Optimal Choice Models for Executing Time to American Options

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Abstract: Based on the structure models of options pricing on non-dividend-paying stock [16], this paper presents the choosing models and methods of optimal time of executing an American options for the first time. By using the models and methods, we can find the choosing criterion and optimal time to exercise the American options, i.e. the product of options price and its occurring probability is at maximum. So we can decide that an American option should be exercised or not in any time. The conclusions in this paper are more important in its consulting effect for single trader and organization investors to make their security market trade.

Key Words: partial distribution; American options; structure pricing; optimal executing; analytic formula

1 Introduction

Options are the very important derivative products for modern financial market. Options could evade the market risk so that they play an important role in regulating market price of commodity and developing the economic market. But options hold the enormous risk from price fluctuation of financial capitals itself. It must be worthy for options traders to know optimal time to exercise an options (whether it is call or put), especially the American options.

In the studies of option pricing, there have been many significant results (F. Black and M. Scholes 1973, R.C. Merton 1976, W.F. Sharpe 1978, R. Whaley 1981, H.E. Johnson 1983, R. Gesk and R. Roll 1984), and approximation methods for American put option (L.W. MacMillan 1986, and M.G. Subrahmanyam 1997). But, Up to now, no exact analytic formula has ever been produced for the value of an American put option on a non-dividend-paying stock (Hull 2000). Fortunately, *DF* structure models for options pricing has been given by F. Dai at 2004[16]; it could price the value of an American put option on a non-dividend-paying stock in an exact analytic way.

But, the further problem is that what is the time or value at which we should exercise the call or put options. R.Whaley, L.W.MacMillan, G.Barone-Adesi,etc had discussed and solved approximately this problem [5~10]. When applying the formulas of L.W.MacMillan, etc, we need to make sure in advance some parameters and use the iterative method, so it is hard to price the real options in some degree.

In order to solve the problems mentioned above, this paper will give an optimal choosing model of executing time for American options based on *DF* structure models for options pricing. By use of this model or method, we shall obtain easily the optimal choosing criterion, i.e. the product of price of options and occurring probability of this price is maximum. So we could make a decision to exercise the American options or not at any time, including the European option of course.

Here, the basic assumptions [16] about the underlying price (stock, spot, etc.) of option come into existence.

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At same time, we suppose that the options could be exercised at any time before its expiration, namely what we discussed is American options.

2 The Basic Assumptions

The basic assumptions [16] we use to define the price of an underlying asset (stock and stock indices) in this paper are as follow:

Basic assumptions. We suppose that the following assumptions are correct:

(i) There are prices (the cost price and the market price) to an underlying assert. The cost price means the average value of all the prices paid by the market traders to buy an underlying asset and the market price is the current trading price of an underlying asset.

(ii) The prices (cost price and market price) have been fluctuating with time. Any price and the fluctuation spread (i.e., the variance) of price are non-negative.

(iii) Both the cost price and the fluctuation of cost price of an underlying are the basic elements of determining the market prices of the underlying; the market prices come into being on the market exchange.

(iv) The possibilities that the market price of underlying is much lower than the cost price, or is much higher than the cost price, will be very small.

(v) The possibility that the price of making a trade steps down gradually along with the market price drifts gradually apart from the cost price.

(vi) All securities are perfectly divisible.

(vii) There are no transaction costs or taxes.

The assumptions above will be regarded as the basis of the following discussion.

3 Partial Distribution and DF Structure

3.1 Partial distribution and partial process

Definition 1 (The Partial Distribution). Let S be a non-negative stochastic variable, and it follows the distribution of density

$$f(x)=\begin{cases} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Big/ \int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx & x\geq 0 \\ 0 & x<0 \end{cases} \tag{1}$$

then S is said to have a Partial Distribution, and denotes $S\in P(\mu,\ \sigma^2)$. The partial distribution is a kind of truncated normal distribution.

