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Optimal Currency Hedging

Rui Albuquerque*
University of Rochester

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Abstract

This paper characterizes optimal currency hedging in several models of downside risk. We consider, in turn, three models of hedging: (i) a firm that chooses its hedging policy in the presence of bankruptcy costs; (ii) an all equity firm that faces a convex tax schedule; and (iii) a firm whose manager is subject to loss aversion. In all these models, and contrary to conventional wisdom, we show that forwards dominate options as hedges of downside risk.

Keywords: Currency hedging, forwards, options, bankruptcy costs, taxes, loss aversion, downside risk.

JEL Classification: F31, G30.

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1 Introduction

The globalization of goods and capital markets means that an increasing number of firms have to make decisions about hedging their foreign exchange exposure. This need is aggravated by extremely volatile foreign exchange markets. There is a large number of derivative securities, including forward and options contracts, that can be used to construct a hedging strategy. How do these instruments interact? And what is the optimal hedging policy? In this paper we provide answers to these questions in several economic models of downside risk.

We focus the analysis on models of optimal hedging in the presence of downside risk. This is for two reasons. First, industry surveys indicate that in designing their hedging policies CFO's seek the selective elimination of risks, instead of the elimination of all risks as dictated by the variance-minimizing framework.[1] In fact, Stulz (1996) argues that this should be the "goal of corporate risk management–namely, the *elimination of costly lower-tail outcomes*" (p. 8). Second, swings in exchange rates have normally been associated with very dramatic industry turnover. Countries such as South Korea and Thailand have seen their banks become insolvent as a result of currency devaluations that greatly increased the local currency value of their dollar denominated liabilities. In the early 1980's some firms found themselves in financial distress for not hedging against the strengthening of the dollar. A famous example discussed in Rawls and Smithson (1993) is Laker Airlines. In September 1992 many European firms were affected by the devaluations associated with the European Monetary Mechanism crisis. Hedging downside risk would have helped banks and firms in these episodes stay financially afloat.

We start by discussing two standard models of downside risk that are consistent with value-maximizing firms. First, we discuss a model of optimal hedging and capital structure choice in which hedging is driven by the desirability to reduce expected bankruptcy costs. Second, we analyze an all-equity, risk neutral firm that faces a piecewise linear convex tax schedule. This model is then generalized to explicitly account for dynamic tax loss carry-forwards. Our third model focuses on managerial loss aversion (Kahneman and Tversky (1979, 1992)). All three models describe the excess sensitivity of a firm to low outcomes, and are appropriate to studying hedging of downside risk. To simplify the analysis we study an environment in which derivative contracts are priced by risk neutral agents, but involve transactions costs.

The paper's main proposition is that, contrary to conventional wisdom (see Stulz (1996) for a discussion), in all three models, currency forwards are better hedges than

[1] Rawls and Smithson (1993) provide a summary of survey evidence on managerial hedging behavior.

options against downside risk. The proof of this result relies on the fact that in efficient markets the net unit payoff of both derivative instruments must have the same expected value. Suppose an exporting firm faces transactions exposure and wishes to eliminate the downside risk associated with an abrupt rise on the value of the domestic currency.[2] This objective can be accomplished by transferring income from high profit states (those with domestic currency depreciation) to low profit states (those with domestic currency appreciation). Because forward contracts pay more than options on the downside (they also make bigger losses on the upside), less currency needs to be sold forward to achieve the same degree of hedging. Thus, total transactions costs paid using forwards must be smaller than those paid using options. This is true even though we have assumed equal transactions costs per unit of foreign currency traded.

Section 2 develops the model with bankruptcy costs. Section 3 deals with the case of tax convexity. Section 4 analyzes hedging when managers exhibit loss aversion. Section 5 presents evidence on transactions costs for forwards and options and section 6 presents the final remarks. Appendices A and B contain proofs of results in sections 2 and 3, respectively. Appendix C provides a discussion of the plausibility of the tax schedule used in section 3 and of other tax issues related with hedging.

2 Model with Bankruptcy Costs

A popular model of downside risk is one in which the firm faces bankruptcy costs when default on debt obligations occurs. Smith and Stulz (1985) have shown that by hedging the firm is able to reduce the likelihood of default by increasing the income it gets in the downside. This leads to lower expected bankruptcy costs (possibly higher debt levels) and higher firm value.[3]

Consider an infinitely lived, all-equity firm with a fixed investment opportunity set and production capacity that exports one unit of output per period. The value in domestic currency of exports' profits is stochastic because of fluctuations in the exchange rate. The revenue from exports denominated in foreign currency is determined one period in advance and normalized to 1. For simplicity of exposition, we assume that there

[2] Transaction exposure reflects the changes in the domestic currency value of cash flows from existing contracts due to changes in the exchange rate. In industry surveys, transaction exposure is typically referred to as the most important reason to hedge (e.g. Millar (1989), Jesswein, Kwok, and Folks (1995)). For a comparative study of currency hedging by German and US corporations see Bodnar and Gebhardt (1998).

[3] Bessembinder (1991), Ross (1996), and Leland (1998) develop models of hedging in the presence of taxes and default costs. Titman (1992) studies how debt maturity is affected by the possibility of buying interest rate swaps in a model of asymmetric information.

are no costs associated with foreign sales. We introduce production costs in the next section. Let s denote the exchange rate quoted as units of domestic currency per unit of foreign currency. The exchange rate follows a first order Markov process and has bounded support: $s \in (0, \bar{s}]$. The behavior of s is described by the conditional cumulative distribution function H. To focus on bankruptcy costs we assume a flat tax rate (τ) on corporate profits.

Pricing of Debt. The firm borrows one-period debt in local currency units.[4] A debt contract is a pair (B, b), where B is the face value of debt, and b is the market value of debt. Hence, $B/b - 1$ is the interest rate paid on debt. Bondholders are risk neutral and discount their future utility at rate r. If the firm defaults, bondholders pay bankruptcy costs in the amount of α.[5] This makes the level of bankruptcy costs independent of hedging. This assumption is realistic since hedging contracts typically have priority over debt contracts. Let primes denote next period's variables and Π be total profits from operations and hedging (which is defined explicitly below). Firms are subject to capital market frictions according to which they cannot borrow against future profits. Thus, the firm defaults if and only if $(1 - \tau) \Pi' < B$. Under these assumptions the market value of debt is given by the zero profit condition:

$$b = \frac{1}{1+r} \int_0^{\bar{s}} \min \left[B; (1 - \tau) \Pi'\right] H \left(ds'; s\right) - \alpha \frac{1}{1+r} \int_{\{s' : (1-\tau)\Pi' < B\}} H \left(ds'; s\right). \quad (1)$$

Equation (1) says that the market value of debt is equal to the discounted value of the minimum between the face value of debt and the firm's net-of-tax profits minus the expected bankruptcy costs. To conform with the fiscal code, taxes have priority over debt claims. The interest rate on debt commands a premium due to bankruptcy costs if the probability of default is positive.

