A synthetic protective put strategy for investment in projects without an outright deferral.

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Abstract

In this paper we propose and computationally demonstrate a synthetic protective put strategy for real options. Specifically, we deal with the problem of *deferral option* when an outright deferral is not permissible due to competitive pressures. We demonstrate that in such a situation an appropriate strategy would be to invest in the new project in phases rather than doing it all at once. By setting the owner's equity in the project equal to the price of a call option on the value of the project, we set up the replicating portfolio for a protective put on the project. Our method is a logical extension of the financial protective put in the real options scenario and is rather simple and practicable for businesses to adopt and apply.

Key words: synthetic protective put, replicating portfolio, deferral option

Problem definition

In the parlance of real options, a *deferral option* comprises of a right to defer the start of a business unit or commencement of a particular project. These are quite realistic in situations where competition is not a crucial issue – e.g. when the business enterprise in question enjoys a certain degree of monopoly power in the market. With an option to defer the start of a project say for a year, a business enterprise would choose to invest in that project only if conditions turned out to be favorable at the end of the year – otherwise it would choose not to. The cost of developing the project could be viewed as the exercise price of the embedded call option on the value of the project and the risk associated with the nature of returns from that particular type of a project could be viewed as the underlying source of uncertainty. Following the *marketed asset disclaimer* (MAD) convention in real options analysis, the present value of the project *without an option to defer* may be used as the initial value of the underlying risky asset when pricing the deferral option (Copeland and Antikarov, 2001).

In an arbitrage-free world, the deferral option can be priced as any other real option using a risk neutral valuation approach. Moreover, all the theoretical underpinnings of the basic pricing scheme for financial options can be also applied to real options as the value that an investor receives from a risky project can follow a random walk (a Gauss-Wiener stochastic process) even though project cash flows may not follow a random walk (Bhattacharya, 1978). This is because when the current value of a project incorporates all information about expected future cash flows from the project, any changes in the current value result from changes in the expectations of future cash flows that are necessarily random (Samuelson, 1965). Random shocks to the expected cash flows will be reflected as random movements in the investor's wealth and in the returns from the project. Since the value of the project may be shown to follow a Gauss-Wiener stochastic process, it can be modeled as a recombining binomial tree. As the number of binomial trials per period tends to infinity, the binomial process approaches the Gauss-Wiener stochastic process as the limiting distribution.

Mathematically it is possible to prove that the net present value (NPV) of a project that can be deferred, when valued using real options analysis, is greater than the maximum of the alternative values that are obtained using the traditional discounted cash flow (DCF) techniques (Copeland, Weston and Shastri, 2005).

That is to say; a traditional DCF technique, if used to evaluate the project with an option to defer, would systematically undervalue its NPV as it would fail to take into account the element of managerial flexibility afforded by the deferral option. The price of the embedded call option gives a better measure of the project's NPV as it gives the requisite value weighting to this element of managerial flexibility i.e. the right to choose to invest in the project only when it was economically favorable to do so (Dixit and Pindyck, 1994).

However what happens when a deferral is too much of a luxury to afford given that the industry in which the business enterprise is operating is extremely competitive? One of the most appropriate examples is the restaurant industry. The business enterprise would have to go ahead and invest right now lest it frittered away an opportunity of staying ahead of the competition. However in doing so it would expose itself to a downside risk in the event that conditions turned out to be economically unfavorable for the project. In such a case one strategy that would certainly hold financial merit is that of using a *protective put*.

In its simplest form, the business enterprise could try and borrow at the lowest possible borrowing rate of interest and "buy" a *protective put* on the project with the borrowings. In real terms the protective put would give the business enterprise a right to hand over the project to a potential buyer ("writer" of the put option) at cost in case it eventually turned out to be an uneconomical investment.

With the protective put in place, in the worst-case scenario, the business enterprise would end up exercising the put option and closing off the debt with the proceeds. In the best-case scenario, the put option would not be exercised and it would expire worthless. The business enterprise would be able to harvest the excess value of the project after paying off the debt.

However the obvious problem would be finding a suitable potential buyer for the project i.e. someone who is willing to "write" such an option in exchange for an acceptable “premium”. In this paper we develop and demonstrate a simple computational strategy by which a synthetic call option could be engineered to replicate the payoff from the protective put, along very similar lines as one would take in replicating a protective put on a financial asset like a stock or a future.

