Published in *Applied Financial Economics*, 1994, 4, 217–224.
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Are There Psychological Barriers in the Dow-Jones Index?

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Abstract. The popular press attaches particular significance to certain numerical values of the Dow-Jones index. These magic numbers are referred to as 'resistance levels' or 'psychological barriers.' In this note we examine 41 years of closing values of the Dow-Jones index to see if it is useful for predicting future stock market returns.

Keywords. Dow-Jones index, psychological barriers, resistance levels, market efficiency

JEL Classification System. G00, G1, C1

On April 17, 1991, the Dow-Jones Industrial Average (DJIA) closed above 3,000 for the first time. This occurrence was widely reported in the financial press; see Norris (1991) or Sease and Dorfman (1991) for examples. Prior to this event, some observers claimed that 3,000 was a 'resistance level' or a 'psychological barrier.' (See, *e.g.*, Torres (1991).)

The financial press contained similar reports when the DJIA crossed 1,000 and 2,000, although the behaviour was somewhat different in each case. The DJIA first touched 1,000 in early 1966; it didn't cross that level again until nearly 17 years later, in late 1982. In 1987 the DJIA passed through 2,000 and continued to 3,000 in 4 years. It appears that 1,000 was a 'resistance level' while 2,000 was not.

Figure 1 shows 2,163 weekly closing prices of the DJIA from 1/52 to 6/93.[1] One could interpret this figure as showing that 500 and 1,000 were 'resistance levels;' however, one could also argue that the behavior exhibited in this figure could have been due to chance alone. In this paper we try to determine which of these two claims are more consistent with the evidence: does the numerical value of the DJIA help predict future returns, or has the historical behavior been due to chance alone?

This is purely an empirical investigation; we want to find out what the evidence says, and we have no particular theory in mind about why 'resistance levels' may or may not occur. All we can offer in this regard is a paragraph from the *Wall Street Journal:*

> "How can a 'resistance level' exist? Because traders believe it is there. Resistance levels in market benchmarks can occur when there's a consensus that the market can't go much higher. Stock index or average levels become sentiment signals. As market barometers approach those levels, stock buyers become less aggressive, fearing a turn in the market,

We benefitted from the comments of an anonymous referee. Eduardo Ley acknowledges financial support from the Spanish DGICYT. Hal Varian acknowledges support from the National Science Foundation.

[1] The data from 1/1/52 to 12/31/85 were obtained from Pierce (1986). The data from 1/1/86 to 14/6/93 were obtained from various issues of the "Daily Stock Price Record, New York Stock Exchange," published by the Standard & Poor's Corporation and from the Dow-Jones online service. (The data is available via anonymous ftp from alfred.econ.lsa.umich.edu.) We have the data from 1885 but in the first 10,000 observations 99.35% of the closing values were less than 100. During the period 1885–1952, 64% of the closing prices were less than 100. We don't think it makes much sense to look at the behaviour of the DJIA close to 100 during that period. Furthermore, even if there were patterns of some sort in the data prior to the 1950s, that might be interesting as matter of economic history, but not very relevant for today's environment.

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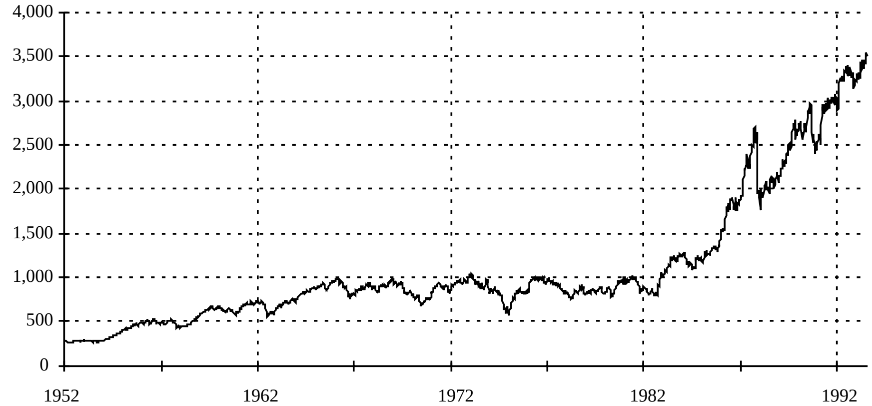

Fig. 1. DJIA (weekly close): January 1952–June 1993.

while sellers need less coaching to drop their prices a notch or two." [Torres (1991).]

