Mark-up and Capital Structure of the Firm facing Uncertainty

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Abstract

This note shows that, with pre-set price and capital decisions of firms facing uncertainty and financial market imperfections, price, mark up and the expected degree of capacity utilization (resp. capital) decreases (resp. increases) with the firm internal net worth.

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1. Introduction

In a recent paper, Chevallier et Scharfstein [1996] provided empirical evidence of a relationship between mark up and leverage and proposed a theoretical underpinning based on the "consumer switching cost" model of Klemperer [1987]. A complementary approach is proposed in this note. I show that a relationship between price, capital and financial structure obtains when the firm faces uncertainty with ex post risk of excess capacity, as in Kahn [1992] and Karlin and Carr [1962]. In this model, price depends on expected "tensions" in the goods markets. The higher the probability of excess demand, the higher the market power which determines the markup. Optimal capital depends on the ratio of the mark-up to the cost of capital. The two decisions are linked.

Therefore, introducing Kiyotaki and Moore [1997] incentive problem leading to a liquidity constraint affects not only investment but also price behaviour. The rise of external finance constraint limits capital and increase the probability of excess demand. Simultaneously, the firm rises the price, which lowers expected demand and the probability of excess demand. In so doing, the loss of investment due to the agency problem is partially offset by an increase of market power.

The paper is organized as follows. Section 2 presents the model. Section 3 solves the perfect capital market case. Section 4 solves the financially constrained case. Section 5 concludes.

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2. The model

The production function is clay-clay, with constant returns to scale for capital and labour. Capital is chosen ex ante and defines productive capacities $YC = K/k$. The ratio capital /capacity is k. The cost of capital is denoted c. The output market is cleared ex post by an adjustment of hours worked, except if demand is higher than productive capacities:

$$L = aY \qquad \text{for } 0 \leq Y \leq YC \tag{2.1}$$

The productivity of labour is $1/a$. The manager is a price-taker for labour and the unit cost of labour is denoted w. The price of capital is taken as numeraire and may be different of the output price.

The entrepreneur faces uncertainty on demand. $g(p)$ is the firm's expected demand. It satisfies the standard general requirements for an unique optimal monopoly price in the certainty case. It is a decreasing function of price p ($g(p) \geq 0$, $g_p(p) < 0$ with a price-elasticity $e(p) = p g_p(p) / g(p) < -1$). Demand is zero ($g(p) = 0$) for all prices such that $p \geq p_{\max}$. The requirement for a positive production is that the maximal price $p_{\max}$ is over the marginal costs of production $p_{\max} > wa + ck > 0$. I assume the function

$$h(p) = (p - wa - ck) g_p(p) + g(p) \tag{2.2}$$

to be continuous and to have a unique zero being the price p^c such as $wa + ck \leq p^c < p_{\max}$. If the maximal price $p_{\max}$ is infinite ($g(p) > 0$ for all prices $p \geq 0$, I assume that $\lim_{p \to +\infty} g(p) = \lim_{p \to +\infty} pg(p) = 0$ and that the function $h(p)$ is continuous and presents a unique zero for the price p^c such as $wa + ck \leq p^c$.

Demand is $ug(p)$, where u is a non-negative random variable of cumulative distribution F, and of a continuous density f, with a mean equal to one ($E[u] = 1$ where E represents the expectation operator).

Ex-post, firm production is set at the minimum of production capacity and of demand, $Y = \min(ug(p), YC)$. This is based on the following assumption on short run rigidities: ex-post goods market price rigidity, the second-hand market for excess investment does not work, investment and hours worked are not substitutable ex-post (Kahn [1992] and Karlin and Carr [1962]).

