Extending the Merton Model: A Hybrid Approach to Assessing Credit Quality

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Abstract

In this paper we have combined fundamental analysis and contingent claim analysis into a hybrid model of credit risk measurement. We have extended the standard Merton approach to estimate a new risk neutral distance to default metric, assuming a more complex capital structure, adjusting for dividend payments, introducing randomness to the default point and allowing a fractional recovery when default occurs. Then, using financial ratios, other accounting based measures and the risk neutral distance to default metric from our structural model as explanatory variables we estimate the hybrid model with an ordered probit regression method. Using the same econometric method, we estimate a model using financial ratios and accounting variables as explanatory variables and a model using our risk neutral distance to default metric as unique explanatory variable. We have found that by enriching the risk neutral distance to default metric with financial ratios and accounting variables into the hybrid model, we can improve both in sample fit of credit ratings and out of sample predictability of defaults. Our main conclusion is that financial ratios and accounting variables contain significant and incremental information, thus the risk neutral distance to default metric does not reflect all available information regarding the credit quality of a firm.

JEL Nos: G11, G12, G13. **Keywords:** credit quality, distance to default, financial ratios, accounting variables

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1 Introduction

Credit risk refers to the risk due to unexpected changes in the credit quality of a counter party or issuer and its quantification is one of the major frontiers in modern finance. The creditworthiness of a potential borrower affects the lending decision, the firm's cost of capital, the credit spread, and the prices and hedge ratios of credit derivatives, since it is uncertain whether the firm will be able to fulfill its obligation. The Basel Committee set up by BIS has been urging banks to develop internal systems and models to measure and manage their credit risk exposure objectively. Credit risk measurement depends on the likelihood of default of a firm to meet its a required or contractual obligation and on what will be lost if default occurs. When one considers the large number of corporations issuing fixed income securities and the relatively small number of actual defaults might regard default as rare event. However, all corporate issuer have some positive probability of default. The "loss given default" factor depends primarily upon security and seniority. Models of credit risk measurement have focused on the estimation of the default probability of firms, since it is the dominant source of uncertainty in the lending decision. Two broad categories of credit risk models may be distinguished on the basis of the analysis they adopt. The first category, the set of traditional models adopt the fundamental analysis. The philosophy of these models that goes back to Beaver (1966) and Altman (1968, 1975) is to find which factors are important in assessing the credit risk of a firm. The second set, called structural models adopt the contingency claim analysis (CCA). The philosophy of these models goes back to Black-Scholes (1973) and Merton (1974) and considers corporate liabilities as contingent claims on the assets of the firm.[1]

Recent empirical studies, such as Kealhofer, Kwok and Weng (1998), Geske and Delianedis (1999, 2001), Leland (2002), Vassalou and Xing (2004), document that the theoretical probability measures estimated from structural default risk models have good predictive power over credit ratings and rating transitions. However, Hillegeist, Keating, Cram and Lundstedt (2004) document that traditional models (updated versions of Altman's Z-Score and Ohlson's O-Score) can provide significant, incremental information and thus, the theoretical probabilities estimated from structural models are not a sufficient statistic of the actual default probability. Thus, the combination of the two major approaches becomes a great challenge in credit risk measurement.

In this paper we have combined fundamental analysis and contingent claim analysis into a hybrid model of credit risk measurement. We have extended the standard Merton approach to estimate a new risk neutral distance to default metric, assuming a more complex capital structure, adjusting for dividend payments, introducing randomness to the default point and allowing a fractional recovery when default occurs. Then, using financial ratios, other accounting based measures and the risk neutral distance to default metric from our structural model as explanatory variables we estimate the hybrid model with an ordered probit regression method. Using the same econometric method, we estimate a model using financial ratios and accounting variables as explanatory variables and a model using our risk neutral distance to default metric as unique explanatory variable. We have found that by enriching the risk neutral distance to default metric

[1]Another widely used category of credit risk models is the reduced form approach where the dynamics of default are given exogenously by an intensity or compensator process. For a review of these models see Jarrow, and Turnbull (1995),Jarrow, Lando and Turnbull (1997), Duffie and Singleton (1999).

with financial ratios and accounting variables into the hybrid model, we can improve both in sample fit of credit ratings and out of sample predictability of defaults. Our main conclusion is that financial ratios and accounting variables contain significant and incremental information, thus the risk neutral distance to default metric does not reflect all available information regarding the credit quality of a firm.

The remainder of this paper is organized as follows: Section 2 provides a brief review of the two main approaches in credit risk measurement: fundamental analysis and contingent claim analysis. In Section 3 we present the original Merton Model (1974) and some of its extensions. In section 4 we describe in details the theoretical foundations of our structural credit risk model. Section 5 demonstrates our hybrid model and our econometric methodology. Details about the sample selection, variable estimation and data collection are reported in section 6. In section 7 we discuss the estimation results and provide empirical tests.Section 8 summarizes and concludes the paper.

2 Credit Risk Measurement

2.1 Fundamental Analysis-Traditional Models

Traditional models adopt fundamental analysis and try to pre-identify which factors such as cash flow adequacy, asset quality, earning performance, capital adequacy, are important in explaining the credit risk of a company. They evaluate the significance importance of these factors, mapping a reduced set of accounting variables, financial ratios and other information into a quantitative score. In some cases, this score can be literally interpreted as a probability of default while in other cases can be used as classification system.[2]

The main characteristic that differentiates traditional models is the econometric method they apply on their estimation procedure. In 1966 the pioneering study of Beaver has introduced the univariate approach of discriminant analysis in bankruptcy prediction. Altman in 1968 has expanded it to a multivariate context and developed the Z-Score model. Multivariate Linear Discriminant Analysis (LDA) is based on a linear combination of two ore more independent variables that will discriminate best between a priori defined groups: the default from non-defaulted firms. It weights the independent variables (financial ratios and accounting variables) and generates a single composite discriminant score. The score is then compared to a cutoff value, which determines the group that the firm belongs to. This is achieved by the statistical rule of maximizing the between group variance relative to the within group variance. The cutoff value is usually defined as the midpoint of the distance between the means of the standardized groups. At that point we must note that the choice of the optimal cut-off score can incorporate changes in economic conditions. Discriminant analysis, does very well provided, that the variables in every group follow a multivariate normal distribution and covariance matrices are equal for every priori defined group. However, empirical studies have shown, that especially defaulted firms violate the normality assumption. It has also proved that the violation of normality assumption affects more the

[2] For a review of traditional models see: Jones (1987) and Cauette , Altman and Naraynan (1998), Saunders (2002).

prediction ability of the method rather than its classification ability. In 1977 Altman, Haldeman and Narrayman have developed the ZETA model, which incorporated several refinements and enhancements to the original Z-Score approach. Until 1980's Multivariate Linear Discriminant Analysis (LDA) was the dominant method on default risk modeling. Then the binary dependent variables models, known as the logit and probit model, have been used in explaining the default risk of firms.[3] Ohlson (1980) used logit methodology to derive a default risk model known as O-Score. Probit (Logit) methodology weights the independent variables and assigns scores in a form of failure and survival probability using the normal (logistic) cumulative function. These models can be also used as a classification system and place the potential borrower into either a good or a bad group according to a cut-off point. The optimal cut-off point is chosen to minimize the noise to signal ratio. Mester (1997) have documented the widespread use of the binary credit risk models: 70 % of banks have used them in their non-listed firm lending procedure.

A new framework in assessing the credit quality of public firms with fundamental analysis, are the ordered dependent variable models (logit, probit) which are based on the pioneering work of McKelvey and Zavoina, (1975). These models apply on credit ratings, allowing an ordered specification for the credit quality of firms where default can be regarding as a special case of credit rating. Credit ratings are ordinal measures of firm's creditworthiness and take into account not only the default probability but also the severity of loss given default. For their assessment, rating agencies use publicly available financial and non-financial information, together with private information obtained during regular discussions with representatives of the firm being rated. As a result, rating agencies have been skeptical about whether models using publicly available information can replicate the professional rating process. Notable contributions in explaining and predicting credit ratings with ordered dependent variable models are Cantor and Packer (1996), Blume, Lim and Mackinlay (1998) and Pottier and Sommer (1999). Despite the skepticism from rating agencies, these models have been successful in explaining and predicting credit ratings .As we will see in a next section of this paper we will use the ordered probit regression to estimate our hybrid model credit risk measurement.