Forward and Put Options Contracts. The firm can hedge its exchange rate risk using forward or options contracts. Both of these contracts are priced by a risk neutral investor who faces transactions costs of ϕ units of local currency per unit of foreign currency traded. Transactions costs are incurred in the period in which the contract is traded. This means that our firm can sell foreign currency forward at the (bid) rate:

$$f = E_s \left(s'\right) - \phi(1 + r), \quad (2)$$

[4] Using covered interest parity we know that any debt contract denominated in foreign currency can be replicated with a forward contract and a debt contract denominated in local currency units.

[5] Several papers estimate the costs of financial distress. For example, Andrade and Kaplan (1998) estimate that 10% to 20% of firm value is lost in direct or indirect costs of financial distress.

where E_s is the conditional expectations operator. The firm can buy one period put options with exercise price s^* at a premium:

$$p = \frac{1}{1+r} \int_0^{s^*} (s^* - s') \, H(ds'; s) + \phi. \tag{3}$$

For simplicity, the notation ignores the dependence of the options premium (p) on the strike price. We consider only options with strike price $s^* < \bar{s}$, since an option with $s^* = \bar{s}$ is equivalent to a forward contract. For future reference it is useful to note that the expected profits associated with both a forward contract and an options contract is negative and equal to $-\phi(1+r)$. The expected value of a forward contract is:

$$E_s(f - s') = -\phi(1+r).$$

The expected value of an options contract (net of the ask price of the option) is:

$$E_s \left[\max(s^* - s', 0) - p(1+r) \right] = -\phi(1+r).$$

Profits. Total profits from operations and hedging activities are:

$$\Pi = s + x_f(f - s) + x_o \max(s^* - s, 0) - p x_o (1+r)$$

where px_o is the cost of the options purchased in the previous period.[6]

Optimal Choice of Hedging Instruments. We now investigate whether the firm should combine forwards and options as part of its hedging strategy or whether it should use a single hedging instrument. Without loss of generality we assume that the firm uses a single type of option. Because of transactions costs it is never optimal to combine options with different strike prices in this setting.[7]

Shareholders discount the firm's after-tax cash flow at interest rate r. Firms maximize the value of equity. The choice of capital structure and hedging policies is simultaneous.[8]

[6] To make the cash flows from options and forwards comparable we assume that the firm must borrow to purchase the options. In order not to favor forwards, we assume that the firm can use risk free debt for this purpose.

[7] In particular, a forward contract can be replicated with two options by put-call parity. However, the synthetic forward produced in this way is dominated by the outright forward because transactions costs are doubled.

[8] This timing assumption is usually nontrivial, in that it affects the amount of hedging done, but it does not affect the choice of which hedging instruments to use. Shareholders may have an incentive to hedge less after debt is in place if the only reason to hedge is to reduce the probability of default. The reason is that although hedging increases firm value it also transfers wealth from shareholders to bondholders, hence potentially making the former reluctant to pursue hedging policies ex-post (see Smith and Stulz 1985).

For simplicity, interest and principal payments benefit from a tax shield. This assumption does not affect our qualitative results. The firm's problem is:

$$\tilde{V}(s) = \max_{B \geq 0, x_o, x_f \in [0,1]} \left\{ b + \frac{1}{1+r} \int_{\{s' : (1-\tau)\Pi' \geq B\}} \left[(1-\tau)(\Pi' - B) + \tilde{V}(s') \right] H(ds'; s) \right\},$$

subject to (1). In this formulation the value of equity given by $V = (1-\tau)(\Pi - B) + \tilde{V}$. We rule out the possibility of gambling with derivative instruments (i.e. $x_f, x_o < 0$, and $x_f, x_o > 1$, are not allowed).

In the absence of debt, the probability of default is zero and hedging does not add value. The following proposition characterizes optimal hedging for a levered firm.

Proposition 1 *For any positive debt level, when bankruptcy costs are positive ($\alpha > 0$), and transactions costs are positive ($\phi > 0$) and identical for forwards and options, the optimal hedging strategy relies solely on forward contracts (i.e. $x_o = 0$, and $x_f \geq 0$).*

Proof. Fix $B > 0$. Suppose the firm chooses to default for values of the exchange rate lower than S_d (hence, $S_d < B/(1-\tau)$). Thus, S_d determines the firm's optimal default probability and expected bankruptcy costs for fixed B. How many forward contracts must the firm buy? This number is given by setting $(1-\tau)\Pi = B$, which, when forwards alone are used, becomes:

$$x_f [f - S_d] + S_d = B/(1-\tau).$$

And, how many options are needed for the same purpose? Obviously, the strike price needs to satisfy $s^* \geq S_d$, otherwise the put option would be useless. This leads to:

$$x_o[s^* - S_d - p(1+r)] + S_d = B/(1-\tau).$$

For any level of debt B, strike price $S_d \leq s^* < \bar{s}$, and any default threshold that the firm chooses, $x_o > x_f$, since $f > s^* - p(1+r)$.[9] Therefore, the total transactions costs needed to achieve the same benefit (i.e., savings in expected bankruptcy costs) are higher with options. The reason for this is illustrated in Figure 1. Both contracts share the same expected value ($-\phi(1+r)$), but forward contracts have lower payoffs than options for high values of s. This means that forward contracts must have higher payoffs than

[9] To see this last step note that

$$\frac{d}{ds^*}[s^* - (1+r)p] = 1 - H(s^*) > 0,$$

and $f = s^* - (1+r)p$, when $s^* = \bar{s}$.

options for low values of s. These higher payoffs for low s mean that we need fewer forward contracts than options contracts to achieve a certain level of hedging. Thus, total transactions costs associated with hedging are always lower when forward contracts are employed. Since the result was proved for any $B > 0$, it also holds for the optimal B. ■

This result runs counter the conventional wisdom that options are an ideal instrument to hedge downside risk (see Stulz 1996). Keeping a low default probability is the goal of the firm's hedging strategy. To do this, hedging instruments must be bought, for which the firm incurs transactions costs. This objective can be achieved by choosing options with an appropriately low strike price. First, these options are cheap since they are unlikely to be exercised. Second, the options payoff has a non-linear shape that can exactly off-set the one-sided risk the firm faces (given an appropriate strike price). Why is hedging with cheap options not optimal? This is because a low price just means that it is more difficult to transfer income across states of nature.

