# Gas Fired Power Plants: Investment Timing, Operating Flexibility and Abandonment

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## Abstract

We analyze investments in gas fired power plants under stochastic electricity and gas prices. We use a real options approach, taking into account the economic information in futures and forward prices. A simple but realistic two-factor model is used for price process, enabling analysis of the value of operating flexibility, the opportunity to sell and abandon the capital equipment, as well as finding thresholds for energy prices for which it is optimal to enter into the investment. Our case study, using real data, indicates that when the decision to build is considered, the plant's flexibility and abandonment option do not have significant value.

Key words: Real options, spark spread, gas fired power plant, forward prices

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## 1 Introduction

In the next 20 years, fossil fuels will account for 75% of all new electric power generating capacity, and 60% of this is assumed to come in the form of gas fired power plants (see, e.g., IEA, 2003). Thus, many companies in the electricity and natural gas industries are considering investments in such plants. At the same time, the restructuring of electricity and gas markets has brought price transparency in the form of easily available spot- and forward prices. This article offers an approach to analyze gas fired power plant investments, using the information available on electricity and gas futures and forward markets.

A gas fired power plant may be interesting not only from the point of view of meeting increased power demand. Consider a company owning an undeveloped gas field at a distance to major gas demand hubs. Most of the world's gas reserves are in such a category of "stranded gas". Building natural gas pipelines is very costly, and the unit costs of gas transportation decreases rapidly with the capacity of the pipeline. Thus, locating a gas fired power plant at the end of a new pipeline, near electricity demand, improves the economy of scale in transmission of natural gas.

The research question addressed here is that of an energy manager having an opportunity to build a gas fired power plant. How high should electricity prices be compared to gas prices, before I start building the plant? Does it matter whether the plant is base load, running whatever the level of electricity and gas prices, or peak load, only running when electricity price is above the fuel cost? How does the opportunity to abandon the plant influence the decision to invest? How do greenhouse gas emission costs affect profitability?

Whether a new power plant will be run as a base load plant, or ramped up and down according to current energy prices, depends more on the state of the local natural gas market than the technical design choice of the plant itself. New gas plants will often be of combined cycle gas turbine (CCGT) type, which can be used both as base load and peak load plants. The operating flexibility is often constrained by the flexibility of the gas inflow. If there is little local storage and/or alternative uses of the natural gas, the plant operator will seldom find it profitable to ramp down the plant.

We use a real options approach (see, e.g., Dixit and Pindyck, 1994). The gas fired power plant's operating cash flows depend on the spark spread, defined as the difference between the price of electricity and the cost of gas used for the generation of electricity. Spark spread based valuation of power plants has been studied in Deng, Johnson, and Sogomonian (2001). Our model makes several extensions to their model. First, by using a two-factor model similar to that of Schwartz and Smith (2000) for the spark spread process, we can incorporate the typical characteristics of non-storable commodity prices, i.e. short-term mean-reversion and long-term uncertainty. Second, our model takes into account the option to postpone investment decisions. Such postponement option analysis originates from the work of McDonald and Siegel (1986).

The long-maturity forwards on electricity and gas, e.g. ten-year forwards, give the exact and certain market value of a constant electricity and gas flow. A base load plant operates with a constant electricity and gas flow, and thus a base load plant can be valued with long-term spark spread forwards. On the other hand, a peak load plant can react to short-term variations in the spark spread by ramping up and down, leading to a non-constant gas and electricity flow. Thus, the short-term dynamics of the spark spread are needed for the valuation of a peak load plant. The short-term dynamics can be estimated from shortmaturity forwards.

Long-term investments, such as gas fired power plants, are never commenced due to nonpersistent spikes in the spark spread. Rather, investment decisions are based on long-term price levels, here called equilibrium prices. We compare the current equilibrium price estimate to a computed investment threshold, reflecting that at this threshold level of equilibrium price, the value of waiting longer is equal to the net present value received if investment is commenced. Thus, when the equilibrium price increases to the investment threshold, the implementation of the power plant project should be started. As it is difficult to precisely characterize the ramping policy of a peak plant, instead of giving an exact value of the plant, we give upper and lower bounds for the plant value. The bounds for the plant value can be used to calculate upper and lower bounds for the investment thresholds. An alternative to using forward prices in the estimation of the parameters of the price dynamics is to focus on spot prices. Deng (2003) studies investment timing and gas plant valuation under electricity and gas price uncertainty by using separate stochastic processes for electricity and gas spot prices. His model is calibrated to historical spot data and it contains jumps and spikes in the spot price process. We do not include jumps or spikes, although these features may very well be present in the spot price history. The reason is that forward prices reflect all important and currently available information about future supply, demand and risk. Forward prices show directly the current market value of future spark spread, and are the risk-adjusted expected future spot price level. Furthermore, ignoring forward price data and only looking at spot price data easily leads to value estimates that are inconsistent with the no-arbitrage principle, i.e. the estimated real asset value can differ from the value dictated by the forward curve.

Our simplifications compared to Deng (2003), omission of price spikes and modeling the spark spread with one price process, mean that our model cannot capture operational efficiency that varies with output or over time. However, that issue is relevant only for optimization of short-term operation, and do not play a significant role when taking a strategic view as we do here. E.g. Deng and Oren (2003) find that for efficient plants, the error can be expected to be small. The main contribution of this paper is to introduce a simple and direct way to capture relevant uncertainty in input and output prices for the investment decision.

We apply our model into the energy market in northern Europe. The electricity markets there have been restructured since the late 1980s, with North Sea gas markets still in transition. Our case study indicates that the difference of a peak and base load plant value is rather small, i.e. the value of being able to ramp up and down is not significant. Our application also indicates that the addition of an abandonment option does not dramatically change the investment threshold. Thus, when investments in gas fired power plants are considered, a good overall view of the investment problem can be made by ignoring the flexibility and abandonment options, whereas the time-to-build option has significant value for the investment threshold. The formulated model enables energy managers to make better decisions, in terms of increasing the market value of their firms, regarding power plant investment opportunities. The model generalizes beyond the case of gas fired power plants. Any investment involving a relatively simple transformation of one commodity to another could be analyzed using this framework. The spread between output price and input costs is then an important source of uncertainty. Examples include transformation of natural gas into liquefied natural gas, a methanol factory, and a biodiesel factory.

The paper is organized as follows. We present the model of price uncertainty in Section 2, where we also argue why it is important to incorporate information in forward prices to real options analyses. In Section 3 upper and lower bounds for the plant value are calculated, whereas in Section 4 the investment problem is studied. In Section 5 we give a real life application of our model. In Section 6 we discuss the results of the application. Finally, Section 7 concludes the study.