It is obvious that an important aspect of traditional models is the selection of the appropriate financial ratios and accounting based measures that will be used as explanatory variables. The two most frequently used methods for variable selection have been the simultaneous (direct) method and the stepwise method. However, none of these methods have been accepted as a basis for a theoretical variable selection. It has been stated that these methods focus solely on the statistical grounds of the variables and ignore their economic importance. Thus, in most failure prediction studies the appropriate financial ratios and accounting variables are selected according to their ability to increase the prediction accuracy and decrease the misclassification rates. Explanatory variables that commonly used in previous studies of corporate credit standing are:

- Liquidity variables. These variables are a measure of quality and adequacy of the current assets of a firm to meet its current liabilities if all become payable simultaneously. Some key variables for examining liquidity are working capital ratio, quick ratio and current ratio.

[3] Jones (1987) in his review of bankruptcy literature, concludes that binary dependent variable models do not led to notable improvements in the predictive power of fundamental analysis when compared to the earlier LDA models.

- Solvency variables. These variables are related to liquidity variables in that they are indented to measure the ability of a firm to service its debt. Most common solvency variables are the interest coverage ratio and the current liabilities service ratio.

- Profitability Variables. They show how successful a firm is in generating returns and profits on its investments. Moreover they show how a firm smooth or manage its earnings. The most common measures of profitability are return on assets, return on equity and internal growth rate.

- Leverage Variables. They show how the capital structure of firm is financed. These variables are related to profitability variables since the capital structure of a firm can be considered as of high quality if the firm has high a return on equity and its modest dividend payout to stockholders results to high internal growth rate. In addition, leverage variables measure firm's vulnerability to business downturns and economic shocks. Key variables to examine leverage are the total leverage ratio and the debt to assets ratio.

- Efficiency Variables. They are designed to evaluate management strategy and performance. Therefore, they are also called asset-management indicators. Specifically, they measure the ability of a firm to turn over its assets, equity, inventories, cash, account receivables or payable. Some important efficiency variables are the asset turnover ratio and the equity turnover ratio.

- Size Variables. They measure the market position and the completive position of a firm. Key variables for the measurement of the size of a firm are total sales and total assets.

Fundamental analysis has been during the last 4 decades an impressive body of empirical and academic research in assessing the default probability of a firm. However, many researchers have questioned the effectiveness of the expected default probabilities generated from traditional models. First of all, their critique relies on the fact that these models do not allow non-linear effects among different credit risk factors. Moreover, they consider that the models are based only on accounting data, which appear at discrete intervals (e.g. yearly) and are formulated under conservative principles. Therefore, it is questionable whether such models can pick up a firm that is rapidly deteriorating (such as during the Asia crisis). Furthermore, they argue that they do no take into account factors such as the market value of assets and the business risk of firms. Two firms with equal liabilities ratios can have different default risk depending on their market value of assets and their business risk. However, these two important factors are unvalued from financial statements and the stock market. The pioneering work of Merton in 1974 has tackled these issues, introducing contingent claim analysis (CCA) in credit risk measurement.

2.2 Contingent Claim Analysis-Structural Models

Over the last 4 decades, a large number of structural models have been developed to estimate and price credit risk.[4] The philosophy of these models, which goes back to Black-Scholes (1973) and Merton (1974), is to consider corporate liabilities(equity and debt) as **contingent claims** on the assets of the firms. Models that adopt contingent claim analysis are also referred as Merton type models and have five primary elements: A fundamental state variable, typically the market value of firm's assets which is assumed to move randomly through time with a specified expected return and volatility, debt, interest rates, a default boundary beneath which promised payments to debtholders are not made and default occurs and a recovery ratio which postulates what debtholders receive in the event of default.[5]

Central to our hybrid model is a variant of Merton's analytical model of firm value. Fundamental, to Merton's model is the idea that equity and debt could be considered as options on the value of the firm's assets. To see this, consider the case of a simple firm with market value of assets $A = (A_t)_{t \geq 0}$, representing the expected discounted future cash flows and a capital structure with two classes of liabilities:equity with market value equal to $S = (S_t)_{t \geq 0}$ and zero coupon debt with face value D^T, maturity at time T. The issue of the debt prohibits the payment of dividends until the face value is paid at maturity T. The contractual obligation of equityholders is to pay D^T back to debtholders at time T. Suppose debt covenants grand debtholders absolute priority: if equityholders cannot fulfill their obligation, then they will find it preferable to exercise their limited liability rights, default on the promised payment and surrender the firm's ownership to debtholders. Hence, we can define the default indicator function as:

$$1_T = 1 \; if \; A_T < D^T$$
$$1_T = 0 \; if \; A_T \geq D^T$$

Assuming also, that the firm is neither allowed to repurchase shares nor to issue any new senior or equivalent claims, we get the following payoffs to equityholders and debtholders at time T.

Event	**Assets**	**Debtholders**	**Equityholders**
No Default	$A_T \geq D^T$	D^T	$A_T - D^T$
Default	$A_T < D^T$	A_T	0

If at T the market value of assets A_T exceeds the face value of debt D^T the debtholders will receive the promised payment D^T and the equityholders will receive the residual claim $A_T - D^T$. However, if the market value of assets A_T does not exceed the face value of debt D^T, the ownership of the firm will be transferred to debtholders (equity is then worthless). These payoffs imply that equity and debt possesses option like features with respect to the solvency of the firm. The payoff of equityholders is equivalent to that of a european call position on the assets of the firm, with strike price equal to D^T (firm's default boundary) and maturity T. Thus, its value at time T is equal to

[4]For a review of structural credit risk models see Nandi (1998), Crouhy, Galai and Mark (2000), Bohn (2000), Giesecke (2004), Leland (2002), Eom, Helwege and Huang (2003).

[5]For details on the five fundamental elements of the structural models see Leland (2002).

$S_T = max(A_T - D^T, 0)$. The payoff of debtholders is equivalent to that of a portfolio composed of default-free debt with face value D^T and maturity at T plus a european put position on the assets of the firm, with strike price D^T and maturity T.Therefore, its value at time T is equal to $D_T^T = min(D^T, A_T) = D^T - max(D^T - A_T, 0)$.

Therefore, equity and debt can be valued as contingent claims on the assets of the firm. Recall that, if financial markets are liquid, have continuous trading, perfect asset divisibility, no transaction costs, no taxes and no arbitrage opportunities, there exists a risk neutral probability measure $\widetilde{P}$, equivalent to the physical measure P , such that the processes of security prices are $\widetilde{P}$-martingales .[6] $\widetilde{P}$ is called also equivalent martingale measure , and $\widetilde{E}[(\cdot)/F_t]$ denotes its corresponding risk neutral expectation operator. So, if Z_t^T denotes the market value at time t of riskless zero coupon debt that pays a unity at maturity T , then according to asset pricing theory the market value of this debt now at time $t = 0$ equals $\widetilde{E}[Z_T/F_0]$. Summarizing, the market value of risky debt D_0^T now at time $t = 0$ with face value D^T and maturity at T can be written as:

$$D_0^T = \widetilde{E}[Z_T D_T^T/F_0] = D^T \widetilde{E}[Z_T/F_0] - \widetilde{E}[Z_T(D^T - A_T)1_T/F_0] \tag{1}$$

The above decomposition implies that, the market value of risky debt equals the risk neutral expected discounted value of riskless debt with face value D^T less the risk neutral expected discounted default loss. If debt was riskless, then this expected loss would be zero. Note, that contingent claim analysis unlike fundamental analysis focuses on the "loss given default" amount, which is important in credit risk measurement. Similarly the market value of equity S_0, now at time $t = 0$ equals to :

$$S_0 = \widetilde{E}[Z_T S_T/F_0] = \widetilde{E}[Z_T(A_T - D^T)1_T/F_0] \tag{2}$$

3 The Merton Approach

It is straightforward from the above analysis that we need a number of assumptions regarding the firm value process and the risk free interest rate process to derive analytically the market value of risky debt and the associated probability that a firm will default on its debt. Merton (1974) involves the Black-Scholes (1973) setting by assuming that the risk free interest rate is constant and identical for borrowing and lending and that the firm value follows a geometric Brownian motion with a constant drift equal to the risk free interest rate r and a constant diffusion rate equal to σ_A:

$$\frac{dA_t}{A_t} = r(A_t, t)dt + \sigma_A dW_{1t} \tag{3}$$

where W_{1t} is a standard Brownian motion.

