Complex Remanence vs. Simple Persistence: Are Hysteresis and Unit Root Processes Observationally Equivalent?

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Abstract

The hysteresis terminology has mainly been used in two fields of economics, unemployment and international trade, with a different meaning however, involving either linear autoregressive macro behaviour or non-linear heterogenous micro behaviour. There may nonetheless be observational equivalence between the "persistence" characterising unit-root processes and the "remanence" created by the aggregation of non-linear dynamics. Stochastic simulations are employed to analyse the properties of the output of an hysteretic system, subject to white noise and random walk inputs. Non-linear hysteretic systems are found to generate a sizeable proportion - two-thirds - of stationary outputs from stationary input, and to possibly generate an output cointegrated with the corresponding input. Such systems therefore appear significantly different from an integrated process. This stresses the specific relevance of a non-linear approach to hysteresis.

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1. INTRODUCTION

The term of hysteresis has been first used for the magnetisation of ferro-magnetic metals by Ewing (1885). This terminology has been ever since widely employed in a broad variety of fields, such as thermodynamics, electricity and, of course, economics. [1] In the latter field, it is usually associated with ideas about "memory of shocks", "multiplicity of equilibria", and "structural change", whether it appears in unemployment theory (Blanchard and Summers, 1986, or Sachs, 1987) or in the international trade theory (Baldwin and Krugman, 1989).
There is a fundamental difference between these two streams, although both have led to a sizeable literature in the meantime - e.g. 204 mentions under "hysteresis and unemployment" as against 102 under "hysteresis and trade" in the Econ Lit CD-rom of September 2000 (for a total of 408 references to "hysteresis"). Unemployment hysteresis - albeit in theory based on path-dependence, habit formation and other elements likely to introduce non-linearities - is usually associated with unit-roots in a fully linear framework, either for the unemployment rate itself or for a whole system of wage-price variables. On the other hand, hysteresis, as employed in international trade, is associated with a non-linear setting where "bistability" is observed, i.e. there may be two possible steady states for the same value of a given parameter. In the case at hand, an exporting firm could be either in or out of the foreign market for the same value of the exchange rate. Empirically, the emphasis has moreover been put in this literature on the differentiated impact of large and small shocks to exchange rates, thereby considering in its own right the hypothesis of non-linearity.
Most of the empirical work related to hysteresis in the labour market - extensively documented e.g. in Cross (1988) and Cross (1995) - has developed very much along the lines of Blanchard and Summers (1986). The focus is therefore on the time-process of unemployment rates, periodically reassessing whether unemployment rates are non-stationary. However, some recent contributions have followed an alternative approach, using non-linear models which are likely to reflect more accurately the theoretical hypotheses underlying the idea of hysteresis. Such models moreover offer the advantage of not imposing an explosive variance in the unemployment rate, as is implied by the description of the latter as a unit-root process. Examples of such an approach are Bianchi and Zoega (1997) or Gordon (1997), who respectively allow for threshold effects and time-varying parameters in the process of unemployment rates.[2] A related approach is Ball (1996), who links changes in the NAIRU term

[1] cf. Cross (1995) and Cross and Allan (1988).
[2] The earlier results of Stock (1989) prodiving evidence of non-linearities in the process for unemployment rates were apparently not taken on board by other researchers in the field.

in the Phillips curve to polynomial terms of degree two in unemployment benefits and duration of disinflation.

On trade, following Baldwin and Krugman (1989), more specific attention has been paid to explain how exchange rate shocks could permanently affect import prices and trade, without necessarily involving threshold effects (e.g. Kollintstas and Zhou, 1992, with staggered import contracts and the role of delivery lags). Alternatively, models have been developed whereby trade shares would depend on a non-linear function of the exchange rate history (e.g. Amable et al., 1994), whereas others tried to test whether firms' market shares depend on their history (e.g. Giovannetti and Samiei, 1996). Another non-linear model which can be seen as related, is the relation between natural resources endowment and the trade balance, proposed by Roberts and Mac Causlan (1999).[3]