## 2 The energy price process

Seasonality in the supply and demand of electricity and gas, combined with limited storage opportunities, causes cycles and peaks in the electricity and gas forward curves. Spark spread measures the contribution margin of a gas fired power plant, thus it is defined as the difference between price of electricity  $S_e$  and the cost of gas used for the generation of electricity

$$S = S_e - K_H S_g, \tag{1}$$

where  $S_g$  is the price of gas and heat rate  $K_H$  is the amount of gas required to generate one MWh of electricity. Heat rate measures the efficiency of the plant: the lower the heat rate, the more efficient the facility. The efficiency of a gas fired power plant varies slightly over time and with the output level. Still, the use of a constant heat rate is considered plausible for long-term analyses (see, e.g., Deng, Johnson, and Sogomonian, 2001). Note that the value of the spark spread can be negative as well as positive.

Both electricity and natural gas are difficult to store, so the usual cash-and-carry arguments determining the relationship between the spot and forward prices do not hold. Thus, they can not be used to determine the risk adjustment that is necessary in the valuation of spark spread dependent assets. However, a reasonable price of risk, i.e. risk adjustment, can be estimated from forward and futures prices. If there are no forward prices available the expected spot price process can be used, but in this case there is no sound theory for the selection of risk adjustment. Often an ad hoc risk-adjusted discount rate is used (see, e.g., Dixit and Pindyck, 1994).

As electricity and gas are often used to same purposes, such as cooling and heating, the seasonality in electricity and gas forward curves have similar characteristics. Hence the seasonality in electricity and gas forward curves decays from the spark spread forward curve. The seasonality left in the spark spread process could be modeled with time dependent drift parameter, but to keep the analysis simple we ignore the seasonality and use constant drift term. The following assumption describes the dynamics of the spark spread process. Schwartz and Smith (2000) use similar price dynamics to evaluate oil-linked assets.

ASSUMPTION 1. The spark spread is a sum of short-term deviations and equilibrium price

$$S(t) = \chi(t) + \xi(t), \qquad (2)$$

where the short-term deviations  $\chi(t)$  are assumed to revert toward zero following an Ornstein-Uhlenbeck process

$$d\chi(t) = -\kappa\chi(t)dt + \sigma_{\chi}dB_{\chi}(t).$$
(3)

The equilibrium price  $\xi(t)$  is assumed to follow an arithmetic Brownian motion process

$$d\xi(t) = \mu_{\xi} dt + \sigma_{\xi} dB_{\xi}(t), \qquad (4)$$

where  $\kappa$ ,  $\sigma_{\chi}$ ,  $\mu_{\xi}$ , and  $\sigma_{\xi}$  are constants.  $B_{\kappa}(\cdot)$  and  $B_{\xi}(\cdot)$  are standard Brownian motions, with correlation  $\rho dt = dB_{\chi}dB_{\xi}$  and information  $F_{t}$ .

Increase in the spark spread attracts high cost producers to the market putting downward pressure on prices. Conversely, when prices decrease some high cost producers will withdraw capacity temporarily, putting upward pressure on prices. As these entries and exits are not instantaneous, prices may be temporarily high or low, but will revert toward the equilibrium price  $\xi$ . The mean-reversion parameter  $\kappa$  describes the rate at which the short-term deviations  $\chi$  are expected to decay. The uncertainty in the equilibrium price is caused by the uncertainty in fundamental changes that are expected to persist. For example, advances in gas exploration and production technology, changes in the discovery of natural gas, improved gas fired power plant technology, and political and regulatory effects can cause changes in the equilibrium price. Other studies where the two factors are interpreted as short- and long-term factors include, for example, Schwartz and Smith (2000), Ross (1997), and Pilipović (1998). Note that the decreasing forward volatility structure, typical for commodities, can be seen as a consequence of the mean-reversion in the commodity spot prices (see, e.g., Schwartz, 1997).

The following corollary expresses the distribution of the future spark spread values.

COROLLARY 1. When spark spread has dynamics as given in (2)-(4), prices are normally distributed, and the expected value and variance are given by

$$E[S(T) | F_t] = e^{-\kappa(T-t)} \chi(t) + \xi(t) + \mu_{\xi}(T-t)$$
(5)

$$Var\left(S(T)\right) = \frac{\sigma_{\chi}^{2}}{2\kappa} \left(1 - e^{-2\kappa(T-t)}\right) + \sigma_{\xi}^{2}(T-t) + 2\left(1 - e^{-\kappa(T-t)}\right) \frac{\rho\sigma_{\chi}\sigma_{\xi}}{\kappa}.$$
(6)

*PROOF*: See, e.g., Schwartz and Smith (2000).

The short-term deviations in the expected value decrease exponentially as a function of maturity, i.e. T-t. The time in which short-term deviations are expected to halve is given by

$$T_{1/2} = -\frac{\ln(0.5)}{\kappa} \,. \tag{7}$$

The spark spread variance decreases as a function of mean-reversion parameter  $\kappa$ . Neither the short-term deviations  $\chi$  nor the equilibrium price  $\xi$  are directly observable from market quotas, but estimates can be obtained from forward prices. Intuitively, the long-maturity forwards give information of the equilibrium price, whereas the short-term dynamics can be estimated from the short-maturity forwards. The estimation of the spark spread process parameters will be considered in Section 5. As the spark spread values are normally distributed the values can be negative as well as positive.

#### 3 Gas plant valuation

In this section we calculate upper and lower bounds for the value of the gas fired power plant. The following assumption gives the operational characteristics of the plant.

ASSUMPTION 2. The gas plant can be ramped up or down according to changes in the spark spread. The costs associated with starting up and shutting down the plant can be amortized into fixed costs.

In a gas fired power plant, the operation and maintenance costs do not vary much over time, thus it is realistic to assume that the fixed costs are constant. The ramping policy of a particular plant depends on local conditions associated with plant design and gas inflow arrangement. Instead of computing an exact value of a plant we give upper and lower bounds. The lower bound  $V_L$  can be calculated by assuming that the plant cannot exploit unexpected changes in the spark spread, i.e. by assuming that the plant produces electricity at the rated capacity independent of the spark spread. Such a plant is often called a base load plant. The following lemma gives the value of a base load plant.

LEMMA 1. At time t, the lower bound of the plant value  $V_L(\chi,\xi) \leq V(\chi,\xi)$  is given by the value of a base load plant

$$V_{L}(\chi,\xi) = \overline{C}\left(\frac{\chi(t)}{\kappa+r} + \frac{\xi(t) - E}{r} + \frac{\mu_{\xi}}{r^{2}} - e^{-r(\overline{T}-t)}\left(\frac{e^{-\kappa(\overline{T}-t)}\chi(t)}{\kappa+r} + \frac{\xi(t) - E}{r} + \frac{\mu_{\xi}\left(r(\overline{T}-t) + 1\right)}{r^{2}}\right)\right) - \frac{G}{r}\left(1 - e^{-r(\overline{T}-t)}\right), (8)$$

where  $\overline{T}$  is the lifetime of the plant,  $\overline{C}$  is the capacity of the plant, and G are the fixed costs of running the plant.

