use our portfolio insurance strategy and sacrifice the premium. The recent rising corporate trend of using leverage to pay dividends may cause an increase in the equity premium, since the premium is an increasing function of the dividend yield, for a given level of the risk-free rate and independently of leverage risk.

Table 2 shows that for various time horizons, our portfolio insurance model predicts an overall decline in the equity premium before the market crash of 2000 with a minimum premium value around 3.5%, and an increase back to about 5% for periods ending in 2004.[28] These estimates are in line with the above-cited literature. The observed decline in the premium compared to the 1926–2001 average of 8.1%, seems to originate from declining trends in dividend yields, marginal dividend income tax rates and stock market volatility along with a rise in the *average* real T-Bill returns since the 1980s.

Because our model works as a predictor of long-term premia, the results shown in Table 2 are only applicable if we assume that these recent trends in dividend yields, marginal tax rates and interest rates represent permanent regime shifts. This latter view may coincide with the Federal Reserve (Fed) recent monetary policy trend, since the Fed appears committed to keep inflation rates *and* discount rates low (although the slide of T-bill rates in the 1 to 2% range is also responsible for larger premia in periods ending in 2004). Regarding tax policy, marginal dividend and capital gains tax rates have reached their lowest levels in history, and hiking rates back up is a virtual political impossibility. On the other hand, dividend policy seems to have shifted towards lower payouts over the 1990s (Fama and French, 2001). However, investors can easily decide to reverse the trend if they perceive that capital gains are exposed to greater downside risk.[29]

[28] Parameter values are annualized monthly averages, except for dividend taxes (yearly averages) and except for the periods starting in 1970, where the real T-Bill is averaged yearly. S&P500 standard deviations are annualized and based on monthly continuously compounded real returns.

[29] Another potential issue is that share repurchases may counter the effect of lower dividend yields. Grullon and Michaely (2002) report that since the mid-1980s, large established U.S. firms did not increase dividends as much as they could have, but rather chose to buy back shares. Thus, in all likelihood, lower expected dividend yields have been associated with greater expected capital gains. In that instance, our portfolio insurance model makes the provision in equation (13) that the expected future value of the S&P500 call option fully reflects these capital gains expectations and thus that the equity premium embodies these trend expectations as well.

One reason why our premium estimates seem to rise slightly (controlling for the slide in T-bill rates) for horizons ending in 2004 compared with 2000 is that market volatility did rise after the market crash of 2000. This effect is accentuated for short horizon periods (15 years or less). To smooth out the effect of the 2000 market crash, an intermediate range of about 20 years may be more appropriate. Table 2 then shows that for horizons starting in 1982 the premium can be estimated in the 4 to 4.5% range.

7. Conclusion

We show that the long-run equity premium is theoretically and empirically consistent with GDP growth and a portfolio insurance motive. We derive the long-run ex-ante equity return and long-run corporate debt and asset returns using a supply-side growth model. We arrive at a macroeconomic generalization of the standard sustainable growth formula used in corporate finance textbooks to determine the long-run average return on stocks.

The long-term average stock return depends on GDP per capita growth, the earnings retention rate, and a premium linked to de-trended earnings volatility and market-to-book volatility and the response of dividend and share repurchase policies to these factors. Our model accurately replicates the arithmetic average historical returns for the S&P500. Our first conclusion is that the equity premium defined as the difference between the S&P500 stock return and 3-month T-Bill is consistent with observed GDP growth and other financial parameters such as marginal income tax rates.

Our result also hinges on the fact that the Flow of Funds data on corporate debt and S&P data on the equity side have an inherent survivor bias. Interestingly, our analysis entails that the corporate debt of *surviving* firms exhibits a long-term average return that is essentially risk-free. In actuality, because investors typically invest in bond portfolios that experience failure rates, a default premium should be added to our estimate.

Our second key result is that the equity premium is consistent with a short-term portfolio insurance motive. We show that the equity premium is closely approximated by a put option premium on a *real* $1 investment in the market index when a long-term investor wishes to insure against *year-to-year* market volatility, by using the average yearly S&P500 volatility over 1926–2001. This result leads us to a new view of corporate asset financing where investors, pursuing a short-term insurance motive, are indifferent between long-run equity returns and the comparatively lower rate on short-term Treasury bonds. Debt holders looking to insure their principal essentially forego long-run equity returns by "paying" an insurance premium equal to the equity premium.

Siegel (1999), Jagannathan, McGrattan and Scherbina (2000), Fama and French (2002), and De Santis (2004) all claim that ex-post returns are a distorted view of expected returns and that a lower equity premium is justified compared with the historical average. Our results suggest that using an 8.1% premium in valuation formulas and capital budgeting problems may be appropriate, since the observed level of the long-run equity premium is fully consistent with the observed steady-state GDP growth and consistent with risk explanations as well. However, if one believes that the recent 1990's trends in dividend yields, interest rates, taxes and inflation represent permanent regime shifts, our model can be parameterized to yield a 3.5% equity premium in line, for example, with Fama and French's (2002) estimate.

Future research will examine the determinants of the equity premium's countercyclical behavior in the short to medium-term. In that respect, using European options on the S&P500 seems to be a promising avenue to characterize the equity premium.

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