

A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-sectional Heterogeneity?*

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Abstract

In this paper we study asset prices in a parsimonious two-agent macroeconomic model with two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution in consumption. The parameter values for the model are taken from the business cycle literature, and in particular, are not calibrated to match financial statistics. The model generates a number of asset pricing phenomena that have been documented in the literature, including a high equity premium and a low risk-free rate; procyclical variation in the price-dividend ratio; countercyclical variation in the equity premium, in its volatility, and in the Sharpe ratio; and long-horizon predictability of returns with high R^2 values. We also show that the similarity of our results to those from an external habit model is not a coincidence: the model has a reduced form representation that is similar to Campbell and Cochrane's (1999) framework for asset pricing. However, the implications of the two models for macroeconomic questions and policy analyses are different.

Keywords: Limited stock market participation, the equity premium puzzle, incomplete markets, habit formation, elasticity of intertemporal substitution.

JEL classification: E32, E44, G12

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1 Introduction

In the last two decades, research on asset prices has uncovered some interesting and puzzling phenomena. For example, Mehra and Prescott (1985) have shown that the equity premium (the excess return on stocks over bonds), which averages about six percent annually in the U.S., was hard to reconcile with a canonical consumption-based asset pricing model, and as it later turned out, with many of its extensions. Another strand of literature has found that the equity premium was predictable by a number of variables including the dividend yield, challenging the long-held view that stock returns follow a martingale (Campbell and Shiller (1988)). Other studies have documented that the expected equity premium, its volatility, and the ratio of the two—the conditional Sharpe ratio—move over time following a (countercyclical) business cycle pattern (Fama and French (1989), Schwert (1989), and Chou, Engle and Kane (1992)).¹

In this paper we ask if these asset pricing phenomena can be explained in a parsimonious macroeconomic model with two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution in consumption (EIS). The limited nature of stock market participation, and the concentration of stock wealth even among stockholders is well-documented. For example, until the 1990's more than two-thirds of U.S. households did not own any stocks at all, while the richest one percent held 48 percent of all stocks (Poterba and Samwick (1995), and Investment Company Institute (2002)). As for the heterogeneity in preferences, the empirical evidence that we review in Section 3 indicates that stockholders (and the wealthy in general) have a higher EIS than non-stockholders (and the poor in general). The interaction of these two features is important as will become clear below.

We choose the real business cycle model as the foundation that we build upon, to provide a contrast with the poor asset pricing implications of that framework that are well-known, which helps to highlight the role of the new features considered in this paper. Specifically, we consider an economy with a neoclassical production technology and competitive markets. There are two (types of) agents. The majority of households (first type) do not participate in the stock market where claims to the firm's future dividends are traded. However, a risk-free bond is available to all households, so the non-stockholders can also accumulate wealth and smooth consumption intertemporally. We also model the capital adjustment costs in production (as in Lucas and Prescott (1971) among others). Finally, we assume that the stockholders have a higher EIS than the non-stockholders. Although with CRRA utility this assumption also implies a different risk aversion for each group, in Section 8 we show that this heterogeneity plays no essential role for the results of the paper. The parameters of the model are then calibrated to values from the business cycle literature, and in particular, are not chosen to match financial statistics.

Here is an overview of our results. First, with a relative risk aversion of 2 for both agents, the model generates an equity premium of 3.4 percent (and 6.1 percent when the risk aversion is 4). The resulting Sharpe ratio is between 0.20 and 0.27, compared to an average of 0.32 in the

¹For excellent surveys of the literature, see Kocherlakota (1996) and Campbell (1999). We discuss some of the related work in Section 4.3.

century-long U.S. data, and 0.42 in the post-World War II period. Second, the volatility of interest rates is very low in the data, which has proved difficult to explain for a number of asset pricing models—a risk-free rate volatility above 15 percent is not uncommon. This volatility is 5.6 percent in our model. Although this figure is still about twice the volatility in the data, it represents a step in the right direction. Moreover, the dynamics of asset prices in the model are consistent with empirical evidence: the expected equity premium, its conditional volatility, and the Sharpe ratio are countercyclical; the price-dividend ratio is pro-cyclical and very persistent, and it predicts future stock returns with high R^2 values. The sensitivity analyses that we perform show that these dynamics are robust to a number of extensions (such as introducing government debt, changing the preference parameters and adjustment costs, etc.) and seem to be mainly driven by limited participation.

These results do not rely on idiosyncratic shocks, substantial borrowing by the stockholders, or binding borrowing constraints. Moreover, the concentration of aggregate *capital* income risk among a small group of households (stockholders)—which is the mechanism most commonly associated with limited participation—only has a modest contribution to the equity premium. Instead, most of the equity premium arises from the interaction of three factors, which results in the concentration of aggregate *labor* income risk among the stockholders. First, in response to labor income shocks the non-stockholders have to exclusively rely on the bond market for smoothing consumption, whereas the stockholders have the additional option of adjusting the level of capital stock. Second, because of the heterogeneity in the EIS, the non-stockholders have a stronger desire for consumption smoothing compared to the stockholders. The combination of these two factors implies that the non-stockholders need the bond market much more than the stockholders. However, and third, the bond market is not a very effective device for consumption smoothing in the face of *aggregate* risk, because it merely *reallocates* the risk rather than vanquishing it, as would be the case if shocks were idiosyncratic. As a result, the non-stockholders’ desire for smooth consumption is satisfied via trade in the bond market, at the expense of higher volatility in the stockholders’ consumption, who then demand a large premium for holding stocks. In Section 5.1 we provide a decomposition of the *average level* of the equity premium and quantify the contribution of this mechanism. We provide a similar decomposition in Section 5.2 for the *time-variation* in the equity premium to identify the source of countercyclicity in the premium.

An implication of the described mechanism is that the stockholders’ consumption growth is considerably more volatile than that of the non-stockholders. While the existing empirical evidence that we discuss in Section 7.1 suggests a higher volatility for the stockholders, the difference is not as large as that implied by the model. At the same time, this empirical evidence is based on micro data sets that contain little information on the richest one percent of U.S. households. Although one percent may seem small, these households own *half* the stock market as mentioned above, and the lack of data on this group makes a conclusive empirical statement on the stockholders’ consumption volatility difficult. For now this remains a microeconomic implication of the model that needs to be reconciled with the data.

In a recent paper Campbell and Cochrane (1999, hereafter CC) study a representative agent

exchange economy with a slow-moving external habit term in the utility function. After choosing the parameters of the habit process to match certain financial moments, they find that the model performs impressively in other dimensions as well. There is an interesting overlap between the results that these authors obtain and those we present in this paper. A natural question is whether these similarities point to a deeper connection between the two frameworks. Indeed, this appears to be the case: the limited participation model has a reduced form which is similar to Campbell and Cochrane’s framework in terms of asset pricing implications. In particular, the exogenous process for the habit stock in that framework corresponds to the consumption process of the non-stockholders in our model.

A simple way to see this point is by considering the Euler equation of the stockholders. Let X be the non-stockholders’ consumption, and C^A be aggregate consumption. Then, the stockholders’ consumption is: $C^h = C^A - X$, and we have:

$$E_t \left[\beta \left(\frac{C_{t+1}^A - X_{t+1}}{C_t^A - X_t} \right)^{-\alpha^h} \left(R_{t+1}^s - R_t^f \right) \right] = 0, \quad (1)$$

where α^h and β are the risk aversion parameter and time discount rate respectively, and R_{t+1}^s and R_t^f are the risky and risk-free rates. Now consider a representative-agent who has external habit preferences of the form $U = (C_t^A - X_t)^{1-\alpha^h} / (1 - \alpha^h)$, where we now let X_t denote the exogenous habit stock. It is clear that the same Euler equation above holds for this representative-agent. But now the part of aggregate consumption accounted for by the non-stockholders (X_t)—which is necessarily omitted in a representative-agent model since there is no limited participation—resurfaces as the habit process in the external habit model. Of course, this argument could only be true if the properties of X_t are very similar in the two models. In Section 6 we show that the statistical properties of the exogenous habit process assumed in CC are quite similar to those of the consumption of the non-stockholders in the current model. In a sense, Campbell and Cochrane’s goal was to identify the key properties that a stochastic discount factor must possess to be consistent with asset pricing facts. This paper can be viewed as a complementary effort to study a model which could potentially provide a microfoundation for the discount factor that these authors identified.

This interpretation also fits well with the seemingly contradictory empirical findings about habit formation from individual- and aggregate-data. Studies using aggregate data typically find evidence in favor of habit formation with large persistence (c.f., Ferson and Constantinides (1991)) whereas individual-level data has so far not revealed such behavior (Naik and Moore (1996); Dynan (2000)). Our findings suggest that even in the absence of strong habit formation at individual-level, it is possible for an economy with limited participation and heterogeneity in the EIS to display aggregate behavior which gives the appearance of external habit formation.

The similarities between the asset pricing implications of the two models should be interpreted with care. For example, this close correspondence does not extend to the macroeconomic behaviors of these models. In a separate paper (Güvenen (2003)) we study the implications of the limited participation model (without adjustment costs) for aggregate quantities and cross-sectional distri-

butions and find that it performs as well as the standard RBC model along several dimensions while improving upon it in certain directions. On the other hand, Lettau and Uhlig (2000) embed the habit specification employed by CC in an RBC model and find it difficult to be reconciled with certain macroeconomic facts. After considering a number of extensions, they conclude that “introducing habit formation in consumption and leisure yields counterfactual cyclical behavior in an otherwise standard real business cycle model.”

The paper is organized as follows. Section 2 introduces the model and the parametrization is discussed in Section 3. We then present the main asset pricing results, and discuss related work in Section 4. Section 5 examines the working of the model, and Section 6 analyzes the connection between this model and the external habit model. Section 7 discusses some microeconomic implications, and Section 8 presents extensions and sensitivity analyses. Section 9 concludes.

2 The Model

The model is an extension of the framework studied in Guvenen (2003). Our modeling goal is to stay as close to the standard real business cycle model as possible and only introduce two key features: limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution.

THE FIRM

There is an aggregate firm producing a single consumption good using capital (K_t) and labor (L_t) inputs according to a Cobb-Douglas technology, $Y_t = Z_t K_t^\theta L_t^{1-\theta}$, where $\theta \in (0, 1)$ is the factor share parameter. The logarithm of the stochastic technology level evolves as an AR(1) process:

$$\log(Z_{t+1}) = \rho_z \log(Z_t) + \varepsilon_{t+1}, \quad \varepsilon \sim N(0, \sigma_\varepsilon^2).$$

The goal of the firm’s managers is to maximize the value of the firm to the owners:

$$P_t^s = \underset{\{I_{t+j}, L_{t+j}\}}{\text{Max}} E_t \left[\sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} \left(Z_{t+j} K_{t+j}^\theta L_{t+j}^{1-\theta} - W_{t+j} L_{t+j} - I_{t+j} \right) \right] \quad (2)$$

subject to the technology constraint which features adjustment costs in investment:

$$K_{t+1} = (1 - \delta) K_t + \Phi \left(\frac{I_t}{K_t} \right) K_t, \quad (3)$$

where P_t^s is the ex-dividend value of the firm, and $\beta^j (\Lambda_{t+j}/\Lambda_t)$ is the discount rate (i.e., the marginal rate of substitution between periods t and $t + j$). The function $\Phi(\cdot)$ is assumed to be concave in investment, which captures the difficulty of quickly changing the level of capital installed in the firm. As a result of these adjustment costs, the price of installed capital is not necessarily equal to the price of the consumption good.