Under the above assumptions on asset and risk free interest rate dynamics, the risk neutral expected default probability now at time $t = 0$ that a firm will defaults on its debt at time T

[6] For further insight see Harrison and Kreps (1979), Harrison and Pliska (1981).

equals to:

$$RNEDP_T = N(-\frac{\ln(\frac{A_0}{D^T}) + (r - \frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}) \tag{4}$$

where N notes the standard normal distribution function. It is straightforward to see, that the risk neutral expected default probability, depends on the market value of assets A_0 now at time $t = 0$ and the asset volatility σ_A. The term $\frac{\ln(\frac{A_0}{D^T})+(r-\frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}$ is called risk neutral distance to default metric and measures the number of standard deviations that the firm's asset value is away from the default point D^T. In order to find the current market value of assets A we will use Black-Scholes call option pricing formula:

$$S_0 = A_0 N(d1) - D^T e^{-rT} N(d2) \tag{5}$$

where $d1 = \frac{\ln(\frac{A_0}{D^T})+(r+\frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}}$ and $d2 = d1 - \sigma_A\sqrt{T}$.
Since the market value of equity S_0 is observable for listed firms from the stock market, equation (5) has two unknowns, the market value of assets A_0 and asset volatility-business risk σ_A. In general, equity and asset volatilities are related through:

$$\sigma_S = \sigma_A \frac{A_0}{S_0} \frac{\vartheta S_0}{\vartheta A_0} \tag{6}$$

The latter equation which can be derived from Ito's lemma provides the equity-implied asset volatility estimate. Equations (5) and (6) is a set of two nonlinear equations with two unknowns that can be solved with numerical recipies. Hence, we are able to value A_0 and σ_A, and estimate the risk neutral expected default probability.

Under the same assumptions, the market value of riskless zero coupon debt now at time $t = 0$ with face value 1 and maturity at T equals: $\widetilde{E}[Z_T/F_0] = e^{-rT}$. Similarly the market value of risky debt now at time $t = 0$ with face value D^T and maturity at T can be written as:

$$D_0^T = A_0 - A_0 N(d1) + D^T e^{-rT} N(d2) \tag{7}$$

Equations (5) and (7) proves the market value identity $A_0 = S_0 + D_0^T$ which implies that the Modigliani-Miller (1958) theorem holds so that the firm asset value is invariant to its capital structure. Therefore, the firm's behavior such as the riskiness of its investments will not be impacted by how close it is to default.[7] Finally, note that the risk neutral expected discounted loss in equation (1) yields the Black-Scholes put option pricing formula:

$$\widetilde{E}[Z_T(D^T - A_T)1_T/F_0] = D^T e^{-rT} N(-d2) - A_0 N(-d1) \tag{8}$$

[7] For further discussion on the independence between the firm asset value and its capital structure see Rubinstein (2003).

4 Our Structural Default-Risk Model

4.1 Assumptions

The quantitative modeling of default risk, initiated by Merton (1974) shows how corporate liabilities (debt and equity) can be priced and the probability of default can be estimated under some specific assumptions. However, some of the assumptions concerning the "nature' (perfectness) of the financial markets, the interest rate dynamics, and the formulation of the issuer's capital structure serve to facilitate the mathematical representation of the model and can be considerable weakened.[8] In this paper, we have made some extensions on some of these assumptions underlying the original model and developed a new Merton type approach to estimate default probabilities and value corporate liabilities. Our approach is summarized at the following points:

We will follow the classic Black,Scholes and Merton assumptions about the firm value process and risk free interest rates. Hence we will assume that risk free interest rates are constant and that the firm's market asset value evolves as a geometric Brownian motion with a constant drift equal to the risk free interest rate r and a constant diffusion rate equal to σ_A:

$$\frac{dA_t}{A_t} = r(A_t, t)dt + \sigma_A dW_{1t} \tag{9}$$

where W_{1t} is a standard Brownian motion. However, will allow cash dividend payments since they can affect the market value of common equity. The total amount of proposed cash dividend payments δ , is assumed to be prepaid now at time $t = 0$. Therefore the change in the firm's market value at time $t = 0$, will be: $d(A_0) = -\delta$.

Merton Model is based on assumptions that in the event of default absolute priority holds, renegotiation is not permitted and liquidation of the firm is costless. These questionable assumptions imply fully recovery rates and default costs equal to zero. We will modify our approach, in order to allow a fractional recovery in case the firm defaults, as we know that direct and indirect costs of financial distress such as lawyer fees, administration expenses or loss opportunities due to the firm's uncertainty can result in debtholders receiving less than the firm value. Additional default costs can arise also from possible deviation on the absolute priority rule, when equityholders gain at the expense of debtholders. If we will let $R \in [0, 1]$, be a random variable expressing the recovery rate as % of the debt's face value, then $(1 - R_T)D^T$ will be the debtholders loss in the event of default at the maturity of debt T. Assuming also that the firm is neither allowed to repurchase shares nor to issue any senior or equivalent claims, we get the following payoffs to equityholders and stockhloders at time T.

Event	**Assets**	**Debtholders**	**Equityholders**
No Default	$A_T \geq D^T$	D^T	$A_T - D^T$
Default	$A_T < D^T$	$R_T D^T$	0

If at T the market value of assets A_T exceeds the face value of debt D^T the debtholders will receive the promised payment D^T and the equityholders will receive the residual claim $A_T - D^T$.

[8]Merton 1974, p.450

However, if the market value of assets A_T does not exceed the face value of debt D^T, equity is worthless and debtholders will receive $R_T D^T$.The above mentioned payoffs imply that equity and debt possesses option like features with respect to the solvency of the firm. Therefore , at time T the value of equity is equal to $S_T = max(A_T - D^T, 0)$ and the value of debt is equal to $D_T^T = min(D^T, R_T D^T) = D^T - max(D^T(1 - R_T), 0)$.

With the payoffs just described the market value of risky debt D_0^T now at time $t = 0$ with face value D^T and maturity at T can be written as:

$$D_0^T = \widetilde{E}[Z_T D_T^T / F_0] = D^T \widetilde{E}[Z_T / F_0] - \widetilde{E}[Z_T(D^T(1 - R_T))1_T / F_0] \tag{10}$$

The above decomposition implies as in Merton's framework that the market value of risky debt equals the risk neutral expected discounted value of riskless debt with face value D^T less the risk neutral expected discounted default loss. If the debt was riskless, then this expected loss would be zero. Following our assumption of a constant risk free interest rate and assuming also that default, riskless interest rates and recovery rates are mutually independent we get that:

$$D_0^T = D^T e^{-rT}(1 - \widetilde{E}[1 - R_T / F_0] RNEDP_T) \tag{11}$$

Therefore, the "loss given default" factor in our structural model is equal to:

$$\widetilde{E}[Z_T(D^T(1 - R_T))1_T / F_0] = D^T e^{-rT} \widetilde{E}[1 - R_T / F_0] RNEDP_T \tag{12}$$

Similarly, the market value of equity S_0 now at time $t = 0$ is equal to :

$$S_0 = \widetilde{E}[Z_T S_T / F_0] = \widetilde{E}[Z_T(A_T - D^T)1_T / F_0] \tag{13}$$

A second serious limitation of Merton Model is the simplicity of the capital structure it assumes. Geske (1977) and others have developed adjustments that allow for the simultaneous existence of multiple debt issues that can differ in maturity, size of coupons and seniority. Following these studies we will enable the capital structure in our approach to handle five classes of liabilities:

- Short term Debt with maturity up to one year and has total face value equal to D^1.
- Other Short term Liabilities with maturity up to one year and total face value STL.
- Long term Debt with total face value LTD and discrete per year maturities.Long term debt is assumed to have maturity buckets of 2, 3, 4 , 5 years with face value D^2, D^3, D^4,and D^5.
- Other Long term Liabilities with total face value LTL. This class incorporates mainly non-interest liabilities such us convertible bonds or us perpetual capitals, which do not have a fixed repayment date. However, it may also allow liabilities with a fixed repayment date.
- Common Equity.

