# An Estimation of U.S. Industry-Level Capital-Labor Substitution Elasticities: Cobb-Douglas as a Reasonable Starting Point?

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# Abstract:

A key parameter that determines the distributional impacts of a policy shift in general equilibrium models is the elasticity of substitution between capital and labor. Despite the importance of this parameter in applied modeling, its identification continues to pose a challenge. Given the structure of most growth models, we posit that the true relationship between capital and labor is likely to be close to Cobb-Douglas. Using a rich new data set from the Bureau of Economic Analysis, we estimate substitution elasticities for 28 industries, which cover the entire economy, and provide an indication of the long- and short-run estimates. We fail to reject the Cobb-Douglas specification in 20 of the 28 industries. These findings lend support to the Cobb-Douglas specification as a transparent starting point in simulation analysis.

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# I. Introduction

A key parameter that determines the distributional impacts of a policy shift in general equilibrium simulations is the elasticity of substitution between capital and labor. In this paper we provide the most comprehensive and up-to-date set of capital-labor substitution elasticity estimates for the U.S. economy. We exploit a rich data source recently released by the Bureau of Economic Analysis (BEA) that include new estimates of gross product by industry over 1947-1998, and represent significant improvements over previous data. Such improvements include a comprehensive revision of the national income and product accounts (NIPA's) and an extension of double deflation techniques, which account for inflation in both input and output markets.<sup>1</sup> We also use the BEA's newly revised estimates for the net stock quantity index of private fixed assets, which include equipment, software and structures. From this new data source, we estimate both the long- and short-run elasticities for 28 industries using established time series techniques.

Given the structure of most growth models, we posit that the true relationship between capital and labor is likely to be close to Cobb-Douglas. Econometric estimation results lend support to the Cobb Douglas specification. Specifically, we fail to reject the Cobb-Douglas specification in 20 of the 28 industries, and for seven of those industries we fail to reject the Leontief specification. We fail to reject Cobb-Douglas for aggregate manufacturing. Also, a comparison of econometric estimates and value-added weighted averages for several aggregations brings into question the common practice of averaging estimates for use in flexible aggregation models.

<sup>&</sup>lt;sup>1</sup> Lum, Moyer, and Yuskavage (2000).

Our objective is to consistently estimate a comprehensive set of capital-labor substitution elasticities for the U.S. economy. The current data only enable estimations at the two-digit level (28 sectors). Using appropriate time-series techniques we distinguish between short-run and relatively higher long-run elasticities. We also estimate elasticities for a few aggregations. We test our prior of a Cobb-Douglas relationship. In addition, we examine the implications of weighted average aggregations of industry level elasticities, because this is a conventional practice relied upon by many modelers. Our estimates provide support for using the Cobb-Douglas specification as a transparent starting point in parameterizing applied models and should be useful for researchers working on simulation and sensitivity analysis.

The paper is organized as follows. In the next section we discuss general issues surrounding parameterization, measurement and calibration, and the problems inherent in elasticity estimation. In section three we present the argument for Cobb-Douglas in the growth literature. In section four we discuss the empirical model, including the specification and the data. In section five we present the estimation results, and in the last section we provide concluding remarks.

#### **II.** Issues Surrounding the Parameterization of the Capital-Labor Relationship

The elasticity of substitution between capital and labor is a key parameter in quantifying distributional impacts of policy. Measurement of this parameter is, however, problematic and controversial. From a structural perspective, capital accumulation is inherently a complex dynamic problem. Once investments are made they may be specific to a given process making reallocation costly. In the historic data it is impossible to identify the portion of capital return that is normal versus that which is due to a productivity realization away from its expected mean.

Furthermore, the misallocation of physical capital in the time series due to adjustment costs cannot be directly identified. Given these realities, it is futile to expect estimations based on our static notion of capital input demand (like those presented below) not to suffer from misspecification. Transparent estimations of the capital-labor relationship based on a static equilibrium include the seminal work on CES functions by Arrow, Chenery, Minhas, and Solow (1961).<sup>2</sup>

Another way to think about the problem is that information sets, about shocks and uncertainty over time, are themselves time dependent. This indicates that forward-looking investments, based on rational expectations at the time they were made, are likely to realize a non-zero economic profit in the historical record. Macro-economists have struggled with these issues for some time, and real business cycle models are a promising area of research.<sup>3</sup> However, for our purpose these models provide little, if any, sectoral detail and are actually partially calibrated relying on assumed elasticities. For example, Kydland and Prescott (1982) and much of the literature that follows assume a Cobb-Douglas relationship between capital and labor in aggregate production.

Like those macro-economists who find calibrated business cycle models appealing for their structural integrity, micro-economists interested in comparative policy analysis face a monumental data shortage relative to the parameter requirements. Sufficient structural detail is necessary in order to capture important features of the economy. At the same time, we require a quantitative context that is not so abstract as to leave the question completely uninformed.

