International Stock-Bond Correlations in a Simple Affine Asset Pricing Model[*]

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Abstract

We use an affine asset pricing model to jointly value stocks and bonds. This enables us to derive endogenous correlations and to explain how economic fundamentals influence the correlation between stock and bond returns. The presented model is implemented for G7 post-war economies and its in-sample and out-of-sample performance is assessed by comparing the correlations generated by the model with conventional statistical measures. The affine framework developed in this paper is found to generate stock-bond correlations that are in line with empirically observed figures.

Key words: Affine Pricing Model, Stock-Bond Correlations, G-7 Countries

JEL: F30, G12, G15

1 Introduction

The research on the pricing of fixed-income securities and equities has traditionally evolved along separate lines. Since each of these two research areas was successful in generating significant innovations, only recently some authors have started to challenge this clear-cut separation by proposing models that make use of a unifying pricing theory for stocks and bonds (See Beltratti and Shiller (1992), Bekaert and Grenadier (2001), Campbell and Viceira (2001), and Mamaysky (2002)). A research topic that naturally conveys both areas of research is the analysis of cross-market correlations. Stock-bond correlations are at the core of many financial decisions such as problems related to risk management and the optimal allocation of financial assets.

In view of the broad spectrum of practical applications and theoretical questions related to the correlation between stocks and bonds, it is not surprising that a number of important articles address very different aspects of this research topic.[1] In general, contributions differ with respect to the scope of the economic foundation and the focus on statistical fit. We identify three major lines of research: econometric articles, papers based on fundamental economic models, and articles that uncover empirical stylized facts of stock-bond correlations.

Prominent contributions that address correlations from an econometric perspective are the Constant Correlation GARCH of Bollerslev (1990), the BEKK GARCH of Engle and Kroner (1995), and the Dynamic Conditional Correlation GARCH of Engle and Sheppard (2001), among others. Focusing on stock and bond returns, Guidolin

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[1] Since this paper focuses on stock-bond correlations, we do not explicitly discuss the rich literature on stock-stock correlations. The most important aspects covered in this literature are financial contagion (e.g. Barberis et al. (2005), Forbes and Rigobon (2002), and Kodres and Pritsker (2002)), asymmetric correlations (e.g. Ang and Chen (2002), Longin and Solnik (2001), and Ribeiro and Veronesi (2002)), and applied topics such as optimal asset allocation decisions (Ang and Bekaert (2002) among others).

and Timmermann (2005) introduce multiple regimes to allow for a state-dependent comovement structure between these asset classes. [2] While financial econometrics has developed powerful and effective tools to analyze, describe, and predict the correlation of individual and aggregate financial series, this line of research has added little to the economic understanding of the factors driving this correlation.

To explain and understand some of the underlying economic linkages between financial assets, some authors have proposed valuation models that jointly price these two assets classes. Barsky (1989) presents the first theoretical model that focuses on the comovement of stock and bond prices. By setting up a simple, yet ambitious, general equilibrium model, he states that the stock-bond comovement, which is driven by productivity shocks and changes in the market risk, crucially depends on the risk-aversion parameter of the representative investor. Beltratti and Shiller (1992) follow a simpler approach that is more suitable for being calibrated on data. To derive theoretical correlations between stock prices and long-term bond yields, Beltratti and Shiller (1992) extend the well known Campbell and Shiller (1988) dividend-ratio model and jointly price stocks and bonds. In the empirical part of their paper, Beltratti and Shiller (1992) analyze the US and UK markets and find that the correlations implied by their model are on average much lower than realized correlations. Using the same framework, Campbell and Ammer (1993) employ a VAR to decompose the variance-covariance matrix of excess stock and bond returns. They identify two components that govern the stock-bond covariance: while unexpected shocks of the real interest rate drive returns of stocks and bonds in the same direction, expected inflation increases excess stock returns and lowers excess bond returns. Campbell and Ammer (1993) investigate the second moments of the innovations in the excess stock returns and excess ten-year bond returns and obtain, for post-war US data, slightly

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[2] In a recent contribution, Andersen et al. (2005) provide an extensive survey on correlation forecasting.

positive correlations. According to these authors, this finding can be explained by the low variability of real interest rates and by increases in expected inflation which drive correlation down. Finally, Fleming et al. (1998) model investors' demand curves based on a mean-variance optimization scheme to investigate the information-driven volatility linkages between stocks and bonds.

Given the limited empirical success of fundamental economic models in explaining observed correlation and in view of the formal difficulties in maintaining analytical tractability, a third line of research has recently emerged. It aims at identifying and understanding stylized facts and historical patterns of the stock-bond correlation by directly focusing on data. Gulko (2002) finds evidence in favor of *decoupling* of stock and bonds during stock market crisis, a phenomenon often referred to as *flight-to-quality.* Similarly, Connolly et al. (2005a) and Connolly et al. (2005b) obtain supportive results for the flight-to-quality hypothesis. In particular, they find that rising stock market uncertainty tends to decrease the comovement between stock and bonds and thus increase the diversification benefits. On the same lines, David and Veronesi (2004) and Li (2002) show that uncertainty about macroeconomic factors (especially expected inflation) has significant predictive power with respect to the covariance and correlation of stock and bond returns. Finally, Andersen et al. (2004) investigate the impact of macroeconomic news on stocks and bonds and find that correlation calculated on high frequency data is higher in expansion periods and lower (and negative) in periods of economic contraction.

In this paper we take an economic approach based on the fundamental valuation of future expected cash flows and contribute to the second line of research. The general approach of this paper is similar to both Beltratti and Shiller (1992) and Campbell and Ammer (1993). However, while those authors apply an extension of the Campbell and Shiller (1988) model, originally developed for pricing equities, to both stocks and bonds, we address the problem from the opposite direction. As in Bekaert and

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Grenadier (2001), Li (2002), and Mamaysky (2002), we use an affine pricing model, traditionally employed for pricing fixed-income securities, to jointly value stock and bond indices. Our contribution is threefold. First, we propose an endogenous formula for the stock-bond correlation developed in an affine asset pricing framework. This correlation formula is purely determined by the dynamics of the underlying economic fundamentals and fully abstracts from historical prices of equities and bonds. The correlation formula is kept as general as possible by allowing all factors to be correlated with each other and with the pricing kernel.[3] Second, we analyze the effect of the model parameters on the stock-bond correlation and provide an intuitive understanding of those relationships. In contrast to Fleming et al. (1998), all the factors in our model are observable, which facilitates a straightforward interpretation of the results. Third, while most model-based correlation studies investigate the US market, we contribute an empirical analysis of stock-bond correlations performed on G7 postwar economies and provide evidence that model correlations are in line with empirical observed figures.

The remainder of the paper is organized as follows. In Section 2 we present the affine asset pricing framework and derive the theoretical relationships used in the empirical investigation. Section 3 introduces the data set used for the empirical analysis. In Section 4 we describe the estimation procedure and discuss the results of the empirical investigation. Section 5 concludes.[4]

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[3] In an independent contribution, Li (2002) derives a formula for the covariance of stock and bond returns in a similar affine setting, using a different modeling of factor correlations.
[4] To economize space, proofs, algebraic derivations, and additional empirical results are provided in the working paper version of this article which is freely downloadable from the authors' webpages.

