

An Empirical Analysis of Structural Models of Corporate Debt Pricing

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Abstract

This paper tests empirically the performance of three structural models of corporate bond pricing, namely Merton (1974), Leland (1994) and Fan and Sundaresan (2000). While the first two models overestimate bond prices, the Fan and Sundaresan model reveals an extremely good performance. When considering the prediction of credit spreads, the three models under-estimate market spreads but, again, Fan and Sundaresan has a better performance. We find rating, maturity and asset volatility effects in the prediction power, as the models under-estimate less the spreads of riskier firms and of bonds with better rating quality and longer maturity. Moreover, our results reveal the existence of a new industry effect. Spread errors are systematically related to some bond- and firm-specific variables, as well as term structure variables.

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Abstract

This paper tests empirically the performance of three structural models of corporate bond pricing, namely Merton (1974), Leland (1994) and Fan and Sundaresan (2000). While the first two models overestimate bond prices, the Fan and Sundaresan model reveals an extremely good performance. When considering the prediction of credit spreads, the three models under-estimate market spreads but, again, Fan and Sundaresan has a better performance. We find rating, maturity and asset volatility effects in the prediction power, as the models under-estimate less the spreads of riskier firms and of bonds with better rating quality and longer maturity. Moreover, our results reveal the existence of a new industry effect. Spread errors are systematically related to some bond- and firm-specific variables, as well as term structure variables.

1. Introduction

The aim of this paper is to test empirically the performance of three structural models of corporate debt pricing: those of Merton (1974), Leland (1994) and Fan and Sundaresan (2000), using a sample of 50 bond prices from firms with simple capital structures, during the period 2001–04. In the case of Fan and Sundaresan (2000) we calibrate particularly its debt-equity swap model¹. With the analysis of prediction errors we evaluate how well these models fit bond prices and credit spreads. We believe that this is important because it allows for a discussion of some “real world features” that are not captured by these models. On the other hand, a comparison of the results of the three models allows us to determine the extent to which some innovations in the models have improved the pricing of risky bonds. We refer specifically to the possibility of early default, coupons, taxes and bankruptcy costs when we compare the Merton (1974) model with the Leland (1994) model and the effect of negotiation features when we compare the Leland (1994) model with the Fan and Sundaresan (2000) model.

Moreover, we evaluate whether there are differences in the performance of the models dependent on rating and maturity of the bonds or on asset volatility and sector of the firms. This analysis is important because previous empirical studies in the field have not been conclusive: while Ericsson and Reneby (2002) report a worst performance of their structural model for speculative grade bonds, Eom *et al.* (2004) do not confirm this pattern in their sample.

Even considering relevant the analysis of rating, maturity and asset volatility effects, the study of a sector or industry effect is probably more important as there is very little empirical evidence regarding this issue, and we believe that the study of a sector effect in the performance of the structural models is one of the main contributions of this paper. If we detect any sector effect, then it will be interesting to analyse which sector characteristics might explain a better or worst performance of the models. We might have models that perform better in some sectors but do a worse job in others, meaning that some of these models have to be adjusted for new industry features.

¹ We refer to the Fan and Sundaresan (2000) debt-equity swap and the Fan and Sundaresan (2000) model interchangeably.

Another important issue that deserves our attention is the study of systematic prediction errors. Are there any bond-specific, firm-specific or market variables that have a systematic relationship with spread errors? Among other factors, we intend to study the influence of size, leverage, maturity, rating, asset volatility and firm-growth opportunities in the performance of the models. This is important not only because the existing empirical literature contains some contradictory results in this analysis but also because we introduce some new explanatory variables, namely the yield to maturity and the market-to-book ratio (as a proxy for growth opportunities).

To our best knowledge, this paper is the first study that calibrates and evaluates the performance of the Fan and Sundaresan (2000) debt–equity swap. There are some calibrations of strategic debt services models such as the one by Mella-Barral and Perraudin (1997) treated in Huang and Huang (2002). However, they do not explicitly calibrate the Fan and Sundaresan (2000) debt–equity swap. The calibration of this model allows us to discuss the importance of a negotiation process between stockholders and debtholders in the distribution of the firm’s claims at liquidation.

The paper is organized as follows. In Section 2 there is a discussion of the existing literature in the field and, in Section 3, a presentation of the theoretical assumptions and valuation formulae of each model. Section 4 presents the empirical implementation, where we explain the process of data gathering and the calibration procedure adopted to implement the models. Section 5 gives the empirical results and a discussion of the performance of the models and the systematic prediction errors. Finally, in Section 6, we summarize the main findings of the paper.

2. Literature Review

Recent years have seen many theoretical developments in the field of credit risk research. Most of this research has concentrated on the pricing of corporate and sovereign defaultable bonds as

the basis of credit risk pricing. These studies can be divided in two main categories: structural models and reduced-form models.

Structural models have their origins in the framework of Merton (1974), which has been the key foundation of corporate debt pricing. Relying on the contingent claims analysis of Black and Scholes (1973), Merton (1974) presents a simplified model that can be used to value each component of the firm's liability mix. In a structural framework, the default process of a company is driven by the value of the company's assets and the firm's default risk is explicitly linked to the variability in the firm's asset value. Under these structural models, all the relevant credit risk elements, including default, are a function of the structural characteristics of the firm: asset volatility (business risk) and leverage (financial risk). Reduced-form models, on the other hand, do not condition default on the value of the firm, and parameters related to the firm's value do not need to be estimated.

Despite the innovative nature of the Merton (1974) model, allowing for the valuation of a firm's debt and equity without a prior knowledge of the real drift of the firm's asset, it presents many shortcomings that are essentially due to its simplifying assumptions about reality. It assumes that the liability structure of the firm consists only of a single class of debt, a non-callable zero coupon bond, and that bankruptcy is not only costless but also cannot be triggered before maturity. In addition, it assumes that the absolute priority rule always holds at maturity, meaning that equityholders can only obtain a positive payoff after debtholders have been totally reimbursed. This is clearly an unrealistic assumption. Franks and Torous (1994) show that the strict absolute priority rule was violated in 78% of the bankruptcies of their sample. Another important stylized version of reality is the assumption of a flat term structure of interest rates.

Many papers, including Black and Cox (1976), Geske (1977), Leland (1994), Leland and Toft (1996), Longstaff and Schwartz (1995), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Fan and Sundaresan (2000) and Collin-Dufresne and Golstein (2001), have extended the original Merton (1974) model to incorporate more realistic assumptions. A new assumption, which is common to all these models, and represents a major improvement of

the Merton framework, is the possibility of early default. In these models the firm can go into bankruptcy before maturity, as soon as a bankruptcy trigger for the asset value is reached.

Leland (1994) extends the Black and Cox (1976) endogenous default model to include the tax advantage of debt and bankruptcy costs. The first “real world friction” works an incentive to increase the leverage (because of the tax benefit of interest payment) and bankruptcy costs as a disincentive. However, the Leland (1994) model still has the limitation of assuming full respect for the absolute priority rule. In fact, the recognition that bankruptcy procedures leave some considerable scope for strategic behaviours from the different claimants involved leads to the appearance of new structural models, usually denominated strategic debt service models. These include the Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) and Fan and Sundaresan (2000) models. Fan and Sundaresan (2000) enhance Anderson and Sundaresan’s (1996) model by considering first, a continuous time-framework, and secondly, corporate taxes. In addition, they introduce a bargaining power parameter, making possible a redistribution of power between debtholders and equityholders.

Several empirical studies have pointed out the weaknesses of the Merton (1974) model, in particular its incapacity to generate the levels of yields spreads observed in the market. These include, among others, the papers of Jones *et al.* (1984), Ogden (1987), Wei and Guo (1997), Lyden and Saraniti (2000), Ericsson and Reneby (2002) and Eom *et al.* (2004).

Jones *et al.* (1984) analysed 177 bonds issued by 15 firms and found that the Merton model overestimates bond prices by an average of 4.5%. They conclude that the model performs better for speculative grade bonds and that prediction errors are systematically related to maturity, equity variance and leverage. Ogden (1987), on the other hand, looked at 57 callable bonds and sinkable corporate bonds and found that the Merton model under-estimates spreads by 104 basis points (bp) on average. These studies suffer from some problems with the inclusion of callable bonds and sinking fund provisions. By considering bonds with these features it is difficult to evaluate whether the under-estimation revealed by the Merton model is due to its assumptions or to the pricing of these features by investors.

In the last decade, the studies of Lyden and Saraniti (2000), Ericsson and Reneby (2002) and Eom *et al.* (2004) represent an improvement in terms of the quality of the bond sample. All these studies not only use firms with simple capital structures but also exclude from the sample bonds with any call or sinking fund provision. Lyden and Saraniti (2000), who compare the performance of the Merton (1974) model with the Longstaff and Schwartz (1995) model, find that both these models under-estimate the credit spread. For the Merton model the average under-estimation in credit spread is between 80 and 90 bp and the errors are systematically related to coupon and time to maturity. Ericsson and Reneby (2002), who implemented a perpetual bond model based on the Black and Cox (1976) framework, found a good performance of the model. They also found that prediction errors are linked to liquidity. There is a greater under-estimation of credit spread for speculative grade bonds, which are perceived to be less liquid.

To date, the most comprehensive empirical study about the performance of corporate debt pricing models is found in Eom *et al.* (2004). They assess the empirical performance of the Merton (1974), Geske (1977), Longstaff and Schwartz (1995), Leland and Toft (1996) and Collin-Dufresne and Goldstein (2001) models using a sample of 182 bond prices during the period 1986–97. For the Merton (1974) model the under-estimation problem is confirmed but for other models, like Leland and Toft (1996), there is an overestimation of credit spread, which they report as due to the accuracy of the calibration process. The prediction power of these models seems to be related to leverage, size, asset volatility and some term structure control variables.

3. Theoretical Models

In this section we summarize the main theoretical assumptions of the Merton (1974) and Leland (1994) models and the Fan and Sundaresan (2000) debt–equity swap. Moreover, there is a presentation of the formulae concerning the firm value, equity, debt and credit spread that are the support of the calibration procedure discussed in Section 4.

3.1 Merton (1974)

Merton (1974), being the seminal paper on structural models, relies on a set of assumptions that constitute the basis for many other models. Among others, it assumes that the dynamics for the value of the assets, V_t , can be described by a diffusion-type process with stochastic differential equation

$$dV_t = (\mu - \delta)V_t dt + \sigma V_t dZ_t \quad (3.1)$$

where μ is the instantaneous expected rate of return on the assets, δ is the constant fraction of value paid to both equityholders and debtholders (payout ratio), σ the constant variance of the return on the underlying asset, and Z_t a standard Wiener process. Even though the original version of the Merton (1974) model assumes no payout ratio, we incorporate this parameter in our model, as most firms pay both interest to bondholders and dividends to equityholders.

The asset value is financed both by equity, E , and one representative zero-coupon noncallable debt contract, D , with maturity T and face value F . The value of the firm and the asset value are identical and do not depend on the capital structure itself. The asset value, V_0 , is thus given by the sum of risky debt and equity. With this framework, equity can be seen as a call option on the value of the firm with strike price F . On the other hand, debtholders have bought a risk-free bond with face value F and given the equityholders the option to sell them the firm's assets for F . Equity and debt values are therefore given by Black and Scholes (1973) formulae:

$$E_0(V_0, T, \sigma, r, F, \delta) = V_0 e^{-\delta T} N(d_1) - F e^{-rT} N(d_2) \quad (3.2)$$

$$D = V_0 e^{-\delta T} N(-d_1) + F e^{-rT} N(d_2) \quad (3.3)$$

with

$$d_1 = \frac{\ln(V_0 / F) + (r - \delta + \sigma^2 / 2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

where $N(\circ)$ is the cumulative standard normal distribution.

One of the most important variables that is analysed in this study is the credit spread, CS . It is defined as the difference between the yield to maturity, ym , and the risk free rate r . The ym is computed as $ym = -\frac{\ln(D/F)}{T}$. Hence, the credit spread formula is

$$CS = ytm - r = -\frac{1}{T} \ln \left[N(d_2) + \frac{V_0}{Fe^{-rT}} N(-d_1) \right] \quad (3.4)$$

where it can be seen that the credit spread is a direct function of the quasi-debt ratio Fe^{-rT}/V_0 , maturity and asset volatility. Intimately related to credit spread is the risk neutral default probability (RNDP), which is, in this case, represented by $N(-d_2)$.

