Wavelet timescales and conditional relationship between higher-order systematic co-moments and portfolio returns: evidence in Australian data

Don U.A. Galagedera* and Elizabeth A. Maharaj
Department of Econometrics and Business Statistics
Monash University

Abstract

This paper investigates association between portfolio returns and higher-order systematic co-moments at different timescales obtained through wavelet multi-scaling- a technique that decomposes a given return series into different timescales enabling investigation at different return intervals. For some portfolios, the relative risk positions indicated by systematic co-moments at higher timescales is different from those revealed in raw returns. A strong positive (negative) linear association between beta and co-kurtosis and portfolio return in the up (down) market is observed in raw returns and at different timescales. The beta risk is priced in the up and down markets and the co-kurtosis is not. Co-skewness does not appear to be linearly associated with portfolio returns even after the up and down market split and is not priced.

Key words: Wavelet multi-scaling, higher-order systematic co-moments, asset pricing

JEL classification: G11, G12

* Correspondence to: Don U.A. Galagedera, Department of Econometrics and Business Statistics, Monash University, PO Box 197, Caulfield East, Victoria 3145, Australia.
E-mail: Tissa.Galagedera@buseco.monash.edu.au

1. Introduction

The capital asset pricing model (CAPM) developed by Sharpe (1964) and Lintner (1965) relates the expected rate of return of an individual security to a measure of its systematic risk known as the beta risk.[1] The CAPM has become an important tool in finance for assessment of cost of capital, portfolio performance, portfolio diversification, valuing investments and choosing portfolio strategy, among others. Building on Markowitz's (1959) work, Sharpe (1964) and Black (1972) developed various versions of the CAPM that can be empirically tested. The last half-century has witnessed the proliferation of empirical studies testing on (i) the validity of CAPM and the stability of beta and (ii) whether or not the cross-asset variation in expected returns could be explained by the market beta alone.

A growing number of studies found that the variation in expected security returns cannot be explained by the market beta alone. It is clear from well-established stylised facts that the unconditional security return distribution is not normal (see, for example, Ané and Geman, 2000 and Chung *et al,* 2001) and the mean and variance of returns alone are insufficient to characterise the return distribution completely. This has led researchers to pay attention to the third moment – skewness – and the fourth moment – kurtosis. Early studies examined the empirical relation of *ex post* returns to total skewness (see, for example, Arditti, 1967). Subsequent studies argue that systematic skewness is more relevant to market valuation rather than total skewness (see, for example, Kraus and Litzenberger, 1976) refuting the usefulness of quadratic utility as a basis for positive valuation theory. The experimental evidence that most

---

[1] The CAPM with respect to security *i* can be written as $E(R_i) = R_f + \beta_i \{E(R_m) - R_f\}$ where, $\beta_i = \frac{\sigma_i r_{im}}{\sigma_m} = \frac{\text{cov}(R_i, R_m)}{\sigma_m^2}$, r_{im} = the correlation between security return, R_i and market portfolio return R_m, σ_i is the standard deviation of security *i* returns and σ_m is the standard deviation of market returns. The β_i (referred to as the beta) can be interpreted as the amount of non-diversifiable risk inherent in security *i* relative to the risk of the market portfolio.

individuals have concave utility displaying absolute risk aversion also supports inclusion of higher-order co-moments in risk-return analysis (see, for example Gordon *et al*, 1972).[2]

Further, empirical evidence suggests that skewness and kurtosis of security returns distribution become prominent when high frequency data is used to model them. Studies have shown that it is possible to obtain different estimates for the beta for the same security if different return intervals are considered (Handa *et al*, 1989; Gençay *et al*, 2003). For further evidence, see Cohen *et al* (1986) and the references there in. In an investigation of impact of return interval and estimation period on beta estimation, Daves *et al* (2000) report that for a given estimation period, daily returns provide a smaller standard error of the estimated beta than do weekly, two-weekly or monthly returns. These revelations suggest that the width of the chosen return interval could influence the results of empirical investigations of the CAPM. For example, Brailsford and Faff (1997) tested the CAPM with daily, weekly and monthly returns and found evidence that supports the CAPM only in the monthly and weekly returns with the latter providing stronger evidence. As we shall see later in Section 6.1, it is also possible to obtain different estimates for higher-order systematic co-moments when different return intervals are used.

The wavelet technique, discussed later in Section 3 is another method to analyse a time series. Wavelets allow the time series to be viewed in multiple resolutions such that each resolution reflects a different frequency. Recently, Gençay *et al* (2003) estimated the beta in a sample of stocks in the US market using the decomposed return series obtained through wavelet analysis. Wavelet analysis decomposes a time series into different time horizons (scales). Gençay *et al* (2003) observe that the relationship between return of a portfolio and its beta becomes stronger as the wavelet scale increases and is nonlinear at lower scales. We argue that

---

[2] Some studies reveal that fundamental variables such as size, book-to-market value, macroeconomic variables and price-to-earnings ratio account for a sizeable portion of the cross-sectional variation in expected returns.

their observations are somewhat hampered due to the aggregation of results in the up market (market return in excess of the risk-free rate is positive) and down market (market return in excess of the risk-free rate is negative).

In this paper we investigate risk-return relationship conditional on the market movements using wavelet timescales in the two-, three- and four-moment asset pricing models. The aim is to estimate beta, systematic co-skewness and systematic co-kurtosis in daily returns and at different levels of resolution and (i) investigate whether they are significantly different in different scales, (ii) investigate their association with portfolio returns and (iii) test whether systematic risks as measured by co-moments are priced or not. In a sample of sixteen Australian industry portfolios we observe that portfolio beta and co-kurtosis have a strong positive (negative) linear association with portfolio returns in the up (down) market. The association between portfolio return and co-skewness appears to be nonlinear.

The paper is organised as follows. In Section 2, a version of higher-order CAPM is presented. The wavelet technique is explained in Section 3. Section 4 gives an outline of the methodology and Section 5 describes the data. The empirical results are reported and analysed in Section 6 followed by concluding remarks.

2. Four-moment CAPM

The following is a brief outline of the Kraus and Litzenberger (1976) version of the four-moment CAPM, in which it is assumed that only the risks measured by systematic variance, systematic skewness and systematic kurtosis are priced.

$$E(R_i) - R_f = \lambda_1 \beta_{im} + \lambda_2 \gamma_{im} + \lambda_3 \theta_{im} \tag{1}$$

where, R_f and R_i are returns on the risk-free asset and risky asset *i* respectively,

$$\beta_{im} = \frac{E[(R_{it} - E(R_i))(R_{mt} - E(R_m))]}{E(R_{mt} - E(R_m))^2} = \text{beta}, \tag{2}$$

$$\gamma_{im} = \frac{E\left[\left(R_{it} - E(R_i)\right)\left(R_{mt} - E(R_m)\right)^2\right]}{E\left(R_{mt} - E(R_m)\right)^3} \text{ = co-skewness (gamma),} \tag{3}$$

and

$$\theta_{im} = \frac{E\left[\left(R_{it} - E(R_i)\right)\left(R_{mt} - E(R_m)\right)^3\right]}{E\left(R_{mt} - E(R_m)\right)^4} \text{ = co-kurtosis (theta).} \tag{4}$$

Due to the desirable properties of the utility function, we expect the market price of beta reduction by one unit to be λ_1, which is expected to be positive as in the conventional CAPM. The market price of co-skewness is λ_2, which is expected to have the opposite sign to the skewness of the market return distribution. The market price of co-kurtosis is λ_3, which is an additional measure of degree of dispersion in returns and is expected to be positive.

A derivation of (1) is available in Hwang and Satchell (1999).