This passage appears to claim that traders may believe that the level of the DJIA helps to forecast future returns in stock prices. If this view is correct, it would be a clear violation of market efficiency.

Donaldson and Kim (1991) have independently investigated some related issues issues involving numerical patterns in the closing prices of the DJIA. However, they do not focus on questions of market efficiency, which is the main concern of this paper. We discuss the relationship between their results and our results in section 4.

1. Distribution of the DJIA's digits

We being by examining the distribution of the DJIA index by the last two digits to the left of the decimal point. Table 1 summarizes the frequency with which the DJIA closed at a value whose last two digits were ij. For example, the entry in row 4, column 3 is $p_{43} = 1.03$; this means that the DJIA closed at a value ending in "43" 1.03 percent of the time.

Table 1. Percent of times that the DJIA Index ends in ij (p_{ij}).
January 1952–June 1993: $N = 10,449$.

$i \downarrow j \rightarrow$	0	1	2	3	4	5	6	7	8	9	$p_{i\cdot}$
0	0.82	0.93	0.96	1.15	1.00	0.99	1.08	1.09	0.91	1.00	9.93
1	0.92	1.04	0.97	1.04	0.92	0.88	1.15	0.87	1.00	0.84	9.63
2	0.98	0.88	1.00	0.93	1.09	0.92	1.06	0.95	0.91	0.88	9.60
3	1.05	0.99	1.03	0.85	0.88	0.96	0.95	1.10	0.98	1.04	9.83
4	1.02	0.83	0.87	1.03	0.89	1.07	1.13	1.05	0.86	0.85	9.62
5	0.94	0.89	0.91	0.82	1.05	1.08	0.81	0.81	1.02	1.15	9.49
6	1.03	0.99	0.97	1.24	0.95	1.12	0.96	0.93	1.17	1.10	10.45
7	1.18	1.19	1.04	1.00	1.26	1.31	1.19	1.12	1.08	1.07	11.44
8	0.84	1.03	1.15	1.13	0.96	1.26	1.23	1.21	0.98	0.84	10.63
9	0.97	1.07	1.17	1.03	0.92	0.80	1.00	0.91	0.74	0.78	9.38
$p_{\cdot j}$	9.75	9.84	10.07	10.23	9.92	10.39	10.56	10.04	9.64	9.56	100.00

We first test the null hypothesis that $p_{ij} = 1$ for all i, j—*i.e.*, that all two-digit terminations are equally likely. The sample chi-square statistic is $\chi^2(99) = 153.57$. This value is more than adequate to reject the null hypothesis. Table 1 shows that the terminations in 60's, 70's, and 80's are too frequent; the most frequent termination of all, 75, occurs 1.31% of the time. Furthermore, the terminations in 89, 98, 99 and 00 are abnormally rare.

We next test the hypothesis that closings in all deciles are equally likely. The chi-square statistic for testing the hypothesis that $p_{\cdot j} = 10$ for all i is $\chi^2(9) = 48.47$. Again, we clearly reject the null hypothesis. On the basis of this evidence we conclude that the closing values of the DJIA are not uniformly distributed: some patterns of digits occur significantly more often than others.

2. Distribution of Returns

We next turn to the relation between stock market returns and the DJIA's last two digits. If century marks (closing prices at 1100,1200, etc.) are "psychological barriers" or "resistance point" then we might expect that the mean return following a close in the 90s would be lower than the mean return following closes in other deciles.

The one-day return, r_t, is defined as

$$r_t = \frac{\ln p_{t+1} - \ln p_t}{d_t} \times 100,$$

where p_t is the DJIA at time t, and d_t is the number of days between trading dates t and $t+1$. If t corresponds to a Friday and Monday is the next trading day, then $d_t = 3$.[2] We want to test whether the last two digits of p_t have any predictive power for r_t. Table 2 shows summary statistics for r_t by the corresponding digit in the 10's position of p_t.

Table 2. One-day percent return statistics by digit in 10's position.