The firm is 100 percent equity financed as commonly assumed in the real business cycle litera-

ture. Abstracting from leverage allows us to focus exclusively on the effects of limited participation and preference heterogeneity. A share in this firm entitles its owner to the entire stream of future dividends given by $D_t = Z_t K_t^\theta L_t^{1-\theta} - W_t L_t - I_t$. The firm does not issue new shares and finances investment through retained earnings. For convenience we normalize the number of shares outstanding to unity so that P_t^s is also the stock price. Finally, competitive labor markets ensure that workers are paid their marginal product: $W_t = (1 - \theta) Z_t (K_t/L_t)^\theta$.

HOUSEHOLDS

The economy is populated by two types of agents who live forever. The population is constant, and is normalized to unity. Let $\mu \in (0, 1)$ denote the measure of the second type of agents (who will be called “stockholders” later). Both agents have time separable expected utility functions defined over future consumption streams: $E_t \left[\sum_{j=0}^{\infty} \beta^j U^i(C_{t+j}) \right]$, for $i = h, n$, where the superscripts h and n denote stockholders and non-stockholders respectively. Regarding the parameterization of the momentary utility function, we have two considerations in mind. On the one hand, we want to keep preferences simple to highlight the effect of limited participation and other endogenous features of the model. This suggests a standard CRRA utility function, $U^i(C) = C^{1-\alpha^i} / (1 - \alpha^i)$, and we adopt it through most of the paper. On the other hand, it is well-known that with this specification the parameter α^i controls both the relative risk aversion (RRA) and the elasticity of intertemporal substitution (EIS) which are different aspects of individuals’ tastes. In Section 8 we disentangle these two parameters using the recursive utility function of Epstein and Zin (1989), to illustrate the different roles played each parameter in the model.

Both agents have one unit of time endowment in each period, which they supply inelastically to the firm. Besides the stock of the firm, there is also a one-period riskless household bond (in zero net supply)² traded in this economy. The difference between the two groups is in their investment opportunity sets: the “non-stockholders” can freely trade the risk-free asset, but they are restricted from participating in the capital market. The “stockholders,” on the other hand, have access to both markets and hence are the sole capital owners in the economy. Following the incomplete markets literature we impose portfolio constraints as a convenient way to prevent Ponzi schemes.

Remark.—It is possible to think of the participation structure assumed here as an endogenous outcome of a model where there is a one-time fixed cost of entering the stock market. With a cost of appropriate magnitude, the group of agents with low risk aversion will enter the stock market whereas the other group will stay out. The resulting equilibrium is identical to the one studied here; see Guvenen (2003) for further discussion. In Section 7.2 we quantify the magnitude of this participation cost in our model.

INDIVIDUALS’ DYNAMIC PROBLEM AND THE EQUILIBRIUM

We study the recursive equilibrium of this economy in which the portfolio holdings of each group together with the exogenous technology shock constitute a sufficient state vector summarizing the relevant information for agents. In a given period, the portfolio of each group can be expressed in

²We allow for a positive supply of bonds in Section 8.

terms of the *beginning-of-period* capital stock, K , the aggregate bond holdings of non-stockholders *after* production, B , and the technology level, Z . Let Υ denote the aggregate state vector (K, B, Z) . The dynamic programming problem of a stockholder can be expressed as follows:

$$\begin{aligned}
V^h(\omega; \Upsilon) &= \max_{C, b', s'} \left\{ U^h(C) + \beta E \left[V^h(\omega'; \Upsilon') \mid Z \right] \right\} \\
&\quad \text{s.t.} \\
C + P^B(\Upsilon) b' + P^s(\Upsilon) s' &\leq \omega + W(K, Z) \\
\omega' &= b' + s' (P^s(\Upsilon') + D(\Upsilon')) \\
K' &= \Gamma_K(\Upsilon) \\
B' &= \Gamma_B(\Upsilon) \\
b' &\geq \underline{B}^h,
\end{aligned}$$

where ω denotes financial wealth; b' and s' are individual bond and stock holdings respectively; the endogenous functions Γ_K and Γ_B denote the laws of motion for the wealth distribution which are determined in equilibrium; and P^B is the equilibrium bond pricing function. Note that each type of agent is facing a constraint on bond holdings with possibly different (and negative) lower bounds. The problem of a non-stockholder can be written as above with $s' \equiv 0$, and the superscript h replaced with n .

A *stationary recursive competitive equilibrium* for this economy is given by a pair of value functions $V^i(\omega^i; \Upsilon)$, $i = h, n$; consumption and bond holding decision rules for each agent, $C^i(\omega^i; \Upsilon)$ and $b^{i'}(\omega^i; \Upsilon)$; a stockholding decision rule for the stockholder, $s'(\omega^h; \Upsilon)$; stock and bond pricing functions, $P^s(\Upsilon)$ and $P^B(\Upsilon)$; a competitive wage function, $W(K, Z)$; an investment function for the firm, $I(\Upsilon)$; laws of motion for aggregate capital and the aggregate bond holdings of non-stockholders, $\Gamma_K(\Upsilon)$, $\Gamma_B(\Upsilon)$; and a marginal utility process $\Lambda(\Upsilon)$, such that:

1) Given the pricing functions and the laws of motion, the value function and decision rules of each agent solve that agent's dynamic problem.

2) Given $W(K, Z)$ and the equilibrium discount rate process obtained from $\Lambda(\Upsilon)$, the investment function $I(\Upsilon)$ and the labor choice of the firm are optimal.

3) Asset markets clear: $\mu b^{h'}(\varpi^h; \Upsilon) + (1 - \mu) b^{n'}(\varpi^n; \Upsilon) = 0$; and $\mu s'(\varpi^h; \Upsilon) = 1$, where ϖ^i denotes the wealth of each type of agent in state Υ in equilibrium; and the labor market clears: $L = \mu \times 1 + (1 - \mu) \times 1 = 1$.

4) Aggregates result from individual behavior:

$$\begin{aligned}
K' &= (1 - \delta) K + \Phi(I(\Upsilon)/K) K, \quad \text{and} \quad B' = (1 - \mu) b^n(\varpi^n, \Upsilon), & (4) \\
\Lambda(\Upsilon) &= C^h(\varpi^h; \Upsilon)^{-\alpha^h}. & (5)
\end{aligned}$$

5) There exists an invariant probability measure \mathbf{P} defined over the ergodic set of equilibrium distributions.

Table 1: BASELINE PARAMETERIZATION

Quarterly Model		
Parameter		Value
β	Time discount rate	0.99
$1/\alpha^h$	EIS of stockholders	0.5
$1/\alpha^n$	EIS of non-stockholders	0.1
μ	Participation rate	0.2
ρ_z	Persistence of aggregate shock	0.95
σ_ε	Standard deviation of shock	0.02
θ	Capital share	0.3
ξ	Adjustment cost coefficient	0.23
δ	Depreciation rate	0.02
\underline{B}^h	Borrowing limit of stockholders	$16\overline{W}$
\underline{B}^n	Borrowing limit non-stockholders	$8\overline{W}$

Notes: The baseline model assumes CRRA utility functions for both agents implying that the relative risk aversion parameter is 2 for the stockholders and 10 for the non-stockholders. Borrowing limits are indexed to the average wage rate, \overline{W} .

3 Quantitative Analysis

We use numerical methods to solve for the equilibrium since an analytical solution is not available. The details of the computational algorithm as well as the accuracy of the solution and related issues are discussed in a computational appendix available from the author’s website.

BASELINE PARAMETERIZATION

A common method for calibrating general equilibrium asset pricing models is to choose a number of parameters to match certain financial statistics, such as the risk-free rate, the equity premium, the persistence of the price-dividend ratio, and so on. Then additional moments of the data serve as overidentifying restrictions to be examined. Instead, here we follow the real business cycle tradition and calibrate the parameters to replicate the long-run macroeconomic facts of the U.S. economy such as the average capital-output ratio, the persistence of the Solow residuals and so on.

Table 1 summarizes our baseline parameterization. The time period in the model corresponds to 3 months of calendar time. The capital share of output (θ) is set equal to 0.3, and the depreciation rate (δ) is set to 0.02. As for the technology shock, we match the persistence of the quarterly Solow residual, $\rho_z = 0.95$, and set the standard deviation of the innovation to 2 percent. Although this latter number is larger than the one reported in Cooley and Prescott (1995) for the post-war period, it is consistent with the estimates obtained by Christiano and Eichenbaum (1992), and the values used in Storesletten, Telmer and Yaron (2001), Danthine and Donaldson (2002) among others. In addition, we will compare the asset prices generated by the model to the U.S. data extending back to 1890. Since output and consumption were more volatile prior to World War II, a higher volatility for the Solow residual seems consistent with this focus. We discretize the AR(1) process for Z_t using a 12-state Markov process following Tauchen and Hussey’s (1991) method as described in Aiyagari

(1994). The approximation is fairly accurate, with an autocorrelation structure (from lag 1 to 5) which tracks that of the AR(1) process closely, and a standard deviation nearly equal to the true value (details are provided in the appendix).³

Following Jermann (1998), the functional form for Φ is specified as $a_1 (I_t/K_t)^{1-1/\xi} + a_2$, where a_1 and a_2 are constants chosen such that the steady state level of capital is invariant to ξ . The curvature parameter ξ determines the severity of adjustment costs. As ξ approaches infinity, Φ becomes linear, and investment is converted into capital one for one (frictionless economy limit). At the other extreme, as ξ approaches zero, Φ becomes a constant function, and the capital stock remains constant regardless of the investment level (exchange economy limit). We set $\xi = 0.23$, as in Jermann (1998) and Boldrin et al. (1999).⁴ This value is near the low end of the empirical estimates for this parameter (i.e., the elasticity of investment with respect to Tobin's q), so in Section 8 we also conduct sensitivity analysis with respect to it.

Participation Rates.—Our model assumes a constant participation rate in the stock market, which seems to be a reasonable approximation for the period before the 1990's when the participation rate was either stable or increasing gradually (Poterba and Samwick (1995, Table 7)). In contrast, during the 1990's participation has increased substantially: from 1989 to 2002 the number of households who owned stocks increased by 74 percent, and by 2002 half of the U.S. households had become stock owners (Investment Company Institute (2002)). Modeling the participation boom in this latter period would require going beyond the stationary structure of our model, so we leave it for future work. In this paper, we exclude this later period (1992–) both when calibrating the participation rate and when comparing the model to the data. We set the participation rate in the model, μ , to 20 percent, roughly corresponding to the average rate from 1962 to 1992 (a period during which participation data is available). Note that even during times when participation was higher, households in the top 20 percent have consistently owned more than 98 percent of all stocks (Poterba and Samwick (1995, Table 9)).