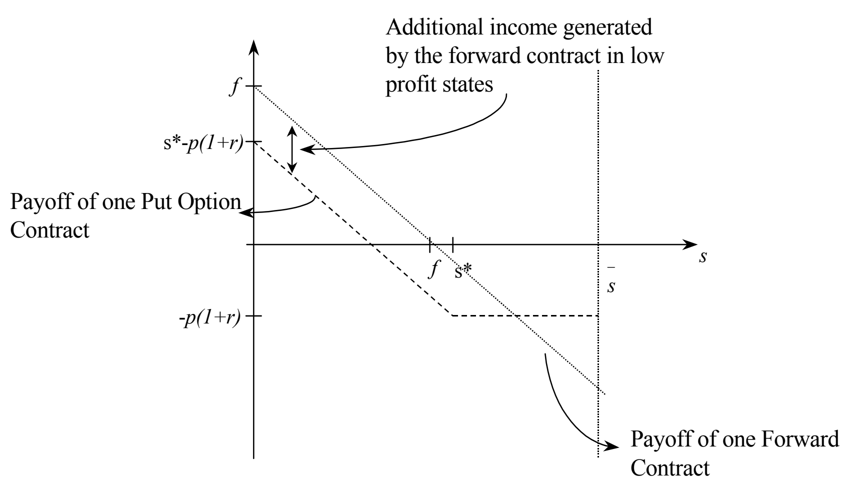

Figure 1: The Forward and Options Payoff.

It is important to note that this result holds very generally and also that we have taken the conservative view that transactions costs are identical for forwards and options (see Section 5).

We can extend this result to a comparison of two currency put options with different strike prices. Contrary to conventional wisdom, expected profits are lower when hedging is implemented with cheaper options. Under our assumptions, all options have the same transactions costs. Because cheaper options have lower payoffs in low states, a higher number of these options is required to achieve a given level of hedging (however, see section 5 for empirical measures of options transactions costs).

Optimal Hedge Ratio and Debt Choice. Setting $x_o = 0$, the first order necessary conditions for interior solutions on x_f and B are:

$$\left(\alpha + \tau B + \tilde{V}(s_d)\right)\left(-\frac{\partial}{\partial x_f}\int_0^{s_d} H(ds'; s)\right) = (1-\tau)\,\phi\,(1+r)\,, \tag{4}$$

and

$$\tau \int_{s_d}^{\bar{s}} H(ds'; s) = \left(\alpha + \tau B + \tilde{V}(s_d)\right)\frac{\partial}{\partial B}\int_0^{s_d} H(ds'; s)\,,$$

where the default threshold is determined by $(1-\tau)\,\Pi(s_d) = B$. The first of these conditions equates the marginal benefit from hedging, obtained through a reduction in the default probability ($\Pr[(1-\tau)\,\Pi < B] = \int_0^{s_d} H(ds'; s)$), to the marginal cost resulting from transactions costs. The second condition says that the firm's optimal debt policy trades-off the expected tax benefit of debt (conditional on not defaulting) against the marginal cost of debt, which arises due to the increased likelihood of paying the bankruptcy costs, the lost tax shield, and continuation value upon default.[10]

If $f - B/(1-\tau) \geq 0$, the firm is able to pay its debt always if it hedges 100% of its exposure. In this case, an increase in the hedge ratio lowers the default probability and makes hedging valuable. If $f - B/(1-\tau) < 0$, an increase in the hedge ratio raises the default probability, and no hedging is done (i.e. $x_f = 0$).

Letting $f - B/(1-\tau) \geq 0$, we can investigate the impact of bankruptcy costs (α) on debt and hedging. For simplicity, consider the case in which s is iid and $h(s)$ is uniform. All the computations are provided in Appendix A. In this model, hedging decreases when bankruptcy costs increase. The reason for this negative impact is that there are decreasing marginal returns to hedging (i.e. $\partial^2 \int_0^{s_d} H(ds'; s)/\partial x_f^2 < 0$). That is, more hedging lowers the default probability, but this impact is smaller the more hedging is done. On the other hand the impact of higher bankruptcy costs on debt is ambiguous. There is a direct negative impact, but there is also an indirect effect because less hedging

[10] Mello and Parsons (1999) also argue that the use of different instruments (in their case forwards and futures) may trigger specific debt convenants. We ignore such issues here.

is done. This is a somewhat unforeseen possibility that is also present in other models of capital structure and risk management (e.g. Leland 1998).

3 Tax Convexity

3.1 A Reduced Form Model of Tax Benefits of Hedging

In this subsection we eliminate the debt choice and bankruptcy costs in order to focus on optimal hedging under tax convexity. To introduce a simple motive for hedging we assume that the firm faces the following tax schedule:

$$T(\Pi) = \begin{cases} \tau_1 \Pi & , \Pi > 0 \\ \tau_2 \Pi & , \Pi \leq 0 \\ \multicolumn{2}{c}{\tau_2 < \tau_1} \end{cases} . \tag{5}$$

Appendix C provides a discussion of fiscal issues related to hedging, and suggests that this is a reasonable representation of the US corporate tax schedule. In the next subsection we present a more general model of hedging with optimal dynamic tax loss carry-forwards.

As before, Π represents the firm's total profits, including any losses or gains from hedging activities.[11] In this section and the next we assume that there is a cost associated with foreign sales of c units of domestic currency. With the elimination of debt, this is required so that profits can be negative. Total profits from operations and hedging activities are now given by:

$$\Pi = s - c + x_f(f - s) + x_o \max(s^* - s, 0) - px_o(1 + r). \tag{6}$$

According to the tax schedule $T(.)$ the firm receives a tax rebate when its profits are negative. However, the tax rate on positive profits is higher than the rebate tax rate. To simplify we assume that profits on exports are treated as domestic income and are taxed only in the country of residence. Thus, there is no double taxation of profits.