2 Model

In modern finance the fair price of any asset is calculated as the conditional expectation of its future payoffs multiplied with a stochastic discount factor, or pricing kernel. Thus, in a discrete time environment, prices can be computed as

$$P_t^* = E_t[W_{t+1}^* M_{t+1}^*], \tag{1}$$

where W_{t+1}^* represents the cash flows generated by the asset in time $t+1$ and M_{t+1}^* is the stochastic pricing kernel. For the time being let us consider prices and payoffs as nominal rather than real (i.e. inflation adjusted) quantities. Here and henceforth, an asterisk denotes nominal variables. The existence of the stochastic discount factor is ensured by assuming that there are no arbitrage opportunities in the economy. Conditions for the uniqueness of the kernel are derived in Harrison and Kreps (1979). By assuming that M^* is conditionally lognormal we can apply standard arguments as described in Campbell et al. (1996) and obtain the following general form for the logarithmic kernel:

$$-m_{t+1}^* = \delta + r_t^* + \epsilon_{t+1}^{m*}, \tag{2}$$

where $\epsilon_{t+1}^{m*} \sim N(0, \sigma_{m*}^2)$ are the i.i.d. shocks of the nominal pricing kernel, $\delta = \frac{1}{2}\sigma_{m*}^2$, and r_t^* is the risk-free interest rate. Due to the relationship between the nominal and the real stochastic discount factor, $m^* = m - \pi$, it is straightforward to obtain the expression for the real pricing kernel. To capture the mean reverting nature of the real short rate, a discrete-time version of the Vasicek (1977) model is adopted: $r_{t+1} = \overline{r} + \alpha_r (r_t - \overline{r}) + \sigma_r \varepsilon_{t+1}^r$, where $\overline{r}$ is the unconditional mean of the real short rate, σ_r is its conditional volatility, and the error $\varepsilon_{t+1}^r \sim N(0, 1)$ is i.i.d. An analogous process is chosen for the inflation rate: $\pi_{t+1} = \overline{\pi} + \alpha_\pi (\pi_t - \overline{\pi}) + \sigma_\pi \varepsilon_{t+1}^\pi$. To account for the interaction with the inflation, we can represent the real interest rate innovation

as:

$$\sigma_\pi \varepsilon_{t+1}^\pi = \beta_\pi \sigma_r \varepsilon_{t+1}^r + \sigma_\varpi \epsilon_{t+1}^\varpi, \tag{3}$$

where β_π is a factor that governs the covariance between r_t and π_t. The error term $\epsilon_{t+1}^\varpi \sim N(0,1)$, i.i.d., represents the part of the inflation shocks which is orthogonal to the real interest rate.

By letting the interest rate and inflation be correlated, we extend the standard affine pricing models as implemented in Bekaert and Grenadier (2001) and Campbell et al. (1996). Further, to price the risk associated with inflation, we allow the inflation process to be correlated with the real stochastic discount factor. On the contrary, Bekaert and Grenadier (2001) impose independence between the real kernel and inflation to obtain neutrality of monetary aggregates. Given the correlation structure among the interest rate, inflation, and the discount factor, we can conveniently represent the innovations of the kernel as $\epsilon^{m*} = \beta \sigma_r \epsilon_{t+1}^r + \beta \sigma_\pi \epsilon_{t+1}^\pi + \sigma_\eta \epsilon_{t+1}^\eta$, where β is the common factor of the shocks that governs the covariance between m^* and r^*. The error term ϵ_{t+1}^η is independently and identically distributed as $N(0,1)$ and conveys those fluctuations of the nominal pricing kernel which are orthogonal to the real interest rate and inflation. Since ϵ_{t+1}^η only affects the average level of the term structure but not its slope, we get the following simplified equation for the logarithmic pricing kernel:
$m_{t+1}^* = -\delta - r_t^* - \beta \sigma_r \epsilon_{t+1}^r - \beta \sigma_\pi \epsilon_{t+1}^\pi$.

2.1 Bonds

A default-free bond price is the sum of a finite stream of known nominal discounted cash flows and, consequently, its fair value is determined by all variables necessary to identify the nominal discount rate: real interest rate and inflation. The affine guess for bond prices we adopt in this paper relates the fair value at time t of a bond with

maturity n to the relevant state variables in the following way:

$$-p_t^{n*} = A_n + B_n r_t + C_n \pi_t. \tag{4}$$

The roots of the above equation assume the following recursive form:

$$A_n = A_{n-1} + \delta + (1 - \alpha_r)\bar{r}B_{n-1} + (1 - \alpha_\pi)\bar{\pi}(C_{n-1} + 1) -$$

$$-\tfrac{1}{2}\left[(B_{n-1} + \beta)^2 \sigma_r^2 + (C_{n-1} + \beta)^2 \sigma_\pi^2 + 2(B_{n-1} + \beta)(C_{n-1} + \beta)\beta_\pi\sigma_r^2\right], \tag{5}$$

$$B_n = \tfrac{(1-\alpha_r^n)}{(1-\alpha_r)}, \text{ and } C_n = \tfrac{(1-\alpha_\pi^n)}{(1-\alpha_\pi)}\alpha_\pi^2.$$

This result proves the validity of the initial affine bond pricing guess in Eq. (4) and enables us to derive bond returns for any maturity. We now apply the simple definition of a one-period logarithmic return to a bond with maturity n and obtain

$$b_{t+1}^{n-1*} = -A_{n-1} - B_{n-1}r_{t+1} - C_{n-1}\pi_{t+1} + A_n + B_n r_t + C_n \pi_t. \tag{6}$$

2.2 Stocks

Stocks can be viewed as an infinite stream of dividends. In contrast to fixed-income securities, the cash-flow stream of equities is not known in advance and has to be estimated for valuation purposes. To obtain theoretical stock returns, it is convenient to consider the real stock price, P_t^s, as an expectation of its real future payoffs at time $t + 1$:

$$P_t^s = E_t[P_{t+1}^s \exp(d_{t+1}) M_{t+1}], \tag{7}$$

where $d_t = \ln\left(1 + \frac{D_t}{P_t^s}\right)$ is the logarithmic dividend yield and D_t is the real dividend in time t. As noted by Lewellen (2004) and other authors, by ruling out financial

bubbles, it is economically reasonable to model the dividend yield as an exogenous mean-reverting stochastic process: $d_{t+1} = \overline{d} + \alpha_d \left(d_t - \overline{d} \right) + \sigma_d \varepsilon_{t+1}^d$.[5] To account for the interaction with the real interest rate, we can represent the dividend yield innovation as:

$$\sigma_d \varepsilon_{t+1}^d = \beta_d \sigma_r \varepsilon_{t+1}^r + \sigma_\nu \epsilon_{t+1}^\nu, \tag{8}$$

where β_d is a factor that governs the covariance between r_t and d_t. The error term ϵ_{t+1}^ν is independently and identically distributed as $N(0,1)$ and represents the orthogonal part of the dividend yield fluctuations with respect to the real interest rate and the pricing kernel.