**PROOF:** The value of a base load plant is the present value of expected operating cash flows

$$V_{L}(\chi,\xi) = \int_{t}^{\overline{T}} e^{-r(s-t)} \left( \overline{C} \left( E[S(s) | F_{t}] - E \right) - G \right) ds =$$

$$= \int_{t}^{\overline{T}} e^{-r(s-t)} \left( \overline{C} \left( e^{-\kappa(s-t)} \chi(t) + \xi(t) - E + \mu_{\xi}(s-t) \right) - G \right) ds$$
(9)
wes (8).
Q.E.D.

Integration gives (8).

The lower bound is just the discounted sum of expected spark spread values less emission and fixed costs. Thus, the lower bound is not affected by the short-term and equilibrium volatilities  $\sigma_{\chi}$  and  $\sigma_{\xi}$ .

An owner of a gas fired power plant can react to adverse changes in the spark spread by temporarily shutting down the plant. The upper bound  $V_U$  of the plant's value can be calculated by assuming that the up and down ramping can be done without delay, i.e. by assuming that the plant produces electricity only when the spark spread exceeds emission costs. Such a plant is often called a peak load plant. The following lemma gives the value of an ideal peak load plant.

LEMMA 2. At time t, the upper bound of the plant value  $V(\chi,\xi) \leq V_U(\chi,\xi)$  is given by the value of an ideal peak load plant

$$V_{U}(\chi,\xi) = \bar{C} \int_{t}^{\bar{T}} e^{-r(s-t)} \left( \frac{v(s)}{\sqrt{2\pi}} e^{\left( -\frac{(E-\mu(s))^{2}}{2v^{2}(s)} \right)} + \left( \mu(s) - E \right) \left( 1 - \Phi \left( \frac{E-\mu(s)}{v(s)} \right) \right) \right) ds - \frac{G}{r} \left( 1 - e^{-r(\bar{T}-t)} \right), \quad (10)$$

where  $\Phi(\cdot)$  is the normal cumulative distribution function, and G are the fixed costs of running the plant. The expected value  $\mu(s) = E[S(s) | F_t]$  and variance  $v^2(s) = Var(S(s))$  for the spark spread are given by Corollary 1.

*PROOF*: See Appendix A.

The upper bound increases as a function of the variance of the spark spread. The value of a gas fired power plant is the discounted sum of expected spark spread values less emission and fixed costs plus the option value of being able to ramp up and down. The value of the operating flexibility is dependent on the response times of the plant, and is maximized when ramping up and down can be done without delay.

To summarize: As we are not able to precisely characterize the response times of the plant, we do not calculate the exact valuation formula for the gas fired power plant, but we give bounds for the plant value. The lower bound is given by the base load plant (Lemma 1) and the upper bound is given by the ideal peak load plant (Lemma 2).

#### 4 Investment analysis

In this section we calculate bounds for the investment thresholds when the gas plant value has the bounds given by Lemma 1 and Lemma 2. The following assumption characterizes the variables affecting the investment decisions.

ASSUMPTION 3. The investment decisions are based on the equilibrium price process, i.e. the short-term deviations are assumed to be zero when investment decisions are made. Moreover, the lifetime of the plant is assumed to be infinite.

Assumption 3 states that when the gas plant investments are considered the decisions are made as a function of the equilibrium price  $\xi$ . Thus, investments are not done due to the current realization of short-term deviations. The short-term dynamics still affect the value of the plant, and thus they also affect the investment decision. In other words, the short-term dynamics are important in the investment decision, even though the particular level is ignored when decisions are made. The omission of the short-term realization is motivated by the fact that gas fired power plants are long-term investments, and a gas plant investment is never commenced due to a non-persistent spike in the price process. This is realistic as long as the expected lifetime of the short-term deviations is considerably smaller than the expected lifetime of the plant. In Section 5 we estimate that in our example data the mean-reversion parameter  $\kappa$  is 8.1, which gives, with (7), that the short-term variations are expected to halve in about one month. Usually, the life time of a gas fired power plant is assumed to be around 30 years. Thus, the omission of the short-term realization in the investment decision is realistic. The infinite lifetime assumption is motivated by the fact that the plant's lifetime is often increased by upgrading and reconstructions, and by downward shifts in the maintenance cost curve (see, e.g., Ellerman, 1998). The plant value as a function of plant's lifetime will be illustrated in Section 5.

Building the plant becomes optimal when the equilibrium price rises to a building threshold  $\xi_H$ . When waiting is optimal, i.e., when  $\xi < \xi_H$ , the investor has an option to postpone the building decision. The value of such a time-to-build option is given by the following lemma.

LEMMA 3. The value of an option to build a gas fired power plant is

$$F_0(\xi) = A_1 e^{\beta_1 \xi} - \frac{W}{r}, \qquad \text{when} \quad \xi \le \xi_H , \qquad (11)$$

where  $A_1$  is a positive parameter and W are constant payments that the firm faces to keep the build option alive. The parameter  $\beta_1$  is given by

$$\beta_{1} = \frac{-\mu_{\xi} + \sqrt{\mu_{\xi}^{2} + 2\sigma_{\xi}^{2}r}}{\sigma_{\xi}^{2}} > 0.$$
(12)

**PROOF:** See Appendix B.

The time-to-build option value increases exponentially as a function of the equilibrium price. The parameter  $A_1$  depends on the value of the plant and on the investment cost. As we are not able to exactly state the gas plant value, we can not state the exact building threshold, but the following proposition gives a method to calculate upper and lower bounds  $\xi_{HL} \leq \xi_H \leq \xi_{HU}$  for the building threshold.

PROPOSITION 1. The lower bound of the building threshold  $\xi_{HL} \leq \xi_H$  is given by

$$F_{0}(\xi_{HL}) = V_{U}(0,\xi_{HL}) - I$$
(13)

$$\frac{\partial F_0(\xi_{HL})}{\partial \xi} = \frac{\partial V_U(0,\xi_{HL})}{\partial \xi}, \qquad (14)$$

whereas the upper bound  $\xi_{\rm H} \leq \xi_{\rm HU}$  is given by

$$F_0(\xi_{HU}) = V_L(0, \xi_{HU}) - I \tag{15}$$

$$\frac{\partial F_0(\xi_{HU})}{\partial \xi} = \frac{\partial V_L(0,\xi_{HU})}{\partial \xi}.$$
(16)

*PROOF:* This is a special case of Proposition 2 and the proof will be omitted.

The equations in Proposition 1 cannot be solved analytically but a numerical solution can be attained. The more valuable the plant becomes, the more eager the firms are to invest, thus the lower bound for the building threshold is given by the upper bound of the plant's value and vice versa.

Next we will consider how the investment decision changes if there is an opportunity to abandon the gas plant and realize the plant's salvage value. In this case, when a decision to build is made the investor receives both the gas plant and an option to abandon the plant. As the lifetime of the plant was assumed to be infinite, there is a constant threshold value  $\xi_L$  for

the abandonment, i.e. abandoning is not optimal when  $\xi_L < \xi$ . The following Lemma states the value of such an abandonment option.