3.2 Leland (1994)

In the Leland (1994) model the firm value still follows (3.1) and the risk-free rate is constant². Leland (1994) models a tax environment in which perpetual continuous coupon payments, C , are tax deductible. Considering a constant corporate tax rate τ , the firm obtains tax shields from its debt at a rate $C\tau$ until default. Bankruptcy occurs when the firm value reaches a threshold V_b . In this case, the firm incurs costs αV_b , where α is defined as the bankruptcy cost parameter or one minus the recovery rate. Because of these new “real world features” the levered firm value, v , is no longer identical to the unlevered firm value V_u . Rather, the firm value increases in the amount of tax shield, TS , and decreases in the amount of bankruptcy cost, BC . Under these new assumptions, the debt value is now

$$D = \frac{C}{r}(1 - P_b) + P_b(1 - \alpha)V_b \quad (3.5)$$

where P_b is $\left(\frac{V_u}{V_b}\right)^\lambda$ and V_b is given by

$$V_b = \frac{C(1 - \tau) - \lambda}{r - 1 - \lambda} \quad (3.6)$$

The parameter λ in the bankruptcy trigger solution is $\frac{1}{2} - \frac{(r - \delta)}{\sigma^2} - \sqrt{\left[\frac{1}{2} - \frac{(r - \delta)}{\sigma^2}\right]^2 + \frac{2r}{\sigma^2}}$.

² Once again we consider the version of Leland (1994) with payout ratio.

P_b can be interpreted as the RNDP and λ as the elasticity of the probability of default with respect to the value of the assets of the firm. As such, it is negative and increases with the volatility of the assets of the firm.

The bankruptcy costs and the tax shields are given by equations (3.7) and (3.8), and the total firm value and equity value by equations (3.9) and (3.10), respectively:

$$BC = P_b \alpha V_b \quad (3.7)$$

$$TS = \frac{\pi C}{r} - \frac{\pi C}{r} \left(\frac{V_u}{V_b} \right)^\lambda \quad (3.8)$$

$$v = E + D = V_u + TS - BC \quad (3.9)$$

$$E = v - D \quad (3.10)$$

The credit spread is

$$CS = \frac{C}{D} - r \quad (3.11)$$

3.3 The Fan and Sundaresan (2000) debt–equity swap

The Fan and Sundaresan (2000) debt–equity swap assumes that at an endogenously determined lower reorganization boundary debtholders are offered a proportion of the firm’s equity to replace the original debt contract. This can be thought of as a distress exchange. At certain trigger point V_b the claimants negotiate not to operate the firm and sell their stake to outsiders who pay them the value of the assets of the firm. It resembles a swap because debtholders swap their debt for equity and then sell the equity to potential buyers.

Unlike Leland (1994), which does not include the possibility of debt renegotiation, Fan and Sundaresan (2000) assume a continuous bargaining power parameter η . When $\eta = 1$, equityholders have all the bargaining power and make take-it-or-leave-it offers to debtholders. On the other hand, when $\eta = 0$, we get the Leland (1994) outcome where debtholders make take-it-or-leave-it offers to equityholders. With this refinement in Leland’s (1994) model the valuation framework is changed as follows. The debt value is now defined as

$$D = \frac{C}{r}(1 - P_b) + P_b(1 - \eta\alpha)V_b \quad (3.12)$$

where the new bankruptcy threshold is

$$V_b = \frac{C(1 - \tau) - \lambda}{r} \frac{1}{1 - \lambda - \eta\alpha} \quad (3.13)$$

and P_b and λ are defined as before. Equity and firm value are given by equations (3.14) and (3.15), respectively:

$$E = V_u - \frac{C(1 - \tau)}{r}(1 - P_b) + \eta\alpha V_b P_b - V_b P_b \quad (3.14)$$

$$v = E + D \quad (3.15)$$

As Fan and Sundaresan (2000) also assumes a continuous perpetual coupon, the credit spread is given by equation (3.11).

4. Empirical Implementation

This section is organized in two parts. First we describe the process of data gathering and secondly we present the calibration procedure used to estimate the parameters of the models. We provide a specific description of the estimation of each model's parameters.

4.1 Data

In order to test empirically the models presented earlier it is important to select a sample of companies with simple capital structures. Ideally, we should have companies with zero coupon bonds when testing the Merton (1974) model and companies with perpetual bonds when testing the Leland (1994) and the Fan and Sundaresan (2000) models. However, since it is not always possible to find these "perfect" bonds in the markets, the most reasonable approach consists in selecting bonds that have reliable prices and straightforward cashflows. An attempt to use these models to price corporate debt of firms with complex capital structures would raise doubts as to whether pricing errors are due to the assumptions of the models or to their inability to price this sort of debt.

The first selection criterion used in this study consists in (1) limiting the sample to U.S. non-financial firms with no more than three bonds (issued in U.S. dollars). In addition, the following criteria are applied: (2) consider only coupon bonds with all principal retired at maturity (bullet bonds); (3) do not include bonds with option features like callable, convertible or puttable bonds; (4) do not include floating-rate bonds or bonds with sinking fund provisions; and (5) do not include bonds with time to maturity less than one year, as they are unlikely to trade³.

Bond data were obtained from DATASTREAM but, in order to assure the straight application of these criteria, there was a double check of the characteristic of the bonds by consulting their prospectus on the EDGAR database⁴. Furthermore, in order to assure some reliability of the bond prices, all the bonds with the same quote for more than two months (despite the changes in interest rates) were excluded. As a final criterion there is the requirement that all these companies have publicly traded stock. Stock prices are not only required to compute the market value of equity but also to compute the stock volatility. In the end the sample consisted in 50 bonds. The firms are grouped in a total of six sectors, namely Industrial, Consumer Cyclical, Energy, Basic Materials, Healthcare and Consumer Non-Cyclical. This grouping is based on Thomson ONE Banker sector convention⁵.

The time-period of the study was set as 2001–04. Since DATASTREAM does not provide bond price information prior to 28/09/2001 it was not possible to extend this period. Another important issue of the data selection process is the frequency of the data. As some of the variables of the study rely on accounting data we have to make bond information “compatible” with accounting information. This being so, and trying to maximize the number of observations in the time series, it was decided to use quarterly observations (in the end the pricing performance was carried out using 317 observations).

³ These are necessary but not sufficient conditions. We do not include in our sample all the bonds satisfying these conditions.

⁴ EDGAR database is available at SEC (U.S. Securities and Exchange Commission) web site www.sec.gov.

⁵ Source: <http://banker.analytics.thomsonib.com/ta/>

Appendix A.1 presents summary statistics on the 50 bonds in the sample. There are a total of 10 companies with just one traded bond, 17 companies with two traded bonds and only 2 companies with three traded bonds. The average coupon rate for all bonds is 6.916%, ranging from 4.875 to 8.875%. The Consumer Non-Cyclical sector has the bonds with highest coupons and, not surprisingly, is also the one with the highest average yield spread, namely 279.9 bp. The average yield spread is 221.5 bp for all bonds and most of the bonds (88% of the sample) are investment grade bonds (rated BBB– or higher).

As mentioned earlier, the implementation of the model requires some accounting information, namely information about the liability structure of the firm. Quarterly balance sheets for each company were obtained from EDGAR database. Appendix A.2 lists some descriptive statistics about the firms. These firms are reasonably large, as the average market value of assets is around \$6 billion. For most companies, the market leverage (average 37.4%) is substantially below the book leverage (average 56.1%). A more detailed analysis of the liability structure reveals that, on average, the market value of traded bonds does not represent more than 25.6% of total liabilities. These figures are very similar to previous empirical studies on structural models: for example, see Lyden and Saraniti (2000). The sector with the highest proportion of traded bonds in total liabilities is the Consumer Non-Cyclical with 30.8%. Bond time to maturity ranges from 3.4 to 26.4 years but the average is around 9.7 years. Another interesting statistic is the high stock volatility of these firms (36.2% on average). This feature is somewhat related to the high volatility period and downward trend in the U.S. economy that followed the terrorist attacks of September 11th. Revealing its dependence on the market evolution, the Consumer Cyclical sector presents the highest volatility of the sample (43.5%).

4.2 Parameters

This section provides all the information about the model's calibration. The term structure estimation is discussed in the first instance in a separate section since this information is common to all models. Then, there is a detailed description of the estimation of the parameters specific to each model.

4.2.1 Term Structure of Risk-Free Rate

In order to calibrate the structural model of corporate debt pricing it is necessary to estimate a term structure for the risk-free rate. Several methods can be used to model the risk-free yield curve: for example, the Nelson–Siegel (1987) and the Vasicek (1977) models. In Eom *et al.* (2004) these two models were applied and the term structure estimated was very similar. In this paper the risk-free yield curve is estimated by fitting the Nelson–Siegel (1987) curve (see Appendix A.3 for details about the estimation).

4.2.2 The Merton (1974) Model

The Merton model specification developed in section 3 requires the use of six parameters: asset value, asset volatility, face value of debt, maturity of the debt, risk-free rate and payout ratio. This section demonstrates the estimation process for these parameters.

Merton’s model theoretical framework is adequate to price properly a very specific sort of corporate debt: zero coupon debt. However, all the companies in the sample not only have several kinds of debt but also the only kind of traded debt that they have is coupon debt, with all principal retired at maturity. Therefore, some criteria have to be adopted in order to convert “real debt” into a “synthetic Merton debt”. We started by focusing only on traded debt, which means only on the bonds issued by the firms. Having market prices for these bonds makes the analysis of pricing errors much more reliable since we can directly compare these market prices with the predicted prices given by the model.

We consider the duration of the bonds as a proxy for the maturity of Merton zero-coupon debt. This seems reasonable because the duration of each bond is a weighted average of the maturity of each coupon and the final principal, with the weights being based on the present value of each payment, discounted at the yield to maturity of the bond⁶. For companies with a single bond, this was assumed as the maturity for Merton’s formula and for companies with

⁶ Other approaches have been used by other studies. A recent approach, developed by Cooper and Davydenko (2003) consists in solving Merton’s valuation equation for maturity, obtaining an implied debt maturity. On the other hand, Eom *et al.* (2004) assume each coupon payment as a separate zero and then use Merton’s model to value each zero separately.

more than one bond Merton's model maturity was computed as a weighted average duration of all bonds. With this approach we are implicitly assuming that the liability structure will remain constant over time and that no default can happen before this portfolio duration.

Having obtained the Nelson–Siegel (1987) risk-free yield curve function, the computation of the risk free rate for the portfolio of bonds is straightforward. It is just the risk-free rate corresponding to the duration of the portfolio (maturity of the zero in Merton's model). The face value of the portfolio of bonds to be considered in Merton's estimation is computed as follows: we discount to period zero all the coupons and principal of each bond at the corresponding risk-free rate, and then compound the sum of their present value to the maturity of the zero computed previously. This final value can be considered as a synthetic face value, which replicates the original payments of the bonds.

As regards the payout ratio, a natural proxy for it would be a weighted average of the coupon rate and the dividend yield, with the weights based on the market value of total liabilities (market value of traded debt plus book value of all other liabilities) and the market value of equity. However, since the synthetic face value computed previously for the Merton model already incorporates the coupons of the bonds, these were ignored in this weighted average.

At this stage the challenge is to estimate the last two missing parameters: asset value and asset volatility. We solve simultaneously two equations in order to obtain asset value and asset volatility⁷. The first equation is Merton's equity valuation equation defined in Section 3 as equation (3.2). As we are focusing the implementation of the models only on traded bonds, we cannot assume the total value of equity when using this formula. This value should be based on the proportion of the analysed traded bonds on market value of total liabilities. With this approach, we keep the focus on traded debt and also maintain the original leverage of the firm and consequently the original probability of bankruptcy. The second equation follows from Ito's

⁷ Another recent method is the one proposed by Vassalou and Xing (2004), which uses an iterative process to obtain a time series of asset values that allows further computation of asset volatility. Cooper and Davydenko's (2003) method is also quite reasonable. In this case an iterative process is used to obtain simultaneously the implied maturity and asset volatility but the asset value is not an output of this process.

lemma and the assumptions of the contingent claims analysis. It assumes that the relationship between stock volatility (σ_E) and asset volatility (σ_A) should be

$$\sigma_E = \frac{V_0}{E} \frac{\partial E}{\partial V_0} \sigma_A \quad (4.1)$$

where $\partial E/\partial V_0$ is the partial derivative of the value of equity with respect to the value of the firm. In Merton's model this last figure is equivalent to $N(d_1)$. Using numerical solutions we can easily obtain a time series of asset value and asset volatility for each quarter.