Recall, that in the original model where the capital structure allowed only zero coupon debt the default boundary was equal to its face value. Hence, we need to adjust our complex debt structure to make it fit within the model. Geske and Delianedis (1999, 2001), have assumed complex debt structure in their implementations of Merton (1974) and Geske (1977) models and replaced it with an "equivalent" zero coupon debt structure. In particular, they set the value of the default barrier equal to the value of a zero coupon bond that has the same duration as the given debt structure. Vassallou and Xing (2004) have followed KMV, assumed a capital structure with short term debt and long term debt and set the zero equivalent level of the default boundary equal to the face value of short term debt plus half the value of long term debt. In our model, the initial value of the default point will be estimated from our given complex debt structure by using the above truncated method. Thus, we will take into account in our calculations the face value of all short term liabilities with maturity up to one year plus half the face value of all long term liabilities minus minority interest and deferred taxes, since they do not participate in firm's leverage. Although, there is certain arbitrariness in this truncated method, we agree with the conclusion of Vassallou and Xing[9] and KMV that the method behaves quite well within the model and generates reasonable results.

In the Merton model and most of its modified versions the default boundary is assumed to be constant. Hence, the estimated risk neutral expected default probabilities cannot capture changes in the relationship of asset value to the firm's default point that caused from changes in firm's leverage. As pointed out by KMV (2003) and Vassalou and Xing (2004), these changes are critical in the determination of actual default probability. Moreover, the simplifying assumption of a constant default boundary is the major reason that the model results in unrealistic estimated short term credit spreads that differ from those observed empirically. It is common that firms adjust their liabilities as they are near default. Empirical studies have showed, that the liabilities of commercial and industrial firms increase as they are near default while the liabilities of financial institutions often decrease. This difference reflects the ability of firms to liquidate their assets and adjust their leverage as they encounter difficulties. In order to capture the uncertainty associated with leverage, we will assume that the default barrier evolves as a geometric Brownian motion with a constant drift equal to the risk free interest rate r and a constant diffusion rate equal to λ:

$$\frac{dDP_t}{DP_t} = r(DP_t, t)dt + \lambda dW_{2t} \tag{14}$$

where W_{2t} is a standard Brownian motion.Moreover , we will assume that the source of randomness that drives the default point W_{2t} is independent from the source of randomness that drives the asset value W_{1t}. Finally ,we will assume that the source of randomness that drives the default point W_{2t} is diversifiable in order to ensure the existence of the unique risk neutral probability measure $\widetilde{P}$, equivalent to the physical measure P , such that the processes of security prices are $\widetilde{P}$-martingales.

[9]Vassallou and Xing (2004), p.10

4.2 Derivation of Distance to Default

Following our assumptions, at time horizon T the market value of firm's assets equals $A_T = (A_0 - \delta)e^{(\sigma_A W_{1t} + (r - \frac{\sigma_A^2}{2})T)}$ and the market value of the default point equals to $DP_T = DP_0 e^{(\lambda W_{2t} + (r - \frac{\lambda^2}{2})T)}$. Therefore , default occurs at the maturity of debt T if :

$$(A_0 - \delta)e^{(\sigma_A W_{1t} + (r - \frac{\sigma_A^2}{2})T)} < DP_0 e^{(\lambda W_{2t} + (r - \frac{\lambda^2}{2})T)} (15)$$

Taking the natural logarithm of the above inequality we have:

$$\ln(\frac{A_0 - \delta}{DP_0}) - \frac{(\sigma_A^2 - \lambda^2)T}{2} + \sigma_A W_{1t} - \lambda W_{2t} < 0 \tag{16}$$

Letting X be the left hand side of the above inequality we can define the probability of default up to time T with the following expression:

$$RNEDP_T = \Pr[X < 0] \tag{17}$$

The expected value and variance of X are :

$$E(X) = \ln(\frac{A_0 - \delta}{DP_0}) - \frac{(\sigma_A^2 - \lambda^2)T}{2} \tag{18}$$

$$Var(X) = (\sigma_A^2 + \lambda^2)T \tag{19}$$

Thus, we can rewrite the probability of default (in addition with the normality assumption) as :

$$RNEDP_T = N(-\frac{\ln(\frac{A_0 - \delta}{DP_0}) - \frac{(\sigma_A^2 - \lambda^2)T}{2}}{\sqrt{(\sigma_A^2 + \lambda^2)T}}) \tag{20}$$

Finally, the risk neutral distance to default metric in our framework equals to:

$$RNDD_T = \frac{\ln(\frac{A_0 - \delta}{DP_0}) - \frac{(\sigma_A^2 - \lambda^2)T}{2}}{\sqrt{(\sigma_A^2 + \lambda^2)T}} \tag{21}$$

The current market value of assets A_0, the asset volatility σ_A and the default boundary volatility λ remain unobservable. In the next section, we will describe our calibration method to truncate these unobservable parameters and estimate the risk neutral distance to default metric and the risk neutral expected default probability of a firm.

4.3 Calibrating Model Parametres

As we said in a previous section of this paper, Merton (1974) has involved in his model Black-Scholes settings and used the Black-Scholes Call Option pricing formula and a second equation that relates asset volatility with equity volatility to de-lever the unobservable values of current market value of assets A, asset volatility σ_A. Since we have assumed a random default barrier, we cannot use the Black-Scholes Call Option pricing formula. Our approach, is based on the market value identity:

$$A_0 = S_0 + D_0^{ST} + L_0^{ST} + D_0^{LT} + L_0^{LT} \tag{22}$$

- S_0: is the the market value of Common Equity.
- D_0^{ST} is the market value Short term Debt.
- L_0^{ST} is the market value of other Short term Liabilities.
- D_0^{LT} is the market value of total Long term Debt.
- L_0^{LT} is the market value of other Long term Liabilities.

Since, the market value of equity is observable from the stock market, we need to estimate the market value all other firm's liabilities to back out the market value of assets.

The class of other short term liabilities in our model, allows liabilities with maturity up to one year, which means that their market values are around their book values. If default occurs, short term liabilities are directly required in their face value ($L_0^{ST} = STL$). As a result, we set their market value equal to their face value . However, other long term liabilities are mainly perpetual liabilities with small present values. Hence, we will assume that the market value of other long term liabilities equals half their face value, since they do not affect much short term default risk ($L_0^{LT} = \frac{LTL}{2}$).

Following our decomposition of the market value of risky debt, the market value of short term debt D_0^{ST} equals to:

$$D_0^{ST} = D^1 e^{-r}(1 - \widetilde{E}[1 - R_1/F_0]RNEDP_1) \tag{23}$$

Similarly, the market value of total long term debt (with discrete per year maturities) is given by:

$$D_0^{LT} = \sum_{T=2}^{5} D^T e^{-rT}(1 - \widetilde{E}[1 - R_T/F_0]RNEDP_T) \tag{24}$$

As we see the market value of debt depends on the expected recovery rate on short term debt and long term debt. As a result we need to modify our model, in order to allow a fractional recovery in the case the firm defaults. Many empirical studies such as, Altman (1992), Franks and Torous (1994), Altman, Resti and Sironi (2001), S&P (2001), Credit Grades (2002) estimate the expected recovery rates around 50-60%for long term debt and around 80-90% for short term debt. Moreover, the estimates of the standard deviation of the expected recovery rates are around 22%

for long term debt and 28% for short term debt as documented in the above studies. Following these studies we will set in our model the expected recovery rate on short term debt equal to 80% and the expected recovery rate on long term debt equal to 50%.Finally, we will use the standard deviation of the expected recovery rate on long term debt as a proxy of the default barrier volatility since the expected recovered amounts of short term debt in the case of default are too high.