Definition 2 (The Partial Process**).** If stochastic variable S is related to time, i.e., $\forall\, t\in[0,\infty)$, we have $S(t)\in P(\mu(t),\sigma^2(t))$, then the $\{S(t),\ t\in[0,\infty)\}$ is called a partial process.

In general, the stock price varies with time, therefore we have

Assumption 2. Let $\mu(t)$ be the cost price of stock at the time t, and $\sigma^2(t)$ be the variance of cost price at the time t. If the market prices of stock satisfy the basic assumptions above, thus suppose that $S(t)$, the market price variable, follows the partial distribution at time t, and denotes $S(t)\in P(\mu(t),\ \sigma^2(t))$.

$S(t)\in P(\mu(t),\ \sigma^2(t))$ can be a stock or the market price of the stock. From [16], we have the following theorem 1, theorem 2 and theorem 3:

Theorem 1. Let *S*, the market price variable of a stock, follow the partial distribution $P(\mu, \sigma^2)$, thus

1) The expected value $E(S)$ of *S*, means the average price on market exchange, is as follows

$$E(S) = \mu + \sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^{\infty} e^{-\frac{(x-\mu)^2}{\sigma^2}} dx \tag{2}$$

where, $R(S) = \sigma^2 e^{-\frac{\mu^2}{2\sigma^2}} \Big/ \int_0^{\infty} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$ is the average trading profit.

2) The variance, $D(S)$, of the market price variable *S*, which means the risk of the market price, is as follows

$$D(S) = \sigma^2 + E(S)[\mu - E(S)] \tag{3}$$

Theorem 2. For any $x\in[0,\infty]$, μ and σ are constant ($\mu\geq 0$, $\sigma>0$), then the following equations are correct approximately:

1) $\int_0^x e^{-\frac{t^2}{2}} dt = \sqrt{\frac{\pi}{2}(1 - e^{-\frac{2}{\pi}x^2})}$

2) $\int_0^x e^{-\frac{(u-\mu)^2}{2\sigma^2}} du = \sqrt{\frac{\pi}{2}}\sigma \times (\sqrt{1 - e^{-\frac{2}{\pi}(\frac{\mu}{\sigma})^2}} + \operatorname{sgn}(x-\mu)\sqrt{1 - e^{-\frac{2}{\pi}(\frac{x-\mu}{\sigma})^2}})$

where, $\operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$.

Corollary. For any $x\in[a,\infty]$, a, μ and σ are constant ($a<\mu$, $\mu \geq 0$, $\sigma >0$), then the following equations are correct approximately:

$$\int_a^x e^{-\frac{(u-\mu)^2}{2\sigma^2}} du = \sqrt{\frac{\pi}{2}}\sigma \times (\sqrt{1 - e^{-\frac{2}{\pi}(\frac{\mu-a}{\sigma})^2}} + \operatorname{sgn}(x-\mu+a)\sqrt{1 - e^{-\frac{2}{\pi}(\frac{x-\mu+a}{\sigma})^2}})$$

where, $\operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$.

3.2 DF process and DF structure

Definition 3. (*DF* process)**.** If $\{S(t), t\in[0,\infty)\}$ is a stochastic process, and $\forall\ t\in[0,\infty)$

$$S(t)\in P(\mu(t), \sigma^2(t)t)$$

then $\{S(t), t\in[0,\infty)\}$ is called a *DF* process.

Definition 4. Let *a* and *b* be non-negative constants, If $a>0, b=0$, we define:

$$e^{-\frac{a}{b}} = \lim_{z\to 0^+} e^{-\frac{a}{z}} = 0.$$

Definition 5(*DF* structure)**.** Let *X* be the value of an asset related to stock $S(t)\in P(\mu(t), \sigma^2(t))$, if $\forall\ t\in[0,\infty)$

and $T>t$, $X_S(t,T)\in P(X, D[S(t)](T-t))$, i.e. $X_S(t,T)$ follows the probability density

$$f_{X_S}(x)=\begin{cases}\dfrac{e^{-\frac{(x-X))^2}{2D[S(t)](T-t)}}}{\int_0^{\infty} e^{-\frac{(x-X)^2}{2D[S(t)](T-t)}}dx}, & x\geq 0\\ 0 & , \quad x<0\end{cases}$$

then we call $X_S(t,T)$ the *DF* stochastic structure of X on $S(t)$. $X_S(t,T)$ is called a *DF* structure of X for short.