Optimal Choice of Hedging Instruments. What combination of hedging instruments is better, if any?[12] Firm value conditional on the current level of the exchange rate (s)

[11] Notice that for tax purposes the firm is only allowed to deduct p, the option's premium at historical values, in computing its taxable income. In practice, this feature of the tax schedule places the option at a disadvantage with respect to forwards. This disadvantage can be eliminated by financing the option with risk free debt. The tax benefits associated with debt financing exactly cancel the tax loss implicit in ignoring the time value of money when deducting the option price at book values.

[12] Proposition 2 below holds if instead we allow the firm to write a call option on its export revenue. Though selling a call might seem more appropriate to hedge tax risk, it still does not provide the kind of downside profit that the forward is able to provide.

and hedging policy is:

$$V(x_f, x_o, s^*, s) = \max_{x'_f, x'_o, s^* \geq 0} \left[\Pi - T(\Pi) + \frac{1}{1+r} E_s V(x'_f, x'_o, s^*, s') \right].$$

It is easy to show that if profits are always positive, or $\tau_1 = \tau_2$, it is optimal not to hedge ($x_f = x_o = 0$). In this case hedging brings no benefits. Since there are transactions costs associated with derivative contracts ($\phi > 0$) the expected profits from hedging are negative.

Let us consider the case in which hedging carries benefits. This requires $\tau_2 < \tau_1$ (so losses are treated asymmetrically from profits), and a positive probability that losses will be incurred. The following proposition characterizes optimal hedging.

Proposition 2 *If the tax schedule is convex, $\tau_1 > \tau_2$, and transactions costs are positive ($\phi > 0$) and identical for forwards and options, the optimal hedging strategy relies solely on forward contracts: $x_o = 0$, $x_f \geq 0$. When the exchange rate follows a stationary AR(1) process with positive persistence, there is a region of inaction in which $x_f = 0$. The forward hedge behaves non-monotonically with the exchange rate:*

$$\frac{dx_f}{ds} \begin{cases} \geq 0, & f - c < 0 \\ \leq 0, & f - c > 0 \end{cases}.$$

Proof. Suppose that the firm decides to hedge so that it restricts losses for values of the exchange rate below S (for any $S < c$). How many forward contracts would be necessary to achieve this objective? The answer is given by the equation:

$$x_f(f - S) + S - c = 0.$$

How can we achieve the same objective with options? Obviously, we have to choose a strike price $s^* > S$, so that the option has a positive payoff when $s = S$. In addition, the number of options purchased must be such that the realized profits for $s = S$ are zero:

$$x_o[s^* - S - p(1+r)] + S - c = 0.$$

Using (2) and (3) it is easy to show that $f - S > s^* - S - p(1+r)$, which implies that $x_f < x_o$. (Recall footnote 9.) This means that the cost of achieving an arbitrary level of hedging is always lower with forward contracts than with options. We provide the proof of the second part of the proposition in Appendix B. ■

As before, this result runs counter the common perception that options are an ideal instrument to hedge downside risk. The intuition is similar to the case with bankruptcy costs. The presence of tax convexity makes it desirable for the firm to avoid making losses. Because forward contracts pay more than options on the downside (they also make bigger losses on the upside), less currency needs to be sold forward to achieve the same degree of hedging. Thus, total transactions costs paid using forwards must be smaller than those paid using options.

Optimal Hedge Ratio. Now that we have determined that it is optimal to set $x_o = 0$, we can find the optimal value of x_f. The first order condition for an interior solution to this problem can be written as:

$$\phi(1+r)(1-\tau_1) = (\tau_1 - \tau_2)\int_{\{s':\Pi\leq 0\}} (f - s')H(ds'; s). \tag{7}$$

The left-hand side of this equation is the marginal cost (net of taxes) of entering into an additional forward contract. The marginal benefit corresponds to the income that is transferred by the forward contract from high tax rate states (those with positive profits) to low tax states.

Proposition 2 shows that the effect of an increase in the exchange rate on the dynamic forward hedge depends on the magnitude of the current exchange rate. When the exchange rate is high (and the exchange rate has positive serial correlation), an increase in s produces high future expected exchange rates and hence reduces the probability of having negative profits. As the benefit from hedging is reduced so is x_f. When the exchange rate is low, a marginal increase in s raises the marginal benefit from hedging, and hence x_f.

For some values of the current exchange rate (s) the optimal hedging strategy is to leave the exposure uncovered: $x_f = 0$. When the exchange rate is sufficiently low, the probability of having negative profits next period is too high (with positive serial correlation) and the firm has little probability of incurring losses. When the exchange rate is high enough, the likelihood of positive profits next period is too high and the convexity of the tax schedule also disappears. Thus, even though there are no fixed costs of implementing a hedging program it may be optimal not to hedge. This band of inaction may account for the large percentage of survey respondents that say that they do not hedge because of the reduced significance of their exposures. It may also account for the usual policies of selectively hedging perceived exposures. This effect is exacerbated in cross sectional studies undertaken after periods of high dollar depreciation (when profits from exports are highest).

We prove in Appendix B that the optimal hedge ratio, x_f, is non-monotonic both with respect to costs of exports (c) and to the transactions cost parameter (ϕ). An increase in c produces two effects. An increase in c increases the set of negative profit states, thus making hedging more desirable. However, when c is very high the benefits of hedging decline because profits are low, and thus there is not much income that can be transferred to negative profit states to reduce the taxes paid by the firm.

The optimal hedge is generally not monotonic in transactions costs for $f-c>0$ (this is the interesting case since the firm has positive profits if it fully hedges its exposure). An increase in ϕ increases the marginal cost of hedging, hence lowering the hedge ratio. However, because the forward bid rate decreases with transaction costs a higher hedge ratio is needed to keep a low probability of negative profits. The net effect is ambiguous. This ambiguity disappears when $f-c<0$ where the second effect is eliminated.

3.2 An Explicit Model of Tax Loss Carry-forwards

Here we generalize the analysis of the previous subsection to explicitly model tax loss carry-forwards. As before, we allow for stationary, serially correlated exchange rate processes.

Without loss we assume the firm can carry losses forward indefinitely.[13] Let the tax schedule be given by $T(\Pi(s_t)-L_{t-1})=\tau(\Pi(s_t)-L_{t-1})^+$, where L_{t-1} is the tax loss carry-forward at the beginning of fiscal year t, and the notation $(X)^+=X$ if $X\geq 0$ and $X=0$ otherwise. The law of motion of the carry-forwards is $L_t=(L_{t-1}-\Pi(s_t))^+$.