Borrowing constraints do not play an important economic role in our model (other than preventing Ponzi schemes). However, they are important for computational reasons: the bounds of the grid for B are determined by these constraints, so relaxing them expands the state space over which we need to solve for all the equilibrium functions. Thus, we choose these constraints to be as loose as possible subject to the condition that the model can still be solved without compromising accuracy. The resulting lower bounds are $\underline{B}^h = 16 \times E(W)$, and $\underline{B}^n = 8 \times E(W)$, which rarely bind in our simulation of the baseline model.⁵

Preference Parameters.—The subjective discount factor, β , is set equal to 0.99 in order to match the U.S. capital-output ratio of 2.5. We calibrate the curvature parameter α mainly based on the implied elasticity of intertemporal substitution. There is a large body of empirical work document-

³This appendix (which also contains sensitivity analyses) is made available on the author's website to save space here. Any reference in text to the appendix refers to this online document.

⁴Jermann (1998) chose ξ to improve his model's ability to match some financial moments although he also provided empirical evidence to support this choice. In this sense, there is a partial caveat to the statement above that we do not calibrate any parameter based on financial variables.

⁵Tightening the constraints to $\underline{B}^h = 12 \times E(W)$, and $\underline{B}^n = 2 \times E(W)$, had a negligible effect on the results.

ing heterogeneity in the EIS across the population (see for example, Blundell, Browning and Meghir (1994), Attanasio, Banks and Tanner (2002), and Vissing-Jorgensen (2002). See Guvenen (2003, section 2.1) for a more complete review of the empirical evidence.) In particular, these studies find that stockholders (and the wealthy in general) have a significantly higher elasticity of substitution than non-stockholders (and the poor in general). While it is not straightforward to aggregate the parameter estimates obtained in these different studies, it is reasonable to say that the estimates of stockholders' EIS are generally closer to unity, and those of non-stockholders are closer to zero. In our baseline specification, we set $EIS^n = 1/\alpha^n = 0.1$, but choose stockholders' elasticity to be somewhat lower than some empirical estimates: $EIS^h = 1/\alpha^h = 0.5$, so that the resulting risk aversion for this group is not very low. Notice also that with CRRA utility heterogeneity in the EIS implies heterogeneity in risk aversion. Interestingly, this heterogeneity plays no essential role in our results as we show in Section 8.

4 Model Results

4.1 The Unconditional Moments of Asset Prices

In this section we study the asset pricing implications of the baseline model. We begin by discussing the unconditional moments of stock and bond returns. Table 2 displays the statistics from the simulated model along with their empirical counterparts computed from the historical U.S. data taken from Campbell (1999). The stock return and the risk-free rate are calculated from Standard and Poor's 500 index and the 6-month commercial paper rate (bought in January and rolled over in July) respectively. All returns are real (except where indicated) and are obtained by deflating nominal returns with the consumption deflator series from the same data set. We draw two sub-samples for empirical analysis: The "long sample" corresponds to the period 1890–1991, and the "post-war sample" covers 1947–1991. The long sample has the advantage of providing more observations and consequently more precise estimates of the relevant statistics (especially since some of the variables we study—such as the price-dividend ratio—are very persistent). Moreover, this period includes the Great Depression years. Since one cannot rule out the repeat of such historical episodes, including this period into the sample is important for an accurate characterization of the average properties of asset prices. On the other hand, asset markets have changed significantly over the last century, suggesting that a focus on more recent data could be more relevant for understanding asset markets today. With these considerations in mind, we present empirical statistics from both sub-samples and discuss how our simulations compare to data from each period.

The Mean and Volatility of the Equity Premium.—In the long sample, the equity premium is 6.2 percent with a standard deviation of 19.4 percent yielding a Sharpe ratio of 0.32. The post-war sample provides a harder target with a higher equity premium of 7.2 percent and a lower volatility of 17.0 percent implying a larger Sharpe ratio of 0.42. One explanation for the higher premium in the latter period was suggested by McGrattan and Prescott (2001), who argued that about 1.8 percentage points of the annual return on equity was attributable to favorable changes in the tax

Table 2: THE FIRST TWO MOMENTS OF ASSET RETURNS

	US Data		Baseline Model		RBC
	Long Sample	Post-War	$\alpha^h = 2$	$\alpha^h = 4$	
Panel A: The Equity Premium and the Risk-free Rate					
$E(R^s - R^f)$	6.17	7.21	3.43	6.11	.004
$\sigma(R^s - R^f)$	19.4	17.0	17.2	22.4	0.27
$\frac{E(R^s - R^f)}{\sigma(R^s - R^f)}$	0.32	0.42	0.20	0.27	.014
$E(R^f)$	1.91	1.33	1.98	0.61	4.16
$\sigma(R^f)$	5.44	2.70	5.62	7.31	0.18
$\rho(R^s, R^f)$	-0.02	0.02	0.03	0.02	0.97
Panel B: The Price-Dividend Ratio					
$E(P^s/D)$	22.1	24.7	25.7	29.4	-
$\sigma(\log(P^s/D))$	26.3	27.2	20.1	30.5	-

Notes: The mean and standard deviation of variables are reported in annualized percentages. The data is from Campbell (1999) and covers 1890–1991. The equity return and the risk-free rate are calculated from the Standard and Poor’s 500 index and the commercial paper rate respectively. The RBC model has a single agent with stockholders’ preferences, and the same parameterization as the baseline, except that $\xi = \infty$.

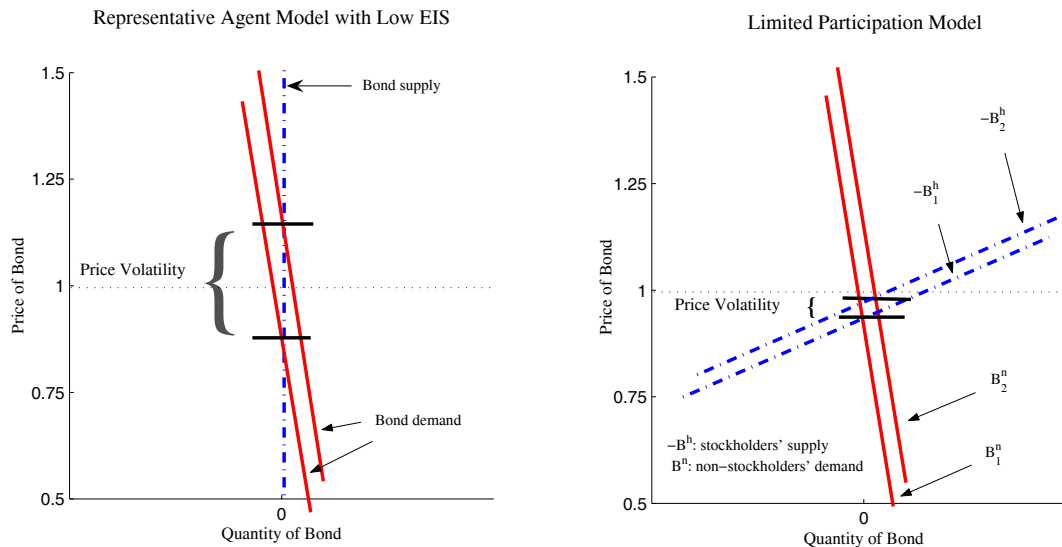
code—specifically to the reduction of dividend and corporate income taxes—from 1960 onward. Using this adjustment from 1960 to 1991 would reduce the equity premium in the post-war sample to 5.9 percent.

The baseline model generates an equity premium of 3.4 percent, which is about half the historical value. One way to increase this number further is by assuming a higher risk aversion for the stockholders: with $\alpha^h = 4$, the equity premium rises to 6.1 percent nearly matching the average value in the long sample. The standard deviation of excess returns is 17.2 percent in the baseline case, which is reasonably close to its empirical counterparts (19.4 and 17.0 percent in each period respectively), but increasing the risk aversion to 4 raises the standard deviation to 22.4 percent, making it somewhat too volatile.

The Sharpe ratio is 0.20 in the baseline model and rises to 0.27 when the risk aversion is 4. The price of risk can be increased further by choosing a larger α^h , and the resulting excessive volatility can be reduced by relaxing the adjustment costs (higher ξ) as we show in Section 8. However, even a curvature of 4 implies an EIS of 0.25 for the stockholders, which is arguably too low in light of the empirical work mentioned in Section 3. Thus, it seems preferable to keep α^h at its relatively low baseline value of 2, and attribute the difference between the Sharpe ratio in the data and the one implied by the model to factors not present in our model (such as idiosyncratic shocks, etc.). Finally, the correlation of stock returns and interest rates is close to zero in the U.S. data as well as in our simulations.

The Mean and Volatility of the Risk-Free Rate.—The average risk-free rate is 2.0 percent in the baseline model, similar to the 1.9 percent in the long sample, and slightly higher than the 1.3 percent in the post-war period. Increasing the risk aversion to 4 increases the equity premium and pushes the interest rate down to 0.6 percent.

Figure 1: DETERMINATION OF BOND PRICE VOLATILITY



A well-documented feature of the interest rate—and as it turns out, a challenging one to explain—is its low volatility. The standard deviation of the real rate is 5.4 percent in the long sample, but even lower, 2.7 percent, in the post-war period. Part of this variability is due to unexpected inflation, so one can alternatively look at the volatility of the nominal rate, which is 2.9 percent and 3.3 percent in the long and post-war samples respectively.

In our baseline model, the standard deviation of the risk-free rate is 5.6 percent. Although this number is close to its counterpart in the long sample, it is about double the volatility in the post-war period, and equally high compared to the variability of the nominal rate. However, models that match the equity premium often have difficulty generating even moderately smooth interest rates. For example, in Boldrin et al. (1999) the variability ranges from 17.4 to 25.4 percent depending on the specification. The volatility is somewhat lower in Jermann (1998), around 12 percent, partly because he assumes a higher risk aversion. Thus, while the interest rate is still too volatile in our model compared to data, it provides a step in the right direction. So, what explains the relatively low variability of interest rates in this model?

To understand the mechanism, consider the bond market diagram in Figure 1. The left panel depicts the case of a representative agent with a low EIS, which is a feature common to the models mentioned above. For example, both the endogenous and the external habit models imply a low EIS (despite differing in the risk aversion implications). In this framework, the interaction of the inelastic (steep) bond demand curve with a bond supply that is *perfectly inelastic* at zero (because of the representative-agent assumption) means that even small shocks to the demand curve will generate large movements in the bond price, and consequently, in the risk-free rate. On the other hand, in the limited participation model the mechanism is different. First, notice that the majority—eighty percent—of the population (the non-stockholders) have a very low EIS as before,

implying very inelastic bond demand (right panel). Turning to bond supply (it is convenient to denote the negative of the stockholders’ bond demand as “bond supply”) the key difference here is that it is not inelastic at all. In fact, the stockholders’ supply curve is rather flat both because of their high intertemporal elasticity ($EIS = 0.5$) and also because they have another asset to substitute for bond. As a result, a shock to the demand curve of similar magnitude as before now results in smaller fluctuations in the interest rate and the rest is reflected in the variability of trade volume.