When $t=T$, $X_S(t,T)=X$. So the real meaning of the *DF* structure, $X_S(t,T)$, is a stochastic value which is equal to that of an cash asset X in the future time T under-taking no discount of the interest rate.

Although the stock $S(t)$ has certain connections with *DF* structure $X_S(t,T)$ in variance, their stochastic movements may have no inevitable relation, so we could suppose that $X_S(t,T)$ and $S(t)$ are independent of each other.

4 Stochastic Structure Model To Price American Option

4.1 The assumption and Notation

Assumption 2.

(i) The basic assumptions are tenable.

(ii) There are no dividends during the life of the derivative.

(iii) The risk-free rate of interest, r, is constant.

(iv)There are no riskless arbitrage opportunities.

(v) Security trading is continuous.

All the following discussions are under Assumption 2. We will use the following notation:

t—the current time.

$S(t)$—market price of the stock at t.

X—strike price of option on $S(t)$.

T—time of expiration of option.

r—risk-free rate of interest to maturity T.

$S(t)e^{r(T-t)}$—forward value of $S(t)$($\hat{E}(S(T))$, the expected value in a risk-neutral world).

$X_S(t,T)$—***DF*** stochastic structure of X on $S(t)e^{r(T-t)}$.

C_S—value of call option to buy one share.

P_S—value of put option to sell one share.

If $S(t)\in P(\mu(t),\sigma^2(t))$ and $X_S(t,T)\in P(X, D[S(t)e^{r(T-t)}](T-t))$, we have the ***DF*** structure models of options pricing (***DF*** model for short) as follows:

4.2 DF structure models of call options pricing

From Definition 1, Definition 5 and Theorem 2, we have

4.2.1 The price of call option at time t is

$$C_S(t)=e^{-r(T-t)}E[\max(S(t)e^{r(T-t)}-X_S(t,T),0)]=e^{-r(T-t)}\int_0^{S(t)e^{r(T-t)}}[S(t)e^{r(T-t)}-x]f_{X_S}(x)dx$$

$$=(S(t)-Xe^{-r(T-t)})\times$$

$$\left[\frac{\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}+\mathrm{sgn}(S(t)e^{r(T-t)}-X)\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right]+\sqrt{\frac{2D[S(t)](T-t)}{\pi}}\left[\frac{e^{-\frac{(S(t)-Xe^{-r(T-t)})^2}{2D[S(t)](T-t)}}-e^{-\frac{(Xe^{-r(T-t)})^2}{2D[S(t)](T-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right] \quad (4)$$

When the call option is executed at any time $\tau\in[t,T]$, the price of underlying stock, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. According to (4) and definition 4, the current value of the option is

$C_S(\tau)=S(\tau)-Xe^{-r(T-\tau)}$, if $S(\tau)>Xe^{-r(T-\tau)}$;

$C_S(\tau)=0$, if $S(\tau)\leq Xe^{-r(T-\tau)}$;

i.e., $C_S(\tau)=max\{S(\tau)-Xe^{-r(T-\tau)}, 0\}$. At this time, the intrinsic value of the call option is $max\ \{S(\tau)-X, 0\}$, thus