4.2.3 The Leland (1994) Model

The calibration procedure adopted in Leland's model is very similar to the one used in Merton's model. Having a proxy for the risk-free rate, coupon, corporate tax rate, bankruptcy cost parameter and payout ratio, the remaining task is to numerically obtain the unlevered asset value and asset volatility.

In Leland's model, we should discount each continuous coupon using a continuous rate. However, for the sake of simplicity, we assume that the flat interest rate in this model is an implied rate that makes the present value of a 30 year annuity (discounting each coupon at the corresponding risk-free rate of the Nelson–Siegel curve) equal to the present value of the same annuity discounted at the unknown risk-free rate r_{Leland} ⁸. This method seems reasonable since the corresponding risk-free rate captures not only the short-term level of the risk-free rate but also the long-term level.

When calibrating Merton's model we had the problem of transforming "real" periodic coupon payments and a final principal payment at maturity into a single payment, denominated face value of debt. Now we have the problem of transforming these original payments into a perpetual coupon payment. The solution to this problem follows the same method used previously. The idea is to make the original debt comparable to Merton's debt and to Leland's debt by making equal the present value of this debt, using the risk free rate as discount rate. As

⁸ The coupon used in this numerical estimation (\$6.914M) was computed using the average coupon rate of all bonds in the sample (6.914%) and the most commonly used nominal value of bonds (\$100M).

we already have the present value of Merton's riskless debt and the risk-free rate at which we will discount the perpetual coupon, Leland's perpetual coupon is determined by solving the equation

$$F e^{-rt} = \frac{C}{r_{Leland}} \quad (4.2)$$

The corporate tax rate and the bankruptcy cost parameter are new in relation to Merton's model. Following Leland (1994) paper we assume a corporate tax rate of 35%. The bankruptcy cost parameter is defined as one minus the recovery rate and is obtained in an industry basis using the Altman and Kishore (1996) study about recovery rates.

Having all the above variables we are now in a position to numerically estimate the unlevered firm value and asset volatility. Again, we have two equations and two unknowns. The first equation is the equity valuation equation (3.10) and the second equation is now

$$\sigma_E = \frac{V_u}{E} \frac{\partial E}{\partial V_u} \sigma_A \quad (4.3)$$

where $\partial E/\partial V_u$ is now the partial derivative of the value of equity with respect to the unlevered firm value. There are two differences between this equation and equation (4.1) used in Merton's calibration. First, instead of asset value V_0 we should consider unlevered firm value V_u . Secondly, the partial derivative $\partial E/\partial V_u$ is not $N(d_1)$ as before but is given by

$$\frac{\partial E}{\partial V_u} = 1 + \left[\frac{C(1-\tau)}{r} - V_b \right] \lambda \frac{V_u^{\lambda-1}}{V_b^\lambda} \quad (4.4)$$

4.2.4 The Fan and Sundaresan (2000) debt–equity swap

As the Fan and Sundaresan debt–equity swap only introduces the bargaining power parameter in Leland's model, its calibration is very similar to Leland's. The difference relies in the partial derivative $\partial E/\partial V_u$ necessary to compute simultaneously the unlevered firm value and asset volatility since the equity value formula has now a different specification. Thus, equation (4.4) of Leland's model is replaced by

$$\frac{\partial E}{\partial V_u} = 1 + \left[\frac{C(1-\tau)}{r} - (1-\eta\alpha)V_b \right] \lambda \frac{V_u^{\lambda-1}}{V_b^\lambda} \quad (4.5)$$

5. Empirical Results

This section is organized in four topics. First, we analyse the distribution of credit spreads and RNDP for the total sample and on a sector basis. Then, in Section 5.2, we evaluate the performance of the Merton and Leland models by interpreting average values of prediction errors in price, yield and spread. In Section 5.3 there is a discussion of the systematic factors that may explain the spread errors of Merton model and finally, in Section 5.4, the Fan and Sundaresan debt/equity swap is discussed. We analyse the results of the Fan and Sundaresan debt–equity swap in a separate section (making a direct comparison with Leland) because we want to highlight the particular feature that distinguishes this swap from Leland model: the negotiation power parameter.

5.1 Distribution of Credit Spreads and RNDP

In the data section we reported information about the credit spread for each bond in the sample and averages values for each sector (presented in Appendix A.1). Now, we are in position to go further in the analysis, by comparing the observed credit spread with the credit spread predicted by Merton and Leland models and an approximation of credit spread based in a Merrill Lynch study, as illustrated in Table 1⁹.

The first important conclusion suggested by the results of Table 1 is that both the Merton and Leland models under-estimate the credit spread. This is true not only for the average values of the total sample but also for the industry averages. The average market spread of the total sample is more than three times higher than the spread predicted by the Merton and Leland models (221.5 bp against 58.9 bp and 57.1 bp, respectively). Furthermore, Table 1 shows that the bonds in the sample also have an average market spread higher than the average spread presented by U.S. firms in the Merrill Lynch study (169.3 bp in this last case).

⁹ Prior to the analysis of these results, we should mention that, for estimation purposes, a rating conversion table was constructed. We assign the number one to the highest rating (AAA+) and the number 23 to the lowest rating (D).

Focusing the analysis on observed credit spread, we verify that the Consumer Non-Cyclical sector has the highest spread (279.9 bp), followed by the Consumer Cyclical (250.3 bp) and the Basic Materials (247.0 bp). The result of the Consumer Cyclical sector is consistent with the highest coupon rates of its bonds (already reported in Appendix A.1) and also with its worst rating quality. The average rating in this sector is 11, which corresponds to BBB-, the cut-off category of investment grade bonds. In the group of bonds with the lowest market spread we have the Energy (209.7 bp), Industrial (191.2 bp) and the Healthcare (171.7 bp) sectors. These are also the sectors with the best rating, which reveals an important negative association between rating and market spread.

In order to improve the analysis of market spread, it is important to have an idea of its distribution. Appendix A.4 displays this distribution for the total sample. This distribution is far from normal. There is a higher concentration of observations between 100 bp and 125 bp and then many more observations to the right of this range than to the left. Between 150 bp and 350 bp the frequency is quite constant and there are some important observations above 700 bp¹⁰.

In addition to the credit spread estimation reported in Table 1 it is important to analyse the risk-neutral default probability predicted by the models, since these two variables are directly related. Table 2 summarizes the results of this last variable. It also reports Moody's one-year default rates for bonds during 1999 based on cross sectional information about rating and maturity. Although this information is not directly comparable with the RNDP it provides an idea of which sectors are likely to present a higher default rate.

As expected, the sectors with the highest predicted RNDP are also the sectors with the highest predicted spread. The Consumer Cyclical assumes the leading of this ranking with a RNDP of 22.9% predicted by the Merton model (predicted spread of 149.3 bp, Table 1) and 17.4% predicted by the Leland model (predicted spread of 108.3 bp). The sector with the lowest

¹⁰ An industry analysis of the distribution of market spread (not reported) reveals some similarities among industries. In none of the sectors does the distribution seem to be normal and most sectors reveal a strong dispersion of credit spreads, with the exception of Energy. In most sectors ranges of high frequency are followed by low frequency and again by high frequency.

predicted RNDP (and lowest credit spread) is the Consumer Non-Cyclical, with 7.1% and 3.6% in the Merton and the Leland's model, respectively.

There seems to be some contradiction between this ranking and the ranking based on the observed market spread. The structural models predict the lowest RNDP and the lowest credit spread for the sector with the highest market spread: Consumer Non-Cyclical. This underpricing issue will be analysed in more detail in the next section but there seems to be a reason for that particular case. The three companies of the Consumer Non-Cyclical sector reveal an historical 250 days stock volatility that is quite low, which resulted in low asset volatility estimation and consequently a high under-estimation of credit spread and RNDP by the structural models.

Finally, taking into account cross sectional information about maturity and rating, we verify that the overall one-year default rate is no more than 0.11%. Given its rating characteristics, the Consumer Non-Cyclical sector is the one which potentially presents a high default rate, between 0.11% and 1.12%. The sector with the lowest default rate (between 0.00% and 0.11%) is the Industrial sector. This ranking is in accordance with the ranking based on the market spread and the Merrill Lynch spread of Table 1.

As a summary of this section we can say that the first results of the structural models reveal an under-estimation of credit spreads and that both the observed credit spread and the predicted spreads are characterized by a high dispersion. The RNDP ranges from 10.5% in the Leland model to 12.3% in the Merton model. Even though the Consumer Non-Cyclical and the Consumer Cyclical sectors have the highest market spreads, the predictions of the models are not always coincident with these results.

5.2 Prediction Errors

In this section we discuss the performance of the models. How well can the models fit the market prices, yields and credit spreads? We decompose the analysis into two parts. First, there is a general overview of model performance by considering all the sample observations (Section 5.2.1) and then we focus the analysis on several categories, according to rating, maturity of the

bonds, asset volatility of the firms and sectors (Section 5.2.2). We test whether the prediction errors are significantly different from zero, if there are differences between Merton and Leland's estimation and whether there are differences in the estimation for the categories mentioned above.

5.2.1 Predicted Errors - Total Sample

Table 3 summarizes the prediction errors for the Merton and Leland models for the total sample. We consider the relative errors to be more informative of model performance since it allows for comparisons between the two models and later on, among categories. Parallel to these predicted errors we also test whether the mean relative errors are different from zero and whether there are differences between the Merton and Leland means. Appendices A.5 and A.6 report the p -values for these tests not only for the total sample, which is analysed in this section, but also for the grouping of relative errors according to sector, rating, maturity and asset volatility, that is discussed in the next section.

The first conclusion that can be drawn from Table 3 is that both models over-estimate bonds prices. Merton's model mean over-estimation is about 11.2% and that of Leland's model is 4.5% (both means are significantly different from zero, as reported in Appendix A.5). The results found for the Merton model show an overestimation higher than the 4.5% found by Jones *et al.* (1984) and the 1.69% by Eom *et al.* (2004). Not surprisingly, we found that Leland's model overprices bonds less than Merton's model (the equality of means does not hold for a 5% significance level). This is essentially due to the consideration of early default and cost of financial distress in Leland's model.

Another important issue that should be discussed when analysing both models' relative pricing errors is the distribution of these errors. These are depicted in Figure 1. There is evidence that the Merton distribution of relative pricing errors is skewed to the right while Leland's distribution is just moderately skewed to the left. This reveals a tendency of the Merton model to overestimate bonds more than Leland's model. Figure 1 also shows that there is a high dispersion of pricing errors, which is more pronounced in Leland's distribution

(standard deviation of 12.3% in Leland against 8.9% in Merton). This pattern of high dispersion is similar to the one found in previous literature.

Regarding the yield and credit spread, we notice, as expected, that both structural models under-estimate these figures. The relative yield error is -30.5% and -3.4% for the Merton and Leland models, respectively, and the relative credit spread error is -76.2% and -75.0% , also respectively (all means are statistically different from zero). Again, we can compare the results for the Merton model with the Eom *et al.* (2004) study. The results found for the yield relative error show less under-estimation than did Eom *et al.* (2004). They found a relative yield error of -91.3% while our's is only -30.5% . However, considering the relative spread error, the conclusion is somewhat different. Our mean of -76.2% shows more under-estimation of credit spread than they found: -54.4% .

The incapacity to generate sufficiently high spreads is one of the main criticisms of structural models. There are several possible explanations for that. Some rely on technical issues and others on theoretical issues. Regarding the technical issues, there seems to be a tractability problem. Recall that both the Merton and Leland models are approximating actual straight coupon bonds with finite maturity with some “synthetic type of debt”. In Merton’s case it is a zero coupon debt and in the Leland model it is perpetual debt, with a continuous coupon payment. The calibration procedure used to convert “real debt” into “synthetic debt” will imply a different relationship between yields and prices in the model and in reality.

From a theoretical point of view, these structural models, which are based on the contingent claim theory, tend to generate low credit spreads because they only capture the default risk component. Besides the credit risk component, actual credit spreads are very likely to include compensation for liquidity (marketability), taxes or systematic risk. There is another feature related to “real” bonds that these two models do not capture: jumps in asset value. These models assume, as we explained in Section 3, a geometric Brownian motion process for the asset value and, therefore, do not admit sudden changes (jumps) in the asset value. Even though these jumps are not so common in practice, there may be a small proportion of the market spread that compensates for jump risk not priced by the models.