All of this work, however, very often runs into the obstacle of not having tools available for actually discriminating across two types of models exhibiting some strong degree of persistence. The first one is characterised by the existence of multiple equilibria in a non-linear world whereas the other arises from the initial condition dependency of the equilibrium, which appears in systems with zero root in continuous time (cf. Giavazzi and Wyplosz, 1985) or unit roots in discrete time.[4] On the basis of the formal work on hysteresis by Krasnosel'skii and Pokrovski (1990) or Mayergoyz (1991), and considering such dynamics as an "input-output system", it is possible however to make a clearer theoretical distinction between such complex non-linear processes and unit-root systems (see Amable et al., 1994, and Cross, 1994, for illustrations). The gap between hysteresis and a unit-root process can then be described in terms of "persistence" versus "remanence". In the latter case, transitory shocks have a persistent effect on the system itself. The impact of a given shock depends on previous shocks, in the sense that the functional form defining the equilibrium at any point in time is affected by the history of shocks. Contrary to what happens with a random walk, where the input is simply cumulated over time to generate the output, the output of an hysteretic system does not depend on all past values of the input but only on some of them, being moreover a non-linear function of the memorised input. Depending on the level of aggregation considered and on the degree of

[3] Albeit a very general 2 good-2 agent exchange model, Creedy and Martin (1993) could also be used as a microfoundation for hysteresis in this context, since the equilibrium relative price can take three different values, being the root of a cubic function.

[4] A further but even weaker form of hysteresis is not taken into account in what follows, namely the one which has been called "partial" by Layard et al. (1991) or "quasi-" by Vendrik (1993). This kind of phenomenon is related to slow and possibly time-dependent adjustment to a single equilibrium in a linear world. This bears little resemblance with approaches involving structural change and multiple equilibria and is therefore not considered in what follows. Piscitelli et al. (2000) even go as far as calling this form of hysteresis a "bastard" one.

heterogeneity across agents, two forms of hysteresis - "weak" and "strong" - can both be characterised as very specific and non-linear responses to shocks.

Such theoretical differences could nevertheless result in some observational equivalence, since in practice, non-linear dynamics such as those resulting from hysteresis could generate processes closely resembling unit-rooted ones. From a practical point of view, this would clearly weaken the relevance of establishing a clear-cut theoretical distinction between "genuine" hysteresis and unit-root processes. It seems therefore critical to assess the plausibility of such an observational equivalence.

Building on ideas initially put forward in Amable et al. (1994), this paper presents the detailed results of a number of stochastic simulations under a variety of alternative assumptions, with the intention to compare the dynamics resulting from the two types of systems, i.e. unit-root and hysteretic processes - an approach inspired by Campbell and Perron (1991) who assess the observational equivalence between unit-root and trend-stationary processes. We first recall the formula expressing the output of a hysteretic system as a function of the corresponding input. We then show that, in the case of a stochastic input, the algebra needed to derive an explicit formula for the resulting probability distribution would be highly untractable. Accordingly, we take another approach and perform stochastic simulations of a hysteretic system using both stationary and non-stationary series as inputs to the system, deriving the outputs according to the above mentioned function. Finally, the order of integration of the resulting output is analysed through univariate analysis, and also the existence of cointegration between inputs and outputs is investigated. The results, based on two types of tests for cointegration, confirm and extend the much more limited exercise reported in Amable et al. (1994), showing that there is in general no observational equivalence between unit-root and hysteretic processes, to the extent that the latter generate substantially less non-stationarity than the former.

The proposed exercise, by comparing hysteresis to a possibly similar linear process, also complements recent work on how to identify hysteresis as a particular case of non-linearities, e.g. Hughes-Hallet and Piscittelli (1998) who develop and assess the power of tests for hysteresis. It can be viewed, moreover, as an extension of the parallel work by Piscitelli et al. (2000), which suggest using some hysteresis operator to transform macroeconomic variables, as a tool to test for hysteresis. They illustrate the approach on actual UK data for income, wealth and consumption, whereas, in the present paper, such an operator is applied to a large number of simulated artificial series, the time-series properties of which are however in line with actual macroeconomic data.

The remainder of the paper is structured as follows. Section 2 recalls the main ideas underlying the various types of hysteresis in a non-linear setting, with some particular emphasis on the functional form implied in the case of exchange rates and exports (input and output, respectively). Section 3 presents the simulation experiments conducted using this functional form, considering alternatively stationary and non-stationary inputs. Finally, summary and conclusions are presented in Section 4.