$$C_S(\tau)\geq max\{S(\tau)-X, 0\} \quad (5)$$

4.2.2 The price of put option at time t is

$$P_S(t)=e^{-r(T-t)}E[\max(X_S(t,T)-S(t)e^{r(T-t)},0)]$$

$$=e^{-r(T-t)}\int_{S(t)e^{r(T-t)}}^{\infty}[x-S(t)e^{r(T-t)}]f_{X_S}(x)dx$$

$$=(Xe^{-r(T-t)}-S(t))\times$$

$$\times\left[\frac{1-\mathrm{sgn}[S(t)e^{r(T-t)}-X]\sqrt{1-e^{-\frac{2}{\pi}\frac{(S(t)-Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right]+\sqrt{\frac{2D[S(t)](T-t)}{\pi}}\left[\frac{e^{-\frac{(S(t)-Xe^{-r(T-t)})^2}{2D[S(t)](T-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D[S(t)](T-t)}}}}\right] \quad (6)$$

When the put option is executed at any time $\tau\in[t, T]$, the price of underlying stock, $S(\tau)$, becomes a constant to the option contract, thus $D[S(\tau)]=0$. According to (6) and definition 4, the current value of the option is

$P_S(\tau)=Xe^{-r(T-\tau)}-S(\tau)$, if $S(\tau)<Xe^{-r(T-\tau)}$

$P_S(\tau)=0$, if $S(\tau)\geq Xe^{-r(T-\tau)}$

i.e., $P_S(\tau)= max\{Xe^{-r(T-\tau)}-S(\tau), 0\}$. At this time, the intrinsic value of the put option is $max\{X-S(\tau), 0\}$, thus,

$$P_S(\tau)\leq max\{X-S(\tau), 0\} \quad (7)$$

From (4) and (6), we could see that the price of option is made up of two parts. One is the market price of stock, and the other is the fluctuation of stock price.

5 The Optimal Choice Model for Executing Price of American Options

The optimal choice criterion here is the options price at which the product of options value and its occurring probability reaches maximum. According to references [17 ~ 19], we obtain

Theorem 3 We denote the market price of stock (or other underlying assets) at current time t by $S(t)$, and the strike price of option by X. If $S(t)\in P(\mu(t),\sigma^2(t))$, thus

1) For call option, let $L_C(S(t)) = f_S(S(t))\times C_S(t)$ and $L_C(S_C^*(t))=\max\limits_{S(t)>0}\{L_C(S(t))\}$, thus

$$S_C^*(t) = \frac{Xe^{-r(T-t)} + \mu(t) + \sqrt{(Xe^{-r(T-t)} - \mu(t))^2 + 4\sigma^2(t)}}{2} \tag{9}$$

When $S(t)>X$ and $S(t)\geq S_C^*(t)$, we should execute the call option.

2) For put option, let $L_P(S(t)) = f_S(S(t)) \times P_S(t)$ and $L_P(S_P^*(t)) = \max_{S(t)>0}\{L_P(S(t))\}$, thus

$$S_P^*(t) = \frac{Xe^{-r(T-t)} + \mu(t) - \sqrt{(Xe^{-r(T-t)} - \mu(t))^2 + 4\sigma^2(t)}}{2} \tag{10}$$

When $S(t)<X$ and $S(t)\leq S_P^*(t)$, we should execute the put option.

Proof. $S(t)$, $\mu(t)$ and $\sigma^2(t)$ are denoted separately by S, μ and σ^2 for short.

1) $L_C(S(t))$ is denoted by L for short. We need to seek S_C^* and make $L(S_C^*) = \max_S\{L\}$. Because

$$\frac{\partial L}{\partial S} = \left(-\frac{S-\mu(t)}{\sigma^2(t)}\right) L + f_S(S)\int_0^{Se^{r(T-t)}} f_{X_S}\,dx$$

namely

$$\frac{\partial L}{\partial S} = f_S(S) \times \int_0^{Se^{r(T-t)}} \left(-\frac{(S-\mu)}{\sigma^2}(S - xe^{-r(T-t)}) + 1\right) f_{X_S}(x)dx$$

$$= f_S(S)\left\{\left(1 - \frac{S-\mu}{\sigma^2}(S - Xe^{-r(T-t)})\right)\int_0^{Se^{r(T-t)}} f_{X_S}dx + \left(\frac{S-\mu}{\sigma^2}\right)\sqrt{\frac{2D(S)(T-t)}{\pi}}\left[\frac{e^{-\frac{(Xe^{-r(T-t)})^2}{2D(S)(T-t)}} - e^{-\frac{(S-Xe^{-r(T-t)})^2}{2D(S)(T-t)}}}{1+\sqrt{1-e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D(S)(T-t)}}}}\right]\right\}$$