Unlike bonds, the affine pricing guess for stocks involves a recursion to infinity:

$$p_t^s = \lim_{n \to \infty} \left(F_n + G_n r_t + H_n d_t \right), \tag{9}$$

with:

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\begin{aligned}
F_n &= -\delta + F_{n-1} + G_{n-1} \left(1 - \alpha_r \right) \overline{r} + \left(1 - \alpha_d \right) \overline{d} (H_{n-1} + 1) + \\
&\quad + \tfrac{1}{2} \left[\left(G_{n-1} - \beta \right)^2 \sigma_r^2 + \left(1 + H_{n-1} \right)^2 \sigma_d^2 + 2 \left(G_{n-1} - \beta \right) \left(1 + H_{n-1} \right) \beta_d \sigma_r^2 \right], \\
G_n &= \left(G_{n-1} \alpha_r - 1 \right) \Rightarrow G = \lim_{n \to \infty} G_n = -\tfrac{1}{(1 - \alpha_r)}, \\
H_n &= \left(H_{n-1} \alpha_d + \alpha_d \right) \Rightarrow H = \lim_{n \to \infty} H_n = \tfrac{\alpha_d}{(1 - \alpha_d)}.
\end{aligned}
\tag{10}
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[5] By modeling the dividend yield as a mean reverting process, we relate to papers such as Campbell and Shiller (1988), Lewellen (2004), Li (2002), Mamaysky (2002), and Stambaugh (1999), among others. However, several papers (See e.g. Campbell and Yogo (2003) and Lamont (1998)) document the difficulties of rejecting the null hypothesis of a unit root in dividend-yield series. In fact, by performing Dickey and Fuller (1979) and Phillips and Perron (1988) tests using the G7 dividend-yield series, we are able to reject the hypothesis of a unit root only in few cases. We thank an anonymous referee for helping us in clarifying this point.

We derive the expression of nominal stock returns simply by applying the definition of a one-period logarithmic return and adding realized inflation:

$$s_{t+1}^* = F_{n-1} - F_n + G\left(r_{t+1} - r_t\right) + H\left(d_{t+1} - d_t\right) + d_{t+1} + \pi_{t+1}. \qquad (11)$$

Some aspects of the stock pricing model presented above deserve further attention. First, it is worth noting that modeling the dividend yield process instead of the dividend growth process facilitates the derivation of the pricing formula in Eq. (7). Other authors (e.g. Bekaert and Grenadier (2001)) choose to model the dividend growth and consequently obtain an expression for the price-dividend ratio. Second, by modeling the dividend yield process as correlated with the real interest rate, we allow the equity premium to be driven by both interest rate and dividend yield risk. [6]

2.3 Stock-Bond Correlation

To obtain the theoretical formula for the correlation between stock and bond returns, we first use Eqs. (6) and (11) as well as the expectation properties on linear functions to calculate the covariance between s_t^* and b_t^{n*}. By using the standard correlation formula, we get:

$$\frac{-G\mathcal{B}\sigma_r - \mathcal{C}\sigma_\pi - \beta_\pi\sigma_r^2\left(B_{n-1} + GC_{n-1}\right) - B_{n-1}\mathcal{H} - \beta_\pi C_{n-1}\mathcal{H}}{\sqrt{\left(\mathcal{B}^2 + \mathcal{C}^2 + 2\mathcal{B}C_{n-1}\beta_\pi\sigma_r\right)}\sqrt{G^2\sigma_r^2 + \sigma_\pi^2 + (1+H)^2\sigma_d^2 + 2G\mathcal{H} + 2G\beta_\pi\sigma_r^2 + 2\beta_\pi\mathcal{H}}}. \qquad (12)$$

where $\mathcal{B} = B_{n-1}\sigma_r$, $\mathcal{C} = C_{n-1}\sigma_\pi$, and $\mathcal{H} = (1+H)\beta_d\sigma_r^2$.

At this point, it is worth investigating the driving forces of the stock-bond correlation. Not surprisingly, none of the long-term means of the three factors driving the stock and bond prices, $\overline{r}$, $\overline{\pi}$, and $\overline{d}$, have an influence on the correlation. This result is intu-

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[6] For a formal explanation of this result we refer to Eq. (14) in Subsection 4.1 on page 17.

itive because the long-term means affect the expected values but not the variability of the factors.

To analyze in more detail the derived correlation formula, we first consider the covariance in the numerator of Eq. (12) and then the standard deviations of stock and bond returns. The covariance between stocks and bonds can be split into five terms, the first two related to interest rate risk and inflation risk, respectively, and the other three related to the cross covariances among the three sources of uncertainty: real interest rate, inflation, and dividend yield. In the following, we provide a discussion of each term as well as some comparative static results. Assuming a non-explosive process for the interest-rate, the first term of the covariance, $-GB_{n-1}\sigma_r^2$, is always positive. Thus, the covariance between stock and bond returns increases with the volatility of the real interest rate. This result is intuitive since the real interest rate discounts future cash flows of both stocks and bonds and thus affects their prices in the same direction. Further, a higher persistence parameter of the short rate, α_r, has a positive contribution to the stock bond covariance. The impact of the inflation process can be analyzed in a similar way. The second term of the covariance, $-C_{n-1}\sigma_\pi^2$, is always negative. Consequently, the covariance between stock and bond returns tends to fall as inflation shocks get larger. This finding mirrors the different impact of inflation rates on cash flows deriving from stocks and bonds. More precisely, while real stock returns are assumed to be fully hedged against inflation shocks (the dividend is specified as yield and thus in real terms), real bond returns are negatively effected by an unexpected growth of inflation. Similarly, with a high persistence parameter, α_π, the covariance tends to assume negative values.

The remaining terms in the numerator of Eq. (12) arise from the comovement of the three factor processes. The third term of the covariance, $-\beta_\pi\sigma_r^2\left(B_{n-1}+GC_{n-1}\right)$, captures the effect of the correlation between the real interest rate and inflation. The fourth term, $-B_{n-1}\left(1+H\right)\beta_d\sigma_r^2$, refers to the impact of the dividend-yield process

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on the covariance. A positive dividend-yield premium implies a negative β_d and thus a positive contribution of this term on the stock-bond covariance. A higher σ_r increases the magnitude of this influence. Further, the stock-bond covariance increases for higher persistence parameters of both real interest rate and dividend-yield process. Finally, the last term, $-\beta_\pi \beta_d \sigma_r^2 C_{n-1} (1 + H)$, reflects the correlation between the dividend yield and the inflation rate. It is positive as long as the correlation between these factors is positive.

The standard deviations of the stock and bond returns in the denominator in Eq. (12) differ in several ways. First, the volatility of bond returns is easier to understand as it depends solely on the parameters of the interest rate and inflation process. In contrast, the stock return volatility depends additionally on parameters of the dividend-yield process, which adds three more terms to the volatility expression. By letting for the moment the correlation between the processes be zero, we can gain some intuitive insights on the volatility of stock and bond returns.[7] In this setting, the dividend-yield variance only affects the stock-return volatility. The inflation variance has a different impact on the two volatilities, although the sign is always positive. For a high persistence parameter, α_π, the influence of the inflation variance is stronger on bond returns than on stock returns.[8] The intuition behind this last result is related to the different nature of stock and bond cash flows. Since stock cash flows are expressed in real terms, inflation volatility directly translates into volatility of nominal stock returns (cf. Eq. (11)). On the other hand, bond cash flows are nominal and hence do not depend on the level of inflation. The reason for the importance of the persistence parameter, α_π, for bond prices stems from the fact that present values of future cash flows are affected by the inflation rate. For $\alpha_\pi = 0$, inflation shocks are temporary

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[7] We refer to Fig. 2 on page 28 for an analysis of the impact of the terms referring to the comovement of the three factors.