2. WEAK AND STRONG HYSTERESIS: A FUNDAMENTALLY NON-LINEAR MODEL

As a starting point, it is instructive to illustrate the concepts involved by recalling one standard example of a purely non-linear hysteresis system, taken from trade theory, implying a non-trivial relation between the exchange rate and exports. Such a presentation allows us first to broadly describe such dynamics, whether in a weak form (at the firm level) or in a stronger form (for the aggregate) - the phenomenon appearing much richer when the aggregation is performed on a heterogeneous set of elementary firms. In addition, more details are provided on the explicit expression of the output with respect to the input and some attempt is finally made to assess how a random input would affect the results. We will consider in what follows the framework of Amable et al. (1995), which generalises the model of Baldwin and Krugman (1989) to heterogeneous firms.[5]

2.1 Weak hysteresis: An example of a so-called "hysteron"

Baldwin and Krugman's (1989) simple model of a firm's exports in the presence of sunk costs can be interpreted as a case of bistability, quite similar to the "fold catastrophe" diagram (see Chart 1). This type of behaviour can be considered as an "hysteron", i.e. the elementary component of aggregate hysteresis.[6] Irrespective on the firm's pricing behaviour, competitive or monopolistic, the decision to export ultimately depends on the value of the foreign exchange rate. However, because of sunk costs associated with the decision - e.g. when setting up a distribution network in a foreign country - a firm's decision to enter or exit

[5] It should be stressed that the objective of this section is not to provide a theoretical presentation of the model, which can be found e.g. in Cross (1994), nor to recall the proof for the formula employed in the simulations - see Amable et al. (1995) for that purpose. The idea is instead to illustrate the basic and specific concepts characterising this particular type of hysteresis, thereby giving some intuition of the key mechanisms involved.

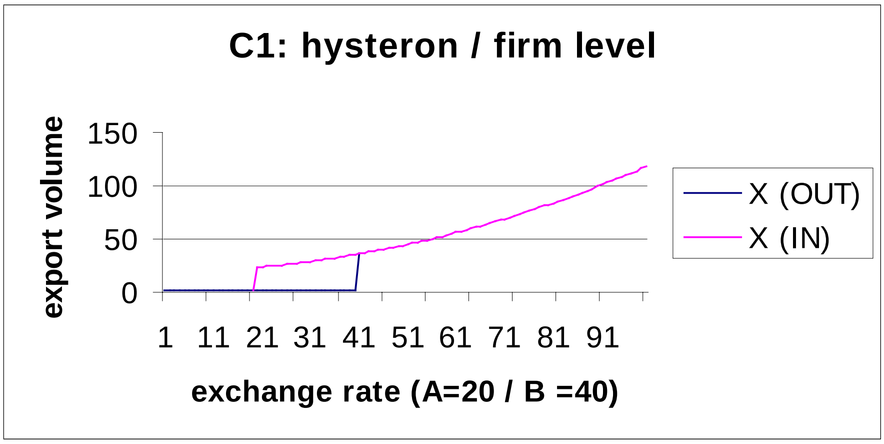
[6] Such "catastrophe" models of hysteresis have also been shown to occur for aggregate labour supply behaviour, cf. Vendrik (1993) and Vendrik (1998). This work involves, however, bandwagon effects, i.e. interdependent preferences across agents whereas the models presented in this paper do not require such micro-interaction to deliver a multiple equilibria configuration. Such interaction relates to some extent to the "avalanche" models, e.g. described in Iori et al. (1998).

from a foreign market will not take place for the same value of the exchange rate. A number of models of investment decision in the presence of fixed costs - e.g. Dixit (1989), (1992) or Lippman and Rumelt (1992) - lead to an optimal behaviour characterised by two stable equilibria or two stable equilibrium loci and by a range of inaction between two threshold values of a certain variable relevant for the decision (prices, income, exchange rate, etc.).

In the case of a single firm, initially out of a foreign market, the firm will enter and then export to this market when the exchange rate rises above a certain threshold, say *B*, making sales abroad profitable. This value of the exchange rate depends on the fixed cost of entry. The export volume of the firm then jumps from zero to the minimum "entry" volume. A further increase in the exchange rate will make exports more profitable and thus lead to an increase in the volume sold abroad. On the other hand, a decrease in the exchange rate below *B* will not lead to an exit decision by the firm, precisely because the sunk costs have already been incurred, but instead the firm will decrease its sales below the minimum entry volume. The firm will however exit from the foreign market when the exchange rate decreases below another threshold value, denoted by *A*. Therefore, a range of inaction exists between *A* and *B*, with respect to the entry or exit decision. When the exchange rate fluctuates within $]A,B[$, firms already exporting stay in that market whereas firms originally out of this market stay out.