LEMMA 4. The value of an abandonment option is

$$F_1(\xi) = D_2 e^{\beta_2 \xi} \qquad \text{when} \quad \xi_L \le \xi \tag{17}$$

where  $D_2$  is a positive parameter. The parameter  $\beta_2$  is given by

$$\beta_2 = \frac{-\mu_{\xi} - \sqrt{\mu_{\xi}^2 + 2\sigma_{\xi}^2 r}}{\sigma_{\xi}^2} < 0.$$
(18)

*PROOF:* The proof is similar to that of the build option (Appendix B), but now the option becomes less valuable as the spark spread increases. Q.E.D.

The abandonment option value decreases exponentially as a function of the equilibrium price. The parameter  $D_2$  depends on the plant's salvage value. Again we are not able to state the exact building and abandonment thresholds, but the following Proposition gives upper and lower bounds for the thresholds, i.e.  $\xi_{HL} \leq \xi_{H} \leq \xi_{HU}$  and  $\xi_{LL} \leq \xi_{L} \leq \xi_{LU}$ .

PROPOSITION 2. The lower bounds for the building and abandonment thresholds  $\xi_{HL} \leq \xi$  and  $\xi_{LL} \leq \xi$  are given by

$$F_0(\xi_{HL}) = V_U(0,\xi_{HL}) + F_1(\xi_{HL}) - I$$
(19)

$$F_{1}(\xi_{LL}) + V_{U}(0,\xi_{LL}) = D$$
<sup>(20)</sup>

$$\frac{\partial F_0(\xi_{HL})}{\partial \xi} = \frac{\partial V_U(0,\xi_{HL})}{\partial \xi} + \frac{\partial F_1(\xi_{HL})}{\partial \xi}$$
(21)

$$\frac{\partial F_1(\xi_{LL})}{\partial \xi} + \frac{\partial V_U(0,\xi_{LL})}{\partial \xi} = 0, \qquad (22)$$

whereas the upper bounds  $\xi \leq \xi_{\rm HU}$  and  $\xi \leq \xi_{\rm LU}$  are given by

$$F_0(\xi_{HU}) = V_L(0,\xi_{HU}) + F_1(\xi_{HU}) - I, \qquad (23)$$

$$F_1(\xi_{LU}) + V_L(0, \xi_{LU}) = D, \qquad (24)$$

$$\frac{\partial F_0(\xi_{HU})}{\partial \xi} = \frac{\partial V_L(0,\xi_{HU})}{\partial \xi} + \frac{\partial F_1(\xi_{HU})}{\partial \xi}$$
(25)

$$\frac{\partial F_1(\xi_{LU})}{\partial \xi} + \frac{\partial V_L(0,\xi_{LU})}{\partial \xi} = 0.$$
(26)

PROOF: See Appendix C.

The equations in Proposition 2 cannot either be solved analytically but a numerical solution can be attained. The less valuable the plant is, the more eager the firms are to abandon the plant. Thus the upper bound of the abandonment threshold is given by the lower bound of the plant value, and vice versa.

To summarize: in this section we have derived a method to calculate lower and upper bounds for the building and abandonment thresholds. If the abandonment option is ignored the building threshold is given by Proposition 1. When both building and abandonment are studied the thresholds are given by Proposition 2.

#### 5 Application

Norwegian energy and environmental authorities have given four licenses to build a gas fired power plant. In this section we illustrate our framework by taking the view of an investor having one of these licenses. Naturally, our method can be applied into other similar investment problems. It is estimated that over the period 2001-2030 about 2000 GW of new natural gas fired power plant capacity will be built (see, e.g., IEA, 2003).

The example consists of four parts. First, we introduce the data used for the valuation including methods to estimate the parameters from the data. Second, we calculate bounds for the plant value and investment thresholds. The sensitivity of the thresholds to some key parameters is illustrated in part three. In the final part we study the effects of carbon emission costs to the installation of  $CO_2$  capture technology, by assuming that a plant with  $CO_2$  capture technology does not face emission costs.

The costs of building and running a natural gas fired power plant are estimated by Undrum, Bolland, Aarebrot (2000). A plant in Norway, with an exchange rate of 7 NOK/\$, costs approximately 1620 MNOK, and the maintenance costs G are approximately 50 MNOK/year. We estimate that the costs of holding the license W are 5% of the fixed costs of a running a plant. In their estimate approximately 35% of the investment costs are used for capital equipment. We assume that if the plant is abandoned all the capital equipment can be realized on second hand market, i.e. the salvage value of the plant D is 570 MNOK. The estimated parameters are for a gas plant whose maximum capacity is 415 MW. We assume that the capacity factor of the plant is 90%, thus we use a production capacity of 3.27 TWh/year. Table 1 contains a summary of the gas plant characteristics.

Table 1: The gas plant parameters

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Parameter	W	$\bar{C}$	G	Ι	D
Unit	MNOK/year	TWh/year	MNOK/year	MNOK	MNOK
Value	2.5	3.27	50	1620	570

We calculate the spread process from electricity and gas prices by adjusting the gas prices with the heat rate so that a unit of gas corresponds to 1 *MWh* of electricity generated. The efficiency of a combined cycle gas fired turbine is estimated to be 58.1%, thus the heat rate  $K_H$  is 1.72.

We use Kalman filtering techniques (see, e.g., Harvey, 1989 and West and Harrison, 1996) to estimate the volatility and mean-reversion parameters from the short-maturity forwards. The Kalman filter facilitates the calculation of the likelihood of observing a particular data series given a particular set of model parameters. Hence we use maximum likelihood method to estimate the volatility and mean-reversion parameters (i.e.  $\kappa$ ,  $\sigma_{\chi}$ , and  $\sigma_{\xi}$ ). For more about the estimation procedure see Schwartz and Smith (2000). The equilibrium drift  $\mu_{\xi}$  is estimated with linear regression from long-maturity forward prices. In Figure 1 the short-term data, used for the volatility and mean-reversion estimation, are illustrated together with the expected value and 68% confidence interval over the period 2002-2008. The expected value and confidence intervals are given by Corollary 1. The short-term data is based on quotes of seasonal contracts with 1-year maturity. The electricity data is from Nord Pool and gas data is from International Petroleum Exchange IPE. For the long-term data yearly contracts from Nord Pool and IPE together with 10 year contracts traded bilaterally are used The estimate of the equilibrium spark spread at the end of the year 2002 is 35 NOK/MWh. Table 2 summarizes the spark spread parameter estimates and the risk-free interest rate.