2.1 Four-moment conditional model

When testing the two-moment CAPM, Pettengill *et al* (1995) argue that the use of the realized return in the market model instead of the expected can induce some form of bias in the estimates due to aggregation of results in the up and down markets. They point out that in the up market (down market), portfolio betas and returns should be positively (negatively) related. Galagedera *et al* (2004) in a study of higher-order CAPMs suggested that in the down market the beta, gamma and theta and returns should be inversely related. To test whether beta, gamma and theta are priced or not, Galagedera *et al* (2004) estimate the cross-sectional regression model given by

$$\begin{aligned} R_{it} = {} & \delta\lambda_0^U + (1-\delta)\lambda_0^D + \lambda_1^U \delta\beta_{im} + \lambda_1^D (1-\delta)\beta_{im} + \lambda_2^U \delta\gamma_{im} + \lambda_2^D (1-\delta)\gamma_{im} \\ & + \lambda_3^U \delta\theta_{im} + \lambda_3^D (1-\delta)\theta_{im} + \varepsilon_{it} \end{aligned} \tag{5}$$

where $\delta = 1$ for up market, $\delta = 0$ for down market and $\varepsilon_{it} \sim N\left(0, \sigma_i^2\right)$ for each day in the testing period. They refer to (5) as the conditional four-moment risk-return relationship.[3]

2.2 Hypotheses of interest

Galagedera *et al* (2004) postulate that in the time periods where the market return in excess of the risk-free rate is negative, it is reasonable to infer inverse relationships between realized return and beta, gamma and theta. In order to see if there is supportive empirical evidence of a conditional relationship between expected return and higher-order co-moments, the following pairs of hypotheses are tested.

Test for a systematic conditional relationship between beta and realized returns: $\{ H_0 : \overline{\lambda}_1^U = 0, H_A : \overline{\lambda}_1^U > 0 \}$ and $\{ H_0 : \overline{\lambda}_1^D = 0, H_A : \overline{\lambda}_1^D < 0 \}$. If the null hypotheses in both are rejected, then a systematic conditional relationship between beta and realized return is supported.

Test for a systematic conditional relationship between gamma and realized returns when the up market return distribution is positively skewed: $\{ H_0 : \overline{\lambda}_2^U = 0, H_A : \overline{\lambda}_2^U < 0 \}$ and $\{ H_0 : \overline{\lambda}_2^D = 0, H_A : \overline{\lambda}_2^D > 0 \}$. If the null hypotheses in both are rejected, then a systematic conditional relationship between gamma and realized return is supported.

Test for a systematic conditional relationship between gamma and realized returns when the up market return distribution is negatively skewed: $\{ H_0 : \overline{\lambda}_2^U = 0, H_A : \overline{\lambda}_2^U > 0 \}$ and $\{ H_0 : \overline{\lambda}_2^D = 0, H_A : \overline{\lambda}_2^D < 0 \}$. If the null hypotheses in both are rejected, then a systematic conditional relationship between gamma and realized return is supported.

---

[3] When θ_{im} is assumed zero, we obtain the conditional three-moment risk-return relationship and when θ_{im} and γ_{im} are assumed zero, we obtain the conditional two-moment risk-return relationship.

Test for a systematic conditional relationship between theta and realized returns: { $H_0 : \overline{\lambda}_3^D = 0$, $H_A : \overline{\lambda}_3^D < 0$ } and { $H_0 : \overline{\lambda}_3^U = 0$, $H_A : \overline{\lambda}_3^U > 0$ }. If the null hypotheses in both are rejected, then a systematic conditional relationship between theta and realized return is supported.

Assuming that the market movements (up or down) do not have asymmetric effects on risk premiums, we can obtain the symmetric model, which is (5) with $\lambda_1^U = \lambda_1^D$, $\lambda_2^U = \lambda_2^D$ and $\lambda_3^U = \lambda_3^D$. We estimate the symmetric and asymmetric models and compare the results.

3. Wavelet analysis

Wavelet analysis is applicable to any type of time series and is a windowing technique with variable size regions. It allows the use of long time intervals when more precise low-frequency information is needed, and short time intervals when more precise high-frequency information is needed. In the time domain, if we want to examine the features of a daily time series in different time intervals such as weekly or monthly, the series will have to be aggregated and this would result in loss of useful data. In wavelet analysis, this can be done without aggregation and hence no data will be lost (Gençay *et al*, 2003). In what follows, a brief description of the wavelet analysis of a time series is presented. See Percival and Walden (2000) for more details.

Given a signal represented by $\{x(t), -\infty < t < \infty\}$, the collection of coefficients $\{ W(\lambda, t) : \lambda > 0, -\infty < t < \infty \}$ known as the continuous wavelet transform (CWT) of *x*(.), is such that

$$W(\lambda, t) = \int_{-\infty}^{\infty} \psi_{\lambda,t}(u)\, x(u)\, du \qquad (6)$$

and

$$\psi_{\lambda,t}(u) \equiv \frac{1}{\sqrt{\lambda}} \psi \left(\frac{u-t}{\lambda} \right) \qquad (7)$$

where λ is the scale associated with the transformation and t is its location. The function $\psi(.)$ is a wavelet filter that satisfies the properties

$$\int_{-\infty}^{\infty} \psi(u) du = 0 \qquad (8)$$

and

$$\int_{-\infty}^{\infty} \psi^2(u) du = 1 \ . \qquad (9)$$

In an application of wavelet analysis to a series $x(t)$ observed over a discrete set of times $t = 1,2,\ldots,T,$ we would be interested in the discrete wavelet transform (DWT). The DWT can be thought of as sensible sub-sampling of $W(\lambda,t)$ where a number of 'dyadic' scales are viewed with a varying number of wavelet coefficients at each scale. This means that we need to pick a scale λ_j to be of the form 2^{j-1}, $j = 1,2,\cdots,J$ where J is the number of scales and then within a given dyadic scale 2^{j-1}, we pick T_j observation points in time that are separated by multiples of 2^j. For a series of length T and the scale λ_j corresponding to 2^{j-1}, there are $T_j = T/2^j$ observation points at which wavelet coefficients can be defined. For example, consider a time series of length $T = 256 = 2^8$. We would then have eight dyadic scales available: 2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6 and 2^7 and at these scales there are 128, 64, 32, 16, 8, 4, 2 and 1 wavelet coefficients respectively. The wavelet coefficients for the eight scales account for the DWT coefficients, the number of which is equal to one less than the length of the time series. The single remaining coefficient is known as the scaling coefficient. In practice, we may choose to decompose a time series using a fewer number of scales depending on the length of the series. For example, in a time series of length 256 if we pick 5 scales (that is, 2^0, 2^1, 2^2, 2^3 and 2^4) there will be 128, 64, 32, 16 and 8 wavelet coefficients totalling 248 and the remaining 8 coefficients would be the scaling coefficients.

The wavelet coefficients are associated with the frequencies of $x(t)$. The wavelet coefficients at scale λ_j are associated with frequencies in the interval $[1/2^{(j+1)}, 1/2^{j}]$. Hence at the first scale λ_1, the wavelet coefficients are associated with frequencies in the interval [1/4, 1/2], whereas at the second scale λ_2, the coefficients are associated with frequencies in the interval [1/8, 1/4] and so on. If the time series under consideration consists of daily data, then the first scale captures the behaviour of the time series within a 2-4 day period, the second scale captures the behaviour of the time series within a 4-8 day period and so on.

The wavelet coefficients are proportional to the differences of averages of the time series observations at each scale, whereas the scaling coefficients are proportional to the averages of the original series over the largest scale. The scaling coefficients reflect long-term variations, which would exhibit a trend similar to that in the original series. Long timescales give more low-frequency information about the time series whereas short timescales give more high frequency information about the time series. DWT re-expresses a time series in terms of coefficients that are associated with a particular time and a particular dyadic scale. These coefficients are fully equivalent to the information contained in the original series in that a time series can be perfectly reconstructed from its DWT coefficients.