Consider hedging with forward contracts only. A firm with tax loss carry-forwards of $L_{t-1}\geq 0$, current hedge ratio of $100x_{t-1}\%$, and level of exchange rate s_t solves:

$$V(x_{t-1},L_{t-1};s_t)=\max_{x_t,L_t}\left[\Pi(s_t)-T(\Pi(s_t)-L_{t-1})+\frac{1}{1+r}E_sV(x_t,L_t;s_{t+1})\right].$$

If $L_t=0$ then no hedging is done. The first order necessary condition when $L_t>0$ is (after some simplifications):

$$\begin{aligned}&\phi(1-\tau)(1+r)+\frac{1}{1+r}\int_{\{\Pi(s_{t+1})-L_t<0\}}V_L(x_{t+1},L_{t+1};s_{t+1})(f_{t+1}-s_{t+1})H(ds_{t+1};s_t)\\ =\;&\tau\int_{\{\Pi(s_{t+1})-L_t<0\}}(f_{t+1}-s_{t+1})H(ds_{t+1};s_t),\end{aligned}$$

with

$$V_L(x_{t-1},L_{t-1};s_t)=\tau\mathbf{1}_{(\Pi(s_t)-L_{t-1}\geq 0)}+\frac{1}{1+r}\mathbf{1}_{(\Pi(s_t)-L_{t-1}<0)}E_sV_L(x_t,L_t;s_{t+1}),\qquad(8)$$

[13] Imposing finite carry-forward of losses limits the usefulness of options even more since with options it is more likely that some of the losses will never be used to offset gains.

where $V_L(.)$ is the partial derivative of the value function with respect to L (the marginal value of tax loss carry-forwards), and $\mathbf{1}(\omega)$ is an indicator function taking the value of 1 if the event ω is true and zero otherwise. The first term on the left-hand-side and the term on the right-hand-side are the same as in equation (7) above. There is a new term: It characterizes the value of postponing the exercise of the carry-forwards. Rewrite the first order condition as:

$$\phi(1-\tau) = \frac{1}{1+r} \int_{\{\Pi(s_{t+1})-L_t<0\}} \left[\tau - \frac{1}{1+r} V_L(x_{t+1}, L_{t+1}; s_{t+1})\right] (f_{t+1} - s_{t+1}) H(ds_{t+1}; s_t). \tag{9}$$

Condition (9) is very similar to the one we have derived in the main text for the reduced form tax schedule (equation 7). Using (8) it is easy to see that $V_L(x_{t+1}, L_{t+1}; s_{t+1}) \leq \tau$. Intuitively, postponing the tax shield from the tax loss carry-forward cannot yield a greater benefit than using the tax shield today when interest rates are positive.

Consider now the choice between forwards and options. Since with forwards the firm achieves the largest reduction in tax loss carry-forwards today, the firm is better off by doing so than postponing the realization of the tax shield, which the option does in greater proportion. Thus, forwards dominate options.

4 Loss Aversion

Loss aversion plays a central role in the prospect theory developed in Tversky and Kahneman (1979) and Kahneman and Tversky (1992). Loss aversion describes the excess sensitivity of a firm to low outcomes. Mathematically, the firm's utility function displays a kink. This leads to risk neutrality with respect to profits' volatility above a certain threshold and risk aversion for volatility of outcomes below that threshold. Thus, the model is appropriate to studying hedging of downside risk, i.e. the elimination of these undesirable outcomes.

Let manager's compensation be a linear function of profits. The lifetime utility of the firm's manager can be written as:

$$U(x_f, x_o, s^*, s) = \max_{x'_f, x'_o \geq 0} \left[u(\Pi) + \frac{1}{1+r} E_s U(x'_f, x'_o, s^*, s')\right]$$

where the utility function u is given by:

$$u(\Pi) = \begin{cases} \Pi, & \Pi > 0 \\ \lambda\Pi, & \Pi \leq 0 \end{cases}, \tag{10}$$

and Π is given by (6). The parameter $\lambda > 1$ measures the degree of loss aversion (e.g. Benartzi and Thaler, 1995), which refers to the psychological tendency to be more sensitive to low outcomes than to high ones. Note that the same formulation is valid for a different threshold level, in which case profits (Π) would have to be adjusted accordingly. With this in mind we refer to negative profits as the firm's downside.

It is now obvious that the model of a risk neutral firm that maximizes its value while facing the tax structure (5), presented in section 3, is formally equivalent to the model with loss aversion if one sets $\lambda = (1 - \tau_2) / (1 - \tau_1) > 1$.

5 Empirical Evidence on Transactions Costs

In the model, transactions costs on forwards and options are equal, proportional to the amount traded, and constant across strike prices. This section shows that we have taken a conservative view of identical transactions costs for forwards and options in favor of options, independently of the strike price. Thus, our results are robust to modelling transactions costs on options that change with the strike price (which is a better description of the data) once we account for higher options transactions costs.

We analyze data on transactions costs for American options and forwards for three exchange rates: the Yen/US\$, Euro/US\$, and BP/US\$. The contract size is 10,000 BP and Euros, and 1,000,000 Yen. The data was collected from March/31/99-May/27/99. There are 20 days in the sample, and as many observations per business day as there are traded strike prices for 3- and 6-month options contracts. Bid and ask spreads were obtained from Bank One for forward rates, and from the Philadelphia Stock Exchange for put options. These quotes are roughly at the same time of day, and measure only the cost of establishing a position.

Figure 2 plots the annualized transactions costs for 3-month put options, in excess of that for forwards, measured in basis points for the three currencies considered. Figure 3 reproduces the annualized excess transactions costs for 6-month contracts.[14] Average annualized transactions costs for 3-month forward contracts are roughly 30 basis points across the currencies, with a minimum of 8.5 b.p. and a maximum of 40.8 b.p. Since transactions costs are measured by the bid-ask spread, these numbers compare to the theoretical value of $2\phi(1 + r)$. To get a sense of these numbers, a firm buying a one year put option for 1,000,000 BP would pay 1,000 pounds over the forward in transactions costs alone (half spread, or ϕ).

[14] There are no put options on the Euro at 6-months due to absence of trades in the sample. Note that the illiquidity of these contracts is also an obvious cost to them.