Chart 1 shows a firm's export volume function, $X(E)$, characterised by a range of inaction between $E=A=20$ and $E=B=40$. For instance, if the firm is OUT of the market and the initial exchange rate is $(A+B)/2=30$, an increase in the exchange rate until $2B=80$ followed by a decrease back to its initial value, i.e. 30, would have the firm end up being IN the market. Therefore, a transitory shock on the exchange rate has a permanent effect on this firm's situation. In addition, the exchange rate would have to decrease below its initial value to get back to a situation where the firm is OUT again.

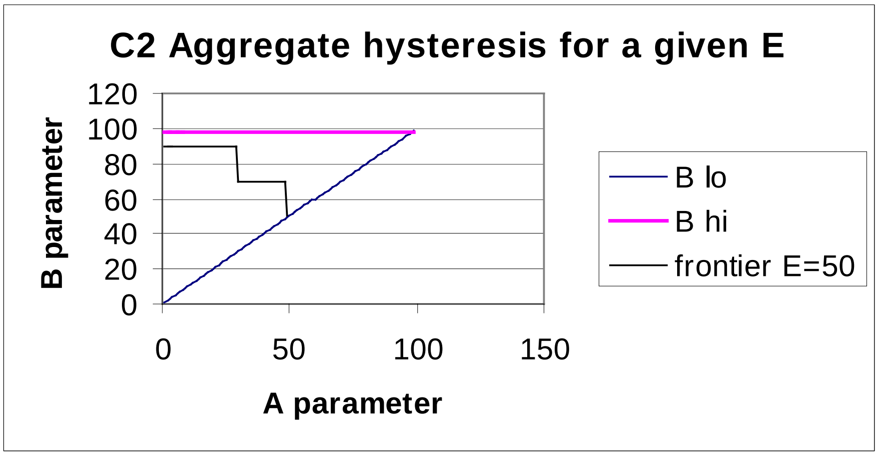
Such a property whereby temporary shocks permanently affect the structure of the system is called "remanence". Strictly speaking, the shock considered above, whereby a certain variable is modified from a given initial value and taken back to this initial value, is akin to a "loading-unloading" sequence in a control parameter (i.e. a controlled exogenous variable) rather than a stochastic change in a fully random variable (therefore not controllable). This property, albeit analogous, therefore differs from the well-known concept of "persistence" in time-series analysis, for e.g. autoregressive processes where exogenous random shocks either perfectly cumulate if the process has a unit-root, or persist with some decay rate in the case of a stationary process. A shock followed by its opposite would have no lasting effects in such a linear stochastic framework.



This structural property is consistent with what can be called "weak" hysteresis. At any time *t*, the position of a firm does not depend only on the current value of the exchange rate, as long as the latter lies between *A* and *B*. The past (or history) of the exchange rate has to be known in order to say whether the firm would be IN or OUT of the market. This is an hysteretic phenomenon, called "weak" because of the following two qualifications. First, the memory which is required is of a limited content, namely information on the number of times the exchange rate has crossed the critical values and on the initial position of the firm is sufficient to explain the firm's position (IN or OUT) at any point in time. Second, not all shocks on the exchange rate would create remanence. If the exchange rate moves from $A/2$ to $2B$ and back, i.e. on both sides of the range of inaction, this would have no impact on the firm, which remains OUT of the market. Remanence can occur only for some range of exchange rates where some bistability exists, namely for any value between A and B (e.g. 30) where there exist two locally stable equilibria (IN and OUT).