### [Figure 1 about here]

Parameter	K	$\mu_{\xi}$	ρ	$\sigma_{\chi}$	$\sigma_{_{\xi}}$	$\xi_0$	ľ
Unit		NOK/MWh		NOK/MWh	NOK/MWh	NOK/MWh	
Value	8.1	0.2	-0.3	42.1	9.6	35	6%

Table 2: Spark spread parameter estimates

When emission costs E are assumed to be zero, and the plant's lifetime  $\overline{T}$  is assumed infinite, the lower bound for the plant value  $V_L$ , given by Lemma 1, is 1256 *MNOK*. Correspondingly, the upper bound for the plant value  $V_U$ , given by Lemma 2, is 1440 *MNOK*. The plant value as a function of the lifetime  $\overline{T}$  is illustrated in Figure 2. In Figure 2 the value of the plant gradually stabilizes to a given level as the lifetime increases.

#### [Figure 2 about here]

Proposition 1 gives that the building threshold  $\xi_{H}$  when abandonment is not considered is somewhere between [68.4; 70.3] NOK/MWh. When also the abandonment option is taken into account the building threshold  $\xi_{H}^{A}$  is on an interval [66.5; 66.6] NOK/MWh, and the abandonment threshold  $\xi_{L}^{A}$  is between [-5.0; -3.7] NOK/MWh. In the latter case the thresholds are given by Proposition 2. If there is an option to abandon some of the investment costs can be returned when the investment turns to be unprofitable, and thus the addition of abandonment option makes earlier investment more favorable. The abandonment option also narrows the gap between upper and lower bound of the building threshold. In other words, the abandonment makes the flexibility in the plant less valuable as the possibility to abandon partly compensates the value of being able to temporarily shut down. The bounds of the plant value and investment thresholds are summarized in Table 3.

Variable	$V(0,\xi_0)$	$\xi_{\scriptscriptstyle H}$	$\xi_{H}^{A}$	$\xi^A_L$
Unit	MNOK	NOK/MWh	NOK/MWh	NOK/MWh
Value	[1256; 1440]	[68.4; 70.3]	[66.5; 66.6]	[-5.0; -3.7]

Table 3: Plant value and investment thresholds

For comparison we calculate the thresholds with a traditional discounted cash flow method, i.e. we assume that the plant is built when the expected value of the plant is equal to investment costs and the abandonment is done when the plant value is equal to salvage value. The discounted cash flow method gives that the investment threshold  $\xi_{H}^{NPV}$  is on the interval [38.8; 41.7] NOK/MWh and the abandonment threshold  $\xi_{L}^{NPV}$  is on the interval [15.2; 22.4] NOK/MWh. In the discounted cash flow method the option to postpone the investment decisions are ignored. The options to postpone have positive value and thus the building threshold increases and the abandonment threshold decreases when the options to postpone are included.

Figure 3 illustrates the option values  $F_0$  and  $F_1$  and the plant value V as a function of equilibrium price  $\xi$ . The solid lines represent the upper bounds, and the lower bounds are indicated by the dashed lines. Also the bounds for the investment thresholds are shown. The value of the build option increases exponentially as a function of the equilibrium price until it is optimal to build the plant. The gap between the bounds of the build option is so small that they are seen as one line in Figure 3. The owner of a gas plant has also an abandonment option whose value decreases exponentially as a function of equilibrium price. The peak load plant can react to decreasing prices by ramping down the plant. Therefore, the difference between the bounds of the plant value increases as the equilibrium price decreases. As the bounds for the option values are determined by the bounds of the plant value, the upper and lower bound of the abandonment option also diverge when equilibrium price decreases.

[Figure 3 about here]

Next we study how the thresholds change as a function of some key parameters. In Figure 4 the thresholds as a function of equilibrium volatility  $\sigma_{\xi}$  are illustrated. An increase in the equilibrium volatility increases the building threshold, but at the same time the abandonment threshold decreases, i.e. uncertainty makes waiting more favorable. When the equilibrium volatility approaches zero, the thresholds converge to the thresholds calculated with discounted cash flow method. In Figure 4 the gap between upper and lower bound of the thresholds also increases as function of uncertainty. An increase in the equilibrium volatility does not change the value of a base load plant, but it increases the value of a peak load plant. Thus, as the market becomes more volatile the more valuable the peak load plant is compared to the base load plant, and the broader is the gap between bounds of the investment thresholds.

### [Figure 4 about here]

Figure 5 illustrates the thresholds as a function of emission costs E. In Figure 5 the unit of emission costs is NOK/MWh, whereas it usually is quoted in \$/ton. The CO<sub>2</sub> production of a gas fired power plant is 363 kg/MWh. With an exchange rate of 7 NOK/\$, an emission cost of 10 NOK/MWh corresponds 3.94 \$/ton. In Figure 5 the thresholds increase linearly, with slope one, as a function of emission costs. Thus, if the emission costs are increased by one NOK/MWh, both thresholds are also increased by one NOK/MWh. This is a consequence of a normally distributed equilibrium price. Change in emission costs can be seen as a change in initial value of the equilibrium price. Even though we have used constant emission costs, there is uncertainty in future levels of emission costs. An easy way to model the uncertainty in emission costs is to increase the equilibrium uncertainty. Thus, not just increase in the expected value of emission costs, but also uncertainty in emission costs postpones investment decisions, i.e. increases the building threshold and decreases the abandonment threshold.

### [Figure 5 about here]

Undrum, Bolland, Aarebrot (2000) evaluate different alternatives to capture  $CO_2$  from gas turbine power cycles. They estimate that costs to install equipment to capture  $CO_2$  from exhaust gas using absorption by amine solutions are 2140 *MNOK*. Thus, the costs of a gas power plant with CO<sub>2</sub> capture technology are 3760 *MNOK*. Figure 6 illustrates the thresholds as a function of investment costs when the salvage value is 35% of the investment costs (i.e. D = 0.35I). The resale value of a plant with CO<sub>2</sub> capture technology is 1316 *MNOK*.

### [Figure 6 about here]

In Figure 6 the threshold to build a gas turbine with CO<sub>2</sub> capture equipment is about 108 NOK/MWh. Figure 5 indicates that once the emission costs are 42 NOK/MWh the building threshold for a plant without CO<sub>2</sub> capture equipment is about 108 NOK/MWh. By assuming that all emission costs are caused by CO<sub>2</sub>, and by ignoring the reduced efficiency of the plant when the greenhouse gas capture equipment is in place and uncertainty in CO<sub>2</sub> emission costs are greater than 16.5 \$/ton (i.e., 42 NOK/MWh).

The current estimate is that emission costs will be somewhere between 5%/ton and 20%/ton, where the lower range is most likely. When emission costs are 8%/ton, the threshold to build a plant without CO<sub>2</sub> capture equipment is about 87 NOK/MWh. The building threshold for the plant with CO<sub>2</sub> capture equipment is lowered from 108 NOK/MWh to 87 NOK/MWh if the investment costs are lowered to 2650 MNOK. Thus, if the costs of building a gas plant with CO<sub>2</sub> capture equipment are lowered with 1110 MNOK it is optimal to build gas plants with such equipment.