Computational methodology

Our proposed computational approach of synthetically designing a *real* protective put rests fundamentally on the assumption that the put option is European. Being such, a hypothetical portfolio consisting of a borrowing at the risk-less rate and investment of the proceeds in the project and a put option thereon would, due to the application of the put-call parity, have a payoff same as that of an European call option on the value of the project (i.e. the project with an embedded option to defer). Mathematically this boils down to the following equation:

$$(V_t + P_t) - X\,e^{\{-r\,(T-t)\}} = C_t$$

Here V_t is the net present value of the project (without an option to defer), P_t is the price of the put option at time t, X is the exercise price (i.e. cost of the project), r is the risk-less rate of return, T is the terminal date on which the business enterprise must take the decision as to whether or not it wants to invest in the project and C_t is the price of the call option on the value of the project (i.e. the project with an embedded option to defer).

In the worst-case scenario, on the LHS the business enterprise would end up exercising the put and closing off the debt with the proceeds. On the RHS, the call would expire useless. In the best-case scenario, on the LHS the put would expire worthless and the on the RHS the call would have a payoff equal to the value of the project minus the debt payback (Rubinstein and Leland, 1995).

In terms of a project where deferral is allowed, the usual NPV evaluation procedure using ROA simply calculates the fair price of the embedded call option on the value of the project. This involves only the RHS of the put-call relationship. However the same analytical approach may be reverse-engineered to work from the LHS of the put-call parity relationship as well; and may offer a particularly interesting and reasonably practicable solution in cases where a complete deferral is not possible due to extremely intense competition. We assume that there is an opportunity of starting off small and scaling up when conditions are favorable and scaling down when conditions are unfavorable. As already observed, a restaurant chain is probably a rather appropriate example.

The proposed solution approach is surprisingly simple as it is a straightforward application of the LHS of the put-call parity relationship. The LHS of the put-call parity relationship may actually be viewed as a synthetic protective put. If viewed as such, it is the same as investing in the project and at the same time holding a put option thereon akin to a standard portfolio insurance strategy involving a financial asset such as a stock portfolio.

In the context of a capital budgeting decision the solution then is to simply keep investing an amount in the project that equals its NPV i.e. equal to the fair value of the call option. If the option value goes up the project will have to be upscaled, if it goes down the project will have to be downscaled. This investment strategy would work (albeit to a limited extent) even when there are "limits" to the extent to which a project can be upscaled or downscaled. It is however implicitly assumed that the firm can procure funds for the project (when upscaling) and re-apply funds elsewhere (when downscaling) at the same rate i.e. the firm's *weighted average cost of capital* equals the project's *opportunity cost of capital*.

The cost of borrowings should theoretically be the risk-free rate. However in real life it is not possible for a business enterprise to borrow at a “risk-free” rate given that it would employ the borrowed funds in an inherently risky business project. But a protective put portfolio consisting of an investment in the project as well as a put thereon is by definition, protected from downside risk. Therefore, if the lending institution can be adequately satisfied that the borrowed funds would be employed in “acquiring” a portfolio that is theoretically risk-free, the business enterprise should certainly be able to procure the borrowings at the lowest cost.

We formulate a discrete time model with origin time t = 0 and terminal time T = 1. We assume the possibility of taking a bank loan B_t (equivalent to issuing in a zero-coupon bond). The interest rate is $r \geq 0$, i.e. $B_1 = B_0\ e^{(rT)} = B_0\ e^r$. The project has only one *node* at time 0 with present value V0, and two nodes at time 1, with present values $V_1^{(+)}$ and $V_1^{(-)}$ such that $V_1^{(+)} > V_1^{(-)}$. There are probabilities p and (1- p) for the PV of the project to move to $V_1^{(+)}$ or $V_1^{(-)}$ respectively. We further assumed that $e^{(-rT)} V_1^{(-)} < V_0 < e^{(-rT)} V_1^{(+)}$. If this relationship does not hold, then it would imply an arbitrage opportunity which cannot exist in a perfect market.

The business enterprise is able to employ the borrowed funds in the project and thereby acquires a portfolio whose value is given by $I_0 = kV_0 + k' B_0$ which means that at time t = 0 it invests k amount in the project and has a bank loan to the tune of k′. The vector [k, k′] is the *strategy vector* at time t = 0. Then, at time T = 1 the portfolio will cost $I_1 = kV_1 + k'B_1$ which equals $kV_1^{(+)} + k'B_0 e^r$ with probability p and $kV_1^{(-)} + k'B_0 e^r$ with probability 1 – p. The *expected value* of this random variable is given by the following equation (Rendleman and Bartter, 1979):

$$E(I_1) = k\{V_1^{(+)} p + V_1^{(-)} (1 - p)\} + k'B_0 e^r$$

Now, to imitate the call option, one needs to construct a portfolio which has the same payoff as the call option on the value of the project i.e. the portfolio will mimic or *replicate* the call. A strike price X = Cost of project > 0 is fixed for the call option. The exercise price must satisfy the relationship $V^{(-)} < X < V^{(+)}$. The expiry is at t = T. The problem is to find such a vector (k, k′) such that the payoff of this portfolio is $(V_1 - X)$. If we find such a vector it will constitute a *synthetic call* which will replicate the payoff from our call option on the value of the project if it had an embedded deferral option. Hence their prices coincide i.e. we get $C_0 = I_0$.