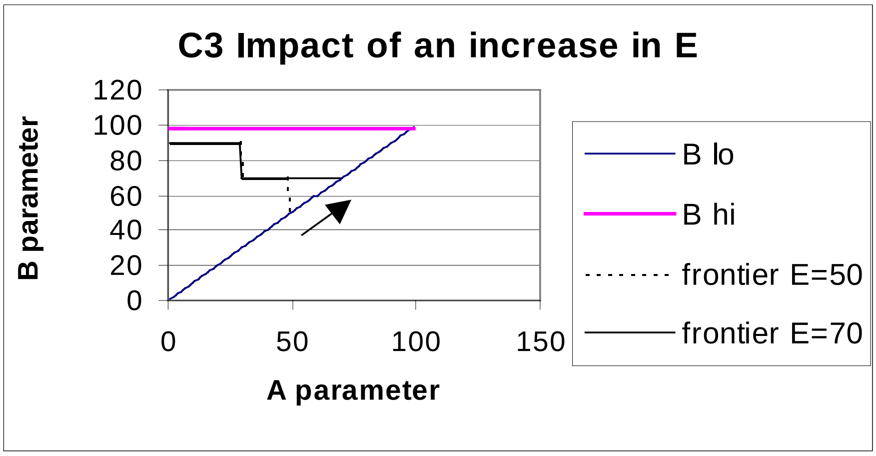
2.2 Strong hysteresis: The aggregation of heterogeneous hysterons

We now turn to the case described in detail in Amable et al. (1995), where there exist a large number, for simplicity a continuum F, of firms characterised by the just described behaviour. We further assume that some degree of heterogeneity exists, so that firms can be distinguished according to their specific range of inaction, i.e. by the two parameters A and B representing values of the exchange rate for which the firm will enter the market if initially OUT (B) or exit from the market if initially IN (A). Thus, each firm i in F is characterised by the coordinates (A_i, B_i) - with $A_i \leq B_i$. These structural firm-specific parameters are determined by the sunk costs associated with the entry decision on the foreign market. $B_i - A_i$ is determined by the gap between entry and maintenance costs. The average of the two parameters, $(B_i + A_i)/2$, can in turn be interpreted as a measure of "comparative advantage", being the average value of the many exchange rates for which the firm can be IN or OUT of the market, depending on its history.

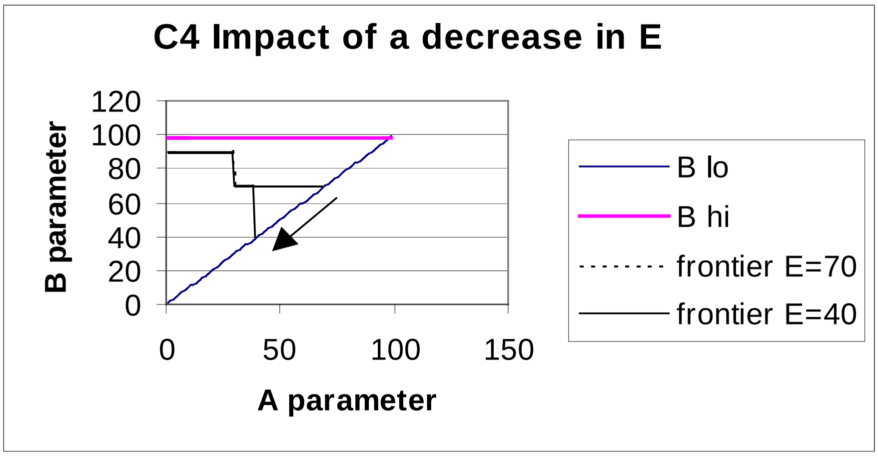


Performing the aggregation on such a continuum of firms, the macro behaviour becomes much more intricate than its elementary counterpart. To get a geometric illustration of the implied macro behaviour, take the diagram in the space (*A, B*) in Chart 2. The upper part of the first quadrant above the diagonal (i.e. $B \geq A$) represents the continuum of firms considered. Assuming in addition that *B* is a parameter with an upper bound, the whole set of firms is represented in the chart by the triangular area delimited by the *B* axis and the two lines *B lo* and *B hi*. Firms are finally identified within this triangle with the help of a boundary which separates firms IN or OUT of the market. At each time *t*, the "frontier" (L) divides the triangle in two parts: above this borderline firms are OUT of the foreign market, below it they are IN the foreign market.

The boundary / frontier is a key element, since its shape can be affected by any change in input. It can be shown that (L) is always a staircase line, the steps of which depend on historical extrema of the exchange rate: vertical lines for minima and horizontal lines for maxima, the values of which can be read on the A and the B axis, respectively. (L) also intersects the 45-degree line at the current value of the exchange rate, where, by construction, $A=B=E$. In addition, the relevant "memory" at each time t comprises only a sequence of decreasing maxima and increasing minima - the so-called "dominant extrema" of the past values of the input (see Cross, 1994, for a detailed analysis). On Charts 2 to 5, this property is reflected in the fact that decreasing (increasing) values are associated with the horizontal (vertical) segments, which implies that the staircase line always corresponds to a downward sloping curve, starting from the B axis, so that B is a decreasing function of A for all points located on (L).