Thus, that the current market value of assets in our approach equals to :

$$A_0 = S_0 + STL + \frac{LTL}{2} + \sum_{T=1}^{5} D^T e^{-rT}(1 - \widetilde{E}[1 - R_T/F_0]RNEDP_T) \tag{25}$$

The equity-implied asset volatility in our structural model can be obtained from Ito's lemma and is equal to :

$$\sigma_S = \frac{1}{S_0}\sqrt{(\frac{\vartheta S_0}{\vartheta A_0})^2 \sigma_A^2 A_0^2 + (\frac{\vartheta S_0}{\vartheta DP_0})^2 \lambda^2 DP_0^2} \tag{26}$$

Recall that the risk neutral expected default probability now at time $t = 0$ that a firm will default at time T equals :

$$RNDD_T = \frac{\ln(\frac{A_0 - \delta}{DP_0}) - \frac{(\sigma_A^2 - \lambda^2)T}{2}}{\sqrt{(\sigma_A^2 + \lambda^2)T}} \tag{27}$$

With equations (25), (26) and (27), we have a set of 3 non-linear equations with 3 unknowns which can be solved with Newton-Raphson iterative method. Once we back out the unknowns A_0,σ_A, we can estimate the risk neutral distance to default metric which will be the basic explanatory variable in our hybrid model of credit risk measurement.

5 Hybrid Model

Our modified version of Merton Model, is based like all structural default risk models on the idea that corporate liabilities (debt and equity) can be valued as contingent claims on the firm's assets. The model is forward looking since it uses current market information regarding the future prospects of the underlying firm. Moreover, it relates different credit risk factors in an analytical way and allows non linear effects and interaction among them. Its basic output the risk neutral distance to default equals:

$$RNDD_T = \frac{\ln(\frac{A_0 - \delta}{DP_0}) - \frac{(\sigma_A^2 - \lambda^2)T}{2}}{\sqrt{(\sigma_A^2 + \lambda^2)T}} \tag{28}$$

It is straightforward, that risk neutral distance to default measure and the risk neutral expected default probability depends on:

- The current market value of firm's assets A_0.
- The asset volatility σ_A, which is a measure of business risk.

- The initial level of the default boundary DP_0.
- The default boundary volatility λ, which captures the uncertainty about changes on firm's leverage.
- The continuously compounded risk free rate r.
- The stream of expected cash dividends δ.
- The length of time horizon T.

Unlike, traditional models our risk neutral distance to default metric does not take into account credit risk factors such as liquidity, profitability, efficiency and viability. Moreover, our Merton type model relies like all structural models, on theories about market efficiency. Therefore, equity prices should reflect all relevant and available information about the firm's fundamentals. However, Hillegeist, Keating, Cram and Lundstedt (2004) have documented that the theoretical probabilities estimated from structural models do not capture all available information about the credit risk of a firm. Thus, questionable is if accounting variable and financial ratios can provide significant and incremental information in assessing the credit quality of a firm.

The main purpose of this paper is to investigate whether the combination of fundamental analysis and contingent claim analysis into a hybrid model can be a better way in credit risk measurement rather than traditional econometric models and pure structural models. Thus, we have estimated a hybrid model of credit risk measurement which we refer as *Hybrid Model (HM)* with an ordered probit regression to explain credit ratings and rating transitions using our risk neutral distance to default metric, financial ratios and other accounting based measures as explanatory variables. Using the same econometric methodology, we have also estimated a model with financial ratios and accounting based measures as explanatory variables and a model with our risk neutral distance to default metric as unique explanatory variables. We refer these models as the *Accounting Model (AM)* and the *Distance to Default (DD)* model respectively. The dependent variable in the ordered probit regression is the firm's credit rating and default is regarded as a special case of credit rating. Note, that by using the risk neutral distance to default in an ordered probit regression we map it into a "real world" objective measure of financial distress using only publicly available information.

6 Data & Variable Estimation

Our estimation sample consists of 270 rated industrial and commercial public firms from North America and Canada. The corporate credit ratings are assigned on July 2002. We use corporate credit ratings instead of debt credit ratings since the former reflect the financial health and business risk of a firm rather than its debt-specific features. Credit risk measurement ensures that the default probability of a firm determines the default probability of all firm's obligations. Moreover, we classify into the default class 40 industrial and commercial public firms from North America and Canada that have defaulted during 2002. Data on defaults are obtained from the S&P annual report on Ratings Performance of 2002. Default in our model is regarded as a special case of credit rating. The number of observations in different rating classes is shown in table (1). Notable is that there are few firms in the AAA and AA class and that there are not firms in the CCC, CC, C rating classes. It is apparent, that when calibrating the model, we will have difficulties to estimate the AAA/AA cutoff point. Thus, we have classified all AAA and AA observations into a rating class, which is referred as AAA-AA class. Summarizing our estimation sample contains 270 non-defaulted firms that are assigned in 5 rating classes (AAA-AA, A, BBB, BB, B) and 40 firms that are assigned in the default class (D). Table (1) in section (1) provides a detailed description of our estimation sample.

The financial ratios and accounting variables used as explanatory variables in the *Hybrid Model (HM)* and the *Accounting Model (AM)* are those that one might reasonably expect would influence credit standing. To deduce which variables to include, we draw in previous empirical studies. These studies have examined the determinants of credit ratings (see Blume, Lim and McKinlay (1998), Pottier and Sommer (1999)) and the determinants of corporate failures (see Altman (1968,1975), Ohlson (1980)). In the appendix we provide a detailed list of the financial ratios and accounting variables, we have used as explanatory variables in our models. Their measurement is based on annual financial statements year ending December 2001 since we have corporate credit ratings and defaults of 2002 year. Balance Sheets, Profit & Loss Accounts and Cash Flow Statements are collected from the Computstat database. With the above statements, we have also measured the accounting variables that are involved in our risk neutral distance to default metric. The market data for capitalization and equity volatility that we need to estimate the market value of assets and the asset volatility are obtained from Datastream database. Equity volatility is calculated from the standard deviation of the continuously compounded returns (not annualized) of the last year's trading days.[10] Following Vassalou and Xing (2004), constant treasury bill rates are used as proxies for the risk free interest rates, and they are obtained from Datastream database. Recall, that in order to estimate the market value of assets and the asset volatility and derive the risk neutral distance to default metric, we employ a Newton-Raphson iteration method. Convergence has achieved very quickly. Around 90% of the total estimates take less than 5 iterations to converge.[11]

In addition with the above estimation sample, we have used another sample to test the power of our calibrated models to predict default events. This sample consists of 100 industrial and commercial public firms, where 28 firms have defaulted during 2003 and 78 remained solvent. Data on defaults are obtained from the S&P annual report on Ratings Performance of 2003. In order to implement the models and test their predictive power we have measured again the risk neutral distance to default metric, the financial ratios and the accounting variables following the above stated rules and using the same databases.

Our methodology to estimate the *Hybrid Model(HM)* and the *Accounting Model(AM)* is based on Altman's (1968) idea that considered various combinations of 22 explanatory variables before choosing the five with the highest predictive power. Specifically, we considered for each model using an iterative process all possible combination of our independent variables when taken five

[10]For details see Hull, Options, Futures & other Derivatives,(Prentice-Hall International,2002) p.242-244.

[11]The accuracy level we set for convergence was 0.00001.

at a time. However we do not report the results of our intermediate estimation because of space constraints. Then we reduced the number of combinations to those with variables that have statistical significance at the $p < 0.05$ level, and no correlation among them. This means, that the correlation of all the independent variables in those combinations was less than 0.7 and more than -0.7. Then, for each model we selected the combination that had the highest explanatory power (pseudo-R^2) and the lowest information criteria (Akaike criterion, Schwarz criterion, Hamman-Quinn criterion). Finally, we must note that we could not significantly improve upon our results by adding more variables and no model with fewer variables performed as well.

7 Analysis & Results

In this section we will analyze the characteristics and the performance of the three models: the *Hybrid Model (HM)*, the *Accounting Model (AM)* and the *Distance to Default Model (DD)*. Both in of sample fitting and out of sample predictability are investigated.