When comparing Merton's relative spread error with Leland's relative spread error, we would expect a less negative error for the Leland model, as it incorporates more "real world" features, namely the possibility of early default. However, this is not the case in our study. The p -value found for the equality of means is 0.6779, which reveals that we cannot reject the null hypothesis of equality of means (for a 5% significance level).

Although we do not report here the graphs with the distributions of relative yield and spread errors, we shall say that there is a clear distinction between the distribution of yield errors and the distribution of spread errors. While the distribution of yield errors has some similarities with a normal distribution, the distribution of spread errors shows an extreme concentration of observations in the lower bound. This pattern is more pronounced in Merton's distribution. In this case there are 166 observations (52% of total) in the range between -99% and -92%. But, at the same time, there are also some important observations with very positive spread errors: in the range above 44% there are 10 observations. This reveals the high dispersion of spread errors, usually a characteristic of these empirical studies.

5.2.2 Predicted Errors by Category

In the previous section we discussed the performance of the structural models considering all the observations in the sample. However, there might be differences in the estimation errors depending on the rating category of the bonds, its maturity or even the asset volatility of the firms. In this section we analyse the performance of the models according to this grouping and also according to sector.

To detect any rating effect, we divided the sample in two rating categories: high rating (bonds with a numerical rating conversion below 11 or BBB-) and low rating (all others). This does not correspond to the standard distinction between investment grade bonds and speculative grade bonds because there are only 32 observations of speculative grade bonds in the sample. It would not be reasonable to compare results from a sub-sample of 32 observations with those from a sub-sample of 285 observations of investment grade bonds. The split resulted in 196 observations of high rated bonds and 121 of low rated bonds. As regards the remaining time to

maturity of the bonds, we analyse three sub-samples: short maturity (less than five years), medium maturity (from five to 10 years) and long maturity (above 10 years), corresponding to 49, 169 and 99 observations, respectively. In order to discuss any volatility effects we decompose the sample in low asset volatility (below 20%) and high asset volatility (above 20%), which results in two sub-samples of 152 and 165 observations, respectively.

Appendix A.7 reports the p -values of a two-way ANOVA test that evaluates, as a null hypothesis, no category effects according to rating, maturity of the bonds, asset volatility and sectors. Considering a 5% significance level, we reject the null hypothesis of no effects in Merton's and Leland's relative errors (pricing, yield and spread) according to rating, maturity, asset volatility and sector, except for the maturity in Leland's relative spread error (p -value of 0.1258). To complement this analysis it is important to analyse the values of the relative predicted errors.

Appendix A.8 displays the mean relative errors according to the rating category of the bonds, the maturity, asset volatility of the firms and sector. Both Merton's and Leland's models under-estimate less the spread for high rating categories. Merton's mean spread error is -72.9% for high rating bonds and -81.5% for low rating bonds. Our findings are in accordance with what we should expect from the performance of the models. We should expect a lower capacity of the models to predict spreads of low rating bonds because low rating bonds are usually less liquid. Thus, their spread must show a bigger compensation for liquidity risk, which is not captured by structural models. These models only capture default risk. A comparison to previous studies shows that our results confirm the results found by Ericsson and Reneby (2002). These authors report a better performance of their structural model for speculative grade bonds.

In the Merton model the tendency toward under-estimation of spread appears to be somewhat stronger among short maturity bonds. Merton's relative mean spread error is -97% for short maturity bonds, -77.4% for medium maturities and -63.9% for long maturities. In this case our results are in accordance with previous studies in the field, namely Ericsson and Reneby (2002) and Eom *et al.* (2004).

Another interesting result from our study is that these structural models fit better the prices and spreads of more risky firms. Merton's relative price error is 13.6% for low volatile firms and only 8.9% for high volatile firms. Concerning the spread, there is an extreme under-estimation for low volatile firms, namely -93.4%, while for more risky firms this is only -60.3%. Leland's results show some similarity, especially for the credit spread error. There is also empirical evidence that for high volatile firms the Leland model can fit with extreme precision the prices of the bonds. Leland's relative pricing error for this category is significantly equal to zero.

We have already mentioned that there exists a sector effect in the performance of the models. Now we will analyse which characteristics of the bonds or of the firms belonging to these sectors might lead to a better or worst performance of the structural models. Once again, we should rely on the results of Appendix A.8.

There are two sectors where the Merton model seems to perform better when predicting the credit spread: the Consumer Cyclical and the Energy sector. The relative spread errors have a mean of -58% and -62.5% in these two sectors, respectively, when the mean for the total sample is -76.2%. By reconciling this information with the descriptive statistics on the bonds and the firms reported earlier, we verify that these sectors present some characteristics usually associated with a better prediction power of the Merton model. The bonds in the Consumer Cyclical sector have an average maturity above the average of the total sample and its firms also present asset volatilities and leverage levels above the total sample. We have already established that the Merton model performs better for long maturity bonds and more risky firms. Regarding the leverage, the empirical literature shows that Merton's model usually under-estimates less the spreads for high leverage firms. In relation to the Energy sector we believe that the good performance of the Merton model is probably due to the highest average time to maturity of its bonds (14.2 years for an overall average of 9.7 years).

In the group of sectors with worst predictive power of the Merton model we found the Consumer Non-Cyclical and the Industrial with relative spread errors of -86% and -95%, respectively. The poor performance in the Industrial sector seems to be due to the short average

maturity of its bonds (5.8 years, which is the lowest of the sample) and the low asset volatility of its firms. As regards the Consumer Non-Cyclical sector it seems to be due to the reduced asset volatility of its firms.

We can summarize the analysis of the predictions errors as follows. Both the Merton and Leland models overestimate bond prices and under-estimate credit spreads. Even though in the spread predictions the results are not statistically different from each other, in the prices predictions Merton's overestimation is stronger. We also confirm the Eom *et al.* (2004) results of high dispersion of credit spread errors. The analysis of the prediction errors by category shows that both the Merton and Leland models perform better for bonds with a good rating quality and a longer maturity. Moreover, these models perform better with riskier firms, those that present high asset volatility and high leverage.

5.3 Systematic Prediction Errors

Up to this point we have discussed the performance of the structural models analysing essentially some descriptive statistics of the predicted errors in terms of pricing, yield and spread. We considered the mean relative spread error to be the most informative measure of the ability of the models to fit credit spreads. In this section, we consider in more detail the question of why the models' predictions are inaccurate. With a multivariate regression analysis we examine the relationship between the relative spread error and a set of bond-specific, firm-specific and economy-wide variables. The goal is to identify some systematic factors that cause the weaknesses of the models. This analysis covers the entire sample as well as several categories of rating, maturity, asset volatility and sector.

The methodology used in this section is somewhat similar to the methodology used by Eom *et al.* (2004), Ericsson and Reneby (2002) and Lyden and Saraniti (2000). All these authors perform a multivariate regression analysis instead of a single regression analysis. They argue that a combination of factors leads to higher or lower prediction errors and, therefore, analysis in a multivariate regression setting is more appropriate. Nevertheless, there are some differences in the choice of the dependent variable. Eom *et al.* (2004) use the relative spread

error as the dependent variable while Ericsson and Reneby (2002) and Lyden and Saraniti (2000) use the absolute spread error. But, even the spread error is not defined in the same way by all these studies. While the first two use the definition of error as we do, Lyden and Saraniti (2000) use an inverse definition, which leads to positive errors for these models¹¹. Following the most recent empirical paper in the field (Eom *et al.*, 2004), we use the relative spread error as dependent variable, which makes our findings directly comparable to that study.

In the list of explanatory variables we consider size, leverage, asset volatility, market-to-book ratio and stock return as firm-specific variables. We use the market value of assets as a proxy for size. Leverage is the market leverage, defined as the ratio of the sum of market value of trade debt and book value of non-traded by the market value of assets. We use the definition of market-to-book ratio presented by Rajan and Zingales (1995) as the ratio of market value of assets to book value of assets. This variable is intended to stand for the firm's growth opportunities. The stock return is computed as the annualized stock return of the last 250 days prior to the quarter considered for each firm¹².

As bond-specific variables we use the remaining time to maturity of the bonds, the rating and the observed yield to maturity. Since each regression is estimated using bond prices observed in a variety of interest rate environments we consider two control variables related to term structure. The ten-year yield is used to measure the level of the term structure and the difference between the ten and two year yields to measure the slope.

5.3.1 Credit Spread Regression

As a first check on our explanatory variables, we ran a regression with the market spread as the dependent variable. In this case we only considered a group of six independent variables that we

¹¹ Recall that we define relative spread error as the difference between the predicted spread minus the observed spread divided by the observed spread.

¹² We also evaluate whether we should consider the tangibility of a firm's assets in the list of firm-specific variables. As proposed by Rajan and Zingales (1995), the tangibility can be approximated by the ratio of fixed assets to total assets. However, we decided not to include this variable as it presents a strong association with leverage, one of our explanatory variables. The inclusion of both variables could create multicollinearity problems in our regressions.

consider to be the most relevant for the explanation of observed credit spread, namely size, leverage, years to maturity, asset volatility, rating and market-to-book ratio. This is reported as credit spread regression in Table 4.

From the six parameters considered in the credit spread regression, only maturity and market-to-book ratio are not statistically significant. The others show a sign that is consistent with the results found by Ericsson and Reneby (2002). The negative sign of the size parameter reveals that bigger firms have lower spreads. This happens essentially for two reasons. First, bigger firms are considered to be safer firms and thus have low default risk. This is in some sense reinforced by the negative correlation between rating and size (-0.33), meaning that small firms are more likely to have speculative grade bonds and thus, more default risk. Secondly, bonds belonging to bigger firms are considered to be more liquid than bonds belonging to smaller firms. This means that the market spread should be lower for bigger firms since this spread does not have to provide a high compensation for liquidity, as is the case of smaller firms. Liquidity really seems to be one reason to explain the sign of the size parameter. Our study is also in accordance with Gabbi and Sironi's (2005) which states the great importance of bond rating as a determinant of yield spreads.

Using the Eom *et al.* (2004) definition of safer firms (with low leverage and low asset volatility) we found that safer firms have low credit spreads. Note the high sensitivity of credit spread to leverage and asset volatility, as the parameters for these two variables in regression 2 are 341.318 and 327.076, respectively. In accordance with the previous results we also verify that low quality rating is strongly associated with higher spreads (as the coefficient for rating presents a value of 34.132).

5.3.2 Errors Regressions

Having analysed some explanatory variables of the credit spread we shall now make some considerations about the spread error regressions presented in Table 4. We focus the analysis on regression 2 as these include only significant parameters. The first conclusion that we can draw from these regressions is that these models under-estimate credit spreads. We had already

reached this conclusion when analysing the mean values of the relative spread errors in the previous section and now we confirm it by verifying that the intercept coefficient is negative and statistically significant. The lower coefficient found for the Merton model (-2.111 against -1.661 in Leland) seems to indicate that the Merton model under-estimates the credit spread more than the Leland model but, as we mentioned in the previous section, this difference is not statistically significant.

In the list of firm specific variables, four of the five variables have a systematic relationship with the Merton spread error: namely leverage, asset volatility, market-to-book ratio and stock return. In the Leland model, size also bears some relationship with the errors but, on the other hand, stock return has no influence in explaining these errors. Our results indicate that both models under-estimate less the spread for riskier firms, i.e. firms with high leverage and high volatility. Again, this is more evident for the Merton model.

During the estimation procedure we notice that the adjusted R^2 decreases greatly when we dropped the asset volatility from the list of explanatory variables. This confirms the extreme importance of this variable in explaining the relative spread errors of the Merton model, in line with the findings of Eom *et al* (2004). These authors find that Merton's errors are systematically related to asset volatility in all their regressions. Moreover, this pattern reveals that these structural models have many problems in explaining the observed market spreads due to their simplifying assumptions about reality. All the models discussed in this paper assume constant asset volatility. The introduction of a stochastic process for volatility would probably benefit the performance of these models, as the estimation results depend considerably on the value found (and assumed) for asset volatility.

In Leland's results the under-estimation is also lower for bigger firms, which is in accordance with the earlier discussion about liquidity risk. Furthermore, the negative coefficient of the market-to-book ratio variable indicates that the under-estimation bias is stronger in companies with high levels of growth opportunities. We believe that this is due to the fact that companies with high growth opportunities usually have less leverage and, as previous empirical results also suggest, these models have a worst performance in low leverage firms.