Many families of wavelet filters whose qualities vary according to a number of criteria are available. Some commonly used filters of order *N* are from the Daubechies family abbreviated as DB(N). These filters have length 2*N* and are asymmetric. The Haar filter which is the simplest wavelet filter is a DB(2) filter. Another family of filters which is a modification of the Daubechies family is the least asymmetric family LA(N) and is also of length 2*N*. These filters are nearly symmetric and have the property of aligning the wavelets coefficients very well with the given time series. The coiflets family of filters denoted by CF(N) is of length 2*N* and like

LA(N) possess the property of aligning the wavelets coefficients very well with the given time series. CF(N) are symmetric filters.

Discrete wavelet transform

The DWT of a time series $\{x_t : t = 1,2,\cdots,T\}$ is an orthonormal transform of the original series. If $\{W_n : n = 1,2,\cdots,n\}$ represents the set of wavelet coefficients, then it follows that

$$\mathbf{W} = F\,\mathbf{X} \tag{10}$$

where $\mathbf{X}$ is a column vector of time series elements, $\mathbf{W}$ is a column vector of length 2^J whose n^{th} element is the n^{th} DWT coefficient W_n, and F is a $T \times T$ real-valued matrix defining the DWT. This means that F contains the elements of the filter that transform the time series to wavelet coefficients, such that $F^{'} F = I_T$ where I_T is a $T \times T$ identity matrix.

The elements of the vector $\mathbf{W}$ can be decomposed into J+1 sub-vectors. The first J sub-vectors are denoted by $\mathbf{W}_j$, $j = 1,2,\ldots,J$ and the j^{th} such sub-vector contains all the DWT coefficients for scale λ_j. Each $\mathbf{W}_j$, $j = 1,2,\cdots,J$, is a column vector with $T_j = T/2^j$ elements. The $(J+1)^{\text{th}}$ sub-vector, which is denoted by $\mathbf{V}_J$, contains the scaling coefficients. The wavelet synthesis $\mathbf{X} = F'\,\mathbf{W}$, which is the reconstruction of the time series from the wavelet coefficients, can be expressed as

$$\mathbf{X} = F'\,\mathbf{W} = \sum_{j=1}^{J} F_j'\,\mathbf{W}_j + F_J'\,\mathbf{V}_J \tag{11}$$

where the F_j and F_J matrices partition the rows of F according to the partitioning of W into $\mathbf{W}_1$, $\mathbf{W}_2$, . . . , $\mathbf{W}_J$ and $\mathbf{V}_J$.

Now define $D_j = F_j'\,\mathbf{W}_j$ for $j = 1,2,\ldots,J$, which is a T dimensional column vector whose elements are associated with changes in $\mathbf{X}$ at scale λ_j. This means that $\mathbf{W}_j = F_j\mathbf{X}$ represents the portion of the analysis $\mathbf{W} = F\,\mathbf{X}$ attributed to the scale λ_j, whereas $D_j = F_j'\,\mathbf{W}_j$ is the portion of the synthesis $\mathbf{X} = F'\,\mathbf{W}$ attributable to scale λ_j . By defining $S_J = F_J'\,\mathbf{V}_J$, we obtain

$$\mathbf{X} = \sum_{j=1}^{J} D_j + S_J \tag{12}$$

which defines a multi-resolution analysis of **X**. That is, the time series **X** is expressed as the sum of a constant vector S_J and J other vectors D_j , j = 1, 2, . . . , J, each of which contains a time series related to variations in **X** at a certain scale. D_j is referred to as the j^{th} level wavelet detail. If

$$S_j = \sum_{k=j+1}^{J} D_k + S_J \tag{13}$$

for $0 \le j \le J-1$, then for $j \ge 1$

$$\mathbf{X} - S_j = \sum_{k=1}^{j} D_k . \tag{14}$$

S_j can be regarded as a smoothed version of **X**, since the difference between the two vectors involves only details at scale $\lambda_j = 2^{j-1}$ and smaller. As the index j increases, S_j (which is referred to as the j^{th} level wavelet smooth for **X**) should be smoother in appearance. Similarly, the j^{th} level wavelet rough for **X** is defined as

$$R_j = \sum_{k=1}^{j} D_k \tag{15}$$

for $1 \le j \le J$. Hence, the time series can be expressed as **X** = $S_j + R_j$ for $j = 1,2,\cdots,J$. For example, if we decompose the time series of length 2^8 using only j = 5 scales (that is, up to $\lambda_j = 2^{j-1} = 2^4$) then **X** can be expressed in terms of the 5th level smooth and rough, which gives

$$\mathbf{X} = S_5 + R_5 = S_5 + \sum_{k=1}^{5} D_k . \tag{16}$$

Hence, a time series can be expressed at different resolutions. In this particular case, the time series is expressed at up to five different resolutions. The non-decimated wavelet transformation or the stationary wavelet transform (MODWT) is a modification of the DWT in that while it can be thought of as a sub-sampling of the CWT at dyadic scales, it now deals with all times t and not just multiples of 2^j. For example, if a series of length $T = 256$ is considered, there would be 256 wavelet coefficients at each scale. Retaining all possible times at each scale of the MODWT decomposition has the advantage of retaining the time invariant property of the original series. The MODWT can be used in a similar manner to the DWT in defining a multi-resolution analysis of a given time series. In what follows, we will use the MODWT multi-resolution analysis.

For a detailed discussion on discrete wavelet transform see Percival and Walden (2000).

4. Methodology

The analysis of the risk-return relationship is based on a two-stage procedure. In the first stage of the analysis, the systematic risks beta, gamma and theta, are estimated. In the second stage we test whether the systematic risks are priced or not.

Stage-I: Beta, gamma and theta estimation using time series data

We estimate the beta, gamma and theta in sample portfolios using time series data in the first 256-day (1 year) period.

Stage-II: Estimation of cross-sectional relationship between returns and betas

In each group of 256 days (one year) that follows the sample period used in the estimation of the beta, gamma and theta in *Stage-I*, the daily industry portfolio returns are regressed on the beta, gamma and theta estimates obtained in *Stage-I*, according to the cross-sectional relationships:

$$R_{it} = \lambda_0 + \lambda_1\beta_i + \lambda_2\gamma_i + \lambda_3\theta_i + \varepsilon_{it} \tag{17}$$

where $\varepsilon_{it} \sim N\left(0, \sigma^2\right)$ and the conditional relationship (5). It is assumed here that the sector beta, gamma and theta estimated in *Stage-I* proxy beta, gamma and theta of *Stage-II*. To ascertain whether beta, gamma and theta are priced in the unconditional model, the hypotheses $\left\{H_0 : \overline{\lambda}_i = 0,\ H_A : \overline{\lambda}_i \neq 0\right\}$ for *i=1,2,3* are tested for the averages of the slope coefficients in (17).

The above procedure will uncover possible non-stationarities of the regression coefficients – risk premiums – within the 256-day period. The two-stage estimation procedure is repeated using a rolling window technique, rolling forward one year at a time. Our sample period allows seven repetitions of the two-stage procedure and enables estimation of beta, gamma and theta risk premium in 1792 consecutive days.

5. Data

The data set includes the daily price series of sixteen industry portfolios in Australia. The daily returns are calculated as the change in the logarithm of the closing prices of successive days. Although there is information on twenty-four industry portfolios in Australia, eight were omitted from the analysis due to the non-availability of data for the entire sample period of our study. The time period we investigate is from 28 August 1988 to 29 October 1996. The return series on the Australian All Ordinaries Index is used as a proxy for the market return. Some summary statistics of the return distributions are presented in Table 1. The excess kurtosis of the media sector is 16.34, which is very high compared to the rest. When the media sector is excluded, the excess kurtosis then ranges only from -1.25 to 5.83. The media sector earned the highest and the lowest returns compared to the other portfolios studied here. Ten of the sixteen sector return distributions are negatively skewed. The return distribution of the Australian All Ordinaries Index is also negatively skewed. The risk-free rate is assumed to be the 90-day Treasury bond rate.