When we exercise the call option contract, $S(t)$ is a constant to the contract at same time, so $D(S)=0$. From definition 5, definition 4 and theorem 2, have $f_{X_S}(Se^{r(T-t)})=0$ and $\int_0^{Se^{r(T-t)}} f_{X_S}dx=1$.

Let $\frac{\partial L}{\partial S} = 0$, thus $(S-\mu)(S-Xe^{r(T-t)})-\sigma^2=0$. We obtain

$$S = \frac{Xe^{-r(T-t)} + \mu \pm \sqrt{(Xe^{-r(T-t)} - \mu)^2 + 4\sigma^2}}{2}.$$

The larger S is, the larger $C_S(t)$ is, so we select

$$S_C^* = \frac{Xe^{-r(T-t)} + \mu + \sqrt{(Xe^{-r(T-t)} - \mu)^2 + 4\sigma^2}}{2}.$$

Since

$$\frac{\partial^2 L}{\partial S^2} = -\frac{1}{\sigma^2} L + \left(-\frac{S-\mu}{\sigma^2}\right)\frac{\partial L}{\partial S} + f_S(S) \times \left(-\frac{S-\mu}{\sigma^2}\right)\int_0^{Se^{r(T-t)}} f_{X_S}(x)dx$$

$$+ f_S(S) f_{X_S}(Se^{r(T-t)})\ e^{r(T-t)}$$

and when $D(S)$=0 and $\frac{\partial L}{\partial S} = 0$, $\frac{\partial^2 L}{\partial S^2} = -\frac{f_S(S)}{\sigma^2}\left[C_S(t) + S - \mu\right]$. Because

$$S_C^* - \mu = \frac{Xe^{-r(T-t)} - \mu + \sqrt{(Xe^{-r(T-t)} - \mu)^2 + 4\sigma^2}}{2} > \frac{1}{2}(Xe^{-r(T-t)} - \mu + |\ Xe^{-r(T-t)} - \mu\ |) \geq 0,$$

$\frac{\partial^2 L}{\partial S^2} < 0$, thus $L(S_C^*) = \max_S\{L(S)\}$.

2) $L_P(S(t))$ is also denoted by L for short. We have

$$\frac{\partial L}{\partial S} = \left(-\frac{S-\mu}{\sigma^2}\right) L - f_S(S)\int_{Se^{r(T-t)}}^{\infty} f_{X_S}(x)dx$$

Similar to the proof in 1), have

$$\frac{\partial L}{\partial S} = -f_S(S) \times$$

$$\left\{\left(1 - \frac{S-\mu}{\sigma^2}(S - Xe^{-r(T-t)})\right)\int_0^{Se^{r(T-t)}} f_{X_S} dx + \left(\frac{S-\mu}{\sigma^2}\right)\sqrt{\frac{2D(S)(T-t)}{\pi}}\left[\frac{e^{-\frac{(S-Xe^{r(T-t)})^2}{2D(S)(T-t)}}}{1+\sqrt{e^{-\frac{2}{\pi}\frac{(Xe^{-r(T-t)})^2}{D(S)(T-t)}}}}\right]\right\}$$

Let $\frac{\partial L}{\partial S} = 0$, then

$(S-\mu)(S-Xe^{r(T-t)})-\sigma^2=0$

The smaller S is, the larger $P_S(t)$ is, so we select

$$S_P^*(t) = \frac{Xe^{-r(T-t)} + \mu - \sqrt{(Xe^{-r(T-t)} - \mu)^2 + 4\sigma^2}}{2}$$

Also similar to the proof in 1), then $\frac{\partial^2 L}{\partial S^2} = \frac{f_S(S)}{\sigma^2}\left[S - \mu - P_S(t)\right]$.