As regards the bond-specific variables, only maturity and observed yield to maturity play a role in explaining the spread errors. Longer maturity bonds are subject to less under-estimation of the models. The difficulty these models have in predicting high spreads for short maturity bonds is well known. On the other hand, the regression analysis does not confirm the better capacity of these models to predict spreads of high rating bonds, as we concluded in the predictions errors section.

The only term structure parameter that has a statistically significant relationship with Merton's spread errors is the level. The higher the level of the term structure the lower is the under-estimation of the structural models. Despite this empirical relationship, the reasons for that are not obvious. In a previous study where this variable was used (Eom *et al.*, 2004), it was not found to be statistically significant. This was the only variable in our results that did not match with the results of Eom *et al.* (2004). All other parameters previously discussed presented the same sign and absolute values not so different from theirs. This certainly reinforces our study since we use a smaller sample. Moreover, the high R^2 of our regressions should be noticed.

In addition to the above regressions that apply to the total sample we also ran some regressions according to the categories presented in Section 4, namely different classes of rating, maturities, asset volatility and sectors. The results did not improve the analysis already done for the total sample. Thus, we do not report these estimations.

5.4 The Fan and Sundaresan (2000) debt-equity swap

In our approach to the Fan and Sundaresan debt–equity swap we do not differentiate firms according to the bargaining power of equityholders and debtholders at liquidation. This is clearly a simplistic assumption but makes possible a comparative static's analysis regarding some variables of this model, as presented in Figure 2. It shows the sensitivity of the bankruptcy threshold, debt, equity and firm value to the bargaining power parameter. By assigning more bargaining power to equityholders (as η approximates 1) we benefit the equityholders but the decrease in debt value is such that after a certain point (in this case when η is higher than 0.5)

there is a loss in the firm's value. Thus, the solution presented in Anderson and Sundaresan (1996) where equityholders have all the bargaining power is not the most efficient. The company benefits more if there is some balance between the bargaining power of its claimants.

Our calibration of the Fan and Sundaresan debt/equity swap is made assuming a bargaining power parameter of 0.5, as this is the parameter that maximizes the firm value in our sample. With this new "real world feature" we should expect Fan and Sundaresan's model to outperform the Leland model. This is indeed verified in our estimation, as show the results of Table 5.

The assumption of an equal distribution of bargaining power among firms seems to approximate the Fan and Sundaresan predictions to the market values, as we observe that the mean relative pricing error and the mean relative yield error are almost zero (a mean zero test performed on these means indicates that they are statistically equal to zero, considering a 5% significance level). Furthermore, the relative spread error decreases from -75.0% in the Leland model to -64.9% in the Fan and Sundaresan model. This improvement is just due to the consideration of the bargaining power parameter since Fan and Sundaresan's debt/equity swap is identical to Leland's model when we assume that debtholders have all the power. These results confirm the importance of debt renegotiation in a firm's financing decisions.

6. Conclusion

This paper tests empirically the performance of three corporate bond pricing models using a sample of 50 bonds from companies with simple capital structures between 2001 and 2004. In particular, we implement the models of Merton (1974), Leland (1994), and Fan and Sundaresan (2000) debt–equity swap. We analyse the prediction errors in price, yield and spread as measures of the performance of the models and then we examine whether there are systematic factors that can explain the relative spread errors. The discussion incorporates a decomposition of the companies by sector, which is new in relation to most recent empirical studies in the field.

While the Merton and Leland models overestimate bond prices, Fan and Sundaresan's model does not reveal bias in the estimation of these prices, as we cannot reject the null hypothesis of mean zero in Fan and Sundaresan pricing errors. We find relative price errors of

11.2%, 4.5% and 0.5% for the three models, respectively. These results suggest that the introduction of early default, coupons, taxes and bankruptcy cost in the Leland model and the assumption of a bargaining power parameter in Fan and Sundaresan's model is a major improvement in Merton's pricing framework.

If we rely on relative spread errors as a measure of the model's performance, we conclude that the three models under-estimate credit spreads. We find relative spread errors of -76.2%, -75.0% and -64.9% for the Merton, Leland and Fan and Sundaresan models, respectively. However, this measure does not confirm the differences between Merton and Leland's predictions to be statistically significant. Furthermore, we find a high dispersion of both the observed credit spread and the predicted spreads. These models can either predict very low spreads or very high spreads, depending considerably in the estimation of asset volatility. This might reveal the importance of assuming a stochastic process for asset volatility.

An analysis of the prediction errors by category reveals the existence of important rating, maturity, volatility and sector effects. Both Merton's and Leland's models perform better in bonds with a good rating quality. The lower liquidity of speculative grade bonds seems to be one of the main reasons why these models under-estimate more the credit spreads of these bonds. Our results also clarify the Eom *et al.* (2004) analysis of rating. Furthermore, they confirm Ericsson and Reneby (2002), who report greater bias of their structural model for speculative grade bonds. We confirm the better performance of these models in bonds with longer maturity and in riskier firms (high leverage and high asset volatility). The decomposition of the spread prediction errors by sector allows us to verify that both the Merton and Leland models fit better spreads in the Consumer Cyclical and Energy sectors as the bonds in these sectors have longer maturities and belong to firms with high leverage and high asset volatility. In the bottom line we find the Consumer Non-Cyclical and Industrial sectors.

We find empirical evidence that the market spread is positively related with leverage and asset volatility. In addition, it is higher in bonds with low rating quality. However, we cannot validate an empirical relationship between credit spread and both maturity and market-to-book ratio. Among the firm-specific factors that can explain spread errors we find the leverage, asset

volatility, market-to-book ratio, stock return and size (this last one only in Leland's errors). As regards the bond-specific variables we conclude that the errors are systematically related to maturity and yield to maturity. While the level of term structure has a systematic influence in the spread errors of all models, the slope only has impact in Leland's errors.

In summary, the difficulty these structural models have in accurately predicting bond prices and credit spreads is clear. However, this depends on several bond- and firm-specific features, as well as according to market conditions. A challenge for future research is thus, from a theoretical point of view, the development of tractable structural bond pricing models that are able to better fit credit spreads and bond prices. Future empirical research should try to extend the industrial analysis of the performance of these models, incorporating other industries, and evaluating whether these models perform differently according to country, as the market conditions can vary significantly across countries.

References

- ALTMAN, E. AND V. KISHORE (1996), "Almost Everything You Wanted to Know about Recoveries on Defaulted Bonds," *Financial Analysts Journal*, November/December, 57-64.
- ANDERSON, R. AND S. SUNDARESAN (1996), "Design and Valuation of Debt Contracts," *Review of Financial Studies*, 9, 37-68.
- BLACK, F. AND J. COX (1976), "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance*, 31, 351-367.
- BLACK, F. AND M. SCHOLES (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, 81, 637-59.
- COLLIN-DUFRESNE, P. AND R. GOLDSTEIN (2001), "Do Credit Spreads Reflect Stationary Leverage Ratios?," *Journal of Finance*, 56, 1929-1957.
- COOPER, IAN A., AND S. A. DAVYDENKO (2003), "Using Yield Spreads to Estimate Expected Returns on Debt and Equity," London Business School IFA Working Paper.
- DUFFIE, D. AND K. SINGLETON (2003), *Credit Risk: Pricing, Measurement and Management*, Princeton Series in Finance.
- EOM, Y. H., J. HELWEGE, AND J. HUANG (2004), "Structural Models of Corporate Bond Pricing: An Empirical Analysis," *Review of Financial Studies*, 17, 499-544.
- ERICSSON, J. AND J. RENEBY (2002), "The Valuation of Corporate Liabilities: Theory and Tests," working paper, McGill University and Stockholm School of Economics.
- FAN, H. AND S. SUNDARESAN (2000), "Debt Valuation, Renegotiation and Optimal Dividend Policy," *Review of Financial Studies*, 13, 1057-1099.

- FRANKS, J., AND W. TOROUS (1994), "A Comparison of Financial Recontracting in Distressed Exchanges and Chapter 11 Reorganizations," *Journal of Financial Economics*, 35, 349-370.
- GABBI, G., AND A. SIRONI (2005), "Which factors affect corporate bonds pricing? Empirical evidence from eurobonds primary market spreads," *European Journal of Finance*, Vol. 11, Issue 1, p59.
- GESKE, R. (1977), "The Valuation of Corporate Liabilities as Compound Options," *Journal of Financial and Quantitative Analysis*, 12, 541-552.
- HUANG, J.-Z., AND M. HUANG (2002), "How much of the Corporate-Treasury Yield Spread is Due to Credit Risk?," working paper, Pennsylvania State and Stanford Universities.
- JONES, E.P., S. MASON, AND E. ROSENFELD (1984), "Contingent Claims Analysis of Corporate Capital Structures: An Empirical Investigation," *Journal of Finance*, 39, 611-625.
- LELAND, H. (1994), "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance*, 49, 1213-1252.
- LELAND, H. AND K. TOFT (1996), "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads," *Journal of Finance*, 51, 987-1019.
- LONGSTAFF, F. AND E. SCHWARTZ (1995), "Valuing Risky Debt: A New Approach," *Journal of Finance*, 50, 789-820.
- LYDEN, S., AND D. SARANITI (2000), "An Empirical Examination of the Classical Theory of Corporate Security Valuation," Barclays Global Investors.
- MELLA-BARRAL, P., AND W. PERRAUDIN (1997), "Strategic Debt Service," *Journal of Finance*, 52, 531-566.
- MERTON, R. (1974), "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 29, 449-470.
- NELSON, C., AND A. SIEGEL (1987), "Parsimonious Modelling of Yield Curves," *Journal of Business*, 60, 473-489.
- OGDEN, J. (1987), "Determinants of the Rating and Yields on Corporate Bonds: Tests of the Contingent Claims Model," *The Journal of Financial Research*, 10, 329-339.
- RAJAN, R. G., AND L. ZINGALES (1995), "What Do We Know about Capital Structure? Some Evidence from International Data," *Journal of Finance*, 5, 1421-1460
- VASICEK, O. (1977), "An Equilibrium Characterization of the Term Structure," *Journal of Financial Economics*, 5, 177-188.
- VASSALOU, M., AND Y. XING (2004), "Default Risk in Equity Return," *Journal of Finance*, 59 (2), 831-868.
- WEI, D., AND D. GUO (1997), "Pricing Risky Debt: An Empirical Comparison of the Longstaff and Schwartz and Merton Models," *Journal of Fixed Income*, 7, 8-28.