The described mechanism also shows how the interest rate volatility can be reduced further. Notice from the diagram that, in addition to the shifts in the bond demand curve, interest rate volatility will also be affected by the shifts in the bond supply curve in response to aggregate shocks. One way to make the supply curve more stable is then to relax the adjustment costs faced by the firm. In this case, the level of capital can be adjusted more easily in response to shocks, allowing the stockholders to smooth their consumption without relying on the bond market. As a result the bond supply curve will move less over the business cycle, reducing the interest rate volatility. For example, setting $\xi = 0.5$, reduces the volatility to 4.2 percent, and setting $\xi = 0.8$ reduces it further to 3.4 percent. Relaxing adjustment costs also lowers the volatility of stock returns but has little impact on the dynamics of asset prices as we discuss in Section 8.

The Price-Dividend Ratio.—The average price-dividend ratio in the baseline model is around 25.7 and compares well with the post-war value of 24.7; but it is somewhat higher than the average of 22.1 in the longer sample. The volatility seems a little low, 20.1 percent compared to about 27 percent in the data. Increasing the risk aversion to 4, raises both the mean (29.4) and the volatility (30.4 percent) above their empirical counterparts.

4.2 The Dynamics of Asset Prices

COUNTER-CYCLICAL VARIATION IN CONDITIONAL MOMENTS

There is a large literature in finance on the dynamics of asset prices. These studies document that the price-dividend ratio is pro-cyclical, and exhibits co-movement with macroeconomic conditions not only at business cycle frequencies but also at longer horizons (Fama and French (1989)). Second, the expected equity premium and its conditional volatility both exhibit countercyclical variation over time (see Campbell and Shiller (1988), and Schwert (1989) among others). Moreover, expected excess returns move more than conditional volatility, so the ratio of the two—the conditional Sharpe ratio—is also countercyclical (Chou, Engle and Kane (1992)).

Table 3 displays the cyclical variation in the moments of asset prices.⁶ First, the log price-dividend ratio is strongly pro-cyclical in the baseline model ($corr : 0.90$). This is because when a (persistent) technology shock hits the economy, the stock price capitalizes all the future productivity gains upon impact and thus increases substantially, while the initial response of dividends is muted due to higher investment levels after the shock, making the ratio of the two variables pro-cyclical.

⁶The results presented in the rest of the paper pertain to the baseline model with $\alpha^h = 2$. However, the results are very similar when $\alpha^h = 4$, except for the few cases that we explicitly mention in the text.

Table 3: CYCLICAL BEHAVIOR OF THE EXPECTED EQUITY PREMIUM

	Cross-Correlations with Output		
	U.S. Data		Model
	Long Sample	Post-war	
$p_t - d_t$	0.15	0.42	0.90
$R_{t+1}^s - R_t^f$	0.22	0.15	0.28
$E_t(R_{t+1}^s - R_t^f)$	< 0	< 0	-0.57
$\sigma_t(R_{t+1}^s)$	< 0	< 0	-0.46
$\frac{E_t(R_{t+1}^s - R_t^f)}{\sigma_t(R_{t+1}^s)}$	< 0	< 0	-0.53

Notes: The trend of annual output is removed using a linear trend. Using a Hodrick-Prescott filter with a smoothing parameter of 100 increases the reported correlations slightly. The last three rows of columns 1 and 2 do not provide point estimates since a direct empirical measure of conditional moments is not available to calculate the correlations. The statistics from the model are also calculated from annualized returns.

Although the price-dividend ratio is also pro-cyclical in the U.S. data, the raw correlation with output is much weaker than in our model (*corr* : 0.15 and 0.42 in the long- and post-war samples respectively).

We next discuss the conditional moments of returns (rows 3 to 5). First, the model generates an expected equity premium and conditional volatility that are both countercyclical as in the data. Second, the movements in expected returns are large: the coefficient of variation is 0.35, and the 95 percent confidence interval extends from 0.6 percent to 4.7 percent in the baseline case (and from 1.5 percent to 7.9 percent when $\alpha^h = 4$). The conditional volatility also exhibits significant time variation: the coefficient of variation is 0.19 and the 95 percent confidence interval extends from 8.4 percent to 19.5 percent (and from 11.2 percent to 24.7 percent when $\alpha^h = 4$). The Sharpe ratio is also countercyclical (*corr* : -0.53), as could be anticipated from the fact that expected returns are more variable than conditional volatility. Finally, despite the countercyclical variation in expected returns, the realized equity premium is procyclical in the data, which is also the case in the model (*corr* : 0.28).

PREDICTABILITY OF RETURNS: LONG HORIZON REGRESSIONS

We first regress log stock returns on the log price-dividend ratio using the U.S. data (Table 4). The well-known pattern documented in the literature can be seen here: the coefficients are negative indicating that a high price-dividend ratio forecasts lower returns in the future. Moreover both the coefficients and the R^2 values are increasing with horizon and reach significant levels.

The model counterpart is reported in the last three columns. The coefficient estimates and the R^2 values are similar to empirical results: predictability is modest at one year horizon but increases steadily and reaches 50 percent at 10 year horizon. The coefficients also increase quickly first and then grow more slowly. Furthermore, in the empirical regression lagged dividend growth has almost no predictive power when included as an additional regressor (columns 3 and 6), which is also true in simulated data (column 9): the R^2 values remain virtually unchanged and the coefficients (not reported) are small and statistically insignificant.

Table 4: LONG-HORIZON REGRESSIONS ON PRICE-DIVIDEND RATIO

$r_{t,t+k}^s = a + slope \log(P_t^s/D_{t-1}) + \varepsilon_{t,t+k}$									
	U.S. Data						Model		
	Long Sample			Post-War					
Δd included?	No	No	Yes	No	No	Yes	No	No	Yes
Horizon (k)	<i>Slope</i>	R^2	R^2	<i>Slope</i>	R^2	R^2	<i>Slope</i>	R^2	R^2
1	-0.08	.06	.06	-0.10	.11	.11	-0.13	.12	.12
3	-0.16	.12	.13	-0.18	.31	.32	-0.30	.27	.27
5	-0.25	.21	.22	-0.33	.51	.51	-0.42	.38	.38
7	-0.34	.27	.28	-0.44	.57	.58	-0.49	.44	.44
10	-0.36	.39	.39	-0.64	.73	.73	-0.57	.51	.51

Notes: The coefficients for the regression when Δd is included are very similar to those with dividends left out and hence are not reported. $r_{t,t+k}^s$ is obtained by aggregating the log stock return over k years; k denotes the horizon in years.

Table 5: VARIANCE OF PRICE-DIVIDEND RATIO EXPLAINED BY FUTURE COVARIANCES

Variance explained by	U.S. Data		Model
	Long Sample	Post-War	
$-r_{t+j}^s$	101	137	134
Δd_{t+j}	-10	-31	-28

Notes: Each cell reports the percentage of variance explained by the corresponding variable (that is, $(\sum_{j=1}^{\infty} \gamma^j cov(p_t - d_t, x_{t+j}))/var(p_t - d_t)$ where x is Δd and $-r^s$ in each case). The formula is calculated using 15 lags (years) both in the data and in the model.

An alternative manifestation of return predictability is the excess volatility of stock prices. A simple way to see this is by first decomposing the variance of the log price-dividend ratio following Cochrane (1992). Defining $\gamma \equiv (\bar{P}^s/\bar{D})/(1 + (\bar{P}^s/\bar{D}))$ at the steady state values of P^s and D , we have:

$$var(p_t - d_t) \approx \sum_{j=1}^{\infty} \gamma^j cov(p_t - d_t, \Delta d_{t+j}) - \sum_{j=1}^{\infty} \gamma^j cov(p_t - d_t, r_{t+j}^s)$$

where r_t^s is the log stock return. Both in the U.S. data and in the model (Table 5) a substantial fraction of total volatility is accounted for by the covariance of the log price-dividend ratio with future returns and only a small component is explained by varying expectations of future dividend growth. Moreover, both autocovariances are negative, consistent with the idea that a high P^s/D ratio signals low dividend growth which in turn means low returns in the future.

AUTOCORRELATIONS AND CROSS-CORRELATIONS OF RETURNS

The autocorrelation structures of financial variables display a variety of patterns. For example, the price-dividend ratio is extremely persistent; the risk-free rate is persistent whereas the equity premium has no significant persistence and displays mild mean reversion. At the same time, the absolute value of the risky rate displays positive autocorrelation both at short- and long-horizons

Table 6: AUTOCORRELATION STRUCTURE OF KEY FINANCIAL VARIABLES

		Lag j (years)				
		1	2	3	5	7
		AUTOCORRELATION				
A.	$p - d$					
	Long sample	.79	.59	.52	.35	.32
	Post-war	.85	.69	.60	.23	.03
	Model	.79	.65	.56	.41	.31
B.	$r^s - r^f$					
	Long sample	.03	-.22	.08	-.14	.10
	Post-war	-.08	-.24	.19	.05	.05
	Model	-.04	-.04	-.03	-.02	-.02
C.	$\sum_{i=1}^j \rho[(r^s - r^f)_t, (r^s - r^f)_{t-i}]$					
	Long sample	.03	-.19	-.11	-.29	-.15
	Post-war	-.08	-.31	-.12	-.07	-.02
	Model	-.04	-.08	-.12	-.16	-.18
D.	r^f					
	Long sample (R^*)	.53	.36	.23	.14	.15
	Post-war	.52	.24	.36	.07	-.05
	Long sample (N^*)	.83	.73	.69	.60	.57
	Post-war	.84	.66	.59	.47	.43
	Model	.85	.68	.56	.37	.26
E.	$ r^s $					
	Long sample	.13	.09	.06	.14	.15
	Post war	.03	-.28	.06	-.10	-.16
	Model	.06	.05	.03	.03	.03

Notes: The rows denoted by R^* and N^* report the correlation for real and nominal interest rates respectively. Statistics from the model are calculated from annualized values of each variable, but the stock price is taken to be the value at the end of the year.

indicating clustering of volatility. It is of interest to see what aspects of these autocorrelation structures are captured by the model.

First, the price-dividend ratio is highly persistent in the model, with an annual first order autocorrelation of 0.79. Higher order autocorrelations decay slowly, and track their empirical counterparts in the long sample quite well (Table 6). The persistence is slightly higher in the post-war sample up to the fourth lag, but then falls off abruptly.

Second, the autocorrelations of the equity premium are small and negative in the model, indicating weak mean reversion. Although the general pattern in the data indicates weak mean reversion, the autocorrelations are not uniformly negative. However, because these autocorrelations are close to zero, they are not precisely measured given the modest number of annual observations, so an alternative statistic used in the literature aggregates consecutive autocorrelation coefficients. The sum of these autocorrelations (panel C) shows a stronger pattern of mean reversion in the data, and the simulated data is consistent with the signs and rough magnitudes of these statistics.

Third, the interest rate is highly persistent in the model, with a first order autocorrelation of 0.84, and higher order terms that decay faster than an AR(1) process. Measuring the empirical counterpart is somewhat tricky because in reality bonds are only nominally risk-free due to unanticipated inflation. Using the ex-post real rate is one possible approach (R^* in panel D) but the autocorrelation structure calculated this way is downward biased because of unanticipated inflation. An alternative option is to use the nominal rate which might be a better indicator of the risk-free rate investors anticipate (N^* in panel D). This series is significantly more persistent, with slowly decaying autocorrelations. The model counterpart is closer to the empirical values at the first three lags, but then falls faster than in the data at longer horizons.