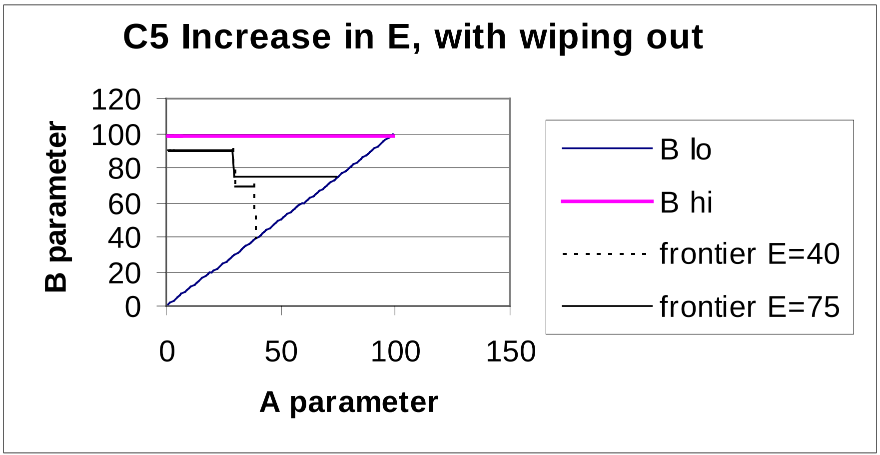


This very particular type of memory results from three properties, related to how changes in the exchange rate alter the shape of (L). First, if the exchange rate increases (see Chart 3, from E=50 to E=70) some new firms enter the market and therefore the boundary separating firms IN from firms OUT will change. The last firm to enter, j, is such that its B_j is equal to the new exchange rate (E=70). All firms characterised by $B_i \leq 70$ will now be IN the market, which is materialised in Chart 3 by the appearance of a new horizontal line for (L), associated with a local maximum for E=70. Conversely, as seen in Chart 4, when the exchange rate decreases, from e.g. E=70 to E=40, some firms leave the market. The last one to leave is firm k, such that A_k=40, also equal to the new exchange rate. All firms with $A_i \geq 40$ will then be OUT of the market. (L) thus gets a new vertical line, as shown in Chart 4, associated with a local minimum for E=40.



Finally, a so-called "wiping-out" effect can be observed, namely if the exchange rate increases (decreases) until a level which is higher (lower) than the former maxima (minima) the corresponding steps of the staircase borderline disappear. For instance, in Chart 5, the steps resulting from the previous three-value sequence for the exchange rate E (50,70,40) disappear when the input increases again from 40 to 75, since all firms with *B* lower than 75 will now be IN. Such phenomenon explains why only dominant extrema eventually define the bordeline (L), and therefore the IN firms.

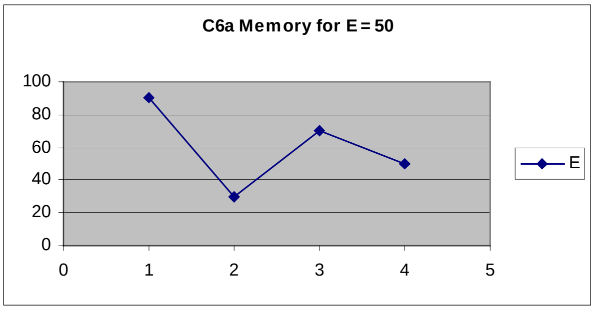
This macro behaviour exhibits "strong" hysteresis, for two reasons. First, the memory which is required to know the structure of the market - the number of firms IN - at each point in time, is much richer than the one which was sufficient at the elementary level. Second, a much wider class of shocks would cause "remanence", to the extent that even small changes to the input would lead to entry and exit decisions by a number of firms, thereby affecting the structure of the market - even in case where no past dominant extrema have been wiped out.