10's digit	quantiles					mean	variance	n
	max	75%	median	25%	min			
0	4.6034	0.3782	0.0252	-0.3231	-4.9349	0.0157	0.6117	1,038
1	4.0476	0.4128	0.0264	-0.3428	-2.5457	0.0365	0.5142	1,006
2	4.1595	0.3392	0.0207	-0.3107	-3.8926	0.0083	0.4583	1,003
3	5.7154	0.4368	0.0191	-0.3000	-2.9985	0.0783	0.6075	1,027
4	9.6662	0.3781	0.0131	-0.3162	-8.5440	0.0342	0.6804	1,005
5	3.9670	0.3788	0.0029	-0.3465	-7.1555	0.0157	0.7032	992
6	3.1101	0.3868	0.0473	-0.3118	-4.0061	0.0337	0.4947	1,092
7	4.5787	0.3529	0.0282	-0.3104	-4.7177	0.0189	0.4740	1,195
8	3.2216	0.3649	0.0234	-0.2953	-3.0376	0.0353	0.4376	1,111
9	4.7814	0.4134	0.0300	-0.2883	-2.4200	0.0894	0.5305	980
All	9.6662	0.3857	0.0245	-0.3140	-8.5440	0.0362	0.5485	10,449

The largest mean return (0.0894%) following a day when the DJIA closes in the 90's; the smallest occurred when the DJIA closed in the 20's. The mean return following a close in the 90's is 2.5 times as large as the unconditional mean return. This is a large effect: on an annualized basis the unconditional mean return was about 14% while the annualized mean return following a close in the 90's was about 38%. Contrary to our initial expectations, the century marks do not seem to represent a "barrier" or a "resistance point;" rather they seem to represent a "launch pad!"

In order to test the significance of the difference of means, we assume that the returns in each decile are independent Normal draws from populations with known variance equal to the estimated variance. As we have seen the mean return if the DJIA ended in the 90's was 0.0894%; the mean return if it ended in any other decile was 0.0307%. The test statistic is

$$\frac{\bar{r}_{90} - \bar{r}_{other}}{\frac{\hat{\sigma}_{90}}{\sqrt{n_{90}}} + \frac{\hat{\sigma}_{other}}{\sqrt{n_{other}}}} = 1.90,$$

[2] Since $p_{t+1} = p_t \exp\{\ln(p_{t+1}/p_t)\}$ it follows that r_t is the continuous-time rate of return for the period between t and $t+1$. Since we want to have periods of equal length, we divide by the number of days between trading days—*i.e.*, whenever we have a holiday or weekend the computed r_t is an *average* rate of return. Out of the 10,449 observations, 8,130 (77.81%) have $d_t = 1$ (1.69% have $d_t = 2$, 18.35% have $d_t = 3$, 2.12% have $d_t = 4$ and 0.04% have $d_t = 5$). We think that calendar returns are the theoretically correct measure of returns, however since trading-day returns are widely used in the literature we also computed all the tables using trading-day returns finding no qualitative differences.

which has a p-value of 0.0287.[3]

3. Robustness of the Results

The results of the last section suggest that the growth of the DJIA accelerates when it moves into the 90's. We found this surprising; even more surprising was the magnitude of the effect. We decided to see if the results stood up to variation in the sample period. Accordingly, we computed the values of p_{ij} in four non-overlapping sub-samples, each with 2,612 observations. While the conclusion that not all terminations occur with the same frequency still holds for each of the four samples, the most and least frequent terminations vary with the sample chosen. This means that the results from the previous section are *not* robust with respect to the sample interval. Table 3 shows a summary.

Table 3. Percent of times that the ten's digit of the DJIA ends in i.
Four non-overlapping samples, each with $N = 2,612$.

i	Sample 1 1/1/52–4/18/62	Sample 2 4/19/62–10/5/72	Sample 3 10/6/72–2/9/83	Sample 4 2/10/83–6/10/93
0	10.38	9.95	9.15	10.22
1	9.57	9.95	8.46	10.53
2	8.50	9.26	10.15	10.49
3	6.55	9.49	12.29	10.99
4	7.62	10.49	11.41	8.96
5	7.01	9.61	11.79	9.57
6	13.06	8.81	9.84	10.11
7	15.39	10.68	10.15	9.53
8	12.86	11.37	8.46	9.84
9	9.07	10.38	8.31	9.76
$\chi^2(9)$	200.04	13.33	49.18	8.16
p-values	0.00	0.15	0.00	0.52

While in the first sample, the DJIA ends in the 60's 13.06% of the time, in the second sample it does so only 8.81% of the time. Similar significant variations can be observed for most digits and positions. The χ^2 statistics reject the hypothesis that the ten's digit is distributed uniformly in two of the four subsamples.