Fourth, a well-documented finding is the persistence of stock return volatility: high volatility is typically followed by more volatility. In the long sample, absolute returns are positively autocorrelated even at long horizons consistent with persistent volatility. On the other hand, the post-war sample reveals some rather large negative autocorrelations. Campbell and Cochrane (1999) also note these negative values and suggest that it may be due to sampling error in a small sample. The model generates absolute returns that are positively correlated at short and long horizons. In addition, one can compute the conditional volatility of stock returns ($\sigma_t(r_{t+1}^s)$) explicitly in the model, which also reveals persistent volatility: its first order autocorrelation is 0.84, and decays slowly; the tenth order autocorrelation is still above 0.30.

4.3 Relation to the Literature

There is a vast literature on the asset pricing puzzles addressed in this paper, and it is not possible to do justice to all the work in the field in this limited space. For detailed surveys of the literature see Kocherlakota (1996) and Campbell (1999). This paper is more closely related to the strand of literature which emphasizes the role of limited participation starting with Mankiw and Zeldes (1991). Saito (1995), and Basak and Cuoco (1998) study general equilibrium models with limited participation in the stock market. To our knowledge, Saito (1995) is the first to draw attention to a possible link between limited participation and habit persistence based on equation (1), though he did not pursue that relation further in his paper. Also, that link is not as strong in his model due to the absence of preference heterogeneity and labor income risk which play essential roles in generating our results. In a recent paper, Guo (2002) studies an exchange economy model with limited participation which generates a number of asset pricing facts. Those results however also rely on large income shocks to each group (36 percent per year) in addition to aggregate shocks, as well as frequently binding borrowing constraints. In an interesting paper, Danthine and Donaldson (2002) study asset prices in a two-agent macro model with an entrepreneur and a worker (where the worker lives hand-to-mouth) and emphasize the role of the operational leverage introduced by labor contracts. Finally, taking slightly different econometric approaches, Attanasio et al. (2002) and Brav et al. (2002) find that Euler equations are less likely to be rejected when only stockholders' consumption is used instead of aggregate consumption. Our paper complements this work by presenting a model that generates the consumption process of stockholders as a general equilibrium

outcome, which in turn generates the asset pricing phenomena examined.

5 Understanding the Sources of Equity Premium

The large and countercyclical equity premium arises from the interaction of (i) limited participation; (ii) preference heterogeneity; and (iii) precautionary savings in response to persistent (aggregate) labor income shocks. The last two components are typically absent from standard models of limited participation (Saito (1995), Basak and Cuoco (1998), and Guo (2002)) and are essential for the results of this paper. In the next subsection we discuss the mechanism generating a high *average* price of risk. Then, in Section 5.2 we address why the price of risk varies (countercyclically) over time.

5.1 The Average Level of Equity Premium

An intuitive explanation of the basic mechanism for the average price of risk is as follows. First, limited participation amplifies the effects of aggregate risk by concentrating capital income among a small fraction of the population. This effect does not depend on (ii) or (iii). Nevertheless, as we shall see, this channel only has a modest contribution to the equity premium for plausible parameterizations of the model.

The second and *major effect* of limited participation works through the bond market, and is a combination of three factors, which reinforce each other. *First*, limited participation creates an asymmetry in consumption smoothing opportunities: facing persistent (aggregate) labor income shocks, the non-stockholders have to exclusively rely on the bond market whereas the stockholders have the additional option of adjusting their stock holdings in response to shocks. *Second*, because of the heterogeneity in the EIS, the non-stockholders have a much stronger desire for a smooth consumption process compared to the stockholders who tolerate fluctuations better. The combination of these two effects imply that the non-stockholders need the bond market much more than the stockholders, which is reflected in the different bond demand elasticities in Figure 1. However, and *third*, the bond market is not a very effective device for consumption smoothing in the face of aggregate risk, because it merely shifts the risk around rather than reducing it, as would be the case if shocks were idiosyncratic.⁷ In equilibrium, the non-stockholders' desire for smooth consumption is satisfied via trade in the bond market, at the expense of higher volatility in the stockholders' consumption. Moreover, since these large fluctuations in the stockholders' consumption are procyclical, they are reluctant to own the shares of the aggregate firm that performs well in booms and poorly in recessions. As a result, they demand a high equity premium. In the rest of this

⁷This distinction is important: in models with idiosyncratic shocks (c.f., Heaton and Lucas (1996)), the two-agent assumption implies that idiosyncratic shocks are perfectly negatively correlated and thus can be virtually eliminated by trading in the bond market. In this case, trade in the bond market can reduce both agents' consumption volatility. In contrast, here income risk arises from aggregate shocks, and the bond market merely *reallocates* this aggregate risk to the agent who is more willing to bear it. As a result, smoothing one agent's consumption comes at the expense of extra volatility in the other agent's consumption.

Table 7: DECOMPOSITION OF CONSUMPTION VOLATILITY AND EQUITY PREMIUM

	Fraction of mean consumption explained by:		
	A^i	a^i	
Stockholders	1.016	-0.016	
Non-stockholders	0.993	0.007	
	Fraction of consumption growth volatility explained by:		
	$\sigma^2(\Delta \log A^i)$	$\sigma^2(\Delta \frac{a^i}{A^i})$	$2\sigma(\Delta \log A^i, -\Delta \frac{a^i}{A^i})$
Stockholders	0.214	0.293	0.495
Non-stockholders	3.30	0.68	-2.99

Note: i=h,n. See the text for the definitions of a^i and A^i .

subsection, we quantify the effect of this mechanism on the equity premium.

It is instructive to begin from the Hansen-Jagannathan inequality which provides a lower bound for the volatility of stockholder's intertemporal MRS, $m \equiv \beta (C_{t+1}^h / C_t^h)^{-\alpha^h}$:

$$\frac{E(R^s - R^f)}{\sigma(R^s - R^f)} \leq \frac{\sigma(m)}{E(m)} \approx \alpha^h \sigma(\Delta c^h) \quad (6)$$

where $c^h = \log C^h$, and the second approximation holds assuming that the consumption growth of the stockholders is log-normal. A large equity premium is then possible with a low risk aversion only if the stockholders' consumption growth is volatile, which is indeed the case: $\sigma(\Delta c^h)$ is 4.8 percent quarterly, generating a quarterly Sharpe ratio of about 10 percent (and $\sqrt{4} \times 0.10 \approx 0.20$ annually). In contrast, this volatility is 1.0 percent for the non-stockholders, and 2.1 percent in a representative-agent model with the same parameterization as in the baseline economy. So, the key to understanding the large equity premium is to understand the sources of the stockholders' consumption growth volatility. And, hence, the main question is: why are the stockholders willing to bear extra fluctuations in their consumption in this environment?⁸

To this end, we use the budget constraint of a stockholder (and substitute the equilibrium conditions, $s' = 1/\mu$, and $b^h = -B/\mu$) to write:

$$C^h = (W + D/\mu) - (B - P^B B') / \mu. \quad (7)$$

Similarly, $X = W + (B - P^B B') / (1 - \mu)$. These expressions provide a useful decomposition: variation in consumption growth may come from *aggregate sources*, such as fluctuations in wage and dividend income (denote $A^h \equiv W + D/\mu$), and from trade in the bond market (denote the *net debt payments* of the stockholders as $a^h \equiv (B - P^B B') / \mu$.) The concentration of aggregate risk is reflected in the scaling factor in D/μ , and the bond market channel is captured by a^h . Notice also that the volatility of stock prices plays no direct role in consumption volatility since in equilibrium

⁸The empirical evidence on stockholders' consumption volatility is discussed in Section 7.

Figure 2: THE EVOLUTION OF THE WEALTH DISTRIBUTION OVER TIME



the stockholders always hold all shares outstanding. Using equation (7), it is easily shown that the stockholders' consumption growth volatility is

$$\text{var} \left(\Delta \log C^h \right) \approx \text{var}(\Delta \log A^h) + \text{var} \left(\Delta \frac{a^h}{A^h} \right) + 2\text{cov} \left(\Delta \log A^h, -\Delta \frac{a^h}{A^h} \right), \quad (8)$$

where we use the approximation $\log(1 + a^h/A^h) \approx a^h/A^h$, noting that this ratio is equal to 0.017 on average with a standard deviation of 0.045. The variance of consumption growth for the non-stockholders can be obtained by replacing A^h and a^h in equation (8) with $A^n \equiv W$, and $a^n \equiv -a^h\mu/(1 - \mu)$ respectively.

Table 7 displays the fraction of variability explained by each of the three terms in equation (8). For the stockholders, only 22 percent of consumption growth variance is attributable to fluctuations in aggregate income ($\text{var}(\Delta \log A^h)$), despite the fact that this component makes up nearly all of their average consumption (row 1). Hence, the concentration of aggregate *capital* income risk, included in A^h , contributes only modestly to consumption fluctuations, and consequently to the equity premium. The main source of volatility for the stockholders comes from the bond market: debt payments made from the stockholders to the non-stockholders, a^h , account for the remaining three-quarters of variance ($0.293 + 0.495$) despite only making up less than 2 percent of average consumption. What is really crucial for this extra volatility is the *timing* of trade: $\text{corr}(\Delta \log A^h, \Delta a^h) = -0.991$, which means that the payments received by the non-stockholders increase exactly when aggregate income falls. This consumption smoothing for the non-stockholders comes at the expense of large fluctuations in the stockholders' consumption, so the covariance term in the third column accounts for half of the total volatility. The flip side of this story is seen in the variance of the non-stockholders: $\text{var}(\Delta x)$ would be 3.3 times higher, were it not for the consumption smoothing provided by the bond market.

Finally, since households continually adjust their asset holdings to smooth consumption fluctuations, it is of interest to ask if the amount of trade in the bond market, and the resulting shifts

in the wealth distribution, are quantitatively plausible. First, fluctuations in the amount of trade in the bond market are quite modest: the standard deviation of the trade volume as a fraction of average bond holdings, $\sigma(B - P^B B') / E(B)$, is 1.3 percent. (Using a slightly different notion of trade volume we have: $\sigma(\Delta B) / E(B) = 0.4$ percent). Second, Figure 2 plots the evolution of the wealth distribution over time, which again shows rather gradual shifts in the wealth shares of each group. These rather modest changes in *aggregate* bond holdings, B , entail large movements in the stockholders' consumption growth because the *per-capita* debt held by these households is large: B/μ is about 6.2 times the stockholders' quarterly consumption (and 1.6 times their annual consumption).

5.2 Countercyclical Variation in Equity Premium

We begin with an intuitive explanation of the basic mechanism. A typical feature of precautionary savings behavior is that agents accumulate wealth gradually in good times, but reduce it as quickly as necessary to prevent a large fall in consumption during downturns.⁹ In the current model, since the non-stockholders can only adjust their wealth through the bond market, this asymmetry in behavior results in *more trade* in the risk-free asset during recessions. This can be seen in the average size of trade (defined as the absolute value of net payments made from one agent to the other, $|a_t^h|$), which is countercyclical: $\rho(|a_t^h|, Y_t) = -0.51$. The discussion in the previous section would then suggest that this increase in trade raises the consumption growth volatility of the stockholders further in recessions, which in turn generates countercyclical variation in the moments of asset prices.