<sup>&</sup>lt;sup>2</sup> Arrow, Chenery, Minhas and Solow (1961) find strong evidence that the capital-labor substitution elasticity is between zero and unity. Also, Harrison, Jones, Kimbell and Wigle (1993) undertook econometric estimation of capital-labor substitution elasticities and report 4 out of 6 sectors to be between zero and unity.

<sup>&</sup>lt;sup>3</sup> See Gregory and Smith (1991) for a survey.

This is an important topic in applied economic analysis, particularly in the policy arena. For example, fundamental questions of competing tax policy are arguably best informed from a general equilibrium perspective (Harberger (1962) and Shoven and Whalley (1972)). Few micro-consistent observations relative to the number of parameters support such a model, if it is to produce anything but trivial quantitative results. Even fewer observations exist across relevant variations in exogenous instruments (alternative tax policies). Thus, reduced-form models are not likely to be accurate in revealing the effects of structural policy shifts especially when most questions concern new untested alternative policy initiatives. The data shortage, in the context of comparative policy studies, has precipitated a movement toward calibrated microeconomic models. Dawkins, Srinivasan, and Whalley (2001) offer a complete perspective on calibration and its role in economics.

Calibration usually follows a method that includes the interaction of a strict theoretic structure with two distinct types of data. The first type of data represents the benchmark equilibrium. In the context of constant-elasticity-of-substitution (CES) forms the first type of data identify exactly the distribution (or share) and efficiency parameters (Uzawa (1962), Rutherford (1995)). The data that determine these parameters are inherently local to the reference solution. So, although they establish a quantitative base for initiating policy experiments, they do little to inform the global properties of the model.

The second type of data indicate the degree of response and are often independent of the local equilibrium.<sup>4</sup> These are data that indicate the elasticity or slope parameters. In most

<sup>&</sup>lt;sup>4</sup> In some cases the benchmark equilibrium and response data are not separable in the calibration process. Rich response data on higher order curvatures (cross elasticities of substitution) require flexible functional forms (Perroni and Rutherford (1996)). In these forms the benchmark equilibrium is explicitly tied to the response data. Even with convenient functions, however, there are cases where elasticities and shares must be considered simultaneously. For example, any number of leisure value shares are consistent with a given uncompensated labor supply elasticity in a benchmark equilibrium. This is true even when a CES is specified between separable-leisure and other consumption, because the choice of labor supply effects income. Balard (1999) makes an important argument that it

applications one compiles a database that includes a point estimate on each of the required parameters. The key question is the source of the estimates. The estimates seldom come from an independent source and rarely are estimated in a way that is consistent with the model structure. The problematic nature of this practice is outlined by the critiques of Jorgenson (1984) and McKitrick (1995).

Examples of models that integrate some elements of consistent econometric estimation include Jorgenson (1984), Jorgenson, Selesnick and Wilcoxen (1992), McKitrick (1995), and McKibbin, Shackleton, and Wilcoxen (1998). Wilcoxen (1988) explains the method used to construct the necessary data for his time series estimation. He constructs consistent annual input-output tables for the years 1947 through 1985. This might appear to be a rich data source, but in fact his primary data only consists of 6 benchmark tables (1947, 1958, 1963, 1967, 1972, and 1977) that often used evolving industry definitions.<sup>5</sup>

It is interesting to note that McKibbin, Shackleton, and Wilcoxen explicitly reject some of their estimates and impose arbitrarily lower production elasticities (on energy sectors in this case). Their explanation for imposing these lower elasticities was to "help the model more accurately track the physical quantities of energy inputs and outputs to the sector" (p.7). We interpret this as their rejection of the econometric point estimates, not because the statistical model failed, but on practical grounds; the estimates imply unrealistic responses when used in the model.

is prudent to consider the interactions between substitution elasticities and value-shares when calibrating labor supply because welfare analysis is sensitive to the implied income elasticity of leisure. Other cases of calibration that blur the line between benchmark equilibrium data and response parameters include merger simulation models (Frobe and Werden (1996)). These procedures combine the market data and elasticities to imply the firms' marginal costs.

<sup>&</sup>lt;sup>5</sup> Our source data also come from the BEA; the primary difference between Wilcoxen's and our data is that the BEA completed the data set by filling in the gaps. Documentation on how the BEA actually constructed the data is provided in Lum, Moyer, and Yuskavage (2000), and *Survey of Current Business* (2001).

Following the lead of the real business cycle literature and a philosophical acceptance of calibration as a method of estimation (Dawkins, Srinivasan, and Whalley (2001)), there is a new direction in the literature to combine aspects of stochastic estimation in structural general equilibrium models (Liu, Arndt, and Hertel (2001), and Francois (2001)). These ideas are in there infancy but appear promising.

This paper offers a set of elasticities using standard econometric techniques that might be useful in the traditional calibrated computational model. Our estimates have the advantage that they update earlier work using the latest data, cover a number of sectors, and provide an indication of the long-run versus short-run elasticities.