Appendix

A.1 – Bonds Summary Statistics

Company	Bond	Face Value (\$M)	Issue Date	Maturity	Coupon	Rating	Yield Spread (bp)
Industrial		219.3			6.803%		191.2
Bemis Co Inc	Bond 1	100	28/06/1995	01/07/2005	6.700%	A	149.7
	Bond 2	250	07/08/2001	15/08/2008	6.500%	A	114.5
CNF Inc	Bond 1	200	03/03/2000	01/05/2010	8.875%	BBB-	374.1
IDEX Corp	Bond 1	150	18/02/1998	15/02/2008	6.875%	BBB	293.3
Pentair Inc	Bond 1	250	30/09/1999	15/10/2009	7.850%	BBB	247.5
Snap-on Inc	Bond 1	200	14/08/2001	15/08/2011	6.250%	A	107.7
	Bond 2	100	28/09/1995	01/10/2005	6.625%	A	119.0
Temple Inland Inc	Bond 1	300	24/02/1999	01/03/2009	6.750%	BBB	238.5
	Bond 2	100	04/09/1992	15/09/2004	7.250%	BBB	301.0
United Techonologies	Bond 1	500	24/04/2002	15/05/2012	6.100%	A	75.7
	Bond 2	400	23/10/2001	01/11/2006	4.875%	A	73.1
USF Corp	Bond 1	150	19/04/2000	15/04/2010	8.500%	BBB+	283.2
	Bond 2	100	29/04/1999	01/05/2009	6.500%	BBB+	236.7
Vulcan Materials	Bond 1	250	07/04/1999	01/04/2009	6.000%	A+	132.6
	Bond 2	240	02/02/2001	01/02/2006	6.400%	A+	121.6
Consumer Cyclical		230.0			6.873%		250.3
Choice Hotels Int. Inc	Bond 1	100	19/10/1998	01/05/2008	7.125%	BBB-	339.7
Knight Ridder Inc	Bond 1	300	23/03/1999	15/03/2029	6.875%	A	148.1
Neiman Marcus Group	Bond 1	125	21/05/1998	01/06/2028	7.125%	BBB	223.5
	Bond 2	125	21/05/1998	01/06/2008	6.650%	BBB	195.8
Nordstrom Inc	Bond 1	300	11/03/1998	15/03/2028	6.950%	A-	200.9
	Bond 2	250	14/01/1999	15/01/2009	5.625%	A-	175.5
Staples Inc	Bond 1	200	07/08/1997	15/08/2007	7.125%	BBB-	224.2
	Bond 2	325	25/09/2002	01/10/2012	7.375%	BBB-	129.2
	Bond 3	325	26/02/2003	01/10/2012	7.375%	BBB-	102.2
Unifi Inc	Bond 1	250	05/02/1998	01/02/2008	6.500%	B+	763.8
Energy		188.9			7.286%		209.7
Energen Corporation	Bond 1	150	19/12/2000	15/12/2010	7.625%	A-	188.0
	Bond 2	100	24/02/1998	15/02/2028	7.125%	A-	234.0
Enesco International	Bond 1	150	20/11/1997	15/11/2027	7.200%	BBB+	171.7
	Bond 2	150	20/11/1997	15/11/2007	6.750%	BBB+	151.5
Murphy Oil Corp	Bond 1	250	29/04/1999	01/05/2029	7.050%	A-	148.7
	Bond 2	350	29/04/2002	01/05/2012	6.375%	A-	110.0
Newfield Exploration	Bond 1	125	17/11/1997	15/10/2007	7.450%	BB+	292.6
	Bond 2	175	21/02/2001	01/03/2011	7.625%	BB+	273.3
	Bond 3	250	08/08/2002	15/08/2012	8.375%	BB+	317.3
Basic Materials		189.3			6.332%		247.0
Lubrizol Corp	Bond 1	100	15/06/1995	15/06/2025	7.250%	BB+	189.8
	Bond 2	200	20/11/1998	01/12/2008	5.875%	BB+	159.4
Nucor Corp	Bond 1	175	07/01/1999	01/01/2009	6.000%	A+	157.5
	Bond 2	350	26/09/2002	01/10/2012	4.875%	A+	67.5
Sensient Technologies	Bond 1	150	22/03/1999	01/04/2009	6.500%	BBB-	240.5
Worthington Industries	Bond 1	150	09/12/1997	01/12/2009	6.700%	BBB	470.5
	Bond 2	200	21/05/1996	15/05/2006	7.125%	BBB	443.6
Healthcare		208.8			6.800%		171.7
Beckman Coulter Inc	Bond 1	235	14/11/2001	15/11/2011	6.875%	BBB	142.0
	Bond 2	100	23/05/1996	01/06/2026	7.050%	BBB	100.8
Guidant Corp	Bond 1	350	11/02/1999	15/02/2006	6.150%	A-	148.6
Watson Pharmaceutical	Bond 1	150	13/05/1998	15/05/2008	7.125%	BBB-	295.5
Consumer Non Cyclical		161.0			7.580%		279.9
Blyth Inc	Bond 1	150	24/09/1999	01/10/2009	7.900%	BBB-	274.4
	Bond 2	100	20/10/2003	01/11/2013	5.500%	BBB-	90.9
Corn Products Int. Inc	Bond 1	200	18/08/1999	15/08/2009	8.450%	BBB-	401.4
	Bond 2	255	28/06/2002	15/07/2007	8.250%	BBB-	340.7
Toro Co	Bond 1	100	15/06/1997	15/06/2027	7.800%	BBB-	292.2
All Bonds							
	<i>Average</i>	205.1			6.916%		221.5
	<i>Sd</i>	92.7			0.852%		125.5
	<i>Max</i>	500			8.875%		763.8
	<i>Min</i>	100			4.875%		67.5

Table A.1 reports descriptive statistics for the bonds in the sample. All the information regarding face value, issue date, maturity date, coupon and yield spread was obtained from DATASTREAM. The yield spread for each bond is an average of the spread over US treasury bills for the sample period. Rating information was obtained from Standard & Poors (www.standardandpoors.com).

A.2 – Firms Summary Statistics

Firm	Market Value of Equity	Market Value of Total Liabilities	Market Value of Assets	Market Leverage	Book Leverage	Bonds as % of Total Liabilities	Avg Bond Time to Maturity	Stock Volatility	Dividend Yield
Industrial									
Bemis Co Inc	2,567.6	1,191.2	3,758.8	31.7%	53.8%	32.1%	4.4	25.9%	2.3%
CNF Inc	1,560.8	2,123.4	3,684.2	57.6%	73.8%	10.2%	7.4	37.0%	1.3%
IDEX Corp	1,107.9	418.0	1,525.9	27.4%	45.3%	37.0%	5.4	33.2%	1.7%
Pentair Inc	2,023.1	1,443.3	3,466.4	41.6%	55.7%	18.9%	6.9	35.9%	2.0%
Snap-on Inc	1,675.4	1,201.1	2,876.6	41.8%	57.5%	26.2%	5.9	28.3%	3.5%
Temple Inland Inc	2,646.9	2,796.4	5,443.3	51.4%	58.3%	15.0%	4.1	31.2%	2.7%
United Technologies	33,857.8	20,998.5	54,856.3	38.3%	68.5%	4.1%	6.9	38.1%	1.6%
USF Corp	846.7	719.4	1,566.1	45.9%	52.2%	37.3%	6.9	41.6%	1.2%
Vulcan Materials	4,231.4	1,856.5	6,088.0	30.5%	52.1%	28.0%	4.9	29.0%	2.3%
<i>Average</i>	<i>5,613.1</i>	<i>3,638.6</i>	<i>9,251.7</i>	<i>40.7%</i>	<i>57.5%</i>	<i>23.2%</i>	<i>5.8</i>	<i>33.3%</i>	<i>2.1%</i>
Consumer Cyclical									
Choice Hotels Int. Inc	988.7	418.4	1,407.0	29.7%	100.0%	24.3%	5.4	45.8%	0.4%
Knight Ridder Inc	5,304.0	2,632.4	7,936.3	33.2%	63.5%	12.0%	26.4	23.6%	1.7%
Neiman Marcus Group	1,018.4	934.0	1,952.5	47.8%	45.6%	27.6%	15.4	34.7%	0.1%
Nordstrom Inc	3,132.5	2,797.9	5,930.5	47.2%	66.7%	19.7%	15.6	42.9%	1.8%
Staples Inc	9,348.2	2,758.3	12,106.5	22.8%	51.0%	19.0%	7.2	44.8%	0.1%
Unifi Inc	359.7	465.9	825.6	56.4%	50.6%	45.2%	5.4	68.9%	0.0%
<i>Average</i>	<i>3,358.6</i>	<i>1,667.8</i>	<i>5,026.4</i>	<i>39.5%</i>	<i>55.5%</i>	<i>24.6%</i>	<i>12.5</i>	<i>43.5%</i>	<i>0.7%</i>
Energy									
Energen Corporation	1,058.7	905.5	1,964.2	46.1%	59.7%	29.6%	16.6	30.4%	2.4%
Enesco International	3,762.5	1,043.2	4,805.7	21.7%	36.1%	31.0%	14.9	48.8%	0.4%
Murphy Oil Corp	4,449.3	2,379.7	6,828.9	34.8%	57.7%	22.4%	17.7	32.4%	1.7%
Newfield Exploration	1,849.2	1,196.3	3,045.4	39.3%	54.1%	38.2%	7.7	31.8%	0.0%
<i>Average</i>	<i>2,779.9</i>	<i>1,381.2</i>	<i>4,161.1</i>	<i>35.5%</i>	<i>51.9%</i>	<i>30.3%</i>	<i>14.2</i>	<i>35.9%</i>	<i>1.1%</i>
Basic Materials									
Lubrizol Corp	1,639.4	975.5	2,614.9	37.3%	52.4%	31.9%	14.1	28.6%	3.3%
Nucor Corp	3,918.2	1,905.1	5,823.4	32.7%	45.0%	18.7%	8.1	42.1%	1.6%
Sensient Technologies	994.8	776.9	1,771.6	43.9%	60.7%	19.9%	6.4	28.7%	2.7%
Worthington Industries	1,275.3	835.5	2,110.7	39.6%	57.5%	40.2%	5.5	39.4%	4.4%
<i>Average</i>	<i>1,956.9</i>	<i>1,123.2</i>	<i>3,080.2</i>	<i>38.4%</i>	<i>53.9%</i>	<i>27.7%</i>	<i>8.5</i>	<i>34.7%</i>	<i>3.0%</i>
Healthcare									
Beckman Coulter Inc	2,700.1	1,649.8	4,349.9	37.9%	71.3%	21.0%	16.1	32.3%	0.9%
Guidant Corp	13,322.7	1,543.9	14,866.5	10.4%	41.2%	24.4%	3.4	40.8%	0.3%
Watson Pharmaceutical	3,807.1	1,012.7	4,819.7	21.0%	35.2%	15.6%	5.4	45.9%	0.0%
<i>Average</i>	<i>6,609.9</i>	<i>1,402.1</i>	<i>8,012.1</i>	<i>23.1%</i>	<i>49.2%</i>	<i>20.3%</i>	<i>8.3</i>	<i>39.6%</i>	<i>0.4%</i>
Consumer Non Cyclical									
Blyth Inc	1,260.2	425.1	1,685.3	25.2%	43.7%	42.5%	8.4	31.4%	0.9%
Corn Products Int. Inc	1,141.0	1,282.8	2,423.8	52.9%	58.8%	30.9%	5.6	27.5%	1.3%
Toro Co	873.0	535.0	1,408.0	38.0%	57.9%	18.9%	24.9	28.8%	0.7%
<i>Average</i>	<i>1,091.4</i>	<i>747.6</i>	<i>1,839.0</i>	<i>38.7%</i>	<i>53.5%</i>	<i>30.8%</i>	<i>13.0</i>	<i>29.2%</i>	<i>1.0%</i>
All Firms									
<i>Average</i>	<i>3,886.9</i>	<i>2,042.1</i>	<i>5,929.0</i>	<i>37.4%</i>	<i>56.1%</i>	<i>25.6%</i>	<i>9.7</i>	<i>36.2%</i>	<i>1.5%</i>
<i>Sd</i>	<i>6,382.3</i>	<i>3,723.3</i>	<i>9,938.2</i>	<i>11.2%</i>	<i>12.7%</i>	<i>10.1%</i>	<i>6.2</i>	<i>9.3%</i>	<i>1.2%</i>
<i>Max</i>	<i>33,857.8</i>	<i>20,998.5</i>	<i>54,856.3</i>	<i>57.6%</i>	<i>100.0%</i>	<i>45.2%</i>	<i>26.4</i>	<i>68.9%</i>	<i>4.4%</i>
<i>Min</i>	<i>359.7</i>	<i>418.0</i>	<i>825.6</i>	<i>10.4%</i>	<i>35.2%</i>	<i>4.1%</i>	<i>3.4</i>	<i>23.6%</i>	<i>0.0%</i>

Table A.2 reports descriptive statistics for the firms in the sample. All the figures are an average of the quarterly observations for each company (third quarter of 2001 until first quarter of 2004). All market values are expressed in million dollars. The market value of equity and dividend yield for each quarter was obtained from DATASTREAM as well as stock prices required to compute stock volatility. Stock volatility for each quarter is the annualised stock volatility. It was computed using a series of daily log returns from the last 250 trading days proceeding each quarter. Daily stock prices were also obtained from DATASTREAM. The market value of total liabilities is the sum of the market value of traded bonds and book value of other liabilities.