To find the strategy vector (k, k′), the *Cox-Ross-Rubinstein one-step model* requires us to solve the following system of linear equations:

$$kV_1^{(+)} + k'B_0\, e^r = V_1^{(+)} - X;\ \text{and}$$

$$kV_1^{(-)} + k'B_0\, e^r = 0$$

The solution to the above system gives the synthetic call which is equivalent to our one-period call option on the project. The value of the call option at time $t = 0$ is $C_0 = I_0 = kV_0 + k'B_0$ with the strategy vector (k, k′). Solving the above system yields (k, k′) as $k = [V_1^{(+)} - X] / [V_1^{(+)} - V_1^{(-)}]$ and $k' = [-kV_1^{(-)}] / B_0\, e^r$ (Cox, Ross and Rubinstein, 1979).

Then a dynamic synthetic real protective put strategy would simply involve making an investment equal to C_0 at the onset. The call option payoff would have to be synthetically replicated by $I_0 = kV_0 + k'B_0$ with kV_0 representing the part of the total project cost that have to be invested at the onset and $(X_0 - kV_0)$ representing the part that is “deferred”. At $t = T_j$, the whole process is re-enacted with the new exercise price being $X_1 = (X_0 - C_0)$. The routine is repeated till such time as $V_j < X_j$ and terminates when $V_j \approx X_j$. At that point $\Sigma_{j=1} (X_j) \approx X_0$. In the next section we give a numerical illustration of our proposed computational approach.

Numerical illustration

The computational scheme:

Step 1: Compute the NPV of the project as the price of a call option on the value of the project at t = 0 using the Cox-Ross-Rubenstein one-step binomial process. Use the NPV of the project calculated by the traditional DCF technique as V_0 and the variance of the returns from the project as the volatility. The cost of development of the project is the exercise price X_0 and T – t = 1. The risk-free rate is proxied by the lowest-cost borrowing rate available to the business.

Step 2: C_0 is the NPV of the project (price of the call option on the value of the project) as calculated in the previous step. Conversely, a portfolio with a current dollar value of C_0e^r at T – t = 1 will decay to a value of C_0 at T – t = 0, $C_0 = I_0 = kV_0 + k' B_0$ is multiplied with e^r on both LHS and RHS to get the actual amount to be invested in the synthetic put portfolio at t = 0. Therefore the actual dollar amount to be invested in the project at t = 0 is C_0e^r out of which kV_0e^r is the "current equity" and $(X – kV_0e^r)$ is the "deferred equity". The actual dollar amount of debt at t = 0 is $k'B_0e^r$. The "deferred" investment is to the tune of $(X_0 – C_0e^r)$ (Kester, 1984).

Step 3: If $C_0 < 0$, the "put option" is exercised i.e. the project is liquidated and the debt is paid off with the proceeds. If $C_0 > 0$ *and* $V_1 \approx X_1$, the process terminates with the balance amount X_1 being invested in full in the project. Else if $C_0 > 0$ *and*

$X_1 = (X_0 - C_0) > V_1$, re-enact Step 1 with X_1 as the exercise price after making adjustments to the project value and volatility estimates if such is necessitated by changed business conditions.

Repeat the steps till such time as $X_j > V_j$. The process terminates when $X_j \approx V_j$. At that point the balance funds are invested into the project and $\Sigma_{j=1}(X_j) = X_0$.