#### **III.** An Argument for Cobb-Douglas in the Growth Literature

Nicholas Kaldor (1963) outlined a number of stylized facts that are often used as a guideline for formulating reasonable models of economic growth (see Robert J. Barro and Xavier Sala-i-Martin, 1995). Kaldor's *facts* illustrate a great deal of stability in the growing economy. For example, the ratio of physical capital to output is nearly constant over a long time series. The stability in the data conveniently limits the theoretic search to those models that possess steady-state characteristics. Models of capital accumulation, at least of developed countries, that do not converge to a constant capital-output ratio in the long run are difficult to defend given the evidence.

Harrod-neutral technical change is a condition that must be placed on production to achieve a steady state. The Cobb-Douglas form is the only form that reduces to Harrod neutrality even when capital or total factor productivity grows over time. So although Cobb-Douglas is a restrictive form, it allows one to envision a number of flexible mechanisms by which technical progress augments growth, in a model consistent with steady state. A formal proof is provided by Barro and Sala-i-Martin (1995, pp. 54-55). The Cobb-Douglas restriction (unitary substitution elasticity) is a testable hypothesis in our econometric model, but first we illustrate how all forms of constant technical change reduce to Harrod neutrality under Cobb-Douglas.

Harrod-neutral technical change is often referred to as *labor-augmenting* because the value added composite, *Y*, in production can be written as:

$$Y = F[K, L \cdot A(t)] \tag{1}$$

where A(t) is an index of technology, which grows at a constant rate over the time index. There are two ways to achieve Harrod neutrality. First, one might assume that technological improvements are truly only applicable to labor. This is not an appealing assumption because it is relatively easy to produce examples of quality improvements in capital over time. Alternatively, if one adopts Cobb-Douglas then the technological improvement can be rearranged in a way that accommodates both Harrod neutrality and capital improvements. That is, any general set of constant productivity changes over time is shown to be Harrod neutral if we place a restriction on the functional form: Cobb-Douglas.

As an example, consider that generic productivity growth is indexed by T(t), capital's productivity index is B(t), and labor's productivity index is C(t). In the general form (which is not necessarily consistent with steady-state) output is represented as:

$$Y = T(t) \cdot F[K \cdot B(t), L \cdot C(t)]$$
<sup>(2)</sup>

and in the special case of Cobb-Douglas:<sup>6</sup>

$$Y = T(t) \cdot \left(K \cdot B(t)\right)^{\alpha} \left(L \cdot C(t)\right)^{1-\alpha}$$
(3)

This reduces to the labor augmenting form if A(t) is defined by:

$$A(t) \equiv C(t) \cdot T(t)^{\frac{1}{1-\alpha}} B(t)^{\frac{\alpha}{1-\alpha}}$$
(4)

No restrictions on the relationship between T(t), B(t), and C(t) are required to achieve a reduced form that exhibits Harrod neutrality. Non-neutral and other forms of neutrality (Hicks neutrality and Solow neutrality) all reduce to Harrod neutrality when we assume Cobb-Douglas.

It is difficult to make a judgment on what restriction to apply. The Cobb-Douglas form is very limiting, and yet it seems reasonable that capital becomes more productive over time. In addition, rejecting Cobb-Douglas might only lead to a minor relaxation. The constant elasticity of substitution form, which is the common alternative, is only one parameter less restrictive. Absent a richer theory that resolves these conflicts, Cobb-Douglas in the value-added nest might be a reasonable starting point for sensitivity analysis in most neoclassical computational models. Furthermore, if steady-state is to be maintained, the domain of the sensitivity analysis is logically restricted to alternative assumptions about capital's productivity under Cobb-Douglas, or varying the substitution elasticity while holding capital's productivity fixed. In the next section we use an econometric model to estimate the substitution elasticities and test the hypothesis that production is Cobb-Douglas at the industry level and at various levels of aggregation.

#### **IV. Empirical Model**

The value added nest of the production function is assumed to take on a constant elasticity of substitution form. Inputs of capital and labor enter in the following fashion:

$$Y = \left[\alpha \cdot K^{(\sigma-1)/\sigma} + (1-\alpha) \cdot L^{(\sigma-1)/\sigma}\right]^{\frac{\sigma}{\sigma-1}}$$
(5)

<sup>&</sup>lt;sup>6</sup> The special case, of Cobb-Douglas, is a necessary condition for steady-state if T(t) and B(t) are not constant over time (again, see the proof provided by Barro and Sala-i-Martin (1995, pp. 54-55)).

where  $\sigma$  is the constant elasticity of substitution between the factor inputs, and  $\alpha$  is the distribution parameter. Constrained optimization of (5) yields the following log linear specification:

$$\ln\frac{K}{L} = \sigma \cdot \ln\frac{\alpha}{1-\alpha} + \sigma \cdot \ln\frac{w}{r}$$
(6)

where w and r are the wage and rental rates, respectively. This equation may be stylized to fit the linear regression equation:

$$\ln y = \beta_o + \beta_1 \ln x + \varepsilon \tag{7}$$

where *y* is the capital-labor ratio, *x* is the wage-rental ratio, and  $\varepsilon$  is the independent and identically distributed (iid) error. The elasticity of substitution between capital and labor is represented by  $\beta_l$ , the coefficient of interest.