A.3 – Nelson-Siegel (1987) model

For each quarter we use price information of the U.S. Treasury Strips obtained from DATASTREAM. This method allows for smoothing the yields of the strips into a continuous yield curve. Given the price of the Treasury Strip P_s and its maturity T , the continuous spot yield R_s is given by

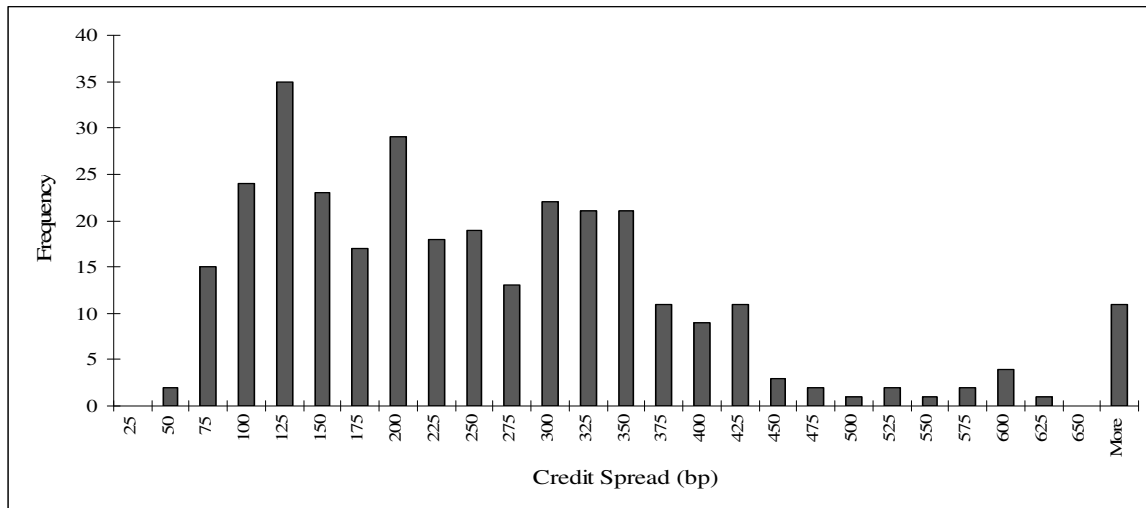
$$R_s = \frac{\ln(100/P_s)}{T} \quad (\text{A.3.1})$$

For the same date, the estimated spot curve given by the Nelson-Siegel (1987) model is

$$R_{NS}(T, \Theta_r) = \beta_0 + (\beta_1 + \beta_2) \frac{(1 - e^{-T/\tau_1})}{T} - \beta_2 e^{-T/\tau_1} \quad (\text{A.3.2})$$

where $\Theta_r = (\beta_0, \beta_1, \beta_2, \tau_1)$. In order to fit the model with the Treasury Strips yields, one chooses the parameters in Θ_r such that the sum of squared errors is minimized, where the error is the difference between the model yield and the observed spot yield.

A.4 - Distribution of Market Spread: Total Sample



A.5 – P-Values to Test Mean Zero of the Merton and Leland Relative Errors

P-values	RELATIVE PRICING ERROR		RELATIVE YIELD ERROR		RELATIVE SPREAD ERROR	
	MERTON	LELAND	MERTON	LELAND	MERTON	LELAND
ALL SAMPLE	0.0000	0.0000	0.0000	0.0186	0.0000	0.0000
INDUSTRIAL	0.0000	0.0000	0.0000	0.0093	0.0000	0.0000
CONSUMER, CYCLICAL	0.0000	0.4869	0.0000	0.0000	0.0000	0.0000
ENERGY	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000
BASIC MATERIALS	0.0000	0.0090	0.0000	0.0498	0.0000	0.0000
HEALTHCARE	0.0000	0.1309	0.0000	0.0065	0.0000	0.0000
CONSUMER NON-CYCLICAL	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HIGH RATING	0.0000	0.0000	0.0000	0.1236	0.0000	0.0000
LOW RATING	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SHORT MATURITY	0.0000	0.3127	0.0000	0.0000	0.0000	0.0000
MEDIUM MATURITY	0.0000	0.0176	0.0000	0.0010	0.0000	0.0000
LONG MATURITY	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
LOW VOLATILITY	0.0000	0.0000	0.0000	0.0016	0.0000	0.0000
HIGH VOLATILITY	0.0000	0.0368	0.0000	0.7574	0.0000	0.0000

Table A.5 reports the P-values for a two-tailed test, which test the following hypothesis for the mean relative error. $H_0: \mu = 0$ and $H_1: \mu \neq 0$. The values in bolt refer to cases where we cannot reject the null hypothesis that the mean relative error is zero for a 5% significance level.

A.6 – P-Values to Test Equality of Means of the Merton and Leland Models Relative Errors

P-values	RELATIVE PRICING ERROR	RELATIVE YIELD ERROR	RELATIVE SPREAD ERROR
ALL SAMPLE	0.0000	0.0000	0.6779
INDUSTRIAL	0.0000	0.0000	0.0538
CONSUMER, CYCLICAL	0.0000	0.0003	0.0728
ENERGY	0.0552	0.0009	0.3472
BASIC MATERIALS	0.0001	0.0000	0.5249
HEALTHCARE	0.0000	0.0000	0.1646
CONSUMER, NON-CYCLICAL	0.5460	0.0000	0.9354
HIGH RATING	0.0000	0.0000	0.7480
LOW RATING	0.0000	0.0000	0.7739
SHORT MATURITY	0.0000	0.0000	0.0000
MEDIUM MATURITY	0.0000	0.0000	0.2162
LONG MATURITY	0.2787	0.0023	0.0195
LOW VOLATILITY	0.0412	0.0000	0.0366
HIGH VOLATILITY	0.0000	0.0000	0.9341

Table A.6 reports the P-values for a two-tailed test, which test the following hypothesis for the differences in Merton and Leland means relative errors. $H_0: \mu^{\text{Merton}} - \mu^{\text{Leland}} = 0$ and $H_1: \mu^{\text{Merton}} - \mu^{\text{Leland}} \neq 0$. The values in bolt refer to cases where the equality of means does hold for a 5% significance level.

A.7 – P-Values to Test No Category Effects in the Merton and Leland Relative Errors

P-values	RELATIVE PRICING ERROR		RELATIVE YIELD ERROR		RELATIVE SPREAD ERROR	
	MERTON	LELAND	MERTON	LELAND	MERTON	LELAND
SECTORS	0.0000	0.0000	0.0000	0.0000	0.0000	0.0009
RATING	0.0000	0.0257	0.0000	0.0000	0.0464	0.0128
MATURITY	0.0119	0.0000	0.0000	0.0000	0.0000	0.1258
ASSET VOLATILITY	0.0000	0.0000	0.0040	0.0454	0.0000	0.0000

Table A.7 reports the P-values for a two-way ANOVA test, which evaluates the following hypothesis for the means of relative errors in the Merton and Leland models.

$H_0: \mu^{\text{High Rating}} = \mu^{\text{Low Rating}} = 0$ and H_1 : otherwise.

$H_0: \mu^{\text{Short Maturity}} = \mu^{\text{Medium Maturity}} = \mu^{\text{Long Maturity}} = 0$ and H_1 : otherwise.

$H_0: \mu^{\text{Low Asset Volatility}} = \mu^{\text{High Asset Volatility}} = 0$ and H_1 : otherwise.

$H_0: \mu^{\text{Industrial}} = \mu^{\text{Consumer Cyclical}} = \mu^{\text{Energy}} = \mu^{\text{Basic Materials}} = \mu^{\text{Healthcare}} = \mu^{\text{Consumer Non-Cyclical}} = 0$ and H_1 : otherwise.

The values in bold refer to cases where there are not category effects for a 5% significance level.

A.8 – Performance of the Merton and Leland Model According to Categories

RATING	MODEL		PRICE		YIELD		SPREAD	
			Absolute Error (\$M)	Relative Error	Absolute Error (bp)	Relative Error	Absolute Error (bp)	Relative Error
HIGH RATING	MERTON	Mean	28.1	8.9%	-155.3	-26.3%	-155.3	-72.9%
		<i>Sd</i>	24.0	7.7%	144.3	18.7%	144.3	41.8%
	LELAND	Mean	7.3	3.3%	-23.4	3.0%	-153.7	-71.6%
		<i>Sd</i>	35.7	9.7%	141.9	27.7%	125.9	35.1%
LOW RATING	MERTON	Mean	34.2	14.8%	-251.3	-37.3%	-251.3	-81.5%
		<i>Sd</i>	26.8	9.6%	143.5	16.9%	143.5	28.7%
	LELAND	Mean	15.1	6.5%	-115.6	-13.8%	-258.8	-80.6%
		<i>Sd</i>	37.3	15.4%	142.3	17.6%	138.4	22.2%

MATURITY	MODEL		PRICE		YIELD		SPREAD	
			Absolute Error (\$M)	Relative Error	Absolute Error (bp)	Relative Error	Absolute Error (bp)	Relative Error
SHORT MATURITY	MERTON	Mean	24.5	8.0%	-206.7	-40.7%	-206.7	-97.0%
		<i>Sd</i>	16.8	6.6%	155.7	11.7%	155.7	6.8%
	LELAND	Mean	1.4	0.9%	45.2	22.4%	-184.3	-82.6%
		<i>Sd</i>	22.4	6.4%	175.8	34.4%	165.3	15.6%
MEDIUM MATURITY	MERTON	Mean	31.4	11.3%	-229.4	-34.5%	-229.4	-77.4%
		<i>Sd</i>	26.2	7.3%	159.6	19.5%	159.6	37.8%
	LELAND	Mean	5.2	2.0%	-70.5	-5.8%	-225.5	-72.4%
		<i>Sd</i>	38.9	11.1%	147.2	22.9%	147.1	35.4%
LONG MATURITY	MERTON	Mean	31.8	12.6%	-120.7	-18.4%	-120.7	-63.9%
		<i>Sd</i>	26.7	11.7%	102.6	13.7%	102.6	41.2%
	LELAND	Mean	23.4	10.6%	-89.6	-12.1%	-144.5	-75.7%
		<i>Sd</i>	34.5	14.1%	111.8	15.0%	93.3	28.5%

(Continues next page)

A.8 – Performance of the Merton and Leland Models According to Categories (cont.)

ASSET VOLATILITY	MODEL		PRICE		YIELD		SPREAD	
			Absolute Error (\$M)	Relative Error	Absolute Error (bp)	Relative Error	Absolute Error (bp)	Relative Error
LOW VOLATILITY	MERTON	Mean	39.6	13.6%	-202.9	-33.6%	-202.9	-93.4%
		<i>Sd</i>	25.2	9.4%	125.4	13.3%	125.4	7.8%
	LELAND	Mean	31.3	11.3%	-72.4	-6.4%	-196.2	-91.4%
		<i>Sd</i>	26.1	10.2%	138.2	25.0%	109.9	8.5%
HIGH VOLATILITY	MERTON	Mean	22.0	8.9%	-181.9	-27.6%	-181.9	-60.3%
		<i>Sd</i>	22.1	7.9%	171.4	22.3%	171.4	46.1%
	LELAND	Mean	-9.1	-1.7%	-45.9	-0.6%	-191.6	-60.0%
		<i>Sd</i>	33.8	10.6%	157.3	26.0%	163.6	36.4%

SECTOR	MODEL		PRICE		YIELD		SPREAD	
			Absolute Error (\$M)	Relative Error	Absolute Error (bp)	Relative Error	Absolute Error (bp)	Relative Error
INDUSTRIAL	MERTON	Mean	27.7	9.4%	-180.3	-32.5%	-180.3	-86.0%
		<i>Sd</i>	14.7	6.4%	108.4	14.0%	108.4	26.7%
	LELAND	Mean	6.7	3.7%	-2.7	7.3%	-170.9	-77.6%
		<i>Sd</i>	27.4	7.1%	135.8	28.0%	109.5	33.4%
CONSUMER, CYCLICAL	MERTON	Mean	27.8	10.7%	-162.1	-22.3%	-162.1	-58.0%
		<i>Sd</i>	27.9	9.8%	178.0	19.7%	178.0	41.8%
	LELAND	Mean	4.4	1.3%	-96.5	-10.8%	-203.2	-68.9%
		<i>Sd</i>	44.4	15.0%	146.1	15.9%	166.9	25.5%
ENERGY	MERTON	Mean	42.8	10.6%	-135.2	-21.8%	-135.2	-62.5%
		<i>Sd</i>	37.6	8.3%	115.8	18.0%	115.8	43.1%
	LELAND	Mean	27.2	6.5%	-69.0	-10.1%	-150.3	-70.2%
		<i>Sd</i>	45.1	11.3%	88.5	13.6%	100.1	33.2%
BASIC MATERIALS	MERTON	Mean	31.4	12.5%	-275.1	-37.7%	-275.1	-80.6%
		<i>Sd</i>	27.1	8.3%	213.5	23.3%	213.5	47.0%
	LELAND	Mean	4.7	4.1%	-97.6	-7.8%	-251.2	-74.6%
		<i>Sd</i>	36.8	10.2%	179.7	25.9%	189.6	40.2%
HEALTHCARE	MERTON	Mean	18.1	6.8%	-143.0	-29.3%	-143.0	-77.1%
		<i>Sd</i>	11.9	4.4%	81.4	16.5%	81.4	33.7%
	LELAND	Mean	-5.4	-2.5%	41.5	14.7%	-118.8	-66.8%
		<i>Sd</i>	24.2	9.5%	102.0	31.1%	65.4	24.9%
CONSUMER, NON-CYCLICAL	MERTON	Mean	38.4	20.5%	-302.5	-43.9%	-302.5	-95.1%
		<i>Sd</i>	23.6	12.0%	101.3	12.6%	101.3	5.8%
	LELAND	Mean	33.1	18.6%	-184.1	-24.1%	-301.6	-95.0%
		<i>Sd</i>	20.1	13.7%	121.5	14.0%	98.8	4.2%

Table A.8 reports the absolute and relative errors in price, yield and spread for the Merton and Leland models for different categories of rating, maturity, asset volatility and sectors. The absolute errors in prices, yields and spreads are calculated as the predicted prices, yields and spreads minus the observed values of these variables. The relative errors are computed as the absolute errors divided by the observed prices, yields or spreads.