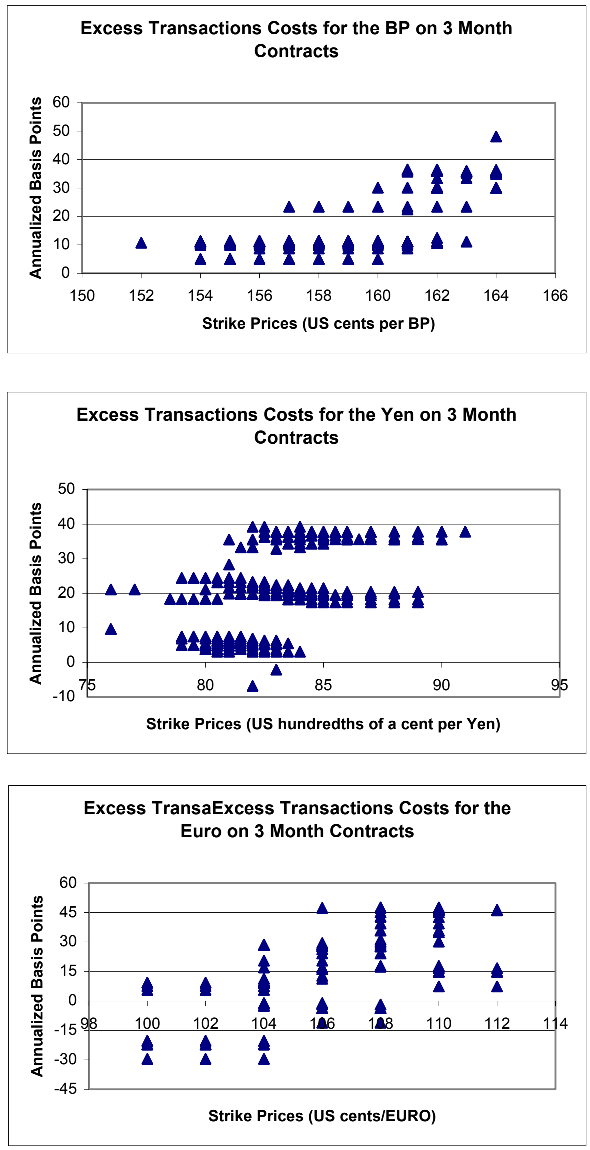

Figure 2: Excess transactions costs for Sterling, the Yen, and Euro on 3-month contracts.

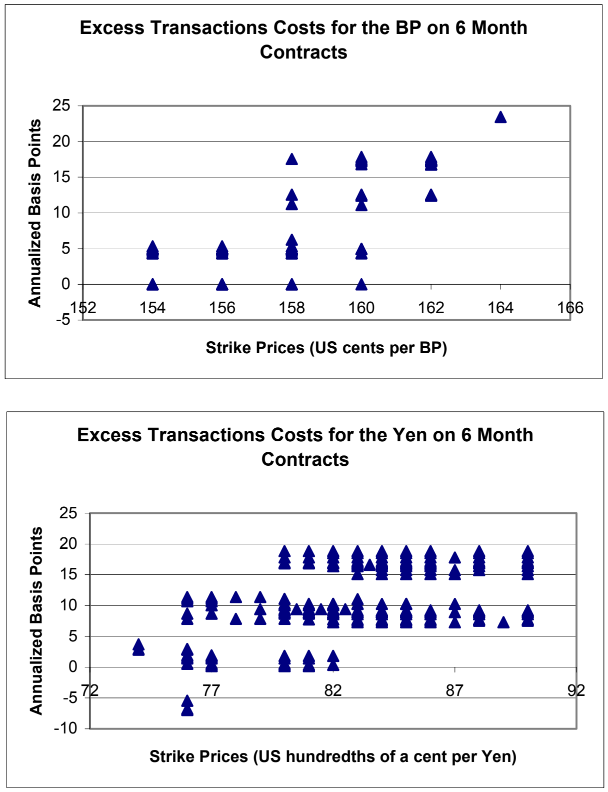

Figure 3: Excess transactions costs for Sterling, and Yen on 6-month contracts.

The evidence collected in these figures says that options transactions costs are higher at any traded strike price, and that transactions costs vary with moneyness of the options.[15] These findings broadly suggest that besides the mechanism that we develop, according to which forwards are better hedges of downside risk for identical costs, the difference in transactions costs is another reason to hedge with forwards.

6 Final Remarks

We have analyzed several models of optimal currency hedging of downside risks. Two of the models consider value maximizing firms that either face (i) bankruptcy costs, or (ii) a piece-wise linear convex tax schedule. The third model focuses on loss aversion, which makes the manager more sensitive towards low outcomes, hence more willing to hedge

[15] Further out-of-the-money options have even larger bid-ask spreads due to, for example, thin markets.

against downside risks.

The main contribution of the paper is to highlight the type of derivative contracts whose payoff structure produces a better hedge against downside risk. In particular, we show that forward contracts dominate options as hedges of transactions exposure when the firm is concerned with downside risks. The main reason for this is the forward's ability to transfer income across states of nature, concentrating the payments on the firm's downside. Thus, a smaller hedge ratio is required, and less costly is risk management.

A preliminary analysis of hedging downside risk in the presence of tax convexities reveals that there are significant non-linearities in hedging behavior. These non-linearities include zones of inaction and non-monotonicity in the relation between hedge ratios and exchange rates. Inaction, or zero hedging, can account for the usual policies of selectively hedging perceived exposures. Also, it is likely that this effect is exacerbated in cross sectional studies/surveys undertaken after periods of high dollar depreciation (when profits from exports are highest).

Indirect evidence from industry survey studies suggests that forwards are more widely used as hedging derivatives than options. Rawls and Smithson (1993) compare several surveys regarding the question of what instruments firms use to hedge their foreign exchange exposure. In Millar (1989), 99% of the respondents indicate that they have used forwards whereas only 48% say they have used options (see also Jesswein, Kwok, and Folks 1995, and Bodnar and Gebhardt 1998). Direct evidence from the gold mining industry indicates that roughly 70% of hedging is done with forward contracts (Adams, 1999). Of course, this evidence does not necessarily provide a conclusive test of our results for two main reasons. First, risk management strategies need not be based on a desire to eliminate downside risk, or may relate to other forms of exchange rate exposure. Second, proportional transactions costs on forwards tend to be smaller than those associated with options. Note that our analysis is not restricted to hedging exchange rate exposure. Given the availability of liquid markets for financial hedging instruments, our results can be applied to corporate risk management in general.