Because $S_P^* - \mu = \frac{Xe^{-r(T-t)} - \mu - \sqrt{(Xe^{-r(T-t)} - \mu)^2 + 4\sigma^2}}{2}$

$< \frac{1}{2}(Xe^{-r(T-t)} - \mu - | Xe^{-r(T-t)} - \mu |) \leq 0$, thus $\frac{\partial^2 L}{\partial S^2} < 0$, namely $L(S_P^*) = \max_S \{L(S)\}$.

The results follow.

6 The Empirical Analysis

Here we shall make an example on the data of MSFT (MICROSOFT CP) and its options, and the data are brought from reference [16].

6.1 Estimating parameters and fitting with Partial Distribution

Time: Jan. 29, 2002 -Dec. 24, 2002.

Trading days: n=230.

Length of each field: Δ=0.467609(US $).

Number of divided fields: m=46.

The estimated results of parameters are as follows:

1) The estimated values of parameters in partial distribution $P(\mu, \sigma^2)$:

$\hat{\mu}$ =53.58500013; $\hat{\sigma}^2$ = 24.62632700.

2) The fiducial test:

χ^2 =58.08095341< $\chi^2_{0.025}$ (43)=62.990.

The result of statistic test indicates that partial distribution could fit with the stock prices better, i.e. partial distribution could describe the prices distribution of MSFT stock.

4.2 The analysis for optimal price to execute the option on MSFT stock

Time: Dec. 25, 2002.

Product: The option contract on *MSFT*.

Maturity: Expires After: Fri, Jan. 16, 2004.

Underlying: Close price of *MSFT* at current date, 53.39$.

The prices on Dec.24, 2002 are contained in table 1, which are the closing prices traded actually in the United States option market (*TP*), the call and put options prices calculated by *DF* structure formulas, (*DF*), and the call and put options prices calculated by *B-S* formulas, (*B-S*).

From table 1, we see that more *DF* prices are closed to real trading prices than *B-S* prices.

Table 1. Contrast of options prices of **MSFT**

Strike prices	Call options prices			Put options prices		
	TP	*DF*	*B-S*	*TP*	*DF*	*B-S*
50.0	11.40	5.406	5.369	7.70	.1399	.1028
55.0	9.20	1.702	1.706	9.90	1.248	1.252
60.0	6.80	.2213	.2466	12.50	4.580	4.605
65.0	5.10	.0212	.0151	15.90	9.192	9.186
70.0	3.80	.0007	.0004	19.50	13.98	13.98

1) The optimal price for executing call option. According to (9) and (4), have

$$S_C^*(t) = \frac{Xe^{-r(T-t)} + \mu(t) + \sqrt{(Xe^{-r(T-t)} - \mu(t))^2 + 4\sigma^2(t)}}{2}.$$

Denote $L_C(S(t))= f_S(S(t))C_S(t)$, $G(t)= L_C(S_C^*(t) + w)$, -3 $\leq w \leq$ 3, and let r=0.07. Thus,

$G(t)\big|_{w=0} = L_C(S_C^*(t))$.

If strike price X=50, X is smaller than current trading price 53.39. At this time, the varying process of $G(t)$ is shown in figure 1. So we see that $S_C^*(t)$ makes $f_S(S(t))\times C_S(t)$ reach maximum. Therefore, if the price of underlying asset is not smaller than $S_C^*(t)$, i.e. $S(t)\geq S_C^*(t)$, we should exercise the call option contract, which is an optimal choice on the occurring probability of the price.