It is useful to quantify the effect of the described mechanism on the variation in the Sharpe ratio. First, note that the Hansen-Jagannathan bound (6) holds approximately as an equality since consumption growth is very highly correlated with excess returns, and this correlation changes very little over time. So, the Sharpe ratio is approximately equal to $\alpha^h \sigma_t(\Delta c^h)$, and since α^h is also constant the only time variation can come from changes in the conditional volatility term. Not surprisingly then, the Sharpe ratio increases by 46 percent from booms¹⁰ to busts, which is nearly matched by a 44 percent increase in the stockholders' consumption growth volatility between these two states. (Table 8 reports the increase in the *variance* of consumption growth, which goes up by $(1 + 0.44)^2 - 1 = 108$ percent.) In contrast, the non-stockholders' volatility increases only by 4 percent during busts.

It is convenient to decompose the stockholders' consumption growth volatility as in equation 8, but without using the approximation $\log(1 + a^h/A^h) \approx a^h/A^h$, which is not accurate for the present calculation. In the last three columns, each entry reports the *percentage increase* in $\text{var}(\Delta c^h)$ from a boom to a bust that is due to the increase in the term at the top of the column (more precisely, the second column for example reports: $(100 \times (\text{var}_{bust}(\Delta \log A^h) - \text{var}_{boom}(\Delta \log A^h)) / \text{var}_{boom}(\Delta c^h))$,

⁹The main reason for this asymmetric behavior is that, with incomplete markets, the interest rate is below the time preference rate, so accumulating precautionary wealth to smooth *future* consumption is costly in terms of foregone *current* consumption.

¹⁰We define a boom (bust) as states when output is one standard deviation or more above (below) its mean.

Table 8: THE SOURCE OF CYCICAL VARIATION IN EXPECTED RETURNS

100 × Change in “Statistic” from Boom to Bust as a fraction of $var_{boom}(\Delta c^h)$			
Statistic:	$\tilde{\Delta} [var(\Delta c^h)]$	=	$\tilde{\Delta} [var(\Delta \log A^h)]$ + $\tilde{\Delta} [var(\Delta \log(1 + a^h/A^h))]$ + $\tilde{\Delta} [2cov]$
	108.3		9.2 49.0 50.2

A “boom” (bust) is defined as states when output is one standard deviation above (below) its mean. For any statistic M , we use $\tilde{\Delta}[M]$ to denote $100 \times (M_{bust} - M_{boom}) / var_{boom}(\Delta c^h)$. With this definition, and using equation (7) it is easily shown that the terms in the last three column add up to $100 \times [var_{bust}(\Delta c^h) - var_{boom}(\Delta c^h)] / var_{boom}(\Delta c^h)$ reported in column 1. In the last column, “cov” is the covariance of the arguments in columns 2 and 3.

and others are defined analogously.) It can easily be shown that the sum of the three entries gives the total increase in consumption growth volatility reported in column 1. As can be seen in Table 8, the change in aggregate volatility, $var(\Delta \log A^h)$, explains only 9 percent of the increase in variance of stockholders’ consumption growth. On the other hand, the increase in the volatility of trade in the bond market during busts—captured by the terms in the last two columns—explains a substantial fraction (99 percent out of the total 108 percent) of the rise in consumption volatility.

6 Habit versus Limited Participation: What is the Connection?

There are some interesting parallels between the asset pricing results of this paper and those obtained by Campbell and Cochrane (1999).¹¹ A natural question to ask then is whether these similarities point to a deeper connection between the two frameworks. To address this question, first recall that the stockholders’ Euler equations can be combined to get:

$$E_t \left[\beta \left(\frac{C_{t+1}^A - X_{t+1}}{C_t^A - X_t} \right)^{-\alpha^h} \left(R_{t+1}^s - R_t^f \right) \right] = 0, \quad (9)$$

where the stockholders’ consumption is written as $C^A - X$. As mentioned in the Introduction, this is the same Euler equation that would be implied by a representative-agent model with external habit preferences, if X is reinterpreted as an exogenous habit stock. Now suppose that, due to limited participation, asset prices are in fact determined by condition (9). In order to explain asset prices in a representative-agent framework, one would need to subtract an amount equal to the non-stockholders’ consumption (X) from the aggregate (C^A) every period to mimic the true Euler equation. Thus omitting limited participation would make it look as if the representative-agent was displaying habit persistence. Clearly, for this to be the case the properties of X_t must be very similar in the two models.

To this end, we define $S_t \equiv (C_t^A - X_t) / C_t^A$, which measures the fraction of consumption above

¹¹In an earlier version of this paper (Guenen (2002), available on the author’s website), we reported a broader set of asset pricing results that are common to both models. We exclude those further results from the current version to save space.

the habit level—called the “surplus consumption ratio”—in the external habit model. In the current framework the same ratio corresponds to the fraction of aggregate consumption accounted for by the stockholders. Now manipulating (9) we obtain:

$$E_t \left[\beta \left(\frac{S_{t+1}}{S_t} \right)^{-\alpha^h} \left(\frac{C_{t+1}^A}{C_t^A} \right)^{-\alpha^h} \left(R_{t+1}^s - R_t^f \right) \right] = 0.$$

This alternative expression also holds in both frameworks, and can be viewed as adding an extra state variable, S_t , to an otherwise standard Euler equation, which is known to have poor asset pricing implications (as in Mehra and Prescott (1985)). Thus, for the success of either model the properties of S_t is key. Stressing this central role CC introduce an AR(1) process for $s_t \equiv \log(S_t)$ with a rich heteroskedastic shock structure:

$$s_{t+1} = (1 - \phi) \bar{s} + \phi s_t + \lambda(s_t) (c_{t+1}^A - c_t^A), \quad (10)$$

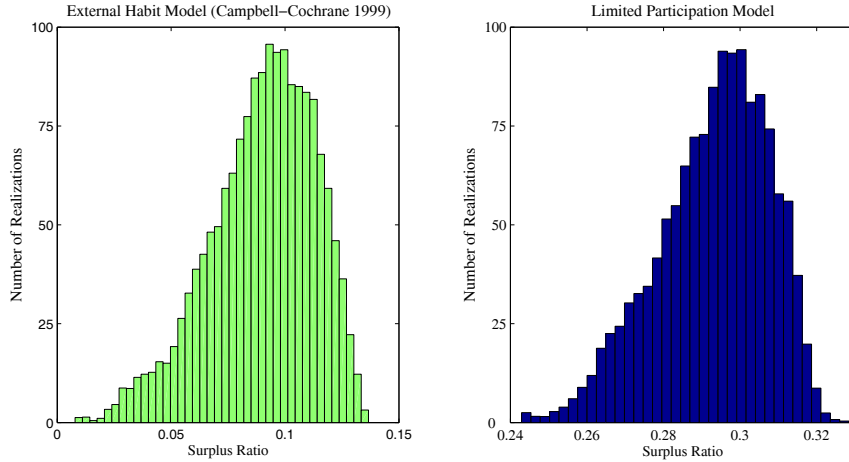
where $c_t^A \equiv \log(C_t^A)$, and choose the parameter value for ϕ and the functional form for $\lambda(s_t)$ to match certain features of asset prices.

To examine the connection between the two models, we compare the statistical properties of S_t as well as the features of $\lambda(s_t)$ assumed in CC with those implied by our model. First, we examine the cyclical behavior of S_t in each framework. In the external habit model, the habit stock evolves very slowly and lags behind actual consumption, making the surplus consumption ratio strongly pro-cyclical. In our model S_t (now, the stockholders’ share of consumption) is also procyclical, with a correlation of 0.92 with output. This correlation is primarily due to preference heterogeneity: the stockholders have a higher EIS, so their consumption rises more than that of the non-stockholders in response to a positive shock, increasing their consumption share in good times. For example, if $\alpha^h = \alpha^n = 2$, the correlation of S_t with output becomes -0.59 .

Second, we compare the densities of S_t displayed in Figure 3. The one in the left panel is obtained by simulating the AR(1) process above (eq. 10) with the parameter choices in CC,¹² and the one in the right panel is the empirical density of (C^h/C^A) in our model. Considering how differently the two ratios are generated, the densities appear quite similar to each other: both of them are negatively skewed, with modes near the upper bound of their respective supports. However, one difference between the two distributions is in their unconditional means: the average surplus ratio is 0.09 in CC (with variable interest rate), compared to 0.29 in the limited participation model. One way to reduce S_t in the our model is by assuming a lower participation rate: for example, setting $\mu = 0.10$, implies an average value of 0.19 for S_t . Thus, one interpretation is that the external habit model corresponds to the limited participation model where the number of stockholders, and

¹²Although the interest rate is constant in the baseline parameterization of CC, it is easy to extend their model (by adding one more parameter) to allow for a variable interest rate (see page 214 of their paper for details). To make the comparison of the two models meaningful, we calibrate this parameter to generate an annual interest rate volatility of 5.6 percent to match the corresponding figure in our model. This extension modifies some of the equations in the external habit model including those that determine $\lambda(s_t)$, and consequently, S_t .

Figure 3: THE EMPIRICAL FREQUENCY DISTRIBUTION OF S_t IN EACH MODEL



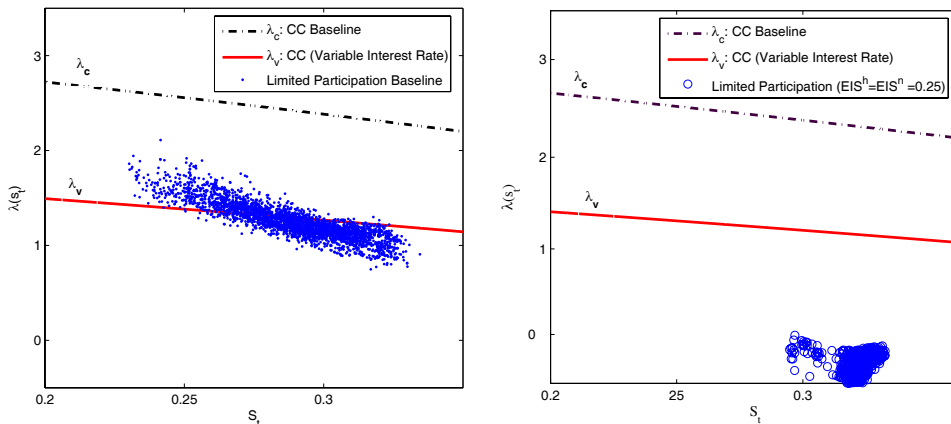
consequently their share in aggregate consumption, is small. (The results of the simulation with $\mu = 0.10$ are reported in the additional appendix.)

Third, the persistence parameter ϕ is calibrated in CC to match the autocorrelation structure of the price-dividend ratio. Their parameter choice implies a quarterly persistence of 0.96 for S_t in their model, compared to a value of 0.951 for (C^h/C^A) in our model.