The entrepreneur sets *ex ante* price and capital while maximizing expected profits denoted $\pi(K,p)$:

$$(K,p) \in Argmax\ \pi(K,p) = (p - wa)E[Y] - cK \tag{2.3}$$

with $K \geq 0$ and $p \geq 0$. Expected production is:

$$E[Y] = E[\min(YC, ug(p))] = g(p) \int_0^x u \cdot dF(u) + YC \int_x^{+\infty} dF(u) \tag{2.4}$$

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where $x = YC/g(p)$ is the capacity/expected demand ratio, measuring the expected "tensions" on the goods market. By integration by parts, as $E[u] = 1$ and as $u \geq 0$ (so that $\int_0^{+\infty} [1 - F(u)] \, du = 1$), one has:

$$E[Y] = g(p)I(x) \text{ where } I(x) = \int_0^x 1 - F(u) \cdot du \tag{2.5}$$

$I(x)$ represents the sum of the probabilities of excess demand up to the level of capital related to x.

Following Kiyotaki and Moore [1997], I add two critical assumptions. First the entrepreneur's technology is idiosyncratic: once his production started at date 0, he is the only agent to have the skill necessary for production to occur. If he withdraws his firm specific labour L between date 0 and date 1, there would remain only durable capital K. Second, he cannot precommit to work. He may therefore threaten his creditors by withdrawing his firm specific labour and repudiate his debt contract. Creditors protect themselves from the threat of repudiation. Hart and Moore [1994] give an argument to suggest that the entrepreneur may be able to negotiate the debt (gross of interest) down to the liquidation value of capital, which eventually incurs a transaction cost τ. At the initial date, the entrepreneur can borrow an amount of external finance $K - W$, where W represents the firm internal net worth, as long as the repayment does not exceed the market value of capital:

$$(1+r)(K - W) \leq (1 - \delta) K \Leftrightarrow K \leq \frac{1+r}{r+\delta} W \tag{2.6}$$

r represents the real interest rate, δ represents the depreciation rate. The cost of capital c is equal to $r + \delta$.

3. The Perfect Capital Market Case

In the perfect capital market case, the first order condition with respect to capital is:

$$(p - wa)(1 - F(x)) - ck = 0. \tag{3.1}$$

The marginal cost of capital is equal to the marginal profits at full capacity utilisation, corrected by the probability of use of this capacity. The price p has to be strictly over the sum of marginal costs $wa + ck$ to have a strictly positive optimal capital (else $0 < p \leq wa + ck \Rightarrow K^* = 0$).

The first order condition with respect to price is:

$$0 = \left[E[Y] + (p - wa) \frac{\partial E[Y]}{\partial g(p)} g_p(p) \right] \frac{p}{g(p)} \frac{g(p)}{E[Y]} \tag{3.2}$$

$$\Rightarrow p = \frac{\eta(x) e(p)}{\eta(x) e(p) + 1} wa \text{ for } x > \eta^{-1}\left(-\frac{1}{e(p)}\right) \tag{3.3}$$

where:

$$0 \leq \eta(x) = \frac{g(p)}{E[Y]} \frac{\partial E[Y]}{\partial g(p)} = 1 - \frac{x I_x(x)}{I(x)} \leq 1. \tag{3.4}$$

The elasticity of expected output with respect to price is the chained elasticity of expected output with respect to expected demand $\eta(x)$ times the the elasticity of expected demand with respect to price. In the certainty case, the elasticity of output with respect to expected demand is indeed equal to 1.

The optimal solution is found by solving the system of the first order conditions. A proof of the existence of optimal price and capital (and therefore of an optimal capacity/expected demand ratio x) for any continuous distribution based on the intermediate value theorem is given in the appendix. Eliminating price provides the optimal ratio x^* in an implicit function form:

$$j(x^*) = 1 - F(x^*) + \frac{ck}{wa} \eta(x^*) e(p) + \frac{ck}{wa} = 0 \tag{3.5}$$

As there exist at least a solution for x, if the function $j(x^*)$ is strictly monotonic for $x > \eta^{-1}\left(-\frac{1}{e(p)}\right)$, then this solution is unique according to the intermediate value theorem. The derivative $j_x = \frac{ck}{wa} e(p) \eta_x(x^*) - f(x^*)$ is strictly negative if $\eta_x(x^*) > 0$. A sufficient condition on the distribution of demand to guarantee $\eta_x(x) > 0$, that we assume to be fulfilled in what follows, is:[1]

$$\forall x \in [0, +\infty[\qquad \frac{f(x)}{1 - F(x)} \geq \frac{\eta(x)}{x} \tag{3.6}$$

Differentiating the function j leads to:

$$\underbrace{(-Ae(p)\eta_x(x^*) + f(x^*))}_{>0} dx^* + \underbrace{(-e(p)\eta(x^*) - 1)}_{>0} dA + \underbrace{(-A\eta(x^*))}_{<0} de(p) = 0 \tag{3.7}$$

where $A = \frac{ck}{wa}$ represents the relative cost of factors corrected by their productivity. The ratio x^* a decreasing function of the real interest rate and of the depreciation of capital and an increasing function of the real wage and of the price elasticity of the demand curve.