Numerical results:

We illustrate our proposed computational scheme with a simple numerical example. The basic input data for the problem are supplied in the following table:

Particulars	**Million $**
Present value of whole project	1.43
Best-case value	2.00
Worst-case value	1.00
Exercise price	1.50
Lowest-cost borrowing rate	5%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table I: Input data (j = 0)

The expected value of the project is obtained as (Best-case value x 0.5) + (Worst-case value x 0.5). The cost of the project is same as its expected value. The present value of the project is obtained by dividing the expected value of the project by $e^{5\%}$. The probability of best-case scenario and the probability of worst-case scenario at each step have been set equal at 0.50 so as to maximize the element of uncertainty (by maximizing the *entropy of the outcomes*). The computational scheme has been implemented on a customized MS-Excel spreadsheet running a VBA routine. The numerical results are summarized in the following tables:

With protective put:	Portfolio	Debt	Equity	ΔEquity
At T - t = 1	0.750000	0.500000	0.250000	-
Best-case at T - t = 0	0.951229	0.475615	0.475615	0.225615
Worst-case at T - t = 0	0.475615	0.475615	0.000000	-0.250000
Without protective put:	Investment	Equity	ΔEquity	
Best-case at T - t = 0	1.500000	1.902459	0.402459	
Worst-case at T - t = 0	1.500000	0.951229	-0.548771	

Table II: Iteration j = 0

Present value of project	$0.71
Best-case	$1.00
Worst-case	$0.50
Exercise price	$0.75
Lowest borrowing/lending rate	5.00%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table III: Input data (j = 1)

With protective put:	Portfolio	Debt	Equity	Δ Equity
At T - t = 1	0.375000	0.250000	0.125000	-
Best-case at T - t = 0	0.475615	0.237807	0.237807	0.112807
Worst-case at T - t = 0	0.237807	0.237807	0.000000	-0.125000
Without protective put:	Investment	Equity	Δ Equity	
Best-case at T - t = 0	0.750000	0.951229	0.201229	
Worst-case at T - t = 0	0.750000	0.475615	-0.274385	

Table IV: Iteration j = 1

Present value of project	$0.36
Best-case	$0.50
Worst-case	$0.25
Exercise price	0.3750
Lowest borrowing/lending rate	5.00%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table V: Input data (j = 2)

With protective put:	Portfolio	Debt	Equity	Δ Equity
At T - t = 1	0.187500	0.125000	0.062500	-
Best-case at T - t = 0	0.237807	0.118904	0.118904	0.056404
Worst-case at T - t = 0	0.118904	0.118904	0.000000	-0.062500
Without protective put:	Investment	Equity	Δ Equity	
-			-	
Best-case at T - t = 0	0.375000	0.475615	0.100615	
Worst-case at T - t = 0	0.750000	0.237807	-0.512193	

Table VI: Iteration j = 2

Present value of project	$0.18
Best-case	$0.25
Worst-case	$0.125
Exercise price	$0.19
Lowest borrowing/lending rate	5.00%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table VII: Input data (j = 3)

With protective put:	Portfolio	Debt	Equity	Δ Equity
At T - t = 1	0.093750	0.062500	0.031250	-
Best-case at T - t = 0	0.118904	0.059452	0.059452	0.028202
Worst-case at T - t = 0	0.059452	0.059452	0.000000	-0.031250
Without protective put:	Investment	Equity	Δ Equity	
-				
Best-case at T - t = 0	0.187500	0.237807	0.050307	
Worst-case at T - t = 0	0.187500	0.118904	-0.068596	

Table VIII: Iteration j = 3

Present value of project	0.0892
Best-case	0.1250
Worst-case	0.0625
Exercise price	0.0938
Lowest borrowing/lending rate	5.00%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table IX: Input data (j = 4)

With protective put:	Portfolio	Debt	Equity	Δ Equity
At T - t = 1	0.046875	0.031250	0.015625	-
Best-case at T - t = 0	0.059452	0.029726	0.029726	0.014101
Worst-case at T - t = 0	0.029726	0.029726	0.000000	-0.015625
Without protective put:	Investment	Equity	Δ Equity	
Best-case at T - t = 0	0.093800	0.118904	0.025104	
Worst-case at T - t = 0	0.093800	0.059452	-0.034348	

Table X: Iteration j = 4

Present value of project	0.0446
Best-case	0.0625
Worst-case	0.03125
Exercise price	0.0469
Lowest borrowing/lending rate	5.00%
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table XI: Input data (j = 5)

With protective put:	Portfolio	Debt	Equity	Δ Equity
				□
At T - t = 1	0.023438	0.015625	0.007813	-
Best-case at T - t = 0	0.029726	0.014863	0.014863	0.007050
Worst-case at T - t = 0	0.014863	0.014863	0.000000	-0.007813
Without protective put:	Investment	Equity	Δ Equity	
-			□	
Best-case at T - t = 0	0.046900	0.059452	0.012552	
Worst-case at T - t = 0	0.046900	0.029726	-0.017174	

Table XII: Iteration j = 5

Present value of project	0.022294
Best-case	0.031250
Worst-case	0.015625
Exercise price	0.023438
Lowest borrowing/lending rate	0.050000
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table XIII: Input data (j = 6)