7.1 In Sample Fitting Comparison

Our analysis starts with the estimation results of the *Accounting Model (AM)*, which are reported in Table (2). The five explanatory variables are free cash flow margin, interest coverage ratio, internal growth rate, basic earning power ratio and asset size. Hence, the model focuses on liquidity, profitability and size in assessing the credit quality of a firm. Internal growth rate and basic earning power ratio were significant in Altman's Z-Score (1968). Moreover, all the explanatory variables except free cash flow margin were also significant in Altman's ZETA Score (1977). In addition, the signs of the coefficients of all explanatory variables are positive and consistent with prediction theories. For instance, the higher internal growth rate means the higher probability of getting a higher rating and the lower the probability to default. Finally, the coefficients of all the variables and cut-off points in the *Accounting Model (AM)* are significant at the $p < 0.01$ level or lower.

Table (3) presents the results of estimating a model using our risk neutral distance to default metric as a unique explanatory variable in an ordered probit regression. The variable is significant at the $p < 0.01$ level and has a positive sign which is consistent with the Merton framework.Risk neutral distance to default metric shows the number of standard deviations that the firm's asset value is away from the default boundary. Therefore, the larger the distance between the asset value and the default boundary, the safer the firm will be. At that point, we must note that we have also run an ordered probit regression using the risk neutral expected probability as unique explanatory variable. The results of this regression are reported in table (4). Comparing, the two models we can see that the risk neutral distance to default has relatively more explanatory power than the risk neutral expected default probability. We believe that the poor explanatory power of the risk neutral expected default probabilities is quite reasonable since they are implied from the normal distribution. Defaults are rare events and occur when the value of a firm substantially drops. Empirical evidence indicates that typical credit returns are likely to follow "fat-tailed" distributions and therefore the fatness of tails become central to default prediction.[12] However, KMV's proposal to use of empirical distributions must be viewed with some skepticism. One cannot, back out the unknown values of asset and asset volatility by assuming normality to calculate the distance to default metric, and then turn to argue that returns are not really normal and estimate the default probabilities using empirical distributions. In addition, empirical distributions require large databases that are not often publicly available. Recall, that by using the risk neutral distance to default metric in an ordered probit regression we map it into a "real world" objective measure of financial distress using only publicly available information. Finally, the superior characteristics of the risk neutral distance to default metric supports our choice to use it as explanatory variable in the *Hybrid Model (HM)*.

Recall that in order to estimate the *Hybrid Model (HM)* we have selected from all possible combinations of the risk neutral distance to default metric with 4 financial ratios and accounting variables, the combination that had the highest explanatory power (pseudo-R^2) and the lowest information criteria (Akaike criterion, Schwarz criterion, Hamman-Quinn criterion). However, we have also considered all possible combination of the risk neutral expected default probability with 4 financial ratios and accounting variables. We find that the *Hybrid Model (HM)* outperforms all these combinations. This suggestion points again the superior performance of the risk neutral distance to default metric and supports our choice to use it as explanatory variable in the *Hybrid Model (HM)*. In table (5) we present the estimation results of the *Hybrid Model (HM)*. The explanatory variables that are significant in addition with the risk neutral distance to default metric are: free cash flow margin, return on assets, asset size and debt ratio. Thus, the model focuses on liquidity, earnings performance, size and leverage. All the explanatory variables and threshold parameters are significant at the $p < 0.01$ level or lower instead of free cash flow margin, which is significant at the $p < 0.05$ level or lower. Debt ratio, which is a degree of leverage, has a negative coefficient. Hence, the higher the ratio of total debt to total assets means the lower probability of getting a higher rating and the higher the probability to default. However, the other explanatory variables have positive coefficients, which is consistent with prediction theories. Return on assets and asset size were also significant in Olhson O-Score (1980). Furthermore, all the explanatory variables except free cash flow margin were also significant in Altman's ZETA Score (1977). Finally, free cash flow margin and asset size are also significant within the *Accounting Model (AM)*.

Table (6) provides a summary of the fitting measures of the three estimated models. The poor fitting results of the *Distance to Default Model (DD)*, indicate that it is not itself a sufficient statistic in assessing the credit quality of a firm. The same holds for the risk neutral expected default probabilities since they have less explanatory power than the risk neutral distance to default metric. This is consistent with the conclusion of Hillegeist, Keating, Cram and Lundstedt (2004) that the estimated default probabilities from structural credit risk models are not a sufficient statistic of the actual default probability. However, according to contingent claim analysis these market based measures should capture all relevant and available information about the future

[12] KMV has demonstrated using historical instances of default that the actual default probability distribution has fatter tail than the normal distribution applies. See Jarrow, Lando and Turnbull (1997) and Crosbie and Bohn (2003) for additional information

credit prospects of a firm. Therefore, we should see with skepticism several assumptions underlying Merton type models. For example, questionable is if corporate debt markets are liquid enough to prevent the crucial assumption of the no arbitrage condition from not holding.[13] Moreover, these models should be used with caution when assessing the credit quality of a firm. Other important factors of fundamental analysis such as liquidity, profitability, efficiency, solidity, size should be also considered. It is obvious from table (6) that the *Hybrid Model (HM)*, which combines contingent claim analysis and fundamental analysis, outperforms the *Accounting Model (AM)* and the *Distance to Default Model (DD)*. It has the highest explanatory power (pseudo R^2) and the lowest information criteria (Akaike criterion, Schwarz criterion, Hamman-Quinn criterion). Hence, our main conclusion is that accounting variables and financial ratios can be incrementally informative to market based measures form structural models when assessing the credit quality of a firm.

It is obvious from the listed fitting measures in table (6) that the *Accounting Model (AM)* has relatively more power than the *Distance to Default Model (DD)* in explaining credit ratings. This is not surprising, although accounting data are by definition backward looking and financial statements are designed under the conservatism and the going concern principle to summarize the state of a firm at a given point in time. Traditional models, while look at historical financial information, adopt a forward-looking approach by focusing on information, which has predictive power to reveal tendencies to the future prospects of a firm. Moreover, in the assignment of credit ratings, rating agencies use publicly available financial information factors such as earnings performance, asset quality, cash flow adequacy in addition with private information Finally, of considerable importance the distance to default metric by definition focuses on the event of default which in our analysis is regarded as a special case of credit rating. In next paragraphs, we will see that although the *Accounting Model (AM)* has relatively more power than the *Distance to Default Model (DD)* to explain credit ratings, it has less relatively power to predict default events.

Note that we have also run the three models by alternatively eliminating one defaulter at a time. The aim of this exercise is to check if a possible outlier drives the above fitting measures. We find that the results did not change substantially and therefore, we discard this possibility.

7.2 Predictability of Credit Ratings

At that point, we will compare the performance of the three models to predict correct credit ratings. Tables (7) and (8) show for each model the percentage of correct rating predictability for each rating class. Rating assignment is computed in two ways. First, we apply the estimated equation from the ordered probit regression and the model generated rating is assigned according to the endogenously estimated threshold parameter. The second way is to calculate the expected probability of a firm falling in each class and the model generated rating is assigned as the one with the highest probability. From tables (7) and (8) we can see that the *Accounting Model (AM)* has more power to predict credit ratings than the *Distance to Default Model (DD)*. Specifically, it classifies more accurately the firms in the middle rating classes than the firms in the highest rating class (AAA-AA). However, the *Distance to Default Model (DD)* predicts more accurately

[13] For further discussion see Sobehart and Keenan (2002)

the firms that are expected to default. That is not surprising, since the risk neutral distance to default metric measures the number of standard deviations that the firm's asset value is away from the default boundary. Moreover, it is straightforward from table (7) and (8) that the *Hybrid Model (HM)* outperforms the other two models in predicting credit ratings and defaults. This fact again suggests that financial ratios and accounting variables can incrementally informative to market based measures form structural models when assessing the credit quality of a firm.