A Some Comparative Statics in the Model with Bankruptcy Costs

For notational convenience let $\Pi^f = f - B/(1-\tau)$. Assume that $\Pi^f > 0$, so that hedging is desirable, and that $x_f < 1$. Let the default threshold (s_d) be

$$s_d = \frac{-\Pi^f + (1-x_f) f}{1-x_f}.$$

Given our assumptions, $\tilde{V}$ exists and is unique, bounded and continuous. Assuming s is iid and follows a uniform distribution, and after some simplifications, the first order necessary conditions can be written as (note that $\tilde{V}$ is constant when s is iid):

$$\left(\alpha + \tau B + \tilde{V}\right) \Pi^f = \bar{s}(1-\tau)\phi(1+r)(1-x_f)^2,$$

and

$$\tau\left((\bar{s}-f)(1-x_f) + \Pi^f\right) = \alpha + \tau B + \tilde{V}.$$

Totally differentiating the first order conditions we obtain the derivatives of B, and x_f, with respect to α:

$$\frac{dx_f}{d\alpha} = \frac{-\frac{\tau\Pi^f + \left(\alpha+\tau B+\tilde{V}\right)}{\tau(1-\tau)}}{\tau\left[(\bar{s}-f)(1-x_f) + \frac{\alpha+\tau B+\tilde{V}}{1-\tau}\right](\bar{s}-f) + 2(2-\tau)\bar{s}\phi(1+r)(1-x_f)},$$

and

$$\frac{dB}{d\alpha} = -\frac{1}{\tau}\frac{1-\tau}{2-\tau} - \frac{1-\tau}{2-\tau}(\bar{s}-f)\frac{dx_f}{d\alpha}.$$

Thus, hedging always decreases with higher bankruptcy costs, whereas debt may decrease or increase.

B Proofs of the Main Results with Tax Convexity

To simplicity the notation define $\sigma^* = s^* - p(1+r)$.

Proof of Proposition 2. Here we prove the second part of the proposition. (An algebraic proof of the first part of the proposition is available upon request.) Given our assumptions, V exists and is unique, bounded and continuous. Because the net profit function is concave in x_f, V is also concave for any fixed s, and so differentiable almost everywhere (see Stokey, Lucas, and Prescott (1989)). The first order necessary and sufficient condition for $0 < x_f^* < 1$ is

$$\phi\,(1+r)\,(1-\tau_1) = (\tau_1-\tau_2)\,\frac{1}{1+r}\int_0^{\tilde{s}_f}(f-s')H(ds';s). \tag{11}$$

A unique solution with positive hedging exists so long as:

$$\phi\,(1+r)\,(1-\tau_1) < (\tau_1-\tau_2)\,\frac{1}{1+r}\int_0^{c}(f-s')H(ds';s). \tag{12}$$

When this condition is violated $x_f^* = 0$. When, $f - c = 0$, and (12) holds, the marginal benefit of an extra unit of forwards $(\tau_1-\tau_2)\,\frac{1}{1+r}\int_0^c(f-s')H(ds';s)$ is invariant with x_f, and so the full hedge is optimal. Let s follow an AR(1) process, with correlation $\rho \in (0,1)$, and unconditional average $\mu = E\,(s')$:

$$s' = \rho s + (1-\rho)\,\mu + v,$$

with v being an iid shock with zero mean and support $(-\mu(1-\rho), (\bar{s}-\mu)(1-\rho))$. This implies that the unconditional support of s is $(0,\bar{s})$. The density function of v is given by $\gamma > 0$. The distribution of s' conditional on s has support:

$$S\,(s) = (\rho s, \rho s + (1-\rho)\,\bar{s})\,.$$

The assumption of serially correlated exchange rate and bounded innovations is equivalent to a model in which the mean of the exchange rate shifts up or down over time. To characterize the impact on the optimal hedge of changes in s, let the first order condition be written as $\varphi\,(x_f, s) = 0$. Because the value function is concave, for an interior optimum the sign of dx_f^*/ds is given by that of $\partial\varphi/\partial s$. First, by change of variables:

$$\varphi\,(x_f,s) = -\phi\,(1+r)\,(1-\tau_1) - (\tau_1-\tau_2)\,\frac{1}{1+r}\int_{-(1-\rho)\mu}^{\tilde{v}}((1+r)\,\phi+v)\,\gamma\,(v)\,dv,$$

where $\tilde{v} = \tilde{s}_f - \rho s - (1-\rho)\,\mu$. It is now easy to see that

$$\frac{\partial\varphi}{\partial s} = -\,(\tau_1-\tau_2)\,\frac{1}{1+r}\rho\frac{f-c}{\left(1-x_f^*\right)^2}\gamma(\tilde{v}).$$

Therefore, with positive persistence ($\rho > 0$):

$$\frac{dx_f^*}{ds}\begin{cases}\geq 0, & f \leq c\\ \leq 0, & f \geq c\end{cases}.$$

■

Some Additional Comparative Statics Aside from when $f - c = 0$, it must be that $x_f^* < 1$. Thus, let $f - c \neq 0$, in the following comparative statics exercise. Totally differentiating (11), and after some simplifications, we obtain:

$$\frac{dx_f^*}{d\phi} = -\frac{1+r}{f-c}\left(1-x_f^*\right)\left[\left(\frac{1-\tau_1}{\tau_1-\tau_2}+H(\tilde{s}_f;s)\right)\frac{\left(1-x_f^*\right)^2}{(f-c)\,h(\tilde{s}_f;s)}-x_f^*\right],$$

which is negative if $f < c$, and

$$\frac{dx_f^*}{dc} = \frac{1-x_f^*}{f-c}\begin{cases} >0, & f-c>0 \\ <0, & f-c<0 \end{cases}.$$