With protective put:	Portfolio	Debt	Equity	Δ Equity
At T - t = 1	0.011719	0.007813	0.003906	-
Best-case at T - t = 0	0.014863	0.007431	0.007431	0.003525
Worst-case at T - t = 0	0.007431	0.007431	0.000000	-0.003906
Without protective put:	Investment	Equity	Δ Equity	
Best-case at T - t = 0	0.023438	0.029726	0.006288	
Worst-case at T - t = 0	0.023438	0.014863	-0.008575	

Table XIV: Iteration j = 6

Present value of project	0.011147
Best-case	0.015625
Worst-case	0.007813
Exercise price	0.007813
Lowest borrowing/lending rate	0.050000
Probability of best-case scenario	0.50
Probability of worst-case scenario	0.50

Table XV: Final position (j = 7)

Conclusion

As the dollar investment in the project doesn't change from j = 6 to j = 7, the process should be terminated at this point. The business enterprise may choose to invest the balance funds in the project at this stage (given that $C_0 \geq 0$) as the gap between the exercise price and the present value of the project has almost been reduced to zero (it is around $571.53 at this point). The following table and graph shows the upscaling of the project over time with a synthetic protective put.

j	C_0	Dollar investment	Cumulative dollar investment
0	0.237807	0.750000	0.750000
1	0.118904	0.375000	1.125000
2	0.059452	0.187500	1.312500
3	0.029726	0.093750	1.406250
4	0.014863	0.046875	1.453125
5	0.007431	0.023438	1.476563
6	0.007431	0.011719	1.488281
7	0.003716	0.011719	1.500000

Table XVI: Upscaling of the project with a synthetic protective put

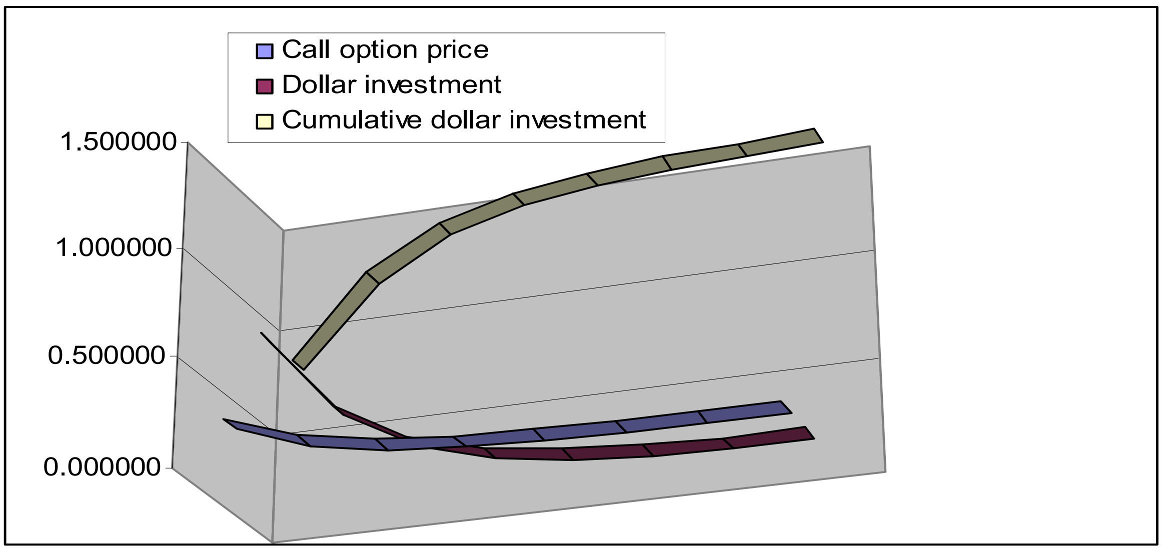

Figure I: Upscaling of the project with a synthetic protective put

The above numerical example illustrates that using a synthetic protective put strategy in a manner as outlined above, a business enterprise can choose to invest in a project in phases rather than doing it all at once; when it cannot defer the commencement of the project outright due to the pressures of competition.

In the numerical example, the input data were chosen so as to make the outcome consistent with the *law of diminishing marginal returns on investment*.

To the best of the author's knowledge, the proposed synthetic proposed put strategy in a real options scenario is unique although it is a commonplace risk management tool when managing a financial portfolio of stocks or futures. The computational procedure of working the hedging scheme is quite simple and applicable in practical situations. An obvious direction of future research would be extending this portfolio insurance concept to include American options as well.

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