Fourth, the specification of $\lambda(s_t)$ is central to the external habit model. One key feature of λ is that it is a decreasing function of s_t , which is responsible for the low volatility of interest rates and the countercyclical variation in the price of risk in that model. To obtain the counterpart of λ in our framework, we substitute the time-series of s_t and c_t^A simulated from our baseline model into equation (10), and back out the implied sensitivity function, which we call $\hat{\lambda}(s_t)$. Notice that there is nothing in our assumptions that mechanically relates the level or the slope of this function across the two models. The left panel in figure 4 plots the sensitivity function from the CC model with a constant interest rate (denoted λ_c) and a variable interest rate (denoted λ_v) together with $\hat{\lambda}$ (the cloud of points) obtained from our baseline model. The function $\hat{\lambda}$ is also downward sloping, but is somewhat steeper than in the habit model. The average level of $\hat{\lambda}$ is nearly the same as λ_v (but lower than λ_c), implying from equation (10) that the response of the surplus ratio to a change in c_t^A is similar in the two models. In contrast, if preference heterogeneity is eliminated, the sensitivity function implied by the limited participation model (the “circles” in the right panel) becomes close to zero and displays no discernible pattern.

Finally, instead of the surplus ratio we can directly compare X_t in each model. In the external habit model X_t evolves slowly, and in particular, it is much less volatile than aggregate consumption. Similarly, non-stockholders’ consumption is very smooth compared to aggregate consumption in our model: $\sigma^2(\Delta c^A)/\sigma^2(\Delta x) = 4.3$. Moreover, CC specify their model so that $C^A - X > 0$ everywhere to make the power utility function well-defined. This is also true in the current framework since stockholders’ consumption ($C^A - X$) is always positive.

Figure 4: A COMPARISON OF $\lambda(s_t)$ ACROSS THE TWO MODELS



Taken together, these comparisons show that the consumption process of the non-stockholders in our model and the habit process in the external habit model share many similarities, which can explain—through equation (9)—why asset prices display similar behavior in the two models.

One important difference between the two models, however, is in the risk aversion of the agents who price the stocks. To see this point, note that with external habit preferences, the RRA parameter is obtained by differentiating all expressions with respect to C_t^A :

$$RRA_t = -\omega \frac{V_{\omega\omega}}{V_\omega} = -C_t^A \frac{U_{c^A c^A}}{U_{c^A}} \psi_t = \frac{\alpha C_t^A}{C_t^A - X_t} \psi_t = \frac{\alpha}{S_t} \psi_t,$$

where ψ_t is the elasticity of consumption with respect to financial wealth. With CC's parameterization $\frac{\alpha}{S_t}$ is approximately 40 at steady state and ψ_t is around 2, resulting in an average risk aversion of 80. On the other hand, with limited participation, the Euler equation (9) only holds for the stockholders, who consume $C_t^h = C_t^A - X_t$, rather than C_t^A . Consequently, taking all derivatives with respect to C_t^h we obtain

$$RRA_t^h = -(C_t^A - X_t) \frac{U_{c^h c^h}}{U_{c^h}} \psi_t = \alpha \psi_t \approx \alpha$$

because ψ_t is around 1 in our framework. Thus in the baseline model $RRA_t^h \approx \alpha = 2$ independent of X_t .

7 Some Microeconomic Implications

7.1 Consumption Volatility

All existing studies that seek to explain the equity premium puzzle by appealing to limited participation require stockholders' consumption to be very volatile, and this paper is no exception. In

Table 9: VOLATILITY OF CONSUMPTION GROWTH IN THE LIMITED PARTICIPATION MODEL

	$\alpha^h = 2$		$\alpha^h = 4$	
	Q	A	Q	A
Stockholders	4.8	7.2	3.4	5.1
Non-stockholders	1.0	1.5	1.3	2.0

Notes: Q refers to quarterly, and A refers to annual statistics, all reported in percentages. Annual consumption growth volatility is calculated by first time-aggregating consumption over four consecutive periods, as is done with actual data.

the baseline model the stockholders' consumption growth volatility is 4.8 percent quarterly, and 7.2 percent annually. (Notice that the annual volatility is lower than $0.048 \times \sqrt{4} = 0.096$ because time-averaging over four consecutive quarters reduces variability in annual data). The corresponding volatility figures for the non-stockholders are lower: 1.0 percent quarterly and 1.5 percent annually. Thus the stockholders' volatility is roughly 5 times higher than that of the non-stockholders.

Turning to empirical evidence, Mankiw and Zeldes (1991) report $\sigma(\Delta c^h) / \sigma(\Delta x) \approx 1.6$, although their consumption measure consists of only food expenditures from the Panel Study of Income Dynamics (PSID). Attanasio, Banks and Tanner (2002) use expenditures on non-durables and services from the U.K. Family Expenditure Survey and calculate stockholders' volatility to be 1.5 to 2 times larger than non-stockholders'. (Moreover, unlike in Mankiw and Zeldes these volatilities are adjusted to take account of the variance induced by sampling error and differences in cell size, which is important since there are significantly fewer stockholders than non-stockholders in these samples.) While this evidence indicates that stockholders have more variable consumption growth than non-stockholders, the difference is not as large as that implied by our model. In this sense, the stockholders' volatility in the baseline model is too high compared to the available data.

One possible way to bring the model closer to data is by assuming a higher risk aversion for the stockholders (Table 9). With $\alpha^h = 4$, the stockholders' annual volatility falls to 5.1 percent (and the ratio of volatilities falls from 4.8 to 2.6). As noted earlier, the model's asset pricing implications are not negatively affected by this change. Although this figure is probably still high compared to the data, quantifying the size of the discrepancy is not very easy. The reason is that existing measures of stockholders' consumption (including those mentioned above) are based on micro data sets that contain few "extremely rich" households. For example, Juster et al. (1999) report that the richest one percent in the PSID data set have less than *one-tenth* of the wealth of the richest one percent in the U.S., because rich households typically choose not to participate in these surveys. But at the same time, the richest one percent own nearly *half* of all stocks, and the top 0.5 percent own 37 percent of all stocks (Poterba and Samwick (1995)), and the lack of reliable information on the consumption of these households makes a definitive empirical statement about stockholders' consumption volatility difficult.

To sum up, more work is needed to reconcile the model with the data along this key dimension. It is of interest to see if the model could be extended to generate a consumption process for the stockholders that is less volatile, while preserving its other time-series properties that delivers

plausible asset pricing implications; or if alternative ways to measure the consumption of the very rich would uncover a higher volatility for the stockholders in the data.

7.2 Quantifying the Participation Cost

We noted earlier that limited participation in the stock market could be supported as an endogenous outcome with a fixed participation cost of appropriate magnitude. Now we quantify the size of this cost. We first calculate the one-time fixed cost, $\tau^F(\Upsilon_0)$, that solves

$$V^n(\omega_0 - \tau^F(\Upsilon_0); \Upsilon_0) = V^{n*}(\omega_0; \Upsilon_0),$$

where ω_0 is the financial wealth of the non-stockholder in aggregate state Υ_0 , and V^{n*} is the value function after entering the stock market. The maximum value of τ^F over the states (in the ergodic set) measures the size of the fixed cost needed to keep current non-stockholders out of the stock market indefinitely. This number is equal to 2.46 times a non-stockholders' average annual income.

An alternative approach is to calculate the welfare gain from participation, expressed as the fraction of consumption a non-stockholder would be willing to give up at every date and state to become a stockholder. If the current aggregate state is Υ_0 , the proportional welfare gain, $\tau^P(\Upsilon_0)$, solves the following equality:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U^n(X_t) \right] = E_0 \left[\sum_{t=0}^{\infty} \beta^t U^n(C_t^{n*} (1 - \tau^P(\Upsilon_0))) \right],$$

where C_t^{n*} is the agent's consumption after entering the stock market. We assume that the agent's wealth at the time of entering the stock market is equal to the average wealth of a non-stockholder in state Υ_0 . The maximum welfare gain (over all possible Υ_0) is 5.2 percent of consumption per year. Hence with a per-period fixed cost equal to this amount, a non-stockholder would never enter the stock market. Loosely speaking, the one time cost, τ^F , can be thought of as the present value of the sequence of per-period fixed costs, τ^P .

By either measure, these costs are clearly large, which shows that non-stockholders have a strong incentive to enter the stock market.¹³ Although in this paper we do not explicitly address what these costs of participation are, we believe that the results of this paper provide further motivation for studying the sources of limited participation in future work.

¹³Note that comparing the average consumption or wealth of the *existing* stockholders to that of the non-stockholders would be misleading. For example, a stockholder owns 27 times more wealth, and consumes 106 percent more than a non-stockholder in the baseline model, which may seem to suggest significantly higher welfare gains from participating in the stock market. Of course these numbers do not account for the fact that non-stockholders have different preferences from stockholders, so when they enter the stock market they will choose not to accumulate a lot of stock wealth, which would result in a very volatile consumption path. Also, the new entrants initially own very little wealth and the transition period to the new steady state wealth level is very long—averaging about 120 years. As a result the relatively high consumption levels realized in the distant future do not significantly affect welfare.

Table 10: WEALTH DISTRIBUTION

	The Fraction Held by the Top 20 percent of the Population:		
	Equity	Financial Wealth	Net Worth
U.S. data	98.2	92.3	83.8
Model	100.0	86.7	86.7

Notes: The equity data is from the 1992 Survey of Consumer Finances (SCF) and is taken from Poterba and Samwick (1995, Table 9). It includes indirect holding through mutual funds and direct contribution pension funds, but excludes direct benefit plans. The data on Financial Wealth and Net Worth is from the 1992 SCF and is taken from Wolff (2000, Table 2).

7.3 Wealth and Debt Distribution

We next turn to the implications of the model for the wealth distribution across the two groups of households. We study the distribution of wealth (and consumption) implied by this model in a separate paper (Güvenen (2003)) and provide a more extensive discussion of the issues summarized here.

First, households in the top 20 percent of the wealth distribution are assumed to hold 100 percent of stocks in our model, and own 98 percent of all the equity in the U.S. data (see Table 10). Second, the model generates substantial wealth inequality: 87 percent of total wealth is held by the stockholders. Before comparing this figure to the data, notice that the model does not explicitly include some important types of assets, such as housing, consumer durables, government debt, and so on, nor does it distinguish between private and publicly traded capital. One approach is to follow the real business cycle tradition and interpret the capital stock in the model more broadly, as including these different types of wealth categories (c.f., Cooley and Prescott (1995, page 17)). With this approach the relevant measure of household wealth is comprehensive (net worth) and includes all types of physical wealth. Column 3 reports that the stockholders own about 84 percent of the net worth. On the other hand, it is fair to say that the non-stockholders in the model hold only a riskless asset, whereas the broader definition of net worth includes categories—most notably housing capital—that have risky returns. Alternatively then, if the model is interpreted more narrowly as being about financial asset portfolios, the empirical counterpart of wealth would be financial assets held by households. By this measure, 92 percent of wealth is held by the stockholders and only 8 percent is held by the non-stockholders. By either definition there is substantial wealth inequality across these two groups in the U.S. data as in the baseline model.