## Data

The four data series that are required to operationalize equation (7) are labor inputs, capital inputs, payments to labor, and payments to capital. A newly released data set by the BEA includes these series, specifically, full-time equivalent employees, compensation of employees, and property type income. Compensation of employees is defined as the sum of wages, salary, and supplements to wages and salaries. Property type income includes corporate profits, proprietor's income, rental income, net interest, private capital consumption allowances, business transfer payments, and government consumption of fixed capital.<sup>7</sup> The BEA data include new estimates of gross product by industry over 1947-1998, and represent significant improvements over previous data, namely, a comprehensive revision of the national income and product

accounts (NIPA's) and an extension of double deflation techniques, which account for inflation in both input and output markets.<sup>8</sup> We use the BEA's newly revised estimates for the net stock quantity index of private fixed assets, which include equipment, software and structures.<sup>9</sup> The estimates provide measures of the value of assets in the prices of the given period, which are end of year for net stocks and annual averages for depreciation. The index uses 1996 as the base year.<sup>10</sup>

The data were compiled by the BEA using two SIC codes. For 1947-1987, data were classified according to 1972 SIC codes, whereas data from 1987-1999 were compiled using 1987 SIC codes. To correct for the discrete change in the time series, the 1987 data from both classifications were compared. Using the proportional difference, we adjust the latter to fit with the earlier data. We use factor input and payments data for 28 two-digit SIC categories. The wage and rental rates were calculated by dividing the compensation to employees by the number of full-time equivalent employees, and property-type income by the net stock quantity index, respectively.

# V. Econometric Results

#### *Specification*

We adopt equation (5) and apply standard time series econometric estimation techniques. We attempt to estimate the long-run elasticities that are appropriate for computable general (and partial) equilibrium models. Capital and labor adjustments to changes in rental and wage rates

<sup>&</sup>lt;sup>7</sup> See Lum, Moyer, and Yuskavage (2000) footnote 8.

<sup>&</sup>lt;sup>8</sup> Lum, Moyer, and Yuskavage (2000).

<sup>&</sup>lt;sup>9</sup> See also *Survey of Current Business* (2001) for formulas to calculate quantity indices.

<sup>&</sup>lt;sup>10</sup> Survey of Current Business (2000).

take time due to the lag involved in accumulating capital and other adjustment frictions. Therefore, we allow for time of adjustment in the estimation procedure.

We use the weighted-symmetric test to determine the order of integration for each series across industries, the ratio of capital to labor inputs, and the corresponding relative factor payments.<sup>11</sup> A group of non-stationary time series is cointegrated if a linear combination of them is stationary; that is, the combination does not have a stochastic trend. We tested for a long-run, stationary relationship between the two ratios for each industry using the Engle-Granger technique when the cointegrating variables had a unitary order of integration, I(1).<sup>12</sup> The cointegration results allowed us to determine whether a single-equation error correction model would be an appropriate specification for each series.

Equation (7) was estimated separately for each industry category, using one of the three specifications laid out below, each utilizing different time-series properties of the series. The first specification is a parsimonious geometric lag model:

$$\ln y_t = \alpha_o + \beta_1 \ln x_t + \beta_2 \ln y_{t-1} + \varepsilon$$
(8)

The autoregressive model of order one (AR(1)) specification is useful here because the long-run and short-run estimates are easily extracted. This estimation procedure generates efficient estimates in the presence of disturbances that exhibit first order serial correlation. The long-run elasticity is calculated as  $\beta_1/(1-\beta_2)$  if  $0 < \beta_2 < 1$ . The short run elasticity is simply  $\beta_1$ .

The second specification is based on using first differences of the dependent and explanatory variables only, and is appropriate for industries with data series that are both I(1) and not cointegrated, or with just one I(1) series:

<sup>&</sup>lt;sup>11</sup> The Weighted Symmetric test is recommended over the Dickey-Fuller test because it has (sometimes only slightly) higher power (see Pantula, Gonzalex, and Fuller, 1994).

<sup>&</sup>lt;sup>12</sup> The theory is set forth in Engle and Granger (1987). The Engle-Granger test is only valid if all the cointegrating variables are I(1).

$$\Delta \ln y_t = \alpha_o + \beta_1 \Delta \ln x_t + \varepsilon_t \tag{9}$$

where  $\Delta \ln y_t = \ln y_t - \ln y_{t-1}$  and  $\Delta \ln x_t = \ln x_t - \ln x_{t-1}$ , and  $\varepsilon$  is an i.i.d. error term. The short run elasticity is  $\beta_{l}$ .

Finally, a single equation error correction model is applicable to industries with data series that are both I(1) and cointegrated:

$$\Delta \ln y_t = \alpha_o + \beta_1 \Delta \ln x_t + \beta_2 \ln y_{t-1} + \beta_3 \ln x_{t-1} + \varepsilon_t$$
(10)

This model allows the data to determine the short-run and long-run responses of factor inputs with respect to factor payments. Specifically, the long-run elasticity is  $-(\beta_3/\beta_2)$  and the short-run elasticity is  $\beta_1$ .