[8] For maturities of over one year, $\alpha_\pi \approx 0.618$ implies an equal impact of the inflation variance on stock and bond volatility.

and thus do not affect the present value of future cash flows. Consequently, in this case, the bond volatility does not depend on the magnitude of inflation shocks. For $\alpha_\pi \geq 0$, a given inflation shock does not immediately disappear but declines over time and affects the bond price. Finally, the different impact of the variance of the real interest rates, σ_r^2, on bonds and stocks is caused by the different time horizon of their cash flows. Since the value of stocks reflects an infinite dividend stream, stocks have a higher duration than bonds and thus a higher interest-rate sensitivity. Only in the limiting case of consols, i.e. bonds with infinite maturity, the influence of σ_r^2 on the respective standard deviation is the same for both instruments.

[INSERT Fig. 1 AROUND HERE]

To investigate the overall impact of the different process parameters on the stock-bond correlation, we provide in Fig. 1 some comparative statics. Starting from a base scenario for the three factor processes, Panels A-D display the stock-bond correlation as key parameters are perturbed. In order to ensure consistent values for the process parameters and isolate the effects of the three factors, it is convenient to assume in Panels A-C that the interest-rate innovations are uncorrelated to both the dividend-yield and inflation shocks.[9] In fact, since Eq. (3) and Eq. (8) imply that $|\beta_\pi| \leq \frac{\sigma_\pi}{\sigma_r}$ and $|\beta_d| \leq \frac{\sigma_d}{\sigma_r}$, setting β_π and β_d equal to zero ensures that these conditions are always satisfied and that the computed correlation is economically meaningful for any combination of σ_r, σ_π, and σ_d.

Fig. 1, Panel A, focuses on the effect of the interest-rate process and finds that volatility increases stock-bond correlation over the whole domain. This is consistent with the previous finding that real interest rate movements drive stock and bond prices in the same direction. Moreover, other things equal, the persistence of the interest

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[9] It is worth noting that the assumption of independence among the factors is made only for the sake of an easier interpretation of the correlation formula but is not pursued further in the remainder of this paper.

rate has a positive impact on the correlation because it increases the variability of the interest rate and thus also the variability of stocks and bonds.

Fig. 1, Panel B, plots the stock-bond correlation for different volatility values of the inflation rate. As mentioned, higher values of the persistence parameter, α_π, and diffusion term, σ_π, lead to lower, and possibly negative, correlations of stock and bond returns.

The influence of the dividend-yield process is displayed in Panel C of Fig. 1. Since a high dividend-yield volatility increases the variability of stocks and has no influence on bond returns, the absolute stock-bond correlation decreases and thus tends to zero, as both σ_d and α_d increase.

Finally, Fig. 1, Panel D, investigates the influence of β_d by letting it vary from -0.02 to 0.02. According to Eq. (8), a larger $|\beta_d|$ implies, other things equal, a larger volatility of the dividend-yield innovations, σ_d. To ensure that the dividend yield volatility remains in the positive domain, β_d and the correlation of the innovations of the real interest rate and the dividend yield, $\rho_{r,d}$, must have the same sign. In Fig. 1, Panel D, $|\rho_{r,d}|$ is held constant at 0.01. Overall, for reasonable parameter values, the impact of β_d on the correlation is not very pronounced. Furthermore, the third term of the covariance, $B_{n-1}\left(1+H\right)\beta_d\sigma_r^2$, is found to have a very limited impact on the stock-bond correlation. Since β_d impacts the correlation almost exclusively through its influence on the dividend yield volatility, the effect strongly resembles the one presented in Panel C.

[INSERT Fig. 2 AROUND HERE]

Fig. 2 analyzes the impact of the correlations $\rho_{r,d}$ and $\rho_{\pi,d}$ on the stock-bond model correlation. As can be easily recognized, model correlations vary widely depending on the values of $\rho_{r,d}$ and $\rho_{\pi,d}$. Moreover, the shape of the correlation surface is greatly affected by the volatility and persistence parameters of the interest rate, inflation, and

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dividend yield. However, for realistic values of the process parameters the correlation between stock and bond returns mostly falls in a plausible range.

3 Data

The financial time series of G7 countries used for the empirical analysis are the Datastream Total Market Indices for stocks and the JP Morgan Government Indices for bonds with the corresponding time series of dividend yields and durations. While all the stock series date back to January 1973,[10] JP Morgan's bond indices begin in the 80's. The macroeconomic time series used for estimating the model are extracted from the IFS-IMF database. For the short-term interest rate the one-month Treasury Bill is used when available, otherwise, the one-month money-market rate is used. The long-term interest rate is the ten-year Treasury Bond yield. The inflation rate is calculated using the Consumer Price Index (CPI). All data has monthly frequency. However, to calculate realized monthly correlations, the corresponding stock and bond indices with daily frequency are employed. A preliminary evaluation of the data shows that in the 80's stock returns are higher than bond returns and this is associated with a higher volatility. For most countries, the risk-return profile in the 90's is fairly similar to the 80's. An important exception is represented by the stock-market downturn in Japan and, to a lesser extent, by the high bond returns in Italy. Starting from the year 2000, the ex-post risk-return profile of international indices changes dramatically due to the global downward movement of equity prices following the new-economy boom.

Having described the raw series of indices, we can now examine the historical evolution of international stock-bond correlations. Historical correlations calculated with

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[10] The stock series of the United Kingdom are provided with a longer history.

an exponentially weighted moving average correlation (EWMA) on the whole data sample display a similar pattern to rolling correlations adjusted for data outliers. In general, we observe that the dates of stock outliers do not coincide with those of bonds. In particular, the data point corresponding to October 1987, clearly influenced by the Black Monday, October 19, happen to be excluded from all but one (Japan) stock indices. Another outlier common to several stock markets (U.S.A., Germany, and Canada) is August 1998. This data point can be easily associated with the Asian crisis that culminated with the float of the ruble and, most importantly, with the restructuring, on August 17, of the Russian debt maturing before January 1, 1999. International stock-bond correlations follow a similar pattern over time. Solely Japan exhibits a more independent evolution: its correlation is typically lower and, starting from 1995, even negative. Many authors (most recently Goetzmann et al. (2005)) have studied the correlations of international stock indices and have reported clear evidence for increasing correlations in the past few decades. It is worth noting that, in spite of this partial erosion of the diversification opportunities among international stock markets, the stock-bond correlation of all G7 countries has not risen but actually, in most recent years, fallen.