Several interesting hypothesis suggested by table 2 are rejected for, at least, three of the four samples considered.[4] It appears that the high mean return following closes in the 90's is only supported by a small fraction of the data, namely, the behavior of the market during the "roaring Eighties." We concluded that this effect does not appear to be a stable representation of the underlying data-generating process. Table 4 shows the descriptive statistics for the return series in each of the four samples. Again, there seems to be no particular pattern that holds across the four subsets.

4. Ignorance and Randomness

We now have an apparent anomaly: the *digits* of the DJIA appear to exhibit certain patterns, while the *returns* conditional on the digit realization are more-or-less random. Perhaps we should reconsider our initial null hypothesis. Is there any reason to expect that the digits of a market index should be uniformly distributed?

To develop some intuition, consider a standardized Normal variable. One might first expect that each of the ten possible values of the first decimal digit, say, is equally likely. However, since the density function is monotonically decreasing when we move away from zero, it follows that the digit most likely to appear in *any* position is 0, then 1, and so on up to 9. Exact probabilities can easily be computed from a standard probability

[3] This is the simplest possible test we could imagine; we also estimated a dummy-variable ARIMA (*i.e.*, intervention) model (Box and Tiao (1975)) which led to the same conclusion.

[4] Both ANOVA and nonparametric (Wilcoxon) tests were done to test various hypothesis regarding different returns, different absolute movements $|r_t|$, different variance of the returns, etc. We couldn't find any appealing regularity across the four samples.

Table 4. One-day return statistics by digit in 10's position:
Four non-overlapping samples.