The optimal price p^* is:

$$p^* = \frac{e(p^*)\eta\left(x^*\left(\frac{ck}{wa}, e(p)\right)\right)}{e(p^*)\eta\left(x^*\left(\frac{ck}{wa}, e(p)\right)\right) + 1} wa. \tag{3.8}$$

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[1]Unimodal distributions such as the lognormal distribution, the uniform distribution and the exponential distribution fulfill this condition.

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It decreases with the ratio x^* and therefore increases with the cost of capital and decreases the price-elasticity of demand and has an ambiguous dependance on the real wage.

The optimal level of capital K^* is:

$$K^* = kg\left(p^*\right)x^*\left(\frac{ck}{wa}, e\left(p^*\right)\right). \tag{3.9}$$

It depends negatively on the cost of capital, positively on the real wage and ambiguously on the elasticity of demand.

4. The Financially Constrained Case

When the finance constraint is binding, the condition giving the optimal stock of capital is now:

$$K^{f*} = \frac{1+r}{r+\delta}W < K^* \Rightarrow x^f = \frac{1+r}{r+\delta}\frac{W}{kg\left(p^f\right)} < x^* \tag{4.1}$$

Capital depends negatively on the interest rate, on the depreciation of capital and positively on the firm internal net worth.

The marginal condition on price is unchanged. Eliminating price provides the optimal ratio x^f in the financially constrained regime (by definition of x, one has: $p = g^{-1}\left(\frac{K}{kx}\right)$):

$$l\left(x\right) = \frac{e\left(p\right)\eta\left(x\right)}{e\left(p\right)\eta\left(x\right)+1}wa - g^{-1}\left(\frac{K^f}{kx}\right) = 0. \tag{4.2}$$

It is easy to prove with the intermediate value theorem that the solution for x is unique as $\lim_{x\to\eta^{-1}\left(-\frac{1}{e\left(p\right)}\right)} l\left(x\right) > 0$, $\lim_{x\to+\infty} l\left(x\right) < 0$ and $l_x\left(x\right) < 0$ (with $\eta_x\left(x\right) > 0$). Differentiating totally $l\left(x\right)$ leads to:

$$0 = \underbrace{\left(\frac{e\left(p\right)\eta_x\left(x\right)wa}{\left[e\left(p\right)\eta\left(x\right)+1\right]^2} + g_p^{-1}\left(\frac{K^f}{kx}\right)\frac{K^f}{kx^2}\right)dx^f}_{<0} \underbrace{-g_p^{-1}\left(\frac{K^f}{kx}\right)\frac{1}{kx}dK^f}_{>0}$$

$$+ \underbrace{\frac{e\left(p\right)\eta\left(x\right)a}{e\left(p\right)\eta\left(x\right)+1}dw}_{>0} + \underbrace{\frac{\eta\left(x\right)wa}{\left[e\left(p\right)\eta\left(x\right)+1\right]^2}de\left(p\right)}_{>0}$$

As in the perfect capital market case, the ratio x^f is a decreasing function of the real interest rate and of the depreciation of capital and an increasing function of the real wage and of the price elasticity of the demand curve. But in the financially constrained regime, it is also an increasing function *of the firm internal net worth.*

The price when the financial constraint binds, denoted p^f, is:

$$p^f = \frac{e\left(p^f\right)\eta\left(x^f\right)}{e\left(p^f\right)\eta\left(x^f\right)+1}wa \tag{4.3}$$