Finally, to investigate whether correlations vary over time, we divide the whole sample into five intervals of equal length and calculate Jennrich (1970) statistics for each of the ten pairs of correlation matrices. Overall, the findings are very similar to the evidence reported in Longin and Solnik (1995) and Kaplanis (1988). In 35 out of a total of 70 cases, the null hypothesis of equal correlation matrices cannot be rejected at a significance level of 5 percent. It is worth noting that the correlation matrix obtained from the last period has the highest rate of rejections (21 rejections out of 28 comparisons). In particular, the last-period correlation matrices of U.S.A., France, and Italy are significantly different from all previous periods at the one percent level. We further observe that the results obtained by testing the equality of the variance-

covariance matrices (instead of the correlation matrices) indicate a much higher rate of rejection of the null hypothesis (65 cases, out of a total of 70 cases). As noted by Kryzanowski and To (1987) and Kaplanis (1988), the rejection rate of the Jennrich (1970) statistics can be affected by the time-varying volatility of the analyzed series.[11]

4 Empirical Results

4.1 Estimation Procedure

To obtain model correlations as outlined in Section 2, the process parameters for the interest rate, inflation, and dividend yield are inferred from time-series data. The parameters of the processes are estimated by maximum likelihood. By making use of Eq. (3) and Eq. (8), we estimate β_π and β_d as $\rho_{r,\pi}\frac{\sigma_\pi}{\sigma r}$ and $\rho_{r,d}\frac{\sigma_d}{\sigma r}$, respectively.[12] In addition to the process parameters of the three factors, it is possible to estimate the parameter β which governs the slope of the term structure curve. In accordance with Campbell et al. (1996), the expected spread between the return on a long-term bond with maturity n and the one-period interest rate in an affine pricing framework is given by $E_t[b_{t+1}^{n*}] - y_t^1 = -Cov_t[b_{t+1}^{n*}, m_{t+1}^*] - \frac{1}{2}Var_t[b_{t+1}^{n*}]$. This can be rewritten as

$$E_t[b_{t+1}^{n*}] - y_t^1 = B_{n-1}\lambda_r\sigma_r + C_{n-1}\lambda_\pi\sigma_\pi - \lambda_{\pi r}\left(B_{n-1} + C_{n-1}\right)\sigma_r^2 - \frac{1}{2}Var_t[b_{t+1}^{n*}], \quad (13)$$

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[11] As noted by Goetzmann et al. (2005), the consistency of the Jennrich (1970) test statistics assumes normally distributed asset returns. We thank an anonymous referee for pointing out this aspect.

[12] To ensure economically meaningful results and increase the numerical robustness of the maximization algorithm, the estimation is performed by restricting the domain of volatilities to positive values and the domain of persistence parameters to the region $(0, 1)$.

where $\lambda_r = -\beta\sigma_r$ and $\lambda_\pi = -\beta\sigma_\pi$ correspond to the market price of interest rate risk and the market price of inflation risk, respectively. We can interpret the values B_{n-1} and C_{n-1} in Eq. (13) as the loadings on these two sources of risk. The third term in Eq. (13), with $\lambda_{\pi r} = \beta\beta_\pi$, arises because of the correlation between the innovations of the inflation rate and the real interest rate. Since β has to be negative to ensure economically meaningful (positive) market prices of risk, a positive correlation between these two factors will increase the overall term spread.

Following a similar procedure we derive the relation between the equity premium and the dividend risk. We can represent the expected spread between the return on a stock and the riskless interest rate as $E_t[s^*_{t+1}] - y^1_t = -Cov_t[s^*_{t+1}, m^*_{t+1}] - \frac{1}{2}Var_t[s^*_{t+1}]$. Using Eq. (8) and substituting $Cov_t[s^*_{t+1}, m^*_{t+1}] = -G\beta\sigma_r^2 - H\beta\beta_d\sigma_r^2$ into the above equation, we get

$$E_t[s^*_{t+1}] - y^1_t = -G\lambda_r\sigma_r + H\lambda_d\sigma_r - \frac{1}{2}Var_t[s^*_{t+1}], \qquad (14)$$

where $\lambda_d = \beta\beta_d\sigma_r$ is the market price of the dividend yield risk. Similarly to the above discussion for the bond premia, we can interpret the values $-G$ and H in Eq. (14) as the loadings of the stock on the interest rate and dividend risk.

We obtain an estimate for β by numerically solving Eq. (13) while using for its left side the difference between the ten-year bond yield and the one-month interest rate. According to Eq. (13), a positive term spread implies a negative β, and the steeper the interest rate curve the smaller the value of β. By multiplying β, which also governs the covariance between m^* and r^*, with the volatility of the short rate, we get the market price of interest-rate risk. For the US economy the average annualized excess return of the ten-year bond over the one-month interest rate is 134 basis points and the standard deviation is 0.33%. During our sample period, all G7 countries but Italy have on average an upward sloping interest rate curve and thus positive market prices

of interest-rate risk.

Similarly, Eq. (14) can be used to extract β_d and obtain the market price of dividend risk λ_d. It would be sufficient to approximate the left side of Eq. (14) with the average historical excess return of stocks and solve for β_d. However, since we are not primarily interested in matching the empirical equity premium, we choose to estimate β_d as previously described.

For the out-of-sample analysis, we estimate process parameters with an alternative procedure that only requires the calculation of a closed-form formula. More precisely, the mean of each of the processes is estimated as a simple average from the relevant time series, the mean-reversion parameters, are estimated as the first order autocorrelation from the relevant time series and the variance is obtained from the data is an unbiased and consistent estimator of the unconditional variance.[13]

4.2 Results

In this section we address both the in-sample correlation fit and the out-of-sample forecasting accuracy of the presented affine model.

[INSERT Tbl. 1 AROUND HERE]

Tbl. 1 compares for all G7 countries the endogenous correlation implied by the affine model with the realized correlation computed on the total-return time series of stock and bond indices. For both the endogenous and the standard Pearson correlation, the distribution of the estimate is non-normal as it is bounded between $[-1, +1]$. Hence, for obtaining confidence intervals, we transform the statistical correlation according

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[13] While this estimation technique does not directly generate standard errors, it is considerably faster and thus very well suited for performing the high number of estimations required in the out-of-sample analysis based on rolling windows. Moreover, the estimation results obtained by this procedure are very similar to those obtained by MLE.

to the method proposed by Fisher (1925).

The distribution of the estimated endogenous correlation is obtained by simulating the estimated parameters and by calculating the correlation implied by the model for each set of simulated parameters. Since, as described in the previous subsection, both the volatility parameters and the persistence parameters are obtained with a restricted estimation, only the transformed parameters may be correctly simulated by means of normal deviates. To test the robustness of the model, we repeat in Panel B, C, and D, the comparison of in-sample model correlations and Pearson correlations for three sub-periods of equal length. By analyzing Tbl. 1, we can draw some preliminary results.

First, the affine model seems to generate correlations that are a priori plausible. This is a positive result since in a similar study Beltratti and Shiller (1992) find that the correlations obtained with an extended version of the Campbell and Shiller (1988) model are much smaller (and very close to zero indeed) than the empirical ones.