6. Empirical results

We begin the analysis by estimating the co-moments: beta, gamma and theta for each industry portfolio using raw returns according to the formulae given in (2-4) in each of the seven 256-day estimation period. Thereafter, we decompose the raw return series of the market and the sixteen industry portfolios by employing the LA(8) filter and obtain wavelet coefficients; wavelet beta, wavelet gamma and wavelet theta as in the case with raw returns. The length of our rolling period is one year (256 days) and therefore wavelet coefficients in scales seven and eight are not used in the analysis. Scale 7 corresponds to 128-256 day dynamics and therefore it is together with scale 8 are inappropriate for the analysis.

6.1 Co-moments and timescales

The betas, gammas and thetas estimated in each of the seven estimation periods separately are averaged and presented in Figure 1. In two portfolios namely, Solid Fuels and Gold the beta estimated with wavelet coefficients at scale 6 is different from the betas estimated with the raw returns and at other scales. Raw returns indicate that Solid Fuel has a low market risk with beta at 0.69 compared to Gold with beta at 0.93. However, wavelet analysis indicates that the beta of Solid Fuels is 1.01 and Gold has a much lower beta risk (beta = 0.45) at dyadic scale 6 which is the 64-128 day period. This suggests that, for Gold and Solid Fuels sectors the relative beta-risk positions assessed by an investor operating at scale 6 would be opposite to the relative beta-risk positions assessed by an investor with raw returns.

The gamma estimated at wavelet scale 5 which is the 32-64 day period is different from the other wavelet gammas and the gamma estimated with the raw returns in five portfolios namely, Chemicals, Diversified Resources, Media, Retail and Oil & Gas. Column 2 entries in Table 1 reveal that four of these five portfolios record the top four mean returns and the other the second lowest mean return in the sample of sixteen portfolios. Panel B in Figure 1 indicates that wavelet gamma in Chemicals, Diversified Resources and Oil &Gas is much lower at scale 5 than those at other dyadic scales and in the Media and Retail sectors wavelet gamma is much higher at scale 5 than those at other dyadic scales.

None of the thetas estimated at different scales and with the raw returns dominates the other estimated thetas in any of the sixteen portfolios. Further, the patterns in the curves plotted in panels (a) and (c) in Figure 1 are similar suggesting that the correlation between beta and theta estimates is high. An interpretation of this observation is that the co-moments of portfolio and market returns that the beta and theta captures have comparable characteristics and as a consequence one co-moment might dominate or complement the other.[4]

In general, at the individual portfolio level there is no significant difference in the co-moments estimated with raw returns and with wavelet coefficients at the lower scales. In some portfolios the co-moments estimated with wavelet coefficients in high scales are different in

[4] The variance and kurtosis both measure dispersion and therefore in some situations kurtosis could become an additional risk measure for assets which variance alone fails to explain (Hwang and Satchell, 1996).

magnitude from those estimated with raw returns and at low wavelet scales. However, these differences are not large enough to affect a significant change in the respective overall means.[5]

6.2 Association between co-moments and returns

For each portfolio, we calculate (i) the average of the beta, gamma and theta estimated in the seven estimation periods and (ii) the average return in the seven risk premium estimation periods. Panels (a)-(c) in Figure 2 shows the scatter plots of average portfolio returns and average beta, gamma and theta computed with raw returns and panels (a)-(c) in Figures 3 and 4 shows the same computed with two sets of wavelet coefficients corresponding to scale 1 and scale 6. These plots indicate that the association between portfolio return and portfolio beta, gamma and theta appears to be non linear. However, when we plot the average beta and theta against the average portfolio up (down) market returns we observe a strong positive (negative) linear association between them. The corresponding scatter plots are displayed in panels (d), (f), (g) and (i) in Figures 2-4. There is no evidence of a linear association between portfolio gamma and return even after the up and down market separation. However the scatter plots in panels (e) and (h) in Figures 2-4 suggests that the hypothesis that gamma is inversely related to returns in the down market appears to hold in our data set. For the sake of brevity we do not report the plots corresponding to wavelet scales 2-5. The results in these cases are similar to those observed at scales 1 and 6.

---

[5] When we perform F-tests on the means: $H_0 : \bar{\beta}_{raw} = \bar{\beta}_{scale1} = ... = \bar{\beta}_{scale6}$ against H_A : at least one of $\bar{\beta}_{raw}, \bar{\beta}_{scale1}, ..., \bar{\beta}_{scale6}$ is different , $H_A : \bar{\gamma}_{raw} = \bar{\gamma}_{scale1} = ... = \bar{\gamma}_{scale6}$ against H_A : at least one of $\bar{\gamma}_{raw}, \bar{\gamma}_{scale1}, ..., \bar{\gamma}_{scale6}$ is different and $H_0 : \bar{\theta}_{raw} = \bar{\theta}_{scale1} = ... = \bar{\theta}_{scale6}$ against H_A : at least one of $\bar{\theta}_{raw}, \bar{\theta}_{scale1}, ..., \bar{\theta}_{scale6}$ is different, the null hypothesis is not rejected in any of the three pairs of hypothesis. This shows that even though notable differences in co-moments in some portfolios are observed (see panel (b), Figure 1) they do not affect the overall mean.

6.3 Pricing of co-moments

Here, adopting the two-stage procedure outlined in Section 4 we examine whether the systematic risks are priced or not in the two-, three- and four-moment unconditional and conditional models. The results in the unconditional models reveal that none of the risk premiums are significantly different from zero at the five percent level of significance suggesting that the systematic risks are not priced in unconditional models.[6] We do not report these results for brevity.

The risk premium estimated in the two moment conditional pricing model is reported in Table 2. As evidenced in Table 2, the beta risk premium in the up (down) market is positive (negative) and significant as expected. This is observed with the raw returns (panel A, Table 1) as well as at different wavelet scales generated from the LA(8) filter (panel B, Table 2).[7]

The risk premium estimated in the three-moment conditional pricing model reported in Table 3 reveals that (i) the beta is priced in the up and down markets and (ii) the gamma risk is priced in the raw return series in the up market and in wavelet coefficients only at scale 5 which is the 32-64 day period.[8,9] A reason for this might be that in each portfolio, there is not much difference among the gamma estimated with raw returns and at timescales 1-4 and 6. This can

---

[6] This is not surprising in the light of the arguments put forward by Pettengill *et al*, (1995) and Galagedera *et al*, (2004) and the empirical observations of many others that aggregation of results in the up and down markets could affect empirical results.

[7] The beta estimated with raw returns and at different wavelet scales are not different from each other in most of the portfolios. This might be the reason for observing similar results in the test of the beta risk premium in panels (A) and (B) in Table 2.

[8] The beta is positive in the up market and negative in the down market and is significant at the one percent level.

[9] In the sample data set the skewness in the market return distribution in the up (down) market is positive (negative). Therefore, gamma risk premium is expected to be negative in the up market and positive in the down market.

be seen in panel (b) in Figure 1. Our data therefore do not provide evidence of pricing of co-skewness in the three-moment pricing model.

Finally, we estimate the conditional four-moment pricing model. The results are reported in Table 4. Here too, the beta is priced in the up and down markets. Inclusion of co-kurtosis does increase the explanatory power as measured by adjusted R-square but the sign of the gamma and theta risk premiums are not always as expected and the estimates are not significant. A reason for the lack of evidence for co-kurtosis as an appropriate additional risk measure may be attributed to the high correlation between beta and theta as discussed in Section 6.1.[10]

Overall our data support the two-moment conditional pricing model and fail to support the unconditional pricing models and the conditional three- and four-moment pricing models. It appears that when the raw data (return) supports a conditional model so does the returns at different timescales corresponding to the LA(8) wavelet filter.