### 6 Discussion

In our case study the upper and lower bound of the plant value are rather close to each other. This indicates that the value of flexibility is rather small in our case study, as the gap between upper and lower bound is the difference of peak and base load plant values. Deng and Oren (2003) report similar findings. Our case study also indicates that the addition of an abandonment option does not change dramatically the building threshold. Thus, as a first

approximation for the investment decision it is plausible to ignore both the plant's flexibility and abandonment option.

In our case study even with zero emission costs it is not optimal to exercise the option to build a gas fired power plant. Regardless, the reality may be different. Some of the firms holding a license to build gas fired power plant in Norway have stated publicly that they are willing to invest, if the government relieves them of emission costs. The building threshold calculated with discounted cash flow method, i.e. [38.8; 41.7] NOK/MWh, is closer to the current equilibrium price estimate, which is around 35 NOK/MWh. Thus, in this particular case it seems that the thresholds calculated with discounted cash flows are closer to "industry practice" than the ones calculated by taking into account the possibility to postpone the investment decision, awaiting better information.

There are also other possible explanations why our results differ from the apparent policies of the actual investors. First, we have used the UK market as a reference for gas. There is lot of natural gas available in the Norwegian continental shelf. Due to the physical distance from the Norwegian coastline to the UK, the gas price at a Norwegian terminal will be equal to the UK price less transportation costs. By using price quotas from IPE we overestimate the gas price for delivery at a Norwegian terminal. Second, there is also a tax issue that has not been considered. Oil and gas companies operating on the Norwegian shelf have a 78% tax rate, while onshore activities are taxed at 28%. If a company invested in a gas power plant, it could sell the gas at a loss with offshore taxation, and buy the same gas as a power plant owner with onshore taxation.

The theory developed rests on an assumption that the energy company has an exclusive license, i.e. a monopoly right to invest. One may be concerned with how (imperfect) competition or other forms of market failure in the electricity or gas markets affect the results. However, as long as the information in efficient market prices of futures and forward contracts are incorporated in the analysis, these concerns are unfounded. Efficient forward prices will reflect any market failure in the cash markets. Of course, in practical cases there will be basis risk, for example due to electricity or gas being delivered or purchased at a different location or quality than that is underlying the forward contracts. Another problem is that long term contracts may not be available. For a discussion of these issues, see e.g. Fama and French (1987).

## 7 Conclusions

We use real options theory to analyze gas fired power plant investments. Our valuation is based on electricity and gas forward prices. We have derived a method to compute upper and lower bounds for the plant value and investment thresholds when the spark spread follows a two-factor model, capturing both the short-term mean-reversion and long-term uncertainty.

In our case study we take the view of an investor having a license to build a gas fired power plant. The example is based on forward prices from Nord Pool and International Petroleum Exchange (IPE). Our results indicate that the abandonment option and the operating flexibility interact so that their joint value is less than their separate values, because an option to permanently shut down compensates for the option to temporarily shut down and vice versa. However, the case study indicates that neither abandonment nor operating flexibility is very important, i.e. the difference between peak and base load plant value is rather small. Moreover, the case study indicates that the addition of abandonment option does not dramatically change the bounds of the building threshold. Thus, when investments to gas fired power plants are considered a good overall view of the investment problem can be made by ignoring the flexibility and abandonment options, whereas the role of the time-tobuild option is significant for the building threshold.

#### References

Deng, S.J. (2003), Valuation of Investment and Opportunity to Invest in Power Generation Assets with Spikes in Power Prices, Working paper, Georgia Institute of Technology

Deng, S.J., Johnson B., Sogomonian A. (2001), Exotic electricity options and the valuation of electricity generation and transmission assets, Decision Support Systems, (30) 3, pp. 383-392

Deng, S.J., Oren S.S. (2003), Valuation of Electricity Generation Assets with Operational Characteristics, Probability in the Engineering and Informational Sciences, forthcoming

Dixit, A.K., Pindyck, R.S. (1994), Investment under Uncertainty, Princeton University Press

Ellerman, D. (1998), Note on the Seemingly Indefinite Extension of Power Plant Lives, A Panel Contribution, The Energy Journal 19 (2), pp. 129-132

Fama, E.F., French, K. (1987), Commodity futures prices: Some evidence on forecast power, premiums, and the theory of storage, Journal of Business 60 (1), pp. 55-73

Harvey, A. C. (1989), Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, Cambridge, U.K.

International Energy Agency (IEA) (2003), World Energy Investment Outlook 2003, Paris, France

McDonald, R., and Siegel D. (1986), The value of waiting to invest, Quarterly Journal of Economics 101 (4), pp. 707-727

Pilipović, D. (1998), Energy Risk: Valuing and Managing Energy Derivatives, McGraw-Hill

Ross, S. (1997), Hedging long run commitments: Exercises in incomplete market pricing, Banca Monte Econom. Notes 26, pp. 99-132

Samuelson, P.A. (1965), Rational theory of warrant pricing, Industrial Management Review 6, pp. 13-31

Schwartz, E.S. (1997), The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging, The Journal of Finance, 52 (3), pp. 923-973

Schwartz, E. & J.E. Smith (2000), Short-Term Variations and Long-Term Dynamics in Commodity Prices, Management Science, vol.46 (7), pp.893-911

Undrum, H., Bolland, O., and Aarebrot, E. (2000), Economical assessment of natural gas fired combined cycle power plant with CO2 capture and sequestration, presented at the Fifth International Conference on Greenhouse Gas Control Technologies, Cairns, Australia West, M., Harrison J., (1996), Bayesian Forecasting and Dynamic Models 2nd ed. Springer-Verlag, New York

#### Appendix A

A peak load plant operates only when the spark spread exceeds emission costs. Thus, a production opportunity in a peak load plant, at time s, corresponds to  $\overline{C}$  European call options on the spark spread with strike price equal to the emission costs E. At time t, the value of such an option, maturing at time s, is

$$c(s) = e^{-r(s-t)} E\left[\max\left(S(s) - E, 0\right) \mid F_t\right] = e^{-r(s-t)} \left(\int_{E}^{\infty} (y - E)h(y)dy\right),$$
(A1)

where y is a normally distributed random variable with mean  $\mu(s)$  and variance  $v^2(s)$  and h(y) is the density function of a normally distributed variable. The integration gives

$$c(s) = e^{-r(s-t)} \left( \frac{v(s)}{\sqrt{2\pi}} e^{-\frac{(E-\mu(s))^2}{2v^2(s)}} + (\mu(s) - E) \left( 1 - \Phi\left(\frac{E-\mu(s)}{v(s)}\right) \right) \right),$$
(A2)

where  $\Phi(\cdot)$  is the normal cumulative distribution function. The value of a peak plant with lifetime  $\overline{T}$  is given by

$$V_U(\chi,\xi) = \int_t^{\overline{T}} \left(\overline{C}c(s) - e^{-r(s-t)}G\right) ds , \qquad (A3)$$

where G denotes the fixed costs of the plant. Equation (A3) gives

$$V_{U}(\chi,\xi) = \bar{C} \int_{t}^{\bar{T}} e^{-r(s-t)} \left( \frac{v(s)}{\sqrt{2\pi}} e^{\left( -\frac{(E-\mu(s))^{2}}{2v^{2}(s)} \right)} + \left( \mu(s) - E \right) \left( 1 - \Phi \left( \frac{E-\mu(s)}{v(s)} \right) \right) ds - \frac{G}{r} \left( 1 - e^{-r(\bar{T}-t)} \right)$$
(A4)