Table 1 – Descriptive Statistics of Spreads: Market, Models and Merrill Lynch

		SPREAD				AVERAGE	
		Market	Merton	Leland	Merril-Lynch	Maturity	Rating
All Firms n = 317	Mean	221.5	58.9	57.1	169.3	9.9	9.6
	<i>Sd</i>	125.5	111.5	74.7		6.4	2.0
Industrial n = 98	Mean	191.2	26.8	36.2	148.0	5.8	8.6
	<i>Sd</i>	92.1	47.2	33.5		1.4	1.7
Consumer Cyclical n = 66	Mean	250.3	149.3	108.3	203.4	12.5	10.3
	<i>Sd</i>	182.0	191.4	119.5		7.6	2.6
Energy n = 44	Mean	209.7	69.9	54.8	169.3	14.8	9.3
	<i>Sd</i>	68.4	83.8	62.0		5.2	1.7
Basic Materials n = 43	Mean	247.0	27.4	51.3	169.3	8.6	9.7
	<i>Sd</i>	141.3	45.4	47.5		3.5	2.3
Healthcare n = 33	Mean	171.7	44.8	69.0	169.3	8.5	9.7
	<i>Sd</i>	73.8	73.8	75.4		6.3	1.3
Consumer Non-Cyclical n = 33	Mean	279.9	14.2	15.2	227.2	12.7	11.0
	<i>Sd</i>	104.3	14.7	12.3		8.8	0.0

While the market spread and the Merton and Leland predicted spreads are averages for the sample period, the Merrill Lynch spread is just an approximation of spreads considering certain intervals of years to maturity and rating of the bonds. The original study in which we based the Merrill Lynch spread presents averages spreads over the period January 1997-August 2003 for U.S. corporate bonds and was obtained from Bloomberg.

Table 2 – Risk Neutral Default Probabilities (RNDP) and Moody’s 1-Year Default Rates

		RNDP		1-Year Default Rates (1999)	AVERAGE	
		Merton	Leland	Moody's	Maturity	Rating
All Companies n = 317	Mean	12.3%	10.5%	0.11%	9.9	9.6
	<i>Sd</i>	13.7%	10.4%		6.4	2.0
Industrial n = 98	Mean	7.8%	7.9%	0,00%-0,11%	5.8	8.6
	<i>Sd</i>	9.2%	6.1%		1.4	1.7
Consumer Cyclical n = 66	Mean	22.9%	17.4%	0,11%-1,12%	12.5	10.3
	<i>Sd</i>	18.6%	15.0%		7.6	2.6
Energy n = 44	Mean	17.1%	10.1%	0.11%	14.8	9.3
	<i>Sd</i>	13.2%	9.3%		5.2	1.7
Basic Materials n = 43	Mean	8.1%	10.6%	0.11%	8.6	9.7
	<i>Sd</i>	8.4%	8.4%		3.5	2.3
Healthcare n = 33	Mean	8.4%	11.8%	0.11%	8.5	9.7
	<i>Sd</i>	11.1%	10.2%		6.3	1.3
Consumer Non-Cyclical n = 33	Mean	7.1%	3.6%	0,11%-1,12%	12.7	11.0
	<i>Sd</i>	7.4%	2.7%		8.8	0.0

Source of Moody’s 1-year default rates: Duffie and Singleton (2003).

Table 3 – Performance of the Merton and Leland Models - Total Sample

ALL SAMPLE		PRICE		YIELD		SPREAD	
		Absolute Error (\$M)	Relative Error	Absolute Error (bp)	Relative Error	Absolute Error (bp)	Relative Error
MERTON	Mean	30.4	11.2%	-191.9	-30.5%	-191.9	-76.2%
	<i>Sd</i>	25.2	8.9%	151.2	18.8%	151.2	37.5%
LELAND	Mean	10.3	4.5%	-58.6	-3.4%	-193.8	-75.0%
	<i>Sd</i>	36.4	12.3%	148.8	25.7%	140.2	31.1%

Table 3 reports the absolute and relative errors in price, yield and spread for the Merton and Leland models. The absolute errors in prices, yields and spreads are calculated as the predicted prices, yields and spreads minus the observed values of these variables. The relative errors are computed as the absolute errors divided by the observed prices, yields or spreads.

Table 4 – Regression of Credit Spread and Relative Spread Errors: Total Sample

Independent Variables	Dependent variables: observed credit spread for "CREDIT SPREAD" and relative spread error of the bond pricing model for "MERTON" and "LELAND"					
	CREDIT SPREAD		MERTON		LELAND	
	Regression 1	Regression 2	Regression 1	Regression 2	Regression 1	Regression 2
Intercept	-290.832 (0.001)	-250.831 (0.000)	-2.065 (0.000)	-2.111 (0.000)	-1.641 (0.000)	-1.661 (0.000)
Size	-0.004 (0.000)	-0.004 (0.000)	0.000 (0.343)		0.000 (0.020)	0.000 (0.014)
Leverage	373.623 (0.000)	341.318 (0.000)	1.927 (0.000)	1.821 (0.000)	1.445 (0.000)	1.452 (0.000)
Yrs to Maturity	1.003 (0.543)		0.019 (0.000)	0.018 (0.000)	0.010 (0.000)	0.010 (0.000)
Asset Volatility	322.267 (0.001)	327.076 (0.000)	3.461 (0.000)	3.470 (0.000)	3.362 (0.000)	3.370 (0.000)
Rating	34.903 (0.000)	34.132 (0.000)	0.008 (0.399)		-0.002 (0.767)	
Mk-to-Book Ratio	6.962 (0.602)		-0.041 (0.005)	-0.041 (0.004)	-0.044 (0.000)	-0.044 (0.000)
Stock Return			0.119 (0.002)	0.118 (0.002)	-0.003 (0.925)	
Observed YTM			-9.765 (0.000)	-8.636 (0.000)	-9.185 (0.000)	-9.318 (0.000)
Level of Term Structure			6.470 (0.001)	5.821 (0.002)	5.928 (0.001)	6.050 (0.000)
Slope of Term Structure			-6.058 (0.079)		-6.239 (0.037)	-6.472 (0.026)
Adj. Rsq	0.35	0.34	0.83	0.83	0.84	0.84
ANOVA F statistics	27.74 (0.000)	41.65 (0.000)	156.96 (0.000)	222.55 (0.000)	166.15 (0.000)	208.96 (0.000)

Table 4 reports regression coefficients and their p-values (in parentheses). In the ANOVA F statistics section all the values in parenthesis also correspond to p-values. Regression 1 includes all explanatory variables (with significant and non-significant parameters) and regression 2 includes only variables with significant parameters, considering a 5% significance level. We always start our estimation with regression 1 and then regression 2 is obtained using a backward elimination strategy. All regressions are corrected for autocorrelation and heteroscedasticity.

Table 5 – Prediction errors: Leland model vs. Fan and Sundaresan Debt-Equity Swap

ALL SAMPLE		PRICE		YIELD		SPREAD	
		Absolute Error (\$M)	Relative Error	Absolute Error (bp)	Relative Error	Absolute Error (bp)	Relative Error
LELAND	Mean	10.3	4.5%	-58.6	-3.4%	-193.8	-75.0%
	Sd	36.4	12.3%	148.8	25.7%	140.2	31.1%
FAN & SUNDARESAN	Mean	-2.8	0.5%	-33.5	0.4%	-168.7	-64.9%
	Sd	39.5	13.0%	143.4	25.8%	133.2	37.0%

Table 5 reports the absolute and relative errors in price, yield and spread for the Merton model and Fan and Sundaresan debt-equity swap. The absolute errors in prices, yields and spreads are calculated as the predicted prices, yields and spreads minus the observed values of these variables. The relative errors are computed as the absolute errors divided by the observed prices, yields or spreads.

Figure 1 – Distribution of Relative Pricing Errors

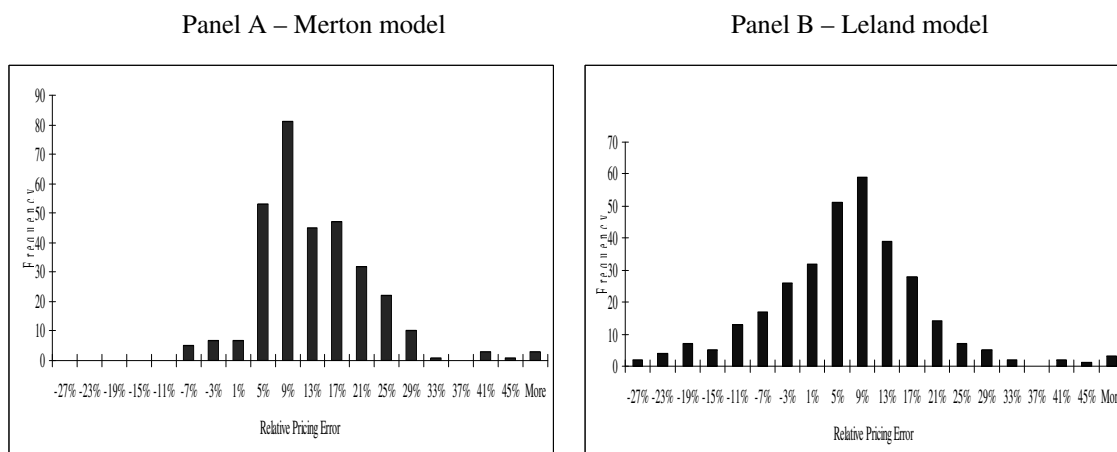
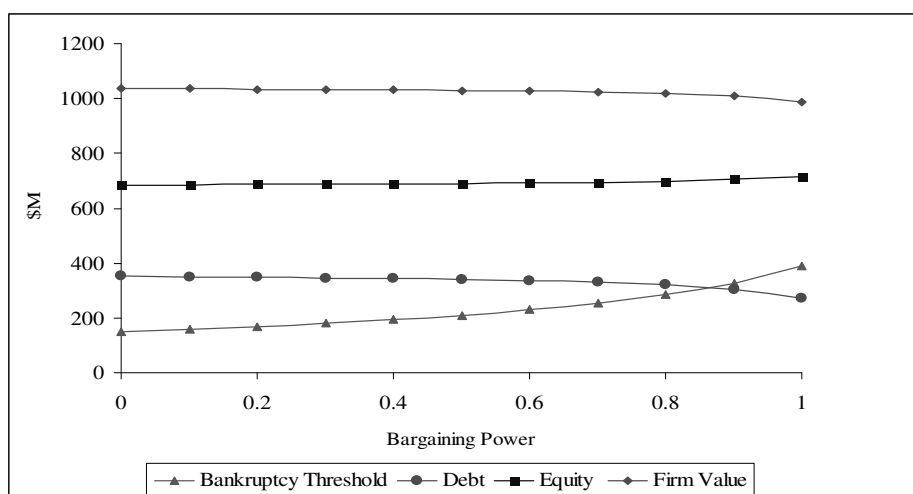


Figure 2 – Fan and Sundaresan Debt-Equity Swap: Sensitivity of Bankruptcy Threshold, Debt, Equity and Firm Value to the Bargaining Power Parameter



We report the predicted mean values for bankruptcy threshold, debt, equity and firm value, using the calibration procedure that estimates simultaneously the unlevered firm value and asset volatility and considers the bargaining power parameter as an input.