Second, the endogenous factor-based correlations are in a reasonable range from actually realized correlations. For the whole sample, the endogenous model correlations and the realized correlations are not statistically different at the 5% level in two out of seven cases: USA and Japan. These results are found to be reasonably robust with respect to the sample period (cfr. Tbl. 1, Panels B-D). For the three sub-periods tested, model correlations fall in the 95% confidence interval in 9 out of 21 cases. Even in the last sub-sample, when all G7 realized stock-bond correlations turn negative, in two out of seven cases (UK and Italy) the model correlation falls into the 95% confidence interval. The fact that the third sub-sample broadly coincides with the period of "irrational exuberance" of the late ninetees is likely to worsen the fit of the model. For instance, if stock prices fully decouple from basic economic fundamentals such as the dividend yield, the presented model can hardly match the empirical stock volatility. In the third sub-sample, the US annual stock market volatility is 17.72% compared to

14.20% in the first sub-sample and 15.04% in the second sub-sample. However, it turns out that the higher stock volatility in the last sub-sample is not backed by a higher dividend-yield volatility, which is overall indicative of an irregular price development of stocks. Since the unexplained stock volatility dampens stock-bond correlations, it is not surprising that our model generates for the US correlations estimates that are higher (in absolute value) than the observed ones. The third sub-sample might further be affected by the 2001:I-2001:IV recession. As pointed out by Andersen et al. (2004) and other authors, one can empirically observe that stock-bond correlations tend to become negative in periods of economic downturn. The model captures this shift in sign for the US and Japan, although the correlation values are distant from actual realizations.

Third, correlations implied by the affine pricing model do not show any systematic bias. In four of the G7 countries, model correlations are higher than observed correlations. This result is confirmed in the sub-periods, where in 9 out of 21 cases model correlations are lower than realized correlations.

Given the mixed evidence regarding the existence of time-varying correlations, a phenomena discussed in Section 3 of this paper and also well documented in the financial literature (e.g. Longin and Solnik (1995), Kaplanis (1988), Ragunathan and Mitchell (1997), and Glabadanidis and Scruggs (2003), among others), we also introduce a time-varying correlation measure obtained by estimating the process parameters based on rolling windows. This enables us to test the out-of-sample predicting performance of the affine model. The model is calibrated on a window of ten years of monthly data that slides over the whole sample generating a time series of expected endogenous stock-bond correlations for the following month.[14]

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[14] To check the robustness of the results, we have calibrated the model employing sliding windows with different lengths. Although the alternative sliding windows tested (60, 80, 100, and 150 months) do not exacerbate the results, we observe that the correlation time series smooth out as the length of the window increases.

In Tbl. 2 we compare these monthly forecasted model correlations with three proxies of the "true" stock-bond correlation: an exponentially weighted moving correlation (Panel A), the realized correlation calculated on stock and bond returns with daily frequency (Panel B), and the in-sample BEKK correlation estimated on all available data (Panel C). To put the results in perspective, each panel also presents the predicting performance of a simple rolling correlation and a Constant Correlation Multivariate GARCH model (CC MV-GARCH) as proposed by Bollerslev (1990). These methods use directly the return series of stocks and bonds and are thus expected to deliver more accurate correlation forecasts. Tbl. 2 presents, as a goodness-of-fit statistic, the mean squared error (MSE). As additional information, we also report the root mean squared error (RMSE) which is intuitive to interpret since it has the same dimension as the correlation itself. As expected, the statistical models systematically outperform model correlations and the differences in MSE are almost always statistically different even at the one percent level. The only two exceptions occur for UK (Panel a and Panel B) and Italy (Panel B). However, the difference in the MSE is never statistically significant.

[INSERT Fig. 3 AROUND HERE]

[INSERT Tbl. 2 AROUND HERE]

Finally, to get a better feeling of the fit of the forecasting ability of the model correlation, we display the data underlying the results presented in Tbl. 2, Panel A. Fig. 3 plots the model and the EWMA correlation for the United States and the United Kingdom. For Japan and France, forecasting ability appears to be poor and upward biased. For four countries (U.S.A., U.K., Germany, and Canada), the graphs show a reasonably good fit. However, for two of them (U.S.A. and Canada), the goodness of fit decreases towards the end of the sample and this could be attributable - as previously argued - to the "irrational exuberance" in the late nineties and to the

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following turmoil on financial markets. In the case of Italy, the performance of the model correlation appears less accurate, but this could be explained by the fact that a large portion of the sample period coincides with the years of the financial market bubble.

5 Conclusion

When modeling the correlation between stocks and bonds researchers face a serious trade-off between empirical accuracy and economic rigor. Currently, important research efforts are directed towards the development of advanced statistical methods to best fit the historical comovement of financial assets. At the other end of the research spectrum, some authors propose simple general-equilibria models that convey strong economic reasoning but can hardly be successfully implemented. This paper takes a middle-way approach by tracing back the correlation of financial assets to the dynamics of some fundamental variables that drive their prices. More precisely, this paper shows how a simple three-factor affine pricing model can value both bonds and stocks and is well suited for generating endogenous correlations based on economic fundamentals. Our model implies that the volatility of the real interest rate increases the correlation between stocks and bonds. This result is intuitive, given that the real interest rate discounts both future dividends and cash flows deriving from fixed-income securities. Inflation shocks tend to reduce the correlation between stocks and bonds, which reflects the fact that in our model stocks provide complete insurance with respect to future inflation. Similarly, a higher variability of the dividend yield boosts the variability of stock returns and reduces the correlation of stocks and bonds. We calibrate the model for G7 economies using post-war monthly data and show that the obtained correlation values are realistic and not very far from conventional statistical measures. This result represents an improvement over previous

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empirical attempts of extracting correlations from a unified pricing model for stocks and bonds.

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Fig. 1. Driving Forces of Stock-Bond Correlation