We do not make any judgement about the dynamic structure and thus do not formally test among the estimation specifications described above. Allowing the data to inform the error structure implicitly assumes that the error structure can inform the dynamics of the model when, in fact, it cannot. Regardless of how well the time series model is fit to the data, it still has no statistical properties that correspond to the actual dynamic model with capital accumulation decisions. We do not submit any one of these as the true specification. However, we note that the estimation results are generally insensitive to specification.

## Estimation Results

In order to analyze the time series properties of the data, unit root and cointegration tests were performed for the capital-labor ratio and wage-rental ratio series.<sup>13</sup> Both series for each industry, with the exception of a few, were found to be stationary in first-differenced form, or

<sup>&</sup>lt;sup>13</sup> Unit root and cointegration tests were not performed for the following industries because of lack of continuous data: metal mining, other transportation equipment, and petroleum and coal products.

I(1).<sup>14</sup> When series were found to be I(1), tests for second-order integration were easily rejected. Results from the Engle-Granger test for cointegration suggest that the series are not cointegrated for any of the industries.

The results from the three specifications—AR(1), first differenced, and single equation error correction—are presented in Table 1. The results from the AR(1) model using the Cochrane-Orcutt procedure is presented in the Table 2. The coefficients of interest are the longand short-run elasticities. Overall, the elasticity estimates do not vary much across specifications either in terms of sign or magnitude.

The AR(1) model using the Cochrane-Orcutt procedure involves estimating the correlation coefficient on the errors and then using these estimates to adjust the data. Thus, the residuals from this new equation are uncorrelated. The adjusted data replace the original data and the equation is re-estimated.<sup>15</sup> This procedure eliminated most of the serial correlation that was present in many of the estimates and produces long-run and short-run estimates. Given the lack of sensitivity of the estimations to the specification and that this procedure eliminated most of the serial correlation, we focus our discussion on these results.

On interpreting statistical significance, testing the null hypothesis that the elasticity estimate is equal to zero is equivalent to a test of the Leontief specification. Testing the null hypothesis that that elasticity estimate is equal to unity is equivalent to a test of the Cobb-Douglas specification. We fail to reject the Cobb-Douglas specification for 20 of the 28 industries (at the five-percent level) and for seven of those industries we fail to reject the Leontief specification. Serial correlation exists in 6 of the 28 individual industry-level

<sup>&</sup>lt;sup>14</sup> Industrial machinery and equipment, motor vehicles and equipment, instruments and related products, and printing and publishing.<sup>15</sup> See Cochrane and Orcutt (1949) for a complete description of this procedure.

regressions. For all of manufacturing industries combined, we reject the Leontief specification, but we cannot reject Cobb-Douglas.

A comparison of direct estimates and weighted averages of disaggregated estimates is shown in Table 3. The VA weighted averages for the industry-wide and manufacturing aggregations are higher than the econometric estimates for those two aggregations. The estimated elasticity for all industries was 0.95 and the VA weighted average was 1.22. Similarly, the estimate for manufacturing was 1.21, and the VA weighted average was 1.32.<sup>16</sup> Also, averages for five independent aggregations were calculated: farming and agriculture, mining and metals, intermediates, durable manufacturing, and nondurable manufacturing. The VA weighted average was higher for three of these five aggregations. These calculations reveal weak evidence of an aggregation bias and bring into question ex-post aggregations that are commonly performed in applied modeling.

However, we show some estimates with very wide confidence intervals and even some negative point estimates. We do not claim to offer estimates that are superior to industry-level studies that look at detailed production functions. Rather, these estimates and their distributions are meant to give the reader a consistent, transparent analysis of this new data source.

#### **VI. Concluding Remarks**

The factor input substitution elasticity is a key parameter that determines the distributional impacts of a policy shift in general equilibrium simulations. Given the structure of most growth models, we posit that the true relationship between capital and labor is likely to be close to Cobb-Douglas. Using a rich new data set by the Bureau of Economic Analysis, we

<sup>&</sup>lt;sup>16</sup> The value-added weighted averages exclude outliers including leather and leather products, food and kindred products, and petroleum and coal products.

present econometric elasticity estimates for 28 2-digit sectors. Our estimates have the advantage over earlier work in that they utilize a richer, more complete data set, cover a larger number of sectors, and provide an indication of the long-run versus short-run elasticities. We fail to reject the Cobb-Douglas specification in 20 of the 28 industries, and for seven of those industries we fail to reject the Leontief specification. We also fail to reject Cobb-Douglas for aggregate manufacturing. Further, value-added weighted averages for various aggregations are compared against the econometric estimates from those aggregations. The calculations reveal the possibility of an aggregation bias and suggest a reconsideration of averaging methods in flexible aggregation models. Our findings lend support to the Cobb-Douglas specification as a transparent starting point in simulation analysis. These results and the arguments we forward should be of interest to those modelers in search of a starting point for specifying a capital-labor substitution rate.