Perhaps one surprising implication of the model is that the stockholders are net borrowers in the bond market despite their substantial total wealth holdings. Given the level of abstraction in the model, it seems more appropriate to interpret this borrowing more broadly as including firm leverage and other types of indirect borrowing of the production sector from the rest of the population. Masulis (1988, Table 1-3) reports that the average debt of U.S. firms (from 1929 to 1986) was 26 percent of firms' market value, and 66 percent of firms' book value. Saito (1995) explicitly models firm leverage in a model with limited participation and shows that with this reformulation the stockholders' portfolio consists of levered equity together with *positive* bond

Table 11: THE UNCONDITIONAL MOMENTS OF RETURNS: VARIOUS PARAMETERIZATIONS

	(1)	(2)	(3)	(4)	(5)	(6)
Difference from Baseline:	Baseline $\alpha^h = 4$	Hom. RRA Het. EIS	Hom. RRA Hom. EIS	Government Debt	Storage technology	$\xi = 0.5$
$E(R^s - R^f)$	6.11	5.98	2.78	6.16	3.24	3.70
$\sigma(R^s - R^f)$	22.4	22.0	15.3	22.7	16.2	16.4
$\frac{E(R^s - R^f)}{\sigma(R^s - R^f)}$	0.27	0.26	0.18	0.27	0.20	0.23
$E(R^f)$	0.61	0.72	2.44	0.55	2.09	1.45
$\sigma(R^f)$	7.31	7.29	5.01	7.33	5.43	5.50
$\rho(R^s, R^f)$	0.01	0.02	-0.03	-0.02	0.03	0.01
$E(P^s/D)$	29.4	29.1	25.9	32.1	26.5	30.2
$\sigma(\log(P^s/D))$	30.5	29.4	15.9	50.3	18.5	33.8

Notes: In column (2), $\text{rra}^h = \text{rra}^n = 4$, $\text{eis}^h = 0.25$, $\text{eis}^n = 0.1$; in column (2), $\text{rra}^h = \text{rra}^n = 4$, $\text{eis}^h = \text{eis}^n = 0.25$. In columns (4) and (5) the supply of the risk-free asset is set to 35 percent of average annual GDP. See the text for more details.

holdings. Furthermore, in the next section we consider two extensions that allow for a positive supply of debt. As we elaborate below, in one of these cases both groups hold positive amounts of bonds, yet there is little change in the asset pricing results.

8 Extensions and Sensitivity Analyses

In this section we consider extensions of the baseline model and variations in the values of some key parameters. As the benchmark, we take the baseline model with $\alpha^h = 4$ to start with a high equity premium. This is important because if the premium falls too much in a specification that we consider below, borrowing constraints could become binding which would make comparison across different specifications difficult. Except where indicated below, these modifications mainly affect the unconditional moments of returns with only minor consequences for the dynamics of asset prices, which appear to be primarily driven by limited participation. To save space, we only report the counterpart to Table 2 here. The counterparts of Tables 3 to 6 (the dynamics of asset prices) for the parameterizations discussed in this section as well as additional sensitivity analyses are included in an appendix available on the author’s website.

The Role of Preference Heterogeneity.—In our baseline model the stockholders and the non-stockholders differ both in their risk aversions and in their elasticities of intertemporal substitution. We now solve the model with Epstein-Zin preferences and disentangle the two parameters. This allows us to examine the role of each kind of heterogeneity for the results of the paper. First, we eliminate the heterogeneity in risk aversion ($RRA^h = RRA^n = 4$), *but* keep the heterogeneity in the elasticities: $EIS^h = 0.25$, $EIS^n = 0.1$. As reported in column 2, the effect of this change on the unconditional moments is very modest. Surprisingly, reducing the risk aversion of the non-stockholders—who constitute 80 percent of the population—from 10 to 4 has very little impact

on asset prices (including the dynamics of asset prices reported in the appendix). Second, we also increase the EIS of the non-stockholders from 0.1 to 0.25, and thus eliminate heterogeneity in preferences so that both agents have *identical* CRRA utility functions (specifically, $RRA^h = RRA^n = 4$, and $EIS^h = EIS^n = 0.25$). The equity premium now falls to 2.78 percent and the volatility falls to 15.3 percent reducing the Sharpe ratio to 0.18 (column 3). The volatility of the price-dividend ratio also falls by half. Thus, the EIS of the non-stockholders has a significant impact on the unconditional moments of returns. In addition, in this case the non-stockholders' consumption becomes slightly more volatile than that of the stockholders (3.4 percent versus 3.3 percent per year), inconsistent with the empirical evidence discussed in Section 7. Furthermore the stockholders' consumption share (S_t) ceases to be procyclical (correlation with output: -0.26), breaking the close link between this model and the external habit model. Overall, these results show that the heterogeneity in the EIS is a key element in our model, while the heterogeneity in risk aversion does not appear to play a significant role.

Government Debt.—To investigate the effect of a positive supply of the risk-free asset on our results, we modify the bond market clearing condition: $\mu b^h + (1 - \mu) b^n = \mathbf{G} > 0$. With this extension every period the household sector receives total interest payments equal to $\mathbf{G}(1 - P_t^b)$ from the issuer of the bond. We consider two possible scenarios for the financing of these payments. First, we interpret this asset as government debt and assume that the government taxes labor income at the required rate to make these interest payments every period. We set \mathbf{G} equal to 35 percent of average annual GDP, corresponding to the average government debt held by the U.S. public during the period since 1962 (as reported on the Congressional Budget Office web site). Column 4 reports the results. There is little change in the unconditional moments (except for a big jump in $\sigma(\log(P^s/D))$ from 30.5 to 50.3 percent), and the dynamics of asset prices are also unaffected (reported in the additional appendix).

The reason is that the financing of debt through taxation transmits the effect of the non-stockholders' cyclical borrowing and lending pattern to stockholders. After a negative productivity shock, the non-stockholders' strong desire to reduce their bond holdings (for consumption smoothing) increases the interest rate, which has two effects: First, a higher interest rate increases the tax rate necessary to finance debt payments, reducing each group's labor income even further in a recession. Second, the net interest payments received by bond owners increase, which primarily benefits the non-stockholders who own 95 percent of government bonds. As a result, the non-stockholders experience both the cost and the benefit of this higher interest rate which partly offset each other, whereas the stockholders only experience the cost (higher income volatility resulting in higher consumption volatility). Thus, the general equilibrium mechanism described in Section 5, which amplifies stockholders' consumption volatility is still at play here, although indirectly transmitted through the government budget constraint. Finally, unlike in the baseline model, the stockholders' average bond holdings is not negative in this case ($\approx +0.05\mathbf{G}$), which suggests that the distribution of debt per se is not critical for the asset pricing results presented.

Access to a Storage Technology.—One restrictive aspect of the previous exercise is that we did not allow the government to smooth taxes by varying the bond supply (\mathbf{G}). Modeling the

government’s behavior is beyond the scope of this paper, so we consider the following (somewhat extreme) case to break the link between interest rate and taxes: We assume that the risk-free asset corresponds to a risk-free (tree) technology with fixed capacity, \mathbf{G} .¹⁴ In this case, (column 5) the equity premium falls to 3.72, a forty percent fall compared to the baseline case. The volatilities of returns are also lower though, so the Sharpe ratio falls by less, to 0.20. It seems reasonable to conjecture that in the intermediate (and arguably more realistic) case—where the government is allowed to change the supply of debt, but has to obey a transversality condition on its borrowing—the effect on asset prices would be somewhere between these two cases considered. Instead, if the supply of safe technologies available to households for consumption smoothing is much larger than what is assumed in these experiments, then clearly the effect of limited participation could be significantly reduced.¹⁵

The Effect of Adjustment Costs.—There is not a general consensus in the empirical literature on the magnitude of the adjustment cost parameter, ξ , so it is useful to examine the behavior of the model as we vary this parameter. Increasing the elasticity of investment, ξ , to 0.5, lowers the equity premium to 3.70, and its volatility to 16.4, resulting in a Sharpe ratio of 0.23. The effect on asset price dynamics is minimal however (reported in appendix).

9 Discussion and Conclusions

In this paper we studied the asset pricing implications of a macroeconomic model with limited stock market participation and heterogeneity in the EIS parameter. This particular two-agent representation is parsimonious compared to traditional (fully) heterogenous-agent frameworks (e.g., Storesletten et al. (2001)), yet it generates a lot of heterogeneity across agents, which turns out to be important for understanding asset prices. The model provides a new intuition for the equity premium. The bulk of the premium results from the timing of trade in risk-free assets, which accommodates the consumption smoothing demand by the population at large at the expense of higher volatility in the consumption of stockholders.

The model generates a number of asset pricing phenomena and seems to capture some important aspects of asset price dynamics. Interestingly, many of these results are also generated by the external habit model of Campbell and Cochrane (1999), and we show that this is not a coincidence: the limited participation model has a reduced form which seems to be closely related to the external habit model. In particular, the part of aggregate consumption accounted for by the non-stockholders—necessarily omitted in a representative-agent framework—resurfaces as the habit process in the external habit model.

¹⁴If we instead assume that the risk-free asset is in *infinite* supply, this will effectively shut down the bond market and make limited participation irrelevant: in this case the non-stockholders’ saving will have no effect on the stockholders’ problem. So we do not consider this case.

¹⁵Another candidate for a storage technology is residential capital, which is the most common type of asset accumulated by households outside of the top 20 percent. But the return on housing capital is hardly risk-free, and large transaction costs in the housing market makes it poorly suited for insuring short-term fluctuations in consumption that is the focus of this paper.

One important element missing from the current framework is growth. One difficulty with introducing growth into an infinite horizon model with heterogeneity in the EIS is that each agent's consumption growth will be proportional to her elasticity of substitution, implying that in the long-run those with high EIS—the stockholders in our model—will end up accounting for all the consumption (and own all the wealth) in the economy. Thus limited participation will have no effect in the long-run. One way to remedy this problem would be to assume an overlapping generations structure and disallow negative bequests. In this case, households with low EIS in each cohort cannot leverage the wealth of their future generations (to increase current consumption) as they do in the infinite horizon case. Hence, the life-time consumption of each cohort would be at least equal to its life-time income, which grows together with the scale of the economy resulting in a non-degenerate distribution of consumption that is needed to make limited participation matter (through equation 9). Of course such an extension would introduce many issues that would have to be resolved, and it is of interest to see how the results of this paper would be affected by this extension.

The assumption that the non-stockholders can only use a risk-free asset for consumption smoothing is not as restrictive as it seems: by abstracting from (individual-level) idiosyncratic income shocks and representing each group with a single agent, we are implicitly assuming the existence of an array of financial assets, which allows households within each group to vanquish *all* idiosyncratic risk and attain perfect risk-sharing. Further, by abstracting from group-level shocks, we are implicitly assuming that any shock that is negatively correlated across the two groups has already been eliminated. The remaining trade that takes place through the bond market arises from the desire to smooth fluctuations resulting from aggregate shocks, a motive whose strength differs across groups because of heterogeneity.

We hope that our results would also encourage further research on the reasons behind limited participation which is not addressed in this paper. Furthermore, given the central role played by limited participation in this model, another important research avenue is to investigate the consequences of the recent trends in participation observed in most countries for asset prices as well as for wealth inequality and welfare.

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