This figure shows the effects of the persistence parameters, volatility parameters, and β_d on the model correlation between a stock and a bond with a duration of ten years. Panel A analyzes the impact of the short-rate parameters α_r and σ_r. Panel B analyzes the impact of the inflation parameters α_π and σ_π. Panel C analyzes the impact of the dividend-yield parameters α_d and σ_d. Panel D focuses on the relationship between β_d and the model correlation. According to Eq. (8), β_d affects the volatility of the dividend yield, σ_d, for a given value of σ_r. The initial parameter values of the basis scenario are $\alpha_r = 0.9$, $\alpha_\pi = \alpha_d = 0.95$, $\sigma_r = \sigma_\pi = 0.005$, and $\sigma_d = 0.0002$. All parameter values refer to monthly frequency.
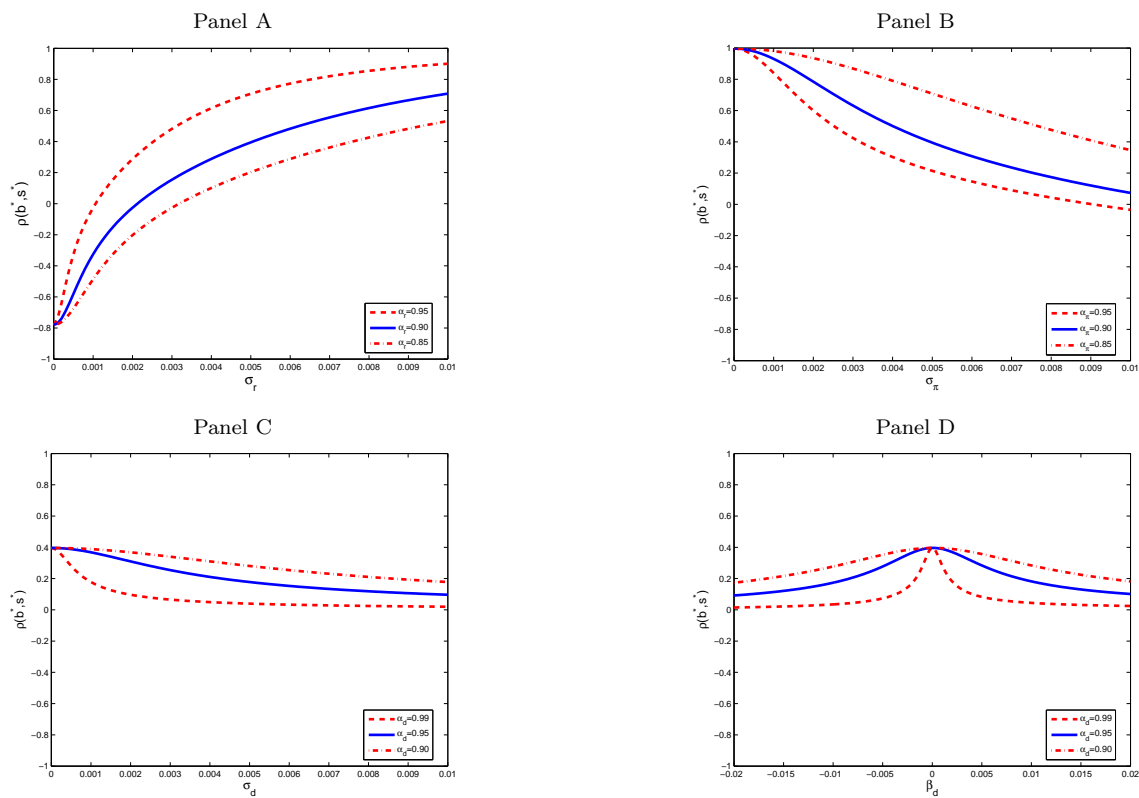

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Fig. 2. Impact of Factor Correlations

This figure shows the effects of different combinations of the factor correlations $\rho_{r,d} \in [-0.95, 0.95]$ and $\rho_{\pi,d} \in [-0.95, 0.95]$ on the model correlation between bond and stock returns. The basic scenario (Panel A) employs parameter values for the factor processes obtained by averaging the estimates for the whole available sample over all countries: $\alpha_r = 0.9686$, $\sigma_r = 0.00033$, $\alpha_\pi = 0.9717$, $\sigma_\pi = 0.00012$, $\alpha_d = 0.9923$, and $\sigma_d = 0.00023$. For this basic scenario the ranges of the factor correlations imply the following ranges for β_π and β_d: $\beta_\pi \in [-0.35, 0.35]$, $\beta_d \in [-0.62, 0.62]$. Panel B-D display the same relationship by solely altering few parameters: Panel B: $\alpha_d = 0.97$; Panel C: $\alpha_r = 0.90$, $\alpha_\pi = 0.99$; Panel D: $\alpha_r = 0.98$, $\alpha_\pi = 0.98$, $\sigma_\pi = 0.0005$. All parameter values refer to monthly frequency.
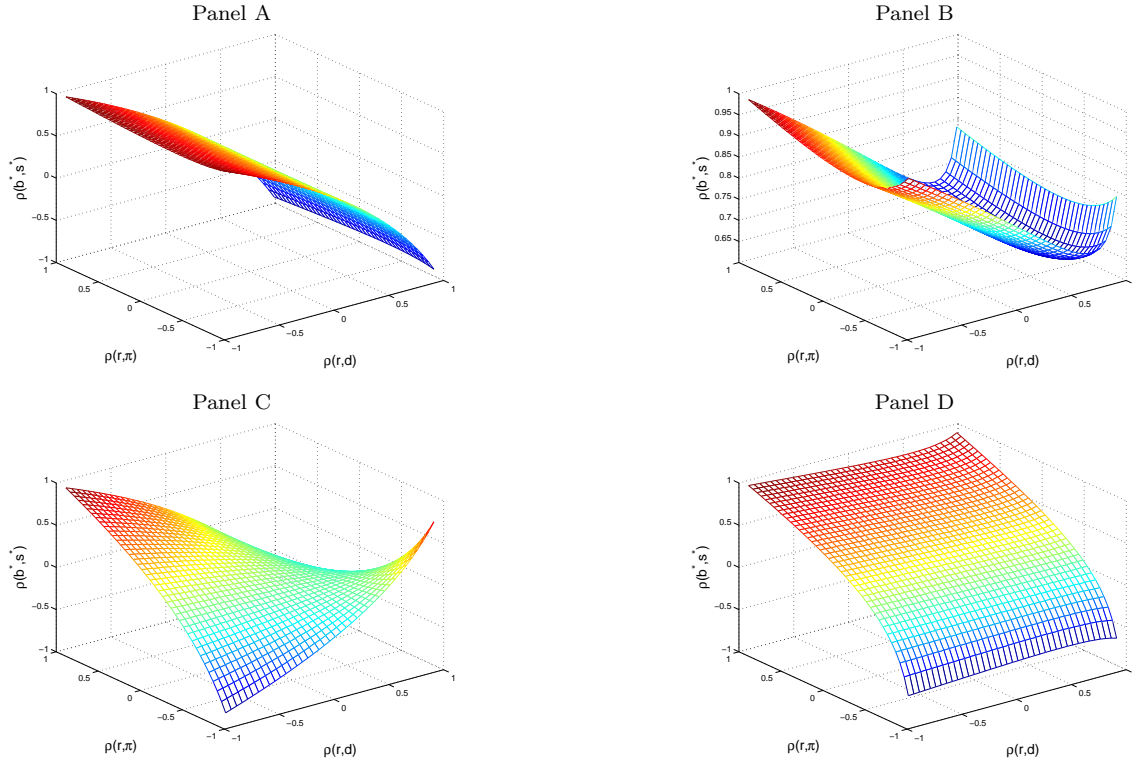

Fig. 3. Out-of-Sample Model Correlations and EWMA - USA and UK

This figure plots the out-of-sample model correlation computed with a 100 months rolling window and the correlation obtained from an exponentially weighted moving average model for both variances and covariances with a weighting factor $\gamma = 0.97$. Panel A refers to the US economy while panel B refers to Great Britain.
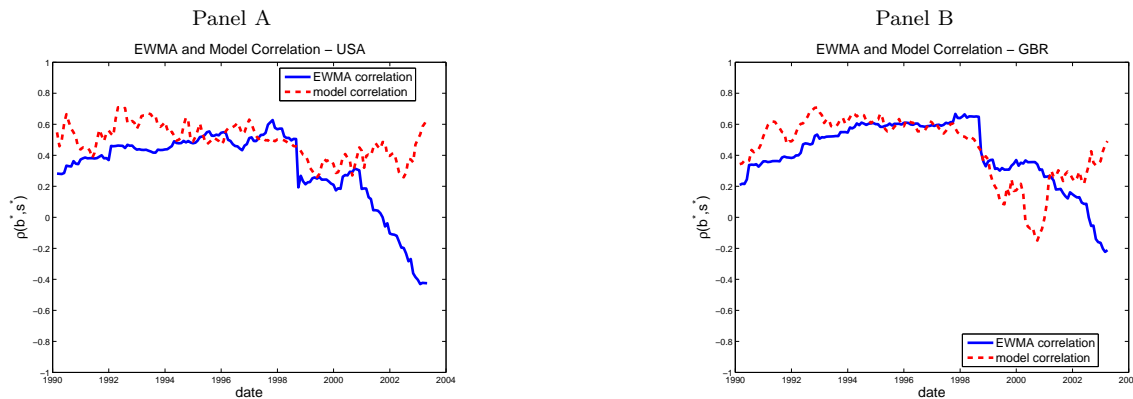
Table 1