We examined the sensitivity of the results with the LA(8) filter to two alternative filter families namely, Daubechies and coiflets. We find that when the Daubechies DB(2), DB(4), DB(6) and DB(8) and coiflets CF(2), CF(4), CF(6) and CF(8) are used in the analysis, the conclusions with the LA(8) filter is largely unchanged.

---

[10] We repeated the analysis with two other conditional pricing models, (1) including the beta and theta as risk measures: $R_{it} = \delta\lambda_0^U + (1-\delta)\lambda_0^D + \lambda_1^U \delta\beta_{im} + \lambda_1^D (1-\delta)\beta_{im} + \lambda_3^U \delta\theta_{im} + \lambda_3^D (1-\delta)\theta_{im} + \varepsilon_{it}$ and (2) including only the theta as a risk measure: $R_{it} = \delta\lambda_0^U + (1-\delta)\lambda_0^D + \lambda_3^U \delta\theta_{im} + \lambda_3^D (1-\delta)\theta_{im} + \varepsilon_{it}$. As expected, the results when only the theta is included are consistent with what is observed in the conditional two-moment pricing model where only the beta is included. In the other case where only the beta and theta are included in the pricing model the results are inconclusive- the signs for the risk premiums obtained with raw returns and at some scales are contrary to what is expected and insignificant.

7. Concluding remarks

Computation of weekly, fortnightly and monthly return results in loss of data due to aggregation. In wavelet analysis however, returns can be examined at different timescales without any loss of data.

In some portfolios systematic co-moments estimated at higher scales indicate risk levels different from what is estimated in raw data suggesting that riskiness is timescale dependent. Therefore, investors operating at a larger timescale should evaluate such portfolios with the risk levels estimated at such timescales.

We examine the association between systematic risks as measured by the beta, co-skewness and co-kurtosis and returns using raw returns and wavelet coefficients obtained through decomposition of the returns with the LA(8) filter. A strong positive linear association between beta and co-kurtosis and portfolio return in the up market and a strong inverse linear association between the beta and co-kurtosis and portfolio return in the down market is observed. These associations are observed in raw returns and at all wavelet scales. The linear association between the systematic risks and portfolio returns without conditioning on market movement is weak. Co-skewness does not appear to be linearly associated with portfolio returns even after the up and down market split.

Tests of risk-return relationship in the two-, three- and four-moment pricing models, reveal that (i) there is no evidence in support of the unconditional pricing models and (ii) when market movement is accommodated in the pricing model, only the two-moment pricing model is supported. There is evidence of a systematic relationship between market beta and portfolio returns in raw returns and at different resolutions of the raw returns. The gamma and theta risk premium in some instances are priced but their sign is often contrary to what is expected. In the sampled data set we find no evidence to suggest co-skewness and co-kurtosis as additional risk measures. Nevertheless, risk as measured by co-moments is timescale dependent and this should be taken into account when testing the validity of asset pricing models.

References

Ané, T. and Geman, H. (2000). Order flow, transaction clock, and normality of asset returns, *Journal of Finance*, 55, 2259-2284.

Arditti, F. (1967). Risk and the required return on equity, *Journal of Finance*, 22, 19-36.

Brailsford, T.J. and Faff, R.W. (1997). Testing the conditional CAPM and the effect of intervaling: a note, *Pacific-Basin Finance Journal*, 5, 527-537.

Chung, Y.P., Johnson, H. and Schill, M.J. (2001). Asset pricing when returns and nonnormal: Fama-French factors vs higher-order systematic co-moments, Working Paper, A.Gary Anderson Graduate School of Management, University of California, Riverside.

Cohen, K., Hawawini, G., Mayer, S., Schwartz, R. and Whitcomb, D. (1986). *The Microstructure of Securities Markets*, Sydney: Prentice-Hall.

Daves, P.R., Ehrhardt, M.C. and Kunkel, R.A. (2000). Estimating systematic risk: the choice of return interval and estimation period, *Journal of Financial and Strategic Decisions*, 13, 7-13.

Galagedera, D.U.A., Henry, D. and Silvapulle, P. (2004). Empirical evidence on the conditional relation between higher-order systematic co-moments and security returns, *Quarterly Journal of Business and Economics*, 42, 121-137.

Gençay, R. and Whitcher, B. (2001). *An Introduction to Wavelets and other Filtering Methods in Finance and Economics*, San Diego, CA: Academic.

Gençay, R., Selçuk, F and Whitcher, B. (2003). Systematic risk and timescales, *Quantitative Finance*, 3, 108-116.

Gordon, M.J., Paradis, G.E. and Rorke, C.H. (1972). Experimental evidence on alternative portfolio decision rules, *American Economic Review*, 62, 107-118.

Handa, P., Kothari, S.P. and Wasley, C. (1989). The relation between the return interval and betas: implications for the size effect, *Journal of Financial Economics*, 23, 79-100.

Hwang, S. and Satchell, S. E. (1999). Modelling emerging market risk premia using higher moments, *International Journal of Finance and Economics*, 4, 271-296.

Kraus, A. and Litzenberger, R.H. (1976). Skewness preference and the valuation of risk assets, *Journal of Finance*, 31, 1085-1100.

Lintner, J. (1965). The valuation of risk assets and selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, 47, 13-37.

Markowitz, H. (1959). *Portfolio Selection*, J. Wiley and Son: New York.

Black, F. (1972). Capital market equilibrium with restricted borrowing, *Journal of Business*, 45, 444-454.

Percival, D.B. and Walden, A.T. (2000) *Wavelet Methods for Time Series Analysis*, Cambridge: Cambridge University Press.

Pettengill, G.N., Sundaram, S. and Mathur, L. (1995). The conditional relation between beta and returns, *Journal of Financial and Quantitative Analysis*, 30, 101-116.

Sharpe, W.F. (1964). Capital asset prices: a theory of market equilibrium under conditions of risk, *Journal of Finance*, 19, 425-442.

Table 1. Descriptive statistics of the distribution of the continuously compounded daily returns of Australian industry portfolios

Industry portfolio	Mean	Max	Min	Standard deviation	Skewness	Kurtosis
(1) Alcohol & tobacco	0.0092	5.1600	-6.0148	1.1798	-0.1765	2.4195
(2) Banks & finance	0.0254	4.4348	-7.1480	1.0464	-0.1974	2.2972
(3) Building materials	0.0041	3.8593	-7.9626	0.9470	-0.2608	3.0867
(4) Chemicals	0.0360	6.5418	-9.1292	1.1518	-0.1494	3.7821
(5) Developers & contractors	0.0158	3.4345	-6.1888	0.9016	-0.3880	3.4065
(6) Diversified resources	0.0396	4.9845	-7.5387	1.1725	-0.0875	1.7466
(7) Engineering	0.0077	5.2928	-8.2022	1.0386	-0.2041	3.2953
(8) Food & household goods	0.0339	10.6032	-6.4921	0.9491	0.6231	8.8350
(9) Media	0.0412	22.9683	-15.4941	1.8414	0.4537	19.3390
(10) Paper & packaging	0.0067	5.3922	-8.1917	1.0553	-0.1349	2.9369
(11) Retail	-0.0043	4.8496	-7.0938	0.9965	-0.3021	3.5141
(12) Transport	0.0090	5.8343	-7.0321	1.0751	-0.0909	2.7925
(13) Solid fuels	0.0206	11.5480	-6.4683	1.3454	0.2025	4.4870
(14) Oil & gas	0.0443	6.0482	-8.4277	1.0000	-0.1574	4.3807
(15) Gold	-0.0003	9.1467	-12.6451	1.4494	0.1144	6.2019
(16) Insurance	-0.0065	7.5473	-10.4533	1.3169	-0.6636	6.7240
All Ords	0.0183	3.7817	-8.4411	0.8017	-0.6046	6.9425

Notes*:* Sample period is 29 August 1988 **-** 29 October 1996. The figures are given as daily percentages. Statistics are based on 2048 observations. Skew is computed as $\frac{1}{T}\sum_{t=1}^{T}\left(\frac{R_t-\overline{R}}{\hat{\sigma}_R}\right)^3$ where R_t is the return in day *i* and $\hat{\sigma}_R$ is the standard deviation of return. Kurtosis is computed as $\left[\frac{1}{T}\sum_{t=1}^{T}\left(\frac{R_t-\overline{R}}{\hat{\sigma}_R}\right)^4\right]$ where R_t is the return in day *i* and $\hat{\sigma}_R$ is the standard deviation of return.