2) The optimal price for executing call option. According to (10) and (6), have

$$S_P^*(t) = \frac{Xe^{-r(T-t)} + \mu(t) - \sqrt{(Xe^{-r(T-t)} - \mu(t))^2 + 4\sigma^2(t)}}{2}$$

And denote $L_P(S(t))= f_S(S(t))P_S(t)$, $H(t)= L_P(S_P^*(t)+w)$, $-3\leq w\leq 3$. Thus,

$H(t)\big|_{w=0} = L_P(S_P^*(t))$

If strike price X=60, X is larger than current trading price 53.39. At this time, the varying process of $H(t)$ is shown in figure 2. So we see that $S_P^*(t)$ makes $f_S(S(t))\times P_S(t)$ reach maximum. Therefore, if the price of underlying asset is not smaller than $S_P^*(t)$, i.e. $S(t)\leq S_P^*(t)$, we should exercise the put option contract, which is an optimal choice on the occurring probability of the price.

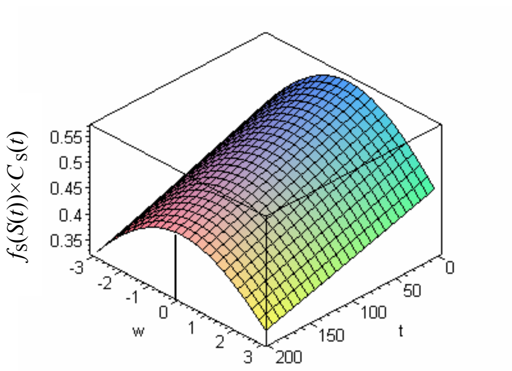

Figure 1 The optimal choice to execute a call option. When w=0, i.e. $L_C(S_C^*(t)) = f_S(S(t))C_S(t)$ =maximum, we get the optimal opportunity to execute a call option, at same time, the product of options price and its occurring probability is maximum.

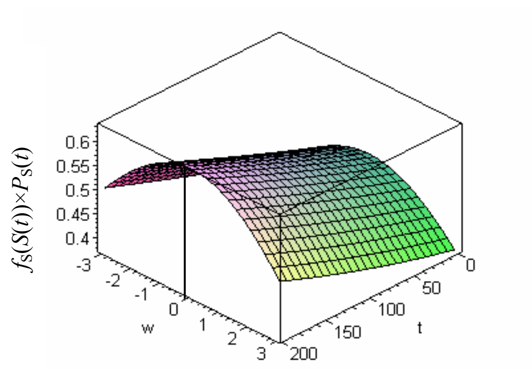

Figure 2 The optimal choice to execute a put option. When w=0, i.e. $L_P(S_P^*(t)) = f_S(S(t))P_S(t)$ =maximum, we get the optimal opportunity to execute a put option, at same time, the product of options price and its occurring probability is maximum.

7 Concluding Remarks

Based on the Partial distribution [17], DF structure pricing for option [16] and the optimal method [18~19], this paper gives the formulas to calculate the optimal criterion for executing the options, include the call option and put option. By the formulas, all of traders could determine whether the options (call and put option) should be exercised or not at any time. The formulas are applicable to both European option and American option. And now, the stochastic structure method for pricing options is more perfect both in theory and practice. So we could say that DF structure model is a better one.

what we need to say are, though the formula (9) is the optimal criterion for traders to execute their call option, it can be taken as the optimal criterion for traders to buy put option; similarly, though the formula (10) is the optimal criterion for traders to execute their put option, it can be taken as the optimal criterion for traders to buy call option.

The current price of call option is larger than the optimal criterion in (4), i.e. $S(t) \geq S_C^*(t)$, does not mean the price reaches maximum limit; similarly, the current price of put option is smaller than the optimal criterion in (4), i.e. $S(t) \leq S_P^*(t)$, does not mean the price reaches minimum limit, but mean the possibility that the price movement keeps in its original trend will be more and more smaller. So the theorem 3 is more important in its consulting effect for organization investors to make their market trade.

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