10's i	Quantiles					Mean	Variance	N
	Max	75%	Median	25%	Min			
Sample 1: 1/2/52—4/18/62								
0	1.8096	0.3689	0.0241	-0.2360	-2.2231	0.0557	0.2922	271
1	4.0476	0.3700	0.0570	-0.2610	-1.8669	0.0731	0.3371	250
2	2.6313	0.2432	0.0533	-0.2019	-2.1451	0.2320	0.3072	222
3	2.4259	0.4677	0.0269	-0.2250	-1.8044	0.0821	0.3576	171
4	1.5840	0.3171	0.0408	-0.2555	-2.0553	0.0049	0.3082	199
5	2.2508	0.4285	0.0565	-0.2349	-2.1700	0.0981	0.3707	183
6	1.8743	0.2917	0.0451	-0.1736	-1.7015	0.0623	0.2851	341
7	1.7264	0.2972	0.0425	-0.1927	-1.6305	0.0536	0.2522	402
8	1.4785	0.3264	0.0534	-0.2682	-2.2553	0.0275	0.2506	336
9	2.0053	0.3607	0.0084	-0.2541	-1.5353	0.0638	0.3140	237
All	4.0476	0.3319	0.0436	-0.2290	-2.2553	0.0531	0.2979	2,612
Sample 2: 4/19/62—10/5/72								
0	1.4688	0.3247	0.0313	-0.2772	-1.9038	0.0160	0.3227	260
1	1.6417	0.3945	0.0350	-0.2784	-2.5457	0.0341	0.3302	260
2	2.1438	0.3000	0.0423	-0.2222	-1.7304	0.0500	0.2551	242
3	4.9517	0.4007	-0.0132	-0.2666	-2.9307	0.0485	0.5020	248
4	1.7469	0.3135	0.0128	-0.2343	-1.9068	0.0665	0.2910	274
5	3.2821	0.3235	-0.0120	-0.2979	-2.0741	0.0358	0.3757	251
6	3.1101	0.3532	0.0035	-0.3166	-2.2613	0.0266	0.4194	230
7	4.5787	0.3030	0.0090	-0.2875	-1.9419	0.0050	0.3926	279
8	2.7125	0.3112	0.0211	-0.2671	-1.7583	0.0189	0.3456	297
9	1.8001	0.2911	0.0176	-0.2483	-2.4200	0.0228	0.3268	271
All	4.9157	0.3301	0.0160	-0.2678	-2.9307	0.0320	0.3541	2,612
Sample 3: 10/6/72—2/9/83								
0	4.6034	0.3653	-0.2800	-0.5205	-2.5630	-0.0107	0.9575	239
1	2.8970	0.5177	-0.0233	-0.5174	-2.4186	0.0010	0.7862	221
2	4.1595	0.4599	-0.0275	-0.4542	-2.1218	-0.0048	0.6178	265
3	3.9443	0.5079	0.0311	-0.3541	-2.8514	0.0842	0.7414	321
4	3.4521	0.5414	0.0204	-0.4968	-2.9340	0.0502	0.7031	298
5	3.9670	0.3930	-0.0749	-0.5601	-2.4773	-0.0135	0.8704	308
6	2.6122	0.4981	0.0197	-0.5177	2.6122	-0.0206	0.6908	257
7	3.0269	0.3976	-0.0373	-0.5213	-2.2675	-0.0459	0.6376	265
8	3.2216	0.4355	0.0192	-0.4644	-3.0376	0.0012	0.6415	221
9	4.7814	0.6102	0.0280	-0.3954	-2.4023	0.1322	0.9046	217
All	4.7814	0.4641	-0.0041	0.4805	-3.1820	0.0175	0.7528	2,612
Sample 4: 2/10/83—6/10/93								
0	4.4665	0.4768	0.0416	-0.3905	-4.9349	-0.0018	0.9146	267
1	3.0541	0.4458	0.0000	-0.3832	-2.5424	0.0340	0.6342	275
2	2.9675	0.3472	0.0265	-0.4523	-3.8926	-0.0277	0.6076	274
3	5.7154	0.4170	0.0160	-0.3172	-2.9985	0.0952	0.7027	287
4	9.6662	0.3790	-0.0090	-0.3735	-8.5440	0.0009	1.4292	234
5	2.7379	0.4009	0.0351	-0.3515	-7.1555	-0.0290	1.0688	250
6	3.0406	0.4643	0.0986	-0.2758	-4.0061	0.0559	0.6413	264
7	2.9127	0.4517	0.0433	-0.3318	-4.7177	0.0476	0.7481	249
8	2.6792	0.4269	0.0142	-0.3257	-2.0704	0.0936	0.6137	257
9	3.1619	0.4685	0.0867	-0.2853	-1.9243	0.1474	0.6261	255
All	9.6662	0.4257	0.0378	-0.3461	-8.5440	0.0420	0.7895	2,612

table. For instance, for the first digit to the right of the decimal point the probabilities for $i = 0, 1, \ldots, 9$ are (in percentages): 13.61, 12.89, 12.07, 11.25, 10.43, 9.55, 8.75, 7.94, 7.14, and 6.38. The frequency of occurrence of the various digits in a Normal random variable is far from uniform!

Of course, a sequence of IID draws of a Normal random variable is not a very good model for the behavior of a stock market index. Instead let us consider the most popular and simplest possible model used for stock returns, a random walk with drift of the form

$$\ln q_t - \ln q_{t-1} = \mu + a_t, \qquad a_t \sim \text{IID } N(0, \sigma_a). \tag{1}$$

This is a Geometric Random Walk Model; it implies that the one-day return, $s_t = \ln q_t - \ln q_{t-1}$, is Normally distributed around its expectation, μ. Equivalently, we can write

$$q_t = q_{t-1} e^{\mu + a_t} \tag{2}$$

We generated five hundred series of 10,000 observations each for q_t according to equation (2) for different values of (μ, σ_a, q_0).[5] We then examined this simulated data to see if there were any "patterns" in the distribution of digits. The simulation results that we present in the next sections are not sensitive to the values of the parameters so we only show the results for $\mu = 0.025/100$ and $\sigma_a = 0.80/100$ (in line with the statistics shown on table 2) and $q_0 = 250$ (since it was approximately the level of the DJIA at the beginning of our sample).