Price is a decreasing function of the ratio x. As $x^f < x^*$, the price when the finance constraint is binding is *higher* than the price chosen in the perfect capital market case. It is a decreasing function of the price elasticity of the demand curve and *of the firm internal net worth* and an increasing function of the real interest rate and of the depreciation of capital. Its dependance on the real wage is ambiguous.[2]

The optimal expected degree of capacity utilisation $E\left[Y\right]/YC = I(x^*)/x^*$ is also a decreasing function of the ratio x, due to the concavity of the function $I(x)$. Therefore, it is an increasing function of the real interest rate and of the depreciation of capital and a decreasing function of the real wage and of the price elasticity of the demand curve, in the perfect capital market case. When the financial constraint binds, it is also a decreasing function *of the firm internal net worth*.

The condition on financial structure for a shift of regimes is obtained as by the solution (x, p, W) of the system of the two first order conditions and of the binding financial constraint. For a sufficiently high level of the firm internal net worth (an implicit function $W^*\left(r, w\right)$), the firms shifts to the uncontrained regime.

5. Conclusion

This note shows that with pre-set price and capital decisions of firms facing uncertainty and financial market imperfections, price, mark up and the expected degree of capacity utilization (resp. capital) decreases (resp. increases) with the firm internal net worth. Further research could consider dynamic general equilibrium extensions of this model to investigate the cyclical properties of mark-up, capital or inventories, the degree of capacity utilization and financial structure.

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5.1. Appendix: Existence of the optimal solution (K, p)

The second order necessary and sufficient condition with respect to capital is always fulfilled except if the density of the distribution is zero for the ratio x^*:

$$\pi_{KK}(K,p) = -f(x^*)\frac{p-wa}{g(p)} < 0 \qquad (5.1)$$

We maximize profit with respect to price incorporating the marginal condition on the choice of capital : $p^* \in Arg\max\pi(K^*,p)$. The intermediate value theorem applied to the first order derivative of expected profits with respect to price π_p helps to prove that *this derivative presents at least a zero which is a local maximum.*

First, when $p = wa + ck$, optimal capital is zero $(x^* = F^{-1}(0) = 0)$ so that expected profits are zero. Second, when the price tends to infinity, expected profits tend to be negative: as $\lim_{x\to+\infty} I(x) = \int_0^{+\infty} [1 - F(u)] \cdot du = 1$, and knowing the hypotheses $\lim_{p\to+\infty} g(p) = \lim_{p\to+\infty} pg(p) = 0$, one has:

$$\lim_{p\to+\infty} \pi(K^*,p) = \lim_{p\to+\infty} g(p)[(p-wa)I(x^*) - ckx^*] \leq 0 \qquad (5.2)$$

To apply the intermediate value theorem, it is now only sufficient to prove that $\pi_p(K, p = wa + ck) > 0$, which is done as follows:

$$\pi_p(K^*,p) = [g_p(p)(p-wa) + g(p)]I(x^*) - g_p(p)ckx^* \qquad (5.3)$$
$$= \underbrace{[(p-wa-ck)g_p(p) + g(p)]}_{=h(p)>0 \text{ for } wa+ck<p<p^c} \cdot \underbrace{I(x^*)}_{>0}$$
$$+ck\underbrace{g_p(p)}_{<0}\underbrace{(I(x^*)-x^*)}_{<0} \qquad (5.4)$$

$I(x) = \int_0^x [1 - F(u)] du \leq x$ is an immediate result. $h(p) = (p-wa-ck)g_p(p) + g(p)$ is the derivative of profits where there is no uncertainty. By assumption, it is zero for the optimal monopoly price p^c. Therefore, $h(p) > 0$ for values of the price such that: $wa + ck < p < p^c$. Hence, $\pi_p(K^*,p) > 0$ for values of the price such that: $wa + ck < p < p^c$. QED.

One remarks that price under certainty is lower than price under uncertainty: $\pi_p(K^c, p^c) > 0$. When the random shock is multiplicative, increasing the price implies a lower standard error on sales: $\sigma_D = \sigma_u \mid g(p) \mid$.

There may be n local maxima (and $n-1$ local minima) which may be related to the modes of the density function f. A sufficient condition for unicity is to have a monotonous elasticity $\eta(x)$.