7.3 Out of Sample Predictability of Default

It is common that for a credit risk model, not only in sample fitting is necessary but out of sample forecasting ability is essential as well. As we said in section 6, we have used another sample to investigate the ability of the three estimated models to rank defaulters and non-defaulters one year later. We have have tested the default prediction power of each model using the two methods described in the previous paragraph. Table (9) and Table (10) provide the results for each method respectively. It is straightforward that the *Hybrid Model (HM)* systematically outperforms the other two models since it has the highest default prediction power. Finally, we see again that the *Distance to Default Model (DD)* is more successful than the *Accounting Model (AM)* in discriminating defaulted from non-defaulted firms.

8 Conclusion

Credit risk measurement is an area of great and renewed interest for both academicians and practitioners. In this paper we have examined the theoretical foundations of fundamental and contingent claim analysis and combine them into a hybrid model of credit risk measurement. We have extended the standard Merton approach to estimate a new risk neutral distance to default metric, assuming a more complex capital structure, adjusting for dividend payments, introducing randomness to the default point and allowing a fractional recovery when default occurs. In these structural models, is inherent the assumption that equity prices should reflect all relevant and available information about the firm's fundamentals since capital markets are efficient. However, according to our results, structural credit risk models estimate market based measures that are not sufficient statistics since they do not capture all available information in assessing the credit quality of a firm. This is consistent with the conclusion of Hillegeist, Keating, Cram and Lundstedt (2004). Therefore, several credit risk factors of fundamental analysis should be also considered. We have found that by enriching the risk neutral distance to default metric from our extended Merton type model with financial ratios and accounting variables into the hybrid model, we can improve both in sample fitting of credit ratings and out of sample predictability of defaults. Hence, our main conclusion is that accounting variables and financial ratios can be incrementally informative to market based measures form structural models when assessing the credit quality of a firm.

9 Appendix

9.1 List of Independent Variables

In this section we provide the description and assign the notation of the independent variables we have used in the *Hybrid Model (HM)*, the *Accounting Model (AM)* and the *Distance to Default Model (DD)*.

Financial Ratios & Accounting Variables

Liquidity Variables

Working Capital Ratio reflects the ability of a firm to finance its assets and is an indicator of the margin of the protection for its creditors.
Formula: $Working\ Capital\ Ratio = \frac{Current\ Assets - Current\ Liabilities}{Total\ Assets}$

Current Ratio expresses the degree to which a company's current assets can cover current liabilities if they become simultaneously payable.
Formula: $Current\ Ratio = \frac{Current\ Assets}{Current\ Liabilities}$

Quick Ratio is a refinement of current ratio and a more conservative measure of liquidity since it indicates the ability of a firm to meet its current liabilities without the need to turn inventories in cash.
Formula: $Quick\ Ratio = \frac{Cash + Cash\ Equivalents + Accounts\ Receivables}{Current\ Liabilities}$

Cash Ratio measures the ability of the firm's most liquid assets such as cash, marketable securities and other short term investments to cover its current liabilities.
Formula:$Quick\ Ratio = \frac{Cash + Cash\ Equivalents}{Current\ Liabilities}$

Free Cash Flow Margin is a measure of the ability of a firm to generate cash available for distribution to its equityholders.
Formula: $Free\ Cash\ Flow\ Margin = \frac{Free\ Cash\ Flow}{Total\ Sales}$

Solvency Variables

Interest Coverage Ratio measures the ability of a firm to service its interest payments.
Formula: $Interst\ Coverage\ Ratio = \frac{EBIT}{Interest\ Expense}$

Current Liabilities Coverage Ratio is a measure of the ability of a firm to to service its current liabilities.
Formula: $Current\ Liabilities\ Coverage\ Ratio = \frac{EBIT}{Interest\ Expense}$

Profitability Variables

Return on Assets indicates the rate of return that a firm is generating on its assets. It shows how effectively a firm utilizes its assets to create profits.
Formula: $Return\ on\ Assets = \frac{Net\ Income}{Total\ Assets}$

Return on Equity determines the rate of return that a firm is generating on its owner's investments.
Formula: $Return\ on\ Equity = \frac{Net\ Income}{Total\ Equity}$

Internal Growth Rate is a measure of firm's cumulative profitability over time. The age of a firm is implicitly considered since a young firm that has not time to build up its cumulative profits will probably have a low internal growth rate.
Formula: $Internal\ Growth\ Rate\ = \frac{Retained\ Earnings}{Total\ Assets}$

Basic Earning Power shows the firm's true productivity without the influence of leverage and tax factors. This ratio is useful for comparing firms with different tax situations and different degrees of leverage.
Formula: $Basic\ Earning\ Power = \frac{EBIT}{Total\ Assets}$

EBIT Margin indicates how much profit is earned on firm's projects without consideration of leverage and tax factors.
Formula: $EBIT\ Margin = \frac{EBIT}{Total\ Sales}$

Leverage Variables

Leverage Ratioindicates the proportion of the firm's assets that are financed by its creditors.
Formula: $Total\ Leverage\ Ratio = \frac{Total\ Liabilities}{Total\ Assets}$

Debt Ratio shows the proportion of the firm's assets that are financed through debt.
Formula: $Debt\ Ratio = \frac{Total\ Debt}{Total\ Assets}$

Debt to Equity Ratio expresses the relationship between the capital invested by the firm's owners and funds provides by firm's creditors.
Formula: $Debt\ to\ Equity\ Ratio = \frac{Total\ Debt}{Total\ Equity}$

Size Variables

Asset Size is a measure of firm's assets diversification.
Formula: $Asset\ Size = \ln(Total\ Assets)$

Sales Size is a measure of firm's market and competitive position in terms of sales revenue.
Formula: $Sales\ Size = \ln(Total\ Sales)$

Efficiency Variables

Asset Turnover Ratio measures the ability of a firm to generate sales through its assets.
Formula: $Asset\ Turnover\ Ratio = \frac{Total\ Sales}{Total\ Assets}$

Equity Turnover Ratio is an indicator of the productive utilization of the firm-owner's investments.
Formula: $Equity\ Turnover\ Ratio = \frac{Total\ Sales}{Total\ Equity}$

Working Capital Turnover Ratio shows how well the working capital of the firm is employed to generate sales.
Formula: $Working\ Capital\ Turnover\ Ratio = \frac{Total\ Sales}{Working\ Capital}$

Inventory Turnover Ratio measures the ability of a firm to generate sales through its inventories.
Formula: $Inventory\ Turnover\ Ratio = \frac{Total\ Sales}{Total\ Inventories}$

Variable from our Structural Model

Risk Neutral Distance to Default measures the number of standard deviations that the firm's market asset value is away from its default point.
Formula: $RNDD_T = \frac{\ln(\frac{A_0-\delta}{DP_0}) - \frac{(\sigma_A^2-\lambda^2)T}{2}}{\sqrt{(\sigma_A^2+\lambda^2)T}}$

9.2 Tables

Table 1: Distribution of Ratings over the Sample

This table presents information about the distribution of ratings over our estimation sample.

Estimation Sample		
Firms	**Actual S&P Rating**	**Rating Used in Paper**
9	AAA	AAA-AA
21	AA	AAA-AA
60	A	A
60	BBB	BBB
60	BB	BB
60	B	B
40	DEFAULT	DEFAULT

Table 2: Estimation Results of the ***Accounting Model (AM)***

This table presents the estimation results of the *Accounting Model (AM)*.

Accounting Model (AM)				
Independent Variables	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
Free Cash Flow Margin	1.300220	0.447130	2.907922	0.0036
Basic Earning Power	4.9114870	1.026821	4.783198	0
Internal Growth Rate	2.032550	0.286162	7.102782	0
Interest Coverage Ratio	0.023065	0.007119	3.239942	0.0012
Asset Size	0.643383	0.058240	11.04703	0
Threshold Parameters	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
γ_1	3.546243	0.464635	7.632317	0
γ_2	5.136140	0.499345	10.28575	0
γ_3	6.396836	0.548600	11.66030	0
γ_4	7.469874	0.577269	12.94002	0
γ_5	8.785159	0.600113	14.63917	0
Fitting Measures	**Coefficient**	**Fitting Measures**	**Coefficient**	
Akaike info criterion	2.225660	**Schwarz criterion**	2.346195	
Log likelihood	-334.9773	**Hannan-Quinn criterion**	2.273845	
Restr. log likelihood	-546.1036	**Avg. log likelihood**	-1.080572	
LR statistic (5 df)	422.2526	**LR index (Pseudo-R^2)**	0.386605	
Probability(LR stat)	0	**Observations**	310	

Table 3: Estimation Results of the ***Distance to Default Model (DD)***

This table presents the estimation results of the *Distance to Default Model (DD)*.