In-Sample Analysis of Model Correlation

This table presents both the endogenous model correlation and the empirical correlation between equity and bond series. For each correlation value, the five percent confidence interval is calculated. 'LCI' indicates the lower confidence interval and 'HCI' the higher confidence interval. Finally, 'z-stat.' values of less than 1.96 indicate that the model correlation falls into the 95% data-correlation confidence interval. Panel A presents the results for the entire sample. In Panels B, C, and D correlations and confidence intervals are calculated for three equally spaced sub-intervals.

	U.S.A.	Japan	Germany	U.K.	France	Italy	Canada
Panel A: Total Sample							
Sample start	Jan80	Jan84	Jan80	Jan80	Feb85	Jan88	Jan85
Sample end	Mar03	Nov02	Mar03	Feb03	Sep02	Mar03	Mar03
Model	0.278	0.049	-0.229	-0.177	0.562	0.530	0.588
LCI	0.260	0.042	-0.231	-0.179	0.556	0.519	0.576
HCI	0.296	0.057	-0.228	-0.176	0.569	0.541	0.602
Data	0.174	0.097	0.126	0.283	0.288	0.354	0.175
LCI	0.058	-0.034	0.009	0.171	0.159	0.220	0.043
HCI	0.286	0.224	0.240	0.388	0.407	0.475	0.300
z-stat	1.818	0.720	5.982	7.799	4.921	2.955	7.328
Panel B: First Sub-Sample							
Sample start	Jan80	Jan84	Jan80	Jan80	Feb85	Jan88	Jan85
Sample end	Sep87	Apr90	Sep87	Sep87	Dec90	Jan93	Jan91
Model	0.182	0.651	0.292	-0.000	0.364	0.915	0.894
LCI	0.158	0.579	0.242	-0.003	0.346	0.904	0.888
HCI	0.206	0.717	0.343	0.003	0.382	0.924	0.898
Data	0.368	0.319	0.332	0.384	0.381	0.578	0.089
LCI	0.177	0.101	0.137	0.195	0.162	0.382	-0.144
HCI	0.532	0.508	0.502	0.545	0.564	0.725	0.313
z-stat	1.921	3.841	0.416	3.835	0.155	6.825	11.305
Panel C: Second Sub-Sample							
Sample start	Oct87	May90	Oct87	Oct87	Jan91	Feb93	Feb91
Sample end	Jun95	Aug96	Jun95	Jun95	Nov96	Feb98	Feb97
Model	0.098	0.593	0.111	0.166	0.945	0.618	0.752
LCI	-0.022	0.593	0.110	0.163	0.938	0.590	0.720
HCI	0.221	0.594	0.112	0.169	0.951	0.646	0.782
Data	0.299	0.035	0.186	0.323	0.612	0.574	0.495
LCI	0.102	-0.192	-0.019	0.127	0.441	0.377	0.299
HCI	0.474	0.258	0.375	0.494	0.739	0.722	0.651
z-stat	1.991	5.538	0.727	1.583	8.858	0.532	3.664
Panel D: Third Sub-Sample							
Sample start	Jul95	Sep96	Jul95	Jul95	Dec96	Mar98	Mar97
Sample end	Mar03	Nov02	Mar03	Feb03	Sep02	Mar03	Mar03
Model	-0.547	-0.827	0.037	0.490	0.086	0.256	0.714
LCI	-0.553	-0.843	0.030	0.457	0.074	0.246	0.673
HCI	-0.540	-0.808	0.044	0.523	0.097	0.265	0.753
Data	-0.185	-0.133	-0.123	-0.016	-0.105	-0.173	0.074
LCI	-0.374	-0.348	-0.319	-0.219	-0.330	-0.407	-0.159
HCI	0.020	0.095	0.083	0.189	0.132	0.083	0.299
z-stat	4.054	8.949	1.527	5.238	1.580	3.322	6.872

Table 2

Forecasting Performance of Model Correlations

This table presents an overview of the forecasting performance of correlations obtained from the affine pricing model described in Section 2. For the sake of comparison, the performance of more traditional approaches that are not supported by an underlying economic model are presented (Rolling Correlation and CC MV-GARCH). For each month, forecasts are compared with correlation measures obtained with the EWMA model (Panel A), the standard Pearson formula applied on daily data (Panel B) and the full BEKK MV-GARCH model (Panel C). The first number is the mean squared error (MSE), the second number is the root mean squared error (RMSE). '$\star$' indicates that the null hypothesis of equal MSE between the rolling correlation/CC MV-GARCH correlation and the model correlation cannot be rejected at a five percent significance level.

	U.S.A.	Japan	Germany	U.K.	France	Italy	Canada
Sample start	Apr81	Jan84	Apr81	Jan80	Feb85	Jan88	Jan85
Sample end	Mar03	Nov02	Mar03	Feb03	Sep02	Mar03	Mar03
Panel A: EWMA							
Model Corr.	0.082	0.881	0.204	0.039	0.391	0.316	0.199
	0.286	0.939	0.452	0.198	0.626	0.562	0.446
Rolling Corr.	0.027	0.043	0.017	0.046$\star$	0.032	0.042	0.040
	0.165	0.207	0.132	0.214	0.178	0.206	0.200
CC MV-GARCH	0.021	0.039	0.014	0.037$\star$	0.028	0.032	0.024
	0.144	0.197	0.118	0.191	0.168	0.178	0.155
Panel B: Realized							
Model Corr.	0.228	0.621	0.310	0.164	0.459	0.472	0.509
	0.478	0.788	0.557	0.405	0.677	0.687	0.713
Rolling Corr.	0.173	0.153	0.162	0.239$\star$	0.216	0.527$\star$	0.288
	0.416	0.391	0.402	0.489	0.465	0.726	0.537
CC MV-GARCH	0.166	0.147	0.155	0.231$\star$	0.228	0.502$\star$	0.285
	0.407	0.384	0.393	0.480	0.477	0.709	0.534
Panel C: BEKK							
Model Corr.	0.242	0.470	0.325	0.082	0.259	0.214	0.272
	0.492	0.686	0.570	0.286	0.509	0.463	0.522
Rolling Corr.	0.111	0.071	0.030	0.027	0.051	0.146	0.041
	0.333	0.266	0.174	0.165	0.225	0.382	0.203
CC MV-GARCH	0.127	0.075	0.032	0.025	0.052	0.138	0.045
	0.357	0.274	0.179	0.158	0.228	0.372	0.213