It is important to recognize that we are *not* claiming that a Geometric Random Walk is a good model for the DJIA. We know that it can be improved upon. We are simply claiming that if an extremely simple model like (1) can reproduce the observed anomalities displayed by the actual DJIA, then we do not have any reason to believe that these anomalities are evidence of market inefficiency—they could easily be due to chance alone.

4.1. Simulation Results

We applied a χ^2 test for a uniform distribution of ten's digit to these 500 series of 10,000 observations. At a 5% significance level we rejected the uniform distribution hypothesis 98.4% of the time. Even though the simulated returns follow a random walk by construction, the ten's digit is distributed in a decidedly non-uniform way! Apparently the observed "pattern" in the distribution of the ten's digit could easily arise if the DJIA followed a pure random walk.

Table 5. Observed distribution of $p_{i\cdot}$ in the simulations.

i	Quantiles					Mean	Std Dev	Observed DJIA $p_{i\cdot}$
	95%	75%	Median	25%	5%			
0	11.69	10.62	9.81	9.05	7.90	9.82	1.17	9.85
1	12.07	10.54	9.74	8.98	7.81	9.79	1.23	9.46
2	11.96	10.55	9.82	9.03	7.85	9.85	1.23	9.53
3	12.50	10.65	9.95	9.18	8.10	10.03	1.27	9.78
4	13.86	10.93	10.10	9.34	8.26	10.18	1.25	9.56
5	12.54	11.04	10.18	9.42	8.29	10.25	1.28	9.48
6	12.25	10.92	10.15	9.35	8.24	10.18	1.23	10.50
7	12.18	10.89	9.98	9.24	8.16	10.08	1.22	11.68
8	11.90	10.75	9.93	9.24	7.95	9.96	1.19	10.74
9	11.94	10.58	9.86	9.08	7.99	9.87	1.15	9.41

What about the high excess return when the DJIA closes in the 90's? Could this be the result of chance alone? We have already shown that we can reject the hypothesis that the mean return conditioned on decile

[5] The routines **ran1** and **gasdev** from Press *et al.* (1986), pp. 714–16, were used to generate the Normal variates.

is the same for all deciles and we saw earlier that the return when the DJIA lands in the 90's is "significantly" greater than when it lands elsewhere.

However, it is not obvious that this is the correct comparison. The one-day expected (*i.e.*, mean) returns conditioned on closing digits of the DJIA comprise a set of 10 random numbers. In our sample, the largest of these numbers was the one associated with closing prices in the 90's and we showed that this was unusually large compared to *average* return. But of course it is unusually large—we performed this test precisely because the return in the 90's was the largest return! The relevant sampling distribution to use is not the distribution of the *average* return, but rather the distribution of the *maximum* of the 10 conditional returns.

It is not difficult to compute this distribution. Let $f(r)$ be a probability density for the one-day return. Draw 10,000 realizations of r and compute the mean one-day returns by decile. Look at the value of the *maximum* of these ten numbers. Now repeat this 500 times and plot the distribution of this maximum. This is the relevant sampling distribution with which to compare the "unusually large" return of 0.0894.

We performed this experiment using two different choices for the density $f(r)$. One was the Normal distribution used in the previous section. The other was the actual frequency distribution of one-day returns. The use of the frequency distribution is essentially a bootstrap method; see Efron and Tibshirani (1986) for discussion of this statistical technique. We used a bootstrap method because it is well-known that the distribution of one-day returns has fatter tails than a Normal distribution, and we were worried that this would affect the distribution of our test statistic.

Table 6 shows the distribution of the *maximum* of the mean simulated returns when the underlying return is taken to be drawn from the Normal distribution and the empirical distribution.

Table 6. Distribution of the maxima of the means by digit from simulation ($N = 500$).