Distance to Default (DD)				
Independent Variables	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
$RNDD_T$	0.913102	0.055519	16.44670	0
Threshold Parameters	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
γ_1	0.590441	0.153359	3.900921	0.0001
γ_2	1.830995	0.161249	11.35510	0
γ_3	2.824692	0.189145	14.93399	0
γ_4	3.930010	0.231528	16.97422	0
γ_5	5.691401	0.337905	16.84321	0
Fitting Measures	**Coefficient**	**Fitting Measures**	**Coefficient**	
Akaike info criterion	2.319169	**Schwarz criterion**	2.391490	
Log likelihood	-353.4712	**Hannan-Quinn criterion**	2.348080	
Restr. log likelihood	-546.1036	**Avg. log likelihood**	-1.140230	
LR statistic (1 df)	385.2647	**LR index (Pseudo-R^2)**	0.352740	
Probability(LR stat)	0	**Observations**	310	

Table 4: Estimation Results of the $RNEDP_T Model$

This table presents the estimation results of the $RNEDP_T$ *Model*.

$RNEDP_T$ Model				
Independent Variables	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
$RNEDP_T$	-15.30010	1.446751	-10.57549	0
Threshold Parameters	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
γ_1	-2.559843	0.198182	-12.91663	0
γ_2	-1.103059	0.098385	-11.21169	0
γ_3	-0.379494	0.084026	-4.516412	0
γ_4	0.264209	0.083649	3.158553	0.0016
γ_5	1.112572	0.104836	10.61248	0
Fitting Measures	**Coefficient**	**Fitting Measures**	**Coefficient**	
Akaike info criterion	2.827877	**Schwarz criterion**	2.900197	
Log likelihood	-432.3209	**Hannan-Quinn criterion**	2.856788	
Restr. log likelihood	-546.1036	**Avg. log likelihood**	-1.394584	
LR statistic (1 df)	227.5655	**LR index (Pseudo-R^2)**	0.208354	
Probability(LR stat)	0	**Observations**	310	

Table 5: Estimation Results of the *Hybrid Model (HM)*

This table presents the estimation results of the *Hybrid Model (HM)*.

Hybrid Model (HM)				
Independent Variables	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
Free Cash Flow Margin	1.256247	0.499905	2.512972	0.012
Return on Assets	2.915002	0.836026	3.486737	0.0005
Asset Size	0.584935	0.061083	9.576081	0
Debt Ratio	-1.670044	0.410535	-4.067973	0
$RNDD_T$	0.736139	0.064937	11.33622	0
Threshold Parameters	**Coefficient**	**Std. Error**	**z-Statistic**	**Prob.**
γ_1	3.395181	0.537253	6.319514	0
γ_2	5.365776	0.569005	9.430097	0
γ_3	6.912424	0.625236	11.05570	0
γ_4	8.323196	0.671630	12.39254	0
γ_5	10.06847	0.714847	14.08479	0
Fitting Measures	**Coefficient**	**Fitting Measures**	**Coefficient**	
Akaike info criterion	1.839908	**Schwarz criterion**	1.960443	
Log likelihood	-275.1858	**Hannan-Quinn criterion**	1.888093	
Restr. log likelihood	-546.1036	**Avg. log likelihood**	-0.887696	
LR statistic (5 df)	541.8356	**LR index (Pseudo-R^2)**	0.496092	
Probability(LR stat)	0	**Observations**	310	

Table 6: Fitting Measures

This table presents a summary of the fitting measures of the *Hybrid Model (HM)*, the *Accounting Model (AM)* and the *Distance to Default Model (DD)*.

Fitting Measures				
Models	Pseudo-R^2	Akaike cr.	Schwarz cr.	Hannan-Quinn cr.
Hybrid Model	0.496092	1.839908	1.960443	1.888093
Accounting Model	0.386605	2.225660	2.346195	2.273845
Distance to Default Model	0.352740	2.319169	2.391490	2.348080

Table 7: Predictability of Correct Rating Assignment

This table presents the predictability of correct rating assignment for the *Hybrid Model (HM)*, the *Accounting Model (AM)* and the *Distance to Default Model (DD)* in actual and in percentage terms. We apply the estimated equation from the ordered probit regression and the model generated rating is assigned according to the endogenously estimated threshold parameter.

Predictability of Correct Rating Assignment							
Models	AAA-AA	A	BBB	BB	B	Default	Total
Hybrid Model	15	37	43	46	45	30	216
Hybrid Model (%)	50%	62%	72%	77%	75%	75%	70%
Accounting Model	12	28	43	41	44	24	192
Accounting Model (%)	40%	47%	72%	68%	73%	60%	62%
Distance to Default Model	14	22	24	29	40	23	152
Distance to Default Model (%)	47%	37%	40%	48%	67%	58%	49%

Table 8: Predictability of Correct Rating Assignment

This table presents the predictability of correct rating assignment for the *Hybrid Model (HM)*, the *Accounting Model (AM)* and the *Distance to Default Model (DD)* in actual and in percentage terms. We calculate the expected probability of a firm failing in each class and the model generated rating is assigned as the one with the highest probability.

Predictability of Correct Rating Assignment							
Models	AAA-AA	A	BBB	BB	B	Default	Total
Hybrid Model	16	37	41	47	47	30	218
Hybrid Model (%)	53%	62%	68%	78%	78%	75%	70%
Accounting Model	13	31	39	41	44	25	193
Accounting Model (%)	43%	52%	65%	68%	73%	63%	62%
Distance to Default Model	14	27	24	20	43	30	158
Distance to Default Model (%)	47%	45%	40%	33%	72%	75%	51%

Table 9: Out of Sample Default Prediction Performance

This table presents the out-of sample default prediction performance for the *Hybrid Model (HM)*, the *Accounting Model (AM)* and the *Distance to Default Model (DD)* in actual and in percentage terms. We apply the estimated equation from the ordered probit regression and the threshold parameters of default class to generate forecasts of corporate distress.

Hybrid Model (HM)			
Model Prediction	**Actual Default**	**Actual-Non Default**	**Total**
Default	22	5	27
Non-Default	6	67	73
Correct%	79%	93%	89%
Incorrect%	21%	7%	11%
Accounting Model (AM)			
Model Prediction	**Actual Default**	**Actual-Non Default**	**Total**
Default	16	3	19
Non-Default	12	69	81
Correct%	57%	96%	85%
Incorrect%	43%	4%	15%
Distance to Default Model (DD)			
Model Prediction	**Actual Default**	**Actual-Non Default**	**Total**
Default	13	1	14
Non-Default	15	71	86
Correct%	46%	99%	84%
Incorrect%	54%	1%	16%

Table 10: Out of Sample Default Prediction Performance

This table presents the out-of sample default prediction performance for the *Hybrid Model (HM)*, the *Accounting Model (AM)* and the *Distance to Default Model (DD)* in actual and in percentage terms. We apply the expected default probabilities to generate forecasts of corporate distress.

Hybrid Model (HM)			
Model Prediction	**Actual Default**	**Actual-Non Default**	**Total**
Default	23	5	28
Non-Default	5	67	72
Correct%	82%	93%	90%
Incorrect%	18%	7%	10%
Accounting Model (AM)			
Model Prediction	**Actual Default**	**Actual-Non Default**	**Total**
Default	16	3	19
Non-Default	12	69	81
Correct%	57%	96%	85%
Incorrect%	43%	4%	15%
Distance to Default Model (DD)			
Model Prediction	**Actual Default**	**Actual-Non Default**	**Total**
Default	19	3	22
Non-Default	9	69	88
Correct%	68%	96%	88%
Incorrect%	32%	4%	12%

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