Table 2. Estimates of risk premium in two-moment conditional pricing model

Model	Up market		Down market	
	$\overline{\lambda}_0^U$	$\overline{\lambda}_1^U$	$\overline{\lambda}_0^D$	$\overline{\lambda}_1^D$
Panel A *Raw return*				
Up market days = 880, down market days = 912, $\overline{R}_{up}^2$ = 0.10 and $\overline{R}_{down}^2$ =0.10				
Estimate	0.1559	0.4587	-0.1128	-0.4503
t-value	4.0424*	10.6989*	-3.0033*	-10.8639*
Panel B *Scales from LA(8) filter*				
Scale 1: up market days = 922, down market days = 870, $\overline{R}_{up}^2$ = 0.10 and $\overline{R}_{down}^2$ =0.09				
Estimate	0.1118	0.4201	-0.1185	-0.4452
t-value	3.4723*	12.052*	-3.7038*	-12.817*
Scale 2: up market days = 917, down market days = 875, $\overline{R}_{up}^2$ = 0.10 and $\overline{R}_{down}^2$ =0.10				
Estimate	0.1992	0.4224	-0.2088	-0.44262
t-value	5.5487*	10.432*	-5.5101*	-10.261*
Scale 3: up market days = 920, down market days = 872, $\overline{R}_{up}^2$ = 0.11 and $\overline{R}_{down}^2$ =0.12				
Estimate	0.3503	0.3118	-0.3696	-0.3289
t-value	9.0449*	7.4083*	-9.2538*	-7.4445*
Scale 4: up market days = 888, down market days = 904, $\overline{R}_{up}^2$ = 0.10 and $\overline{R}_{down}^2$ =0.09				
Estimate	0.3059	0.2961	-0.3005	-0.2909
t-value	11.035*	9.9708*	-10.930*	-10.152*
Scale 5: up market days = 921, down market days = 871, $\overline{R}_{up}^2$ = 0.11 and $\overline{R}_{down}^2$ =0.13				
Estimate	0.3528	0.3264	-0.3730	-0.3452
t-value	12.370*	10.604*	-13.594*	-11.260*
Scale 6: up market days = 964, down market days = 828, $\overline{R}_{up}^2$ = 0.12 and $\overline{R}_{down}^2$ =0.12				
Estimate	0.3249	0.2988	-0.3782	-0.3479
t-value	13.631*	11.315*	-14.008*	-12.157*

Notes: * Significant at the 1 percent level and ** significant at the 5 percent level. The conditional model estimated is $R_{it} = \delta\lambda_0^U + (1-\delta)\lambda_0^D + \lambda_1^U \delta\beta_{im} + \lambda_1^D (1-\delta)\beta_{im} + \varepsilon_{it}$ where $\delta = 1$ for up market (market return in excess of the risk-free rate is positive), $\delta = 0$ for down market (market return in excess of the risk-free rate is negative) and $\varepsilon_{it} \sim N(0, \sigma_i^2)$.

Table 3. Estimates of risk premium in three-moment conditional pricing model

Model	Up Market			Down Market		
	$\overline{\lambda}_0^U$	$\overline{\lambda}_1^U$	$\overline{\lambda}_2^U$	$\overline{\lambda}_0^D$	$\overline{\lambda}_1^D$	$\overline{\lambda}_2^D$
Panel A *Raw return*						
Up market days = 880, down market days = 912, $\overline{R}_{up}^2$ = 0.19 and $\overline{R}_{down}^2$ =0.18						
Estimate	0.1679	0.5544	-0.1035	-0.1007	-0.4923	0.0280
t-value	4.2307*	11.590*	-3.0930*	-2.5608**	-11.111*	0.8541
Panel B *Scales from LA(8) filter*						
Scale 1: up market days = 922, down market days = 870, $\overline{R}_{up}^2$ = 0.19 and $\overline{R}_{down}^2$ =0.18						
Estimate	0.0831	0.4237	0.0246	-0.0881	-0.4490	-0.0261
t-value	2.5548**	12.308*	1.2201	-2.6291*	-12.388*	-1.1819
Scale 2: up market days = 917, down market days = 875, $\overline{R}_{up}^2$ = 0.18 and $\overline{R}_{down}^2$ =0.18						
Estimate	0.1941	0.4959	-0.0630	-0.2034	-0.5197	0.0660
t-value	5.0528*	10.538*	-1.8980	-4.8308*	-10.464*	1.7839
Scale 3: up market days = 920, down market days = 872, $\overline{R}_{up}^2$ = 0.19 and $\overline{R}_{down}^2$ =0.20						
Estimate	0.3707	0.3079	-0.0203	-0.3911	-0.3248	0.0214
t-value	9.5508*	6.7924*	-1.6775	-9.8003*	-6.9112*	1.7638
Scale 4: up market days = 888, down market days = 904, $\overline{R}_{up}^2$ = 0.18 and $\overline{R}_{down}^2$ =0.18						
Estimate	0.3158	0.2801	0.0004	-0.3102	-0.2752	-0.0004
t-value	10.462*	7.5060*	0.0330	-10.335*	-7.2634*	-0.0308
Scale 5: up market days = 921, down market days = 871, $\overline{R}_{up}^2$ = 0.22 and $\overline{R}_{down}^2$ =0.22						
Estimate	0.3729	0.3653	-0.0634	-0.3943	-0.3862	0.0671
t-value	12.420*	11.189*	-5.3251*	-13.882*	-11.8789*	5.4776*
Scale 6: up market days = 964, down market days = 828, $\overline{R}_{up}^2$ = 0.21 and $\overline{R}_{down}^2$ =0.22						
Estimate	0.3252	0.2518	0.0278	-0.3786	-0.2931	-0.0323
t-value	13.265*	8.5577*	1.4056	-13.388*	-10.605*	-1.4428

Notes: * Significant at the 1 percent level and ** significant at the 5 percent level. The conditional model estimated is $R_{it} = \delta\lambda_0^U + (1-\delta)\lambda_0^D + \lambda_1^U \delta\beta_{im} + \lambda_1^D (1-\delta)\beta_{im} + \lambda_2^U \delta\gamma_{im} + \lambda_2^D (1-\delta)\gamma_{im} + \varepsilon_{it}$ where $\delta = 1$ for up market (market return in excess of the risk-free rate is positive), $\delta = 0$ for down market (market return in excess of the risk-free rate is negative) and $\varepsilon_{it} \sim N(0, \sigma_i^2)$.