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dustry	Var.	Est.	Ho: B=0 p-val	Ho: B=1 p-val (C-D)	sc	Var.	Est.	Ho: B=0 p-val	Ho: B=1 p-val (C-D	) SC	Var.	Est.	Ho: B=0 p-val	Ho: B=1 p-val (C- D)	sc
arms	LRE SRE	0.411 0.026	0.179 0.458	<b>0.054</b> 0.000		SRE	-0.048	0.334	0.000	SC	LRE SRE	0.585 ** -0.030	0.000 <b>0.516</b>	0.004	
gricultural	LRE	-0.168	0.332	0.000	SC						LRE	-0.323	0.159	0.000	sc
rrvices, restry, and hing	SRE	-0.017	0.360	0.000		SRE	0.211 **	0.001	0.000	sc	SRE	0.189 **	0.004	0.000	
etal mining	LRE SRE	0.673 ** 0.045 **	0.000 0.008	0.000	SC	SRE	0.036 *	0.099	0.000	SC	LRE SRE	0.683 ** 0.055 **	0.000 0.037	<b>0.084</b> 0.000	sc
oal mining	LRE SRE	1.261 ** 0.101 **	0.000	<b>0.068</b> 0.000		SRE	0.096 **	0.000	0.000		LRE SRE	1.265 ** 0.129 **	0.000	<b>0.119</b> 0.000	sc
l and gas	LRE	0.735 **	0.000	0.000	sc						LRE	0.735 **	0.000	0.000	sc
uraction	SRE	0.268 **	0.000	0.000		SRE	0.224 **	0.000	0.000	sc	SRE	0.273 **	0.000	0.000	
onmetallic	LRE	0.946 **	0.000	0.761	sc						LRE	0.927	0.330	0.938	sc
nerals, cept fuels	SRE	0.060 **	0.031	0.000		SRE	0.101 **	0.000	0.000	sc	SRE	0.105 **	0.000	0.000	
Instruction	LRE SRE	0.827 ** 0.194 **	0.000	<b>0.163</b> 0.000	sc	SRE	0.430 **	0.000	0.000	SC	LRE SRE	0.933 ** 0.420 **	0.000	<b>0.798</b> 0.000	sc

		Mo	del 1AR	(1)			Model 2	first dif	ference		Moc	del 3ECN	
I			Ho: B=0	) Ho: B=1				Ho: B=0	Ho: B=1			Ho: B=0	Ho: B=1
Industry	Var.	Est.	p-val	p-val (C-D)	SC	Var.	Est.	p-val	p-val (C-D) SC	Var.	Est.	p-val	p-val (C-D) SC
Lumber and	LRE	1.006 **	0.000	0.968						LRE	0.748 **	0.013	0.404
wood products	SRE	0.163 **	0.000	0.000		SRE	0.260 **	0.000	0.000	SRE	0.265 **	0.000	0.000
Furniture and fixtures	LRE	0.948 **	0.000	0.752	SC					LRE	0.940 **	0.000	0.662
	SRE	0.097 **	0.011	0.000		SRE	0.036	0.434	0.000	SRE	0.074	0.105	0.000
Stone, clay, and glass products	LRE SRE	0.473 ** 0.050 **	0.000	0.000	SC	SRE	0.068 **	0.002	0.000	LRE SRE	0.424 ** 0.076 **	0.002 0.000	0.000
Primary metal industries	LRE SRE	0.518 ** 0.086 **	0.000	0.000		SRE	0.098 **	0.000	0.000	LRE SRE	0.430 ** 0.109 **	0.021 0.001	0.002 0.000
Fabricated metal products	LRE SRE	1.251 ** 0.108 **	0.000	<b>0.270</b> 0.000		SRE	0.196 **	0.001	0.000	LRE SRE	1.327 ** 0.211 **	0.000	<b>0.256</b> 0.000
Industrial	LRE	0.795 **	0.000	0.000						LRE	0.789 **	0.000	0.000
and equipment	SRE	0.211 **	0.000	0.000		SRE	0.173 **	0.000	0.000	SRE	0.232 **	0.000	0.000
Electronic and	LRE	2.983 **	0.025	0.137						LRE	3.540	0.165	0.319
electric equipment	SRE	0.073 **	0.002	0.000		SRE	0.078 *	0.092	0.000	SRE	0.106 **	0.028	0.000
Motor vehicles and equipment	LRE SRE	0.397 ** 0.049 **	0.001 0.036	0.000	sc	SRE	0.026	0.236	0.000	LRE SRE	0.404 ** 0.048 **	0.000 0.047	0.000
Other	LRE	0.811	0.154	0.740	SC					LRE	0.745 **	0.006	0.344 sc
equipment	SRE	0.027	0.319	0.000		SRE	0.013	0.381	0.000 sc	SRE	0.016	0.557	0.000

Table 1, cont'd.