## Appendix B

When it is not optimal to exercise the build option, i.e. when  $\xi < \xi_H$ , the option to build  $F_0$  must satisfy following Bellman equation

$$rF_0(\xi)dt = E[dF_0(\xi)] - Wdt, \qquad \text{when } \xi < \xi_H.$$
(B1)

Using Itô's lemma and taking the expectation we get following differential equation for the option value

$$\frac{1}{2}\sigma^2 \frac{\partial^2 F_0(\xi)}{\partial^2 \xi} + \alpha \frac{\partial F_0(\xi)}{\partial \xi} - rF_0(\xi) - W = 0, \qquad \text{when } \xi < \xi_H.$$
(B2)

A solution to the differential equation is a linear combination of two independent solutions plus any particular solution (see, e.g., Dixit and Pindyck, 1994). Thus, the value of the build option is

$$F_0(\xi) = A_1 e^{\beta_1 \xi} + A_2 e^{\beta_2 \xi} - \frac{W}{r}, \qquad \text{when } \xi < \xi_H ,$$
(B3)

where  $A_1$ ,  $A_2$  are unknown non-negative parameters and  $\beta_1$  and  $\beta_2$  are the roots of the fundamental quadratic equation, and are given by

$$\beta_{1} = \frac{-\mu_{\xi} + \sqrt{\mu_{\xi}^{2} + 2\sigma_{\xi}^{2}r}}{\sigma_{\xi}^{2}} > 0$$
(B4)

$$\beta_{2} = \frac{-\mu_{\xi} - \sqrt{\mu_{\xi}^{2} + 2\sigma_{\xi}^{2}r}}{\sigma_{\xi}^{2}} < 0.$$
(B5)

The build option value approaches zero as the spark spread decreases, i.e.  $A_2$  must be equal to zero, and thus

$$F_0(\xi) = A_1 e^{\beta_1 \xi} - \frac{W}{r}, \qquad \text{when } \xi < \xi_{H0}.$$
(B6)

### Appendix C

It is optimal to exercise the build option when the option value becomes equal to the values gained by exercising the option

$$F_0(\xi_H) = V(0,\xi_H) - I + F_1(\xi_H).$$
(C1)

Correspondingly, it is optimal to abandon when values gained by abandoning are equal to values lost

$$F_1(\xi_L) + V(0,\xi_L) = D.$$
 (C2)

The smooth-pasting conditions must also hold when the options are exercised (for an intuitive proof see, e.g., Dixit and Pindyck, 1994 and for a rigorous derivation see Samuelson, 1965)

$$\frac{\partial F_0(\xi_H)}{\partial \xi} = \frac{\partial V(0,\xi_H)}{\partial \xi} + \frac{\partial F_1(\xi_H)}{\partial \xi}$$
(C3)

$$\frac{\partial F_1(\xi_L)}{\partial \xi} + \frac{\partial V(0,\xi_L)}{\partial \xi} = 0.$$
(C4)

The building and abandonment thresholds  $\xi_H$  and  $\xi_L$  as well as the option parameters  $A_1$ and  $D_2$  for all plant values V must satisfy (C1)- (C4). It remains to show that increase in the plant value decreases the investment and abandonment thresholds. Let us denote

$$G^{U}(\xi_{H}, A_{1}, D_{2}) = F_{0}(\xi_{H}) - V(0, \xi_{H}) - F_{1}(\xi_{H}) + I$$
(C5)

$$G^{L}(\xi_{L}, A_{1}) = F_{1}(\xi_{L}) + V(0, \xi_{H}) - D, \qquad (C6)$$

where  $A_1$  and  $D_2$  are the parameters of investment and abandonment options and  $\xi_H$  and  $\xi_L$ are the investment thresholds when the plant value is V. By denoting the partial derivatives with subscripts, the value-matching and smooth-pasting conditions for plant value V are

$$G^{U}\left(\xi_{H}, A_{1}, D_{2}\right) = 0 \tag{C7}$$

$$G^{L}\left(\xi_{L},A_{I}\right) = 0 \tag{C8}$$

$$G^{U}_{\xi_{H}}\left(\xi_{H}, A_{1}, D_{2}\right) = 0 \tag{C9}$$

$$G_{\xi_L}^L\left(\xi_L, D_2\right) = 0. \tag{C10}$$

When the plant value V is changed with df differentiation gives

$$G_{A_1}^U(\xi_H, A_1, D_2) dA_1 + G_{D_2}^U(\xi_H, A_1, D_2) dD_2 + G_{\xi_H}^U(\xi_H, A_1, D_2) d\xi_H = df$$
(C11)

$$G_{D_2}^{L}(\xi_L, D_2) dD_2 + G_{\xi_L}^{L}(\xi_L, D_2) d\xi_L = -df.$$
(C12)

Differentiation of the smooth-pasting condition gives

$$G_{\xi_{H}\xi_{H}}^{U}\left(\xi_{H},A_{1},D_{2}\right)d\xi_{H}+G_{\xi_{H}A_{1}}^{U}\left(\xi_{H},A_{1},D_{2}\right)dA_{1}+G_{\xi_{H}D_{2}}^{U}\left(\xi_{H},A_{1},D_{2}\right)dD_{2}=0$$
(C13)

$$G_{\xi_L\xi_L}^L(\xi_L, D_2) d\xi_L + G_{\xi_L D_2}^L(\xi_L, D_2) dD_2 = 0.$$
(C14)

Equations (C10), (C12), and (C14) give for the change of the abandonment threshold

$$d\xi_{L} = \frac{G_{\xi_{L}D_{2}}^{L}(\xi_{L}, D_{2})df}{G_{\xi_{L}\xi_{L}}^{L}(\xi_{L}, D_{2})G_{D_{2}}^{L}(\xi_{L}, D_{2})} = \frac{\beta_{2}df}{G_{\xi_{L}\xi_{L}}^{L}(\xi_{L}, D_{2})}.$$
(C15)

The second equality is obtained by calculating the derivatives of the abandonment option given in (17). Before abandonment, in the value-matching condition,  $G^{L}(\xi_{H}, A_{1})$  approaches zero from above, thus  $G^{L}(\xi, A_{1})$  must be convex in  $\xi$ . When the plant value is increased with positive amount, i.e. df > 0, we get

$$d\xi_L < 0. \tag{C16}$$

Hence when the plant value increases the abandonment threshold decreases. Equations (C9), (C11), (C13) and (C15) give the change of the building threshold

$$d\xi_{H} = \frac{G_{\xi_{H}A_{1}}^{U}\left(\xi_{H}, A_{1}, D_{2}\right) \left(\frac{df + G_{D_{2}}^{U}\left(\xi_{H}, A_{1}, D_{2}\right) \frac{df}{G_{D_{2}}^{L}\left(\xi_{L}, D_{2}\right)}}{G_{A_{1}}^{U}\left(\xi_{H}, A_{1}, D_{2}\right)}\right) + df \frac{G_{\xi_{H}D_{2}}^{U}\left(\xi_{H}, A_{1}, D_{2}\right)}{G_{D_{2}}^{L}\left(\xi_{L}, D_{2}\right)}, \quad (C17)$$
$$= \frac{-\beta_{1}\left(1 + e^{\beta_{2}(\xi_{H} - \xi_{L})}\right) + \beta_{2}e^{\beta_{2}(\xi_{H} - \xi_{L})}}{G_{\xi_{H}\xi_{H}}^{U}\left(\xi_{H}, D_{2}, A_{1}\right)}df$$

where the second equality is obtained by calculating the derivatives of the build and abandonment options given in (11) and (17). Before building, in the value-matching condition,  $G^{U}(\xi_{H}, A_{1}, D_{2})$  approaches zero from above, thus  $G^{U}(\xi, A_{1}, D_{2})$  must be convex in  $\xi$ . When the plant value is increased with positive amount, i.e. df > 0, we get

$$d\xi_H < 0. \tag{C18}$$

## **Figures**

Figure 1: Realization of spread process and expected value with confidence interval

Figure 2: Plant value as a function of the plant's lifetime

Figure 3: Plant and option values

Figure 4: Investment thresholds as a function of equilibrium volatility

Figure 5: Investment thresholds as a function of emission costs

Figure 6: Investment thresholds as a function of investment costs

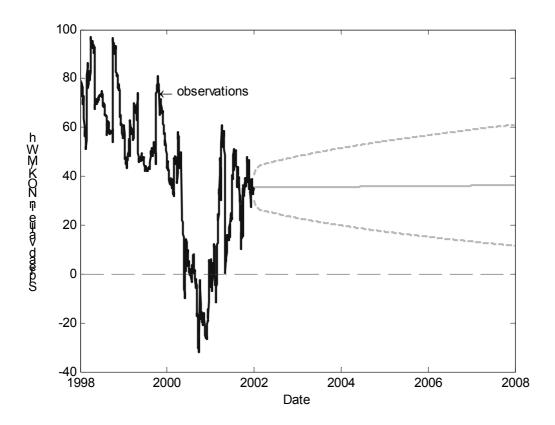


Figure 1

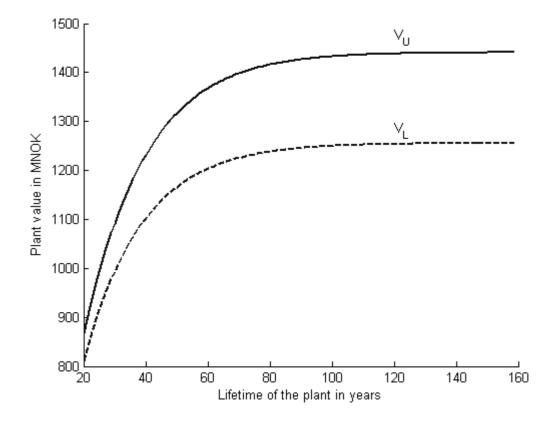
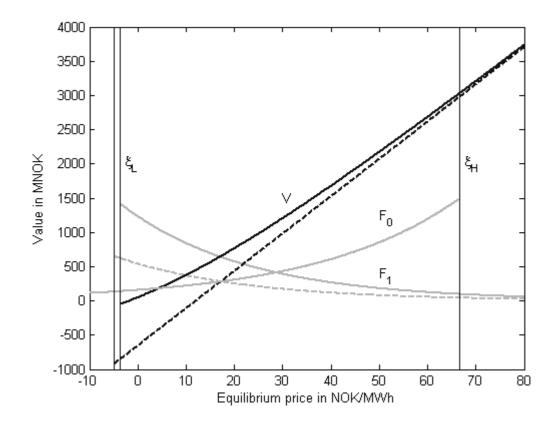


Figure 2





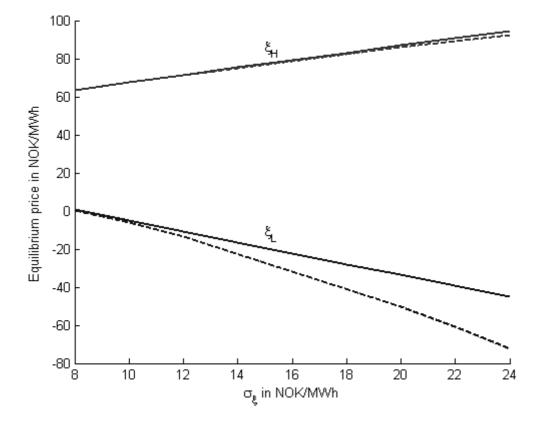


Figure 4

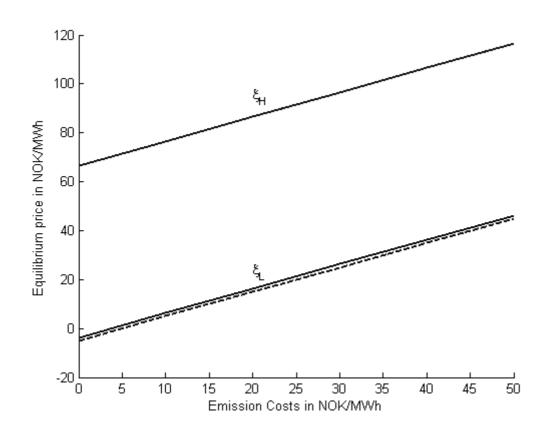


Figure 5

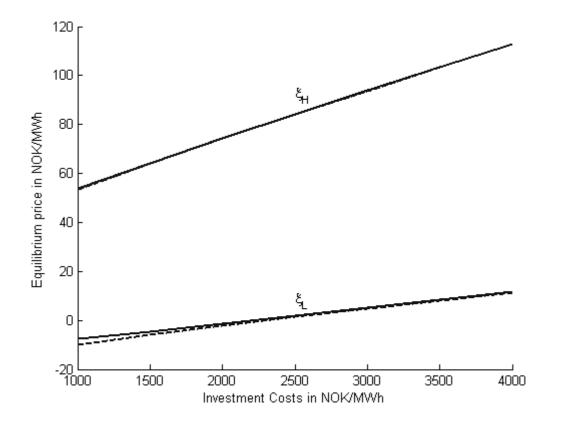


Figure 6