C Discussion of Tax Issues Related to Hedging

Convexity of the US Corporate Tax Schedule. The usual theoretical presentation of the tax hypothesis focuses on strict convexity of the tax schedule, despite the fact that most firms in the US face a flat corporate income tax. In fact, the evidence suggests that the US corporate income tax schedule is convex, but Graham and Smith (1998, page 19) estimate that "the asymmetric treatment of profits and losses drives much of the observed convexity". This asymmetric treatment comes from the use of tax loss carry-backs and carry-forwards. Other features normally associated with the convexity of the tax schedule are: (i) a small interval of progressive taxation (the lowest tax rate is 15% for corporations with positive income lower than \$50,000, and the highest tax rate is 34% for incomes higher than \$100,000); (ii) the foreign tax credit, and general business credit; and (iii) the alternative minimum tax. Another more subtle form of progressive taxation is the possibility of a firm becoming regulated if its value grows unexpectedly large (see Smith and Stulz, 1985). Furthermore, Graham and Smith simulate their model and find that the convexity of the tax schedule can generate significant savings from hedging to an average firm: a reduction on the volatility of taxable income by 5% brings about a 5.4% reduction in the expected tax liability. As of August 1997, the carry-back period of operating losses was shortened from 3 to 2 years and the carry-forward period extended from 15 to 20 years (U.S. Master Tax Guide, 1998). These changes in the tax code were not considered in Graham and Smith (1998). We suspect that the increase in the carry-forward period, being too far off in the future, is not as important as the reduction in the carry-back period. If this is correct these recent changes increased the convexity of the tax schedule. The quantitative importance of the asymmetric treatment of losses versus gains in the U.S. tax schedule has been detected also in past tax codes. Altshuler and Auerbach (1990) show, for the 1982 tax code, that the existence of tax loss carry-forwards alone can significantly lower the marginal income tax rate. Depending on the income range of a particular firm tax loss carry-backs can reduce the marginal income tax rate by 18%-26% whereas tax loss carry-forwards can reduce the marginal income tax rate by 41%-48%.

Reporting of Hedging Profits. Kramer and Heston (1993, page 74) discuss when are gains/losses from hedging reported as ordinary versus capital income. The October 1993 decision by the Supreme Court over *Arkansas Best Corp. vs. Comm'r* determines that "hedges of any 'ordinary' property or obligation will give rise to ordinary gains or losses," as long as the taxpayer is able to identify what is being hedged. Since we are analyzing transaction exposure this is certainly possible. The main difference between capital and income gains/losses is that the capital losses can only be deducted to capital gains. Otherwise, capital gains are treated as ordinary income when computing taxable income (see also Kramer (1997)). The IRS defines hedging transactions as those that reduce the risk of currency fluctuations with respect to ordinary income, written options, and trades that counteract existing hedging transactions (paragraph 1.1221-2 of the 26 CFR, Chapter 1, 1997 edition).

For practical purposes, we are assuming that the firm uses 'hedge accounting' to report these transactions as opposed to 'mark-to-market' accounting. This means that the firm only has to report any gains or losses from trading in forwards at the time the hedged exposure is recorded, instead of reporting any changes in value as they occur (see DeMarzo and Duffie 1995, and Moffett and Skinner 1995).

References

[1] Adam, Tim, "Do Firms Use Derivatives to Reduce their Dependence on External Capital Markets?," Hong Kong University of Science and Technology, working paper, 1999.

[2] Altshuler, Rosanne and Alan J. Auerbach, "The Significance of Tax Law Asymmetries: An Empirical Investigation," *Quarterly Journal of Economics*, 61-86, 1990.

[3] Andrade, Gregor, and Steven Kaplan (1998), "Evidence from Highly Leveraged Transactions that Became Distressed," Journal of Finance, 53: 1443-1494.

[4] Benartzi, Shilomo, and Richard Thaler, "Myopic Loss Aversion and the Equity Premium Puzzle, " *Quarterly Journal of Economics*, 110: 73-92, 1995.

[5] Bessembinder, Hendrik, "Forward Contracts and Firm Value: Investment Incentive and Contracting Effects," *Journal of Financial and Quantitative Analysis*, 26: 519-532, 1991.

[6] Bodnar, Gordon M., and Günther Gebhardt, "Derivatives Usage in Risk Management by US and German Nonfinancial Firms: A Comparative Survey," NBER Working paper 6705, 1998.

[7] DeMarzo, Peter M., and Darrel Duffie, "Corporate Incentives for Hedging and Hedge Accounting," *The Review of Financial Studies*, 8:743-771, 1995.

[8] Engel, Charles, 1996, "The Forward Discount Anomaly and the Risk Premium: a Survey of Recent Evidence," *Journal of Empirical Finance* 3: 123-192.

[9] Graham, John R., and Clifford W. Smith Jr., "Tax Incentives to Hedge," forthcoming Journal of Finance, 1998.

[10] Jesswein, Kurt, Chuck C. Y. Kwok, and W. R. Folks, Jr., "What New Currency Products are Companies Using and Why?," *Journal of Applied Corporate Finance*, 8: 103-114, 1995.

[11] Kahneman, Daniel, and Amos Tversky, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica* 47:263-291, 1979.

[12] Kramer, Andrea S. and J. Clark Heston, "An Overview of Current Tax Impediments to Risk Management," *Journal of Applied Corporate Finance*, 6:73-80, 1993.

[13] Kramer, Andrea S., Financial Products: Taxation, Regulation, and Design," 1997 Cumulative Supplement. John Wiley and Sons, Inc. 1997.

[14] Leland, Hayne E., "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance* 53, 1998.

[15] Mello, A. and J. Parsons (1999), "Hedging and Liquidity, " *Review of Financial Studies*, forthcoming.

[16] Millar, William (1989), "New Directions in Financial Risk Management," Strategic Risk Management. Business International.

[17] Moffet, Michael H., and Douglas J. Skinner, "Issues in Foreign Exchange Hedge Accounting," *Journal of Applied Corporate Finance*, 8:82-94, 1995.

[18] Rawls, S. W., and C. W. Smithson, "Strategic Risk Management," in The New Corporate Finance, Where Theory Meets Practice, Donald H. Chew, Jr. ed., McGraw-Hill, 1993.

[19] Ross, Michael, "Corporate Hedging: What, Why, and How?," Working paper, University of California at Berkeley, 1996.

[20] Smith, Clifford W., and René M. Stulz, "The Determinants of Firm's Hedging Policies," *Journal of Financial and Quantitative Analysis*, 4:391-405,1985.

[21] Stokey, Nancy L., Rober E. Lucas, with Edward C. Prescott, Recursive Methods in Economic Dynamics, Harvard University Press, 1989.

[22] Stulz, René M., "Rethinking Risk Management," *Journal of Applied Corporate Finance*, 9:8-24, 1996.

[23] Titman, Sheridan, "Interest Rate Swaps and Corporate Financing Policies," Journal of Finance XLVII:1503:1516, 1992.

[24] Tversky, Amos, and Daniel Kahneman, "Advances in Prospect Theory: Cumulative Representation of Uncertainty, " *Journal of Risk and Uncertainty* 5:297-323, 1992.

[25] U.S. Master Tax Guide, 1998.