Table 1, cont'd.															
		Model	1AR(1)				Model 2	∹-first diffe	ence			Moc	lel 3EC	Σ	
I			Ho: B=0	Ho: B=1				Ho: B=0	Ho: B=1				Ho: B=0	Ho: B=1	
Industry	Var.	Est.	p-val	p-val (C-D)	SC	Var.	Est.	p-val p	o-val (C-D)	SC	Var.	Est.	p-val	p-val (C-D)	) SC
Instruments and	LRE	0.538	0.657	0.704	sc						LRE	0.695	0.789	0.906	sc
related products	SRE	0.003	0.651	0.000		SRE	0.012 **	0.025	0.000	sc	SRE	0.011 *	0.063	0.000	
Misc. mfg.	LRE	1.226	0.173	0.801							LRE	1.051	0.334	0.963	
eana chilea	SRE	0.043	0.109	0.000		SRE	0.072	0.246	0.000		SRE	0.078	0.132	0.000	
Food and	LRE	22.450	0.870	0.876	sc						LRE	5.388	0.309	0.407	SC
products	SRE	0.023 **	0.022	0.000		SRE	-0.031	0.118	0.000	sc	SRE	-0.012	0.406	0.000	
Tobacco	LRE	1.380 **	0.001	0.347	sc						LRE	0.717	0.311	0.690	
products	SRE	0.046	0.167	0.000		SRE	0.104 *	0.067	0.000	sc	SRE	0.098 *	0.088	0.000	
Textile mill	LRE	0.864 **	0.012	0.694							LRE	0.849	0.430	0.888	
products	SRE	0.056 *	0.058	0.000		SRE	0.107 **	0.004	0.000		SRE	0.115 **	0.006	0.000	
Apparel and	LRE	2.075 **	0.000	0.001							LRE	1.936 **	0.000	0.000	
other textile products	SRE	0.108 **	0.002	0.000		SRE	-0.009	0.811	0.000	sc	SRE	0.067 *	0.054	0.000	
Paper and allied	LRE	0.861 **	0.000	0.292							LRE	0.936 **	0.008	0.855	
products	SRE	0.054 **	0.018	0.000		SRE	0.127 **	0.000	0.000		SRE	0.135 **	0.001	0.000	
Printing and	LRE	1.316 **	0.000	0.169	sc						LRE	1.311 **	0.000	0.213	SC
Billislind	SRE	0.080 **	0.005	0.000		SRE	0.055 *	0.050	0.000	sc	SRE	0.088 **	0.007	0.000	

Table 1, cont'd.														
		Mod	lel 1AR	(1)			Model 2-	-first diff	erence			Mod	lel 3ECI	5
I			Ho: B=0	Ho: B=1				Ho: B=0	Ho: B=1				Ho: B=0	Ho: B=1
Industry	Var.	Est.	p-val	p-val (C-D)	SC	Var.	Est.	p-val	p-val (C-D)	SC	Var.	Est.	p-val	p-val (C-D) SC
Chemicals and	LRE	1.321 **	0.002	0.450							LRE	1.832	0.197	0.558
allied products	SRE	0.035 *	0.060	0.000		SRE	0.078 **	0.005	0.000		SRE	0.089 **	0.002	0.000
Petroleum and	LRE	3.137	0.666	0.769							LRE	2.347	0.653	0.797
products	SRE	0.016 *	0.057	0.000		SRE	0.017 **	0.033	0.000	sc	SRE	0.023 *	0.083	0.000
Rubber and misc.	LRE	0.783 **	0.000	0.127							LRE	0.759 **	0.000	0.207
plastics products	SRE	0.082 *	0.061	0.000		SRE	0.057 *	0.075	0.000		SRE	0.087 **	0.033	0.000
Leather and leather products	LRE SRE	-1.597 0.039 *	0.113 0.055	0.010 0.000		SRE	0.018	0.330	0.000	sc	LRE SRE	-1.533 0.041	0.161 0.104	0.021 0.000
Mfg and Mining	LRE SRE	1.153 ** 0.172 **	0.000	<b>0.146</b> 0.000		SRE	0.302 **	0.000	0.000		LRE SRE	1.233 ** 0.318 **	0.000	<b>0.108</b> 0.000

equivalent to a test of the Leontief specification. The null hypothesis that the elasticity estimates is equal to unity (Ho: B=1) is equivalent to a test of the Cobb-Douglas specification. A failure to reject the null hypothesis at the 5 percent level is indicated by bold p-values. Presence of serial correlation (sc) is indicated Notes: Statistical significance is indicated by \*\* (\*) at the 5 (10) percent level. This null hypothesis that the elasticity estimate is equal to zero (Ho: B=0) is by a "sc" in the SC column.