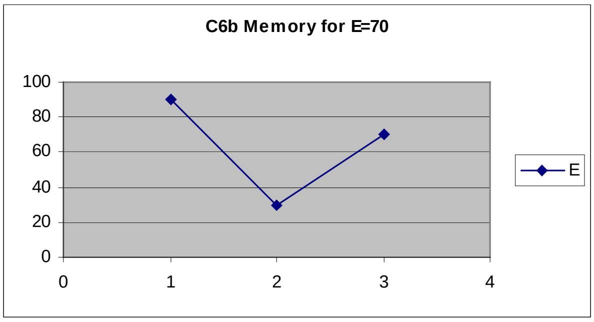


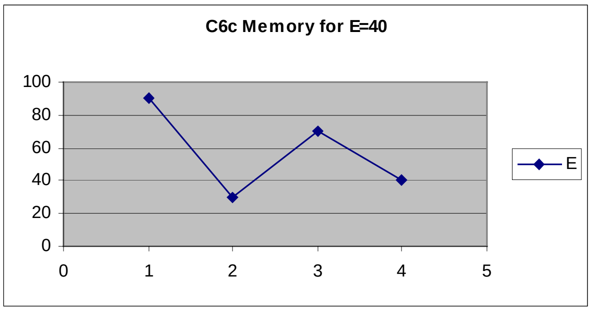
2.3 Closed form expression of the output

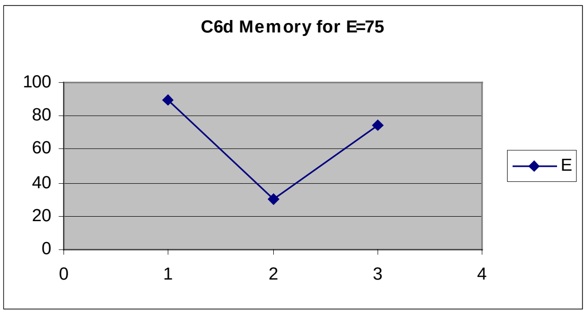
The strong impact of any given change in input on output can be assessed by looking at the expression linking the two, as computed in Amable et al. (1995). For illustration purposes, it is assumed that firms are continuously and uniformly distributed, i.e. there exists a single elementary firm for any specific value taken by the pair of parameters (*A, B*). Assuming moreover that each elementary firm produces the same quantity of output, the overall production on the market at time t is simply proportional to the aggregate number of firms IN the market.[7] Under such hypotheses, the output denoted by N_t becomes a function of the current value of input and of a subset of its past values, i.e. the above mentioned dominant extrema. An illustration of how such a sequence evolves over time is provided on Chart 6, in the case of the sequence of exchange rates E used in the previous section, i.e. (50,70,40,75) .

[7] For simplicity, it is assumed that each elementary firm produces a single unit. In addition the density of the firms' distribution is rescaled so that N_t is exactly the area defined by the B axis, the borderline (L) and the 45 degree line. Such normalising assumptions are neutral to the results.


C6a Memory for E = 50
100
80
60
40
20
0
0
1
2
3
4
5
E


C6b Memory for E=70
100
80
60
40
20
0
0
1
2
3
4
E


C6c Memory for E=40
100
80
60
40
20
0
0
1
2
3
4
5
E


C6d Memory for E=75
100
80
60
40
20
0
0
1
2
3
4
E

The exact expression of N_t as a function of the extrema can now be computed as follows. Denote the sequence of dominant extrema at time t $(E^i_{M,t}, E^i_{m,t})$ - with i=1,...,p(t) in the case where (L) comprises p steps and therefore 2p or 2p+1 segments. Depending on whether E_t is decreasing or increasing, as explained above, $(E^i_{M,t})$ is the decreasing sequence of maxima whereas $(E^i_{m,t})$ is the increasing sequence of minima. The area between (L), the B axis and the 45 degree line can be decomposed into rectangles and triangles involving vertical and horizontal segments, which correspond to the extrema. After some straightforward algebra, the following formula can then be found:

$$N_t = \sum_{i=1,\dots,p(t)} E^i_{M,t}(E^i_{m,t} - E^{i-1}_{m,t}) - (E_t)^2$$

Two different formulations have however to be employed, since the last part of the (L) curve can either be horizontal or vertical, depending on whether the current value of the input E increases or decreases at time t:

$$E_t \geq E_{t-1} \Rightarrow E^{p(t)}_{m,t} = E^{p(t)}_{M,t} = E_t$$
$$E_t \leq E_{t-1} \Rightarrow E^{p(t)}_{m,