Quantiles	Normal	Empirical
5%	0.0565	0.0558
10%	0.0604	0.0590
25%	0.0667	0.0654
50%	0.0742	0.0731
75%	0.0842	0.0826
90%	0.0934	0.0919
95%	0.0989	0.0982
Mean	0.0756	0.0745
Std Err	0.0006	0.0006
Variance	0.0002	0.0002

A remarkable feature of table 6 is that both distributions —Normal and empirical— are very similar. This occurs despite the fact that the observed distribution of returns from which we're sampling is clearly non-Normal. Figure 2 shows a Normal probability graph for the observed returns on the DJIA— the sampled distribution on the horizontal axis and the standard normal on the vertical one.[6] It is evident that the distribution of returns is very leptokurtic. (The empirical distribution has a bigger mass until somewhere between 1.5 and 2 standard deviations from the mean; after that point, the normal has a higher mass in any symmetric interval from the mean.) Apparently the symmetry and the large number of observations are more powerful than the fat tails. In other words, the central limit theorem is at work; at least in this instance a test based on Normal sampling theory is a valid tool, despite the fact that the returns are not Normally distributed.

Table 2 showed that the mean return when the DJIA ended in the 90's was 0.0894. Table 6 shows that a value smaller than this will be observed about 86% of the time. This means that the observed return of 0.0894 is not significant at the the conventional levels of statistical significance. Remember that this mean return is computed using a sample of about ten thousand observations. A test that is not significant with

[6] In a normal probability graph we have values from the sample and from the normal distribution paired by fractiles. If the sampled values had been generated by a normal distribution, we'd expect to see a straight line. Formal statistical tests can be based on the correlation coefficient between the two series.

this many observations must be considered relatively weak evidence against the null hypothesis. Coupled with the fact that the 90's decile did not have unusually high returns in our four subsamples, we have little confidence that the closing prices of DJIA provide much predictive power for future returns.

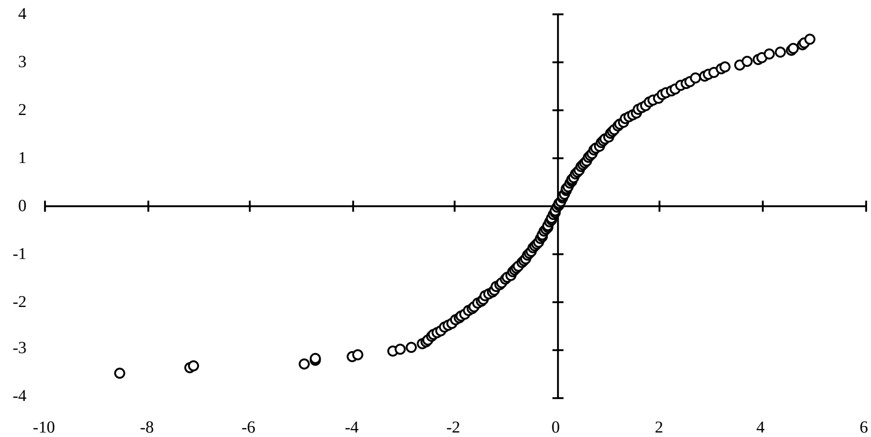

Fig. 2. Normal probability graph for the DJIA return.

Donaldson and Kim (1991) independently examined the question of market efficiency using regression analysis. They found no significant relationship between the returns in period t and the last two-digits of the closing price in period $t-1$. They conclude that "... knowing what value the DJIA closed yesterday ... does not help *predict* the return R_t for today." This is certainly consistent with our findings. However, our findings are considerably stronger: the Donaldson-Kim regression analysis assumes a linear relationship between closing prices and next-day returns, while our analysis allows for an arbitrary relationship.

5. Conclusion

In our initial investigation we found that the distribution of the Dow-Jones' digits was decided non-uniform, and that the mean return conditional on ending in the ninth decile was three times as large as the mean return elsewhere. We then asked whether this evidence was significant in the statistical sense.

First we showed that the phenomenon was not robust in subsamples: the mean return varies with the closing decile, but in a decidedly non-uniform way. When we looked at the data from a simulated random-walk model we found that it looked very much like the actual data: the distribution of the closing decile is simply not uniform. The observed distribution of closing values of the DJIA does not appear to be unusual compared to the distribution resulting from a geometric random walk.

We then examined the distribution of the returns. Although we could easily reject the hypothesis that the returns were the same for each decile, which return was largest seems to vary with the sample. The observed mean return of 0.0894 was large compared to the average return, but it was not significant at the usual confidence level using the distribution of the maximum of the returns in the ten deciles. Our conclusion from all this is that, contrary to initial impressions, there is little if any predictive power in the closing values of the DJIA.

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