Table 2. Elasticity Estimation	Result:	s from th	e Co	chrane-Orc	utt Procedure	
		AR(1	) usiı	ng Cochran	e-Orcutt	
			Ť	o: B=0	Ho: B=1	
Industry	Var.	Est.		p-val.	p-val.	SC
Farms	LRE	0.307		0.523	0.148	
	SRE	0.015		0.677	0.000	
Agricultural services,	LRE	0.364	* *	0.007	0.000	
forestry, and fishing	SRE	0.225	* *	0.000	0.000	
Metal mining	LRE	0.670	* *	0.003	0.141	
	SRE	0.058	* *	0.000	0.000	
Coal mining	LRE	1.271	* *	0.000	0.105	
	SRE	0.101	* *	0.000	0.000	
Oil and gas extraction	LRE	0.735	* *	0.000	0.000	
	SRE	0.268	* *	0.000	0.000	
Nonmetallic minerals,	LRE	0.893	* *	0.000	0.659	
except fuels	SRE	0.071	* *	0.011	0.000	
Construction	LRE	0.893	* *	0.000	0.442	sc
	SRE	0.383	* *	0.000	0.000	
Lumber and	LRE	1.120	* *	0.000	0.457	
wood products	SRE	0.208	* *	0.000	0.000	
Furniture and fixtures	LRE	1.007	* *	0.000	0.967	
	SRE	0.097	* *	0.000	0.000	
Stone, clay, and	LRE	0.501	* *	0.000	0.000	sc
glass products	SRE	0.066	*	0.000	0.000	

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Table 2, cont'd.			•		:	
		AR(1	H H	ng Cochrar o: B=0	ne-Orcutt Ho: B=1	
Industry	Var.	Est.		p-val.	p-val.	SC
Primary metal	LRE	0.533	* * * *	0.000	0.000	
Industries	UXC UXC	1.80.0		0.000	0000	
Fabricated metal	LRE	1.393	* *	0.000	0.065	
products	SRE	0.113	* *	0.000	0.000	
Industrial machinery	LRE	0.815	* *	0.000	0.000	
and equipment	SRE	0.226	* *	0.000	0.000	
Electronic and	LRE	3.736		0.109	0.241	SC
other electric equipment	SRE	0.075	* *	0.001	0.000	
Motor vehicles and	LRE	0.400	* *	0.000	0.000	
equipment	SRE	0.047	* *	0.000	0.000	
Other transportation	LRE	0.322		0.508	0.163	SC
equipment	SRE	0.012		0.509	0.000	
Instruments and	LRE	0.599		0.226	0.417	sc
related products	SRE	0.011		0.270	0.000	
Misc. mfg. industries	LRE	1.684	*	0.063	0.450	
	SRE	0.054	*	0.050	0.000	
Food and kindred	LRE	-34.6		0.968	0.968	
products	SRE	-0.034	* *	0.019	0.000	
Tobacco products	LRE	1.226	* *	0.000	0.349	
	SRE	0.061		0.105	0.000	

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		AR(1	l) usi	ng Cochran	e-Orcutt		
		Ho: B	e		Ho: B=1		
Industry	Var.	Est.		p-val.	p-val.	SC	
Textile mill products	LRE	1.138	**	0.004	0.729		
	SRE	0.052	*	0.058	0.000		
Apparel and other	LRE	2.051	**	0.000	0.000		
textile products	SRE	0.125	* *	0.000	0.000		
Paper and allied products	LRE	0.907	* *	0.000	0.538	SC	
	SRE	0.057	* *	0.008	0.000		
Printing and publishing	LRE	1.510	* *	0.000	0.098		
	SRE	0.084	* *	0.004	0.000		
Chemicals and	LRE	1.448	* *	0.001	0.297		
allied products	SRE	0.046	* *	0.017	0.000		
Petroleum and coal	LRE	76.3		0.983	0.983		
products	SRE	0.020	* *	0.025	0.000		
Rubber and misc.	LRE	0.806	**	0.000	0.257		
plastics products	SRE	0.072	* *	0.040	0.000		
Leather and	LRE	-1.532		0.133	0.013		
leather products	SRE	0.040	*	0.061	0.000		
Mfg and Mining	LRE	1.229	* *	0.000	0.024		
	SRE	0.202	* *	0.000	0.000		
Notes: Statistical significance	is indicat	ted by ** ( R-0) is p	(*) at	the the 5 (10	) percent leve	el. This nu ef snecific	ull hypothesis that the
hypothesis that the elasticity e	stimates	is equal t	inu o	ty (Ho: B=1)	is equivalent t	o a test o	f the Cobb-Douglas
specification. A failure to reject	ct the null	hypothe:	sis at	the 5 percen	nt level is indic	ated by b	old p-values.
Presence of serial correlation	is indicate	ed by a "s	s.c." ii	n the S.C. co	lumn.		

Disaggregated Estimates		
1	Direct	'alue-Added Weighted
Aggregation	Estimate	Average Estimate
All	0.951	1.215
Mfg.	1.213	1.318
Farming and Agriculture	0.083	0.325
Mining and Metals	0.988	0.789
Intermediates	0.502	1.049
Durable Mfg. Goods	0.918	1.721
Nondurable Mfg. Goods	1.315	1.256

Table 3. Comparison of Direct Estimates and Weighted Averages of