Table 4. Estimates of risk premium in conditional four-moment pricing model

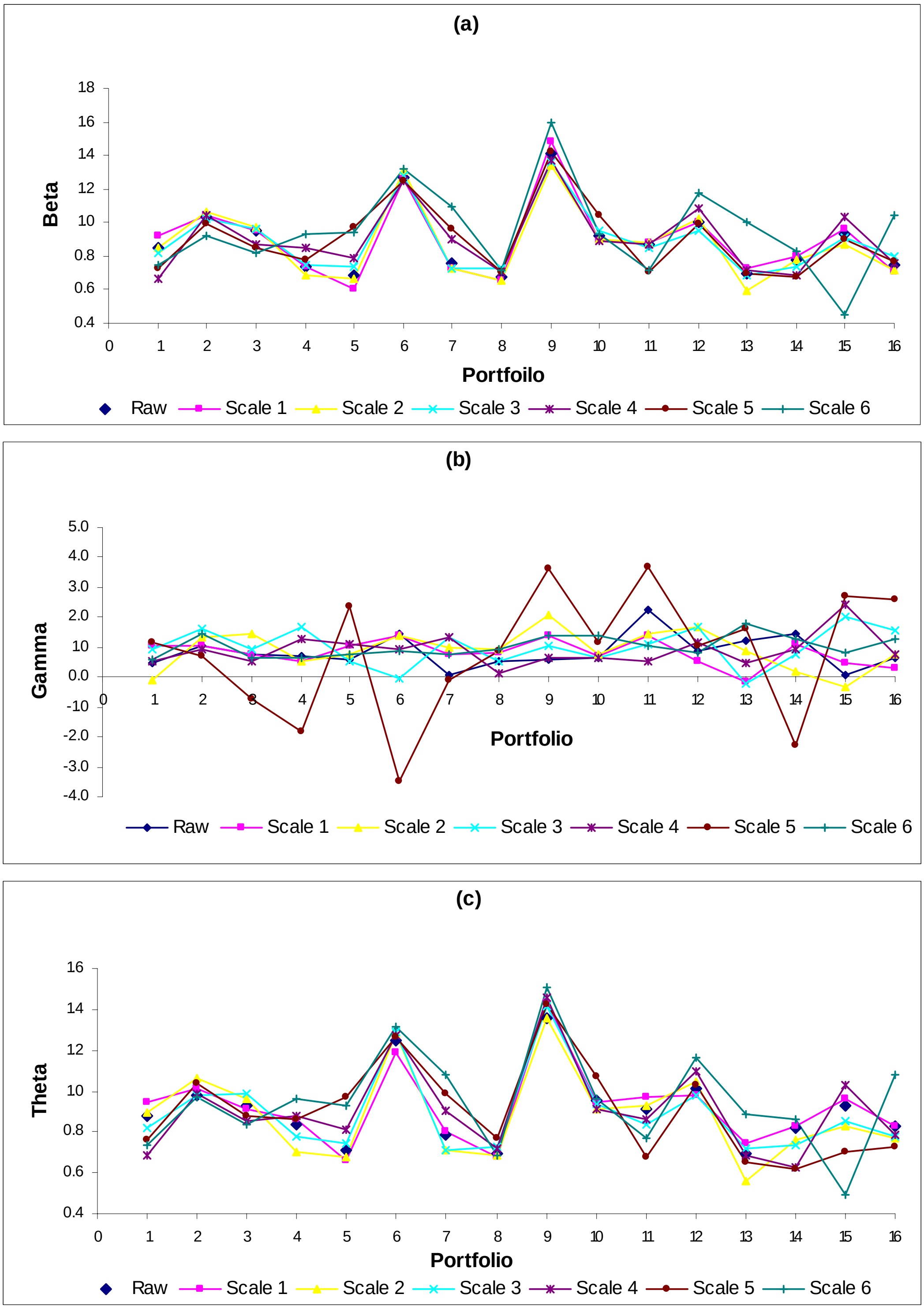
Model	Up Market				Down Market			
	$\overline{\lambda}_0^U$	$\overline{\lambda}_1^U$	$\overline{\lambda}_2^U$	$\overline{\lambda}_3^U$	$\overline{\lambda}_0^D$	$\overline{\lambda}_1^D$	$\overline{\lambda}_2^D$	$\overline{\lambda}_3^D$
Panel A *Raw return*								
Up market days = 880, down market days = 912, $\overline{R}_{up}^2$ = 0.26 and $\overline{R}_{down}^2$ =0.26								
Estimate	0.1950	0.5903	0.4842	-0.6591	-0.1028	-0.5086	-0.3696	0.4203
t-value	5.1218*	4.9378*	2.9760*	-3.3375*	-2.7126*	-4.7429*	-2.7426*	2.5412**
Panel B *Scales from LA(8) filter*								
Scale 1: up market days = 922, down market days = 870, $\overline{R}_{up}^2$ = 0.27 and $\overline{R}_{down}^2$ =0.26								
Estimate	0.1026	0.6836	0.0250	-0.2725	-0.1087	-0.7244	-0.0265	0.2888
t-value	2.8810*	7.1978*	1.1988	-2.8358*	-3.1238*	-7.2374*	-1.1972	2.8716*
Scale 2: up market days = 917, down market days = 875, $\overline{R}_{up}^2$ = 0.25 and $\overline{R}_{down}^2$ =0.24								
Estimate	0.1767	0.7757	-0.0404	-0.2819	-0.1852	-0.8129	0.0423	0.2954
t-value	4.6217*	5.4321*	-1.0825	-2.0470**	-4.3847*	-5.5589*	0.9925	2.1011
Scale 3: up market days = 920, down market days = 872, $\overline{R}_{up}^2$ = 0.26 and $\overline{R}_{down}^2$ =0.27								
Estimate	0.3952	-0.0186	-0.0342	0.3239	-0.4170	0.0196	0.0361	-0.3417
t-value	9.7871*	-0.1284	-2.4147**	2.2627**	-10.050*	0.1339	2.5494**	-2.3587**
Scale 4: up market days = 888, down market days = 904, $\overline{R}_{up}^2$ = 0.25 and $\overline{R}_{down}^2$ =0.25								
Estimate	0.3137	0.2244	0.0285	0.0334	-0.3082	-0.2205	-0.0280	-0.0328
t-value	10.350*	2.4550**	1.5606	0.3406	-10.234*	-2.3786**	-1.5098	-0.3337
Scale 5: up market days = 921, down market days = 871, $\overline{R}_{up}^2$ = 0.31 and $\overline{R}_{down}^2$ =0.31								
Estimate	0.4210	0.9245	-0.0736	-0.6157	-0.4452	-0.9776	0.0778	0.6510
t-value	11.823*	7.3102*	-6.5788*	-4.7216*	-13.761*	-7.4974*	7.0739*	4.9214*
Scale 6: up market days = 964, down market days = 828, $\overline{R}_{up}^2$ = 0.31 and $\overline{R}_{down}^2$ =0.30								
Estimate	0.3528	0.3198	0.0844	-0.1384	-0.4107	-0.3724	-0.0983	0.1611
t-value	14.773*	2.1671**	5.3239*	-0.9007	-14.506*	-2.8461*	-4.5866*	1.1212

Notes: * Significant at the 1 percent level and ** significant at the 5 percent level. The conditional model estimated is

$$R_{it} = \delta\lambda_0^U + (1-\delta)\lambda_0^D + \lambda_1^U \delta\beta_{im} + \lambda_1^D (1-\delta)\beta_{im} + \lambda_2^U \delta\gamma_{im} + \lambda_2^D (1-\delta)\gamma_{im} + \lambda_3^U \delta\theta_{im} + \lambda_3^D (1-\delta)\theta_{im} + \varepsilon_{it}$$

where $\delta = 1$ for up market (market return in excess of the risk-free rate is positive), $\delta = 0$ for down market (market return in excess of the risk-free rate is negative) and $\varepsilon_{it} \sim N(0, \sigma_i^2)$.

Figure 1. Portfolio beta, gamma and theta estimated with raw data and wavelet coefficients



Note: The portfolios are described in Table 1.

Figure 2. Average daily portfolio returns versus average portfolio beta, gamma and theta with raw data

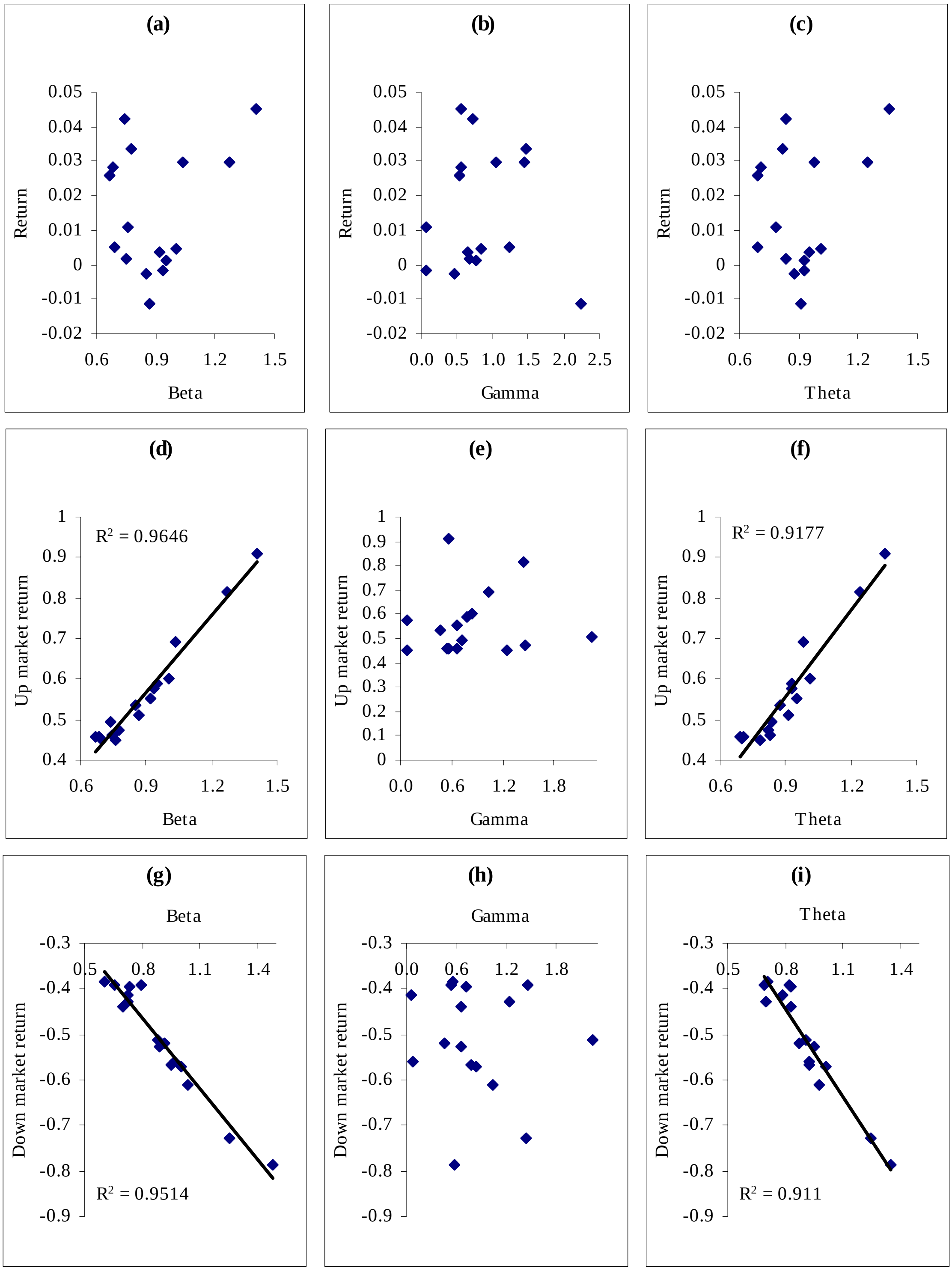


Figure 3. Average daily portfolio returns versus average portfolio beta, gamma and theta with LA(8) scale 1 coefficients

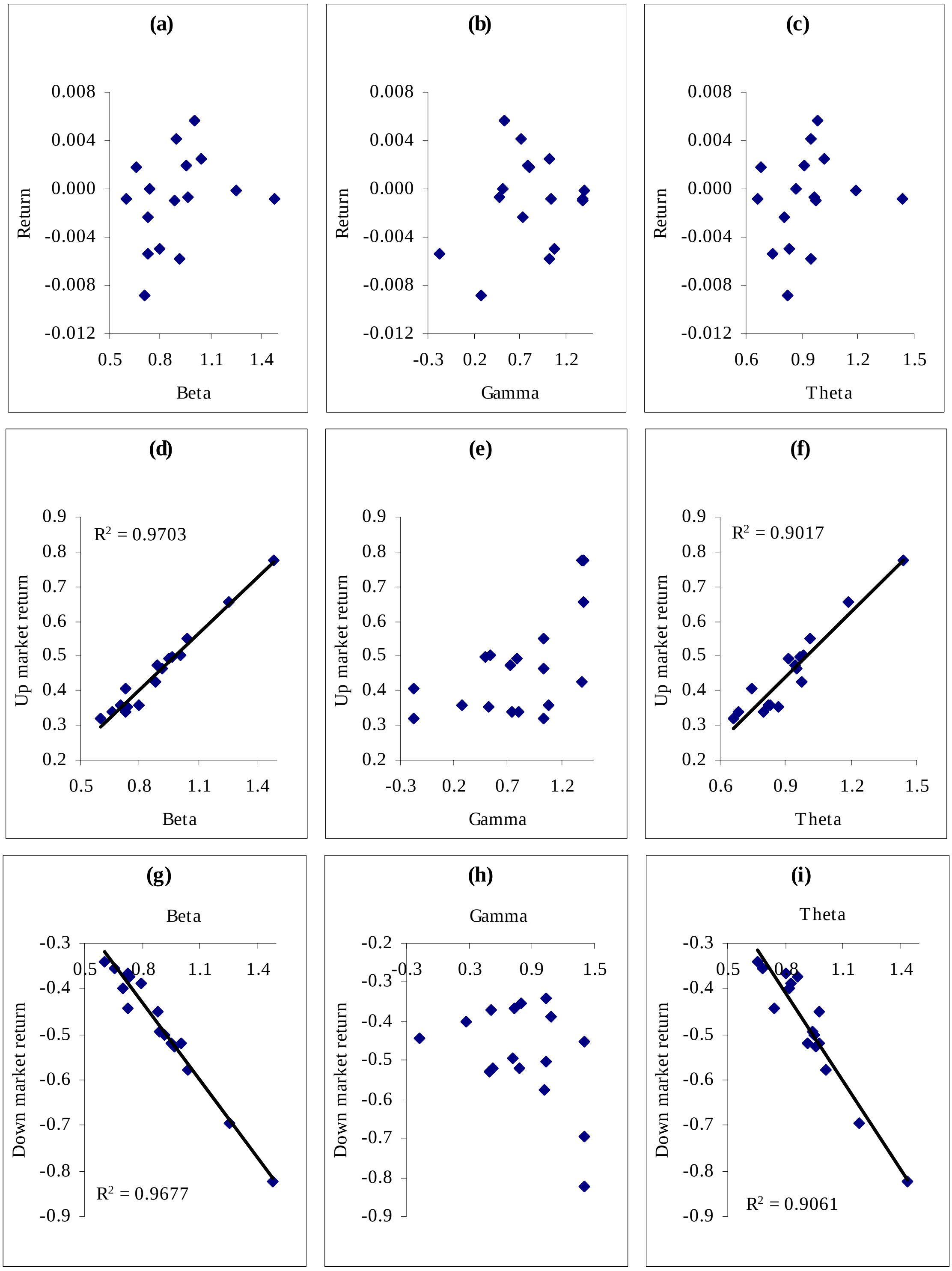


Figure 4. Average daily portfolio returns versus average portfolio beta, gamma and theta with LA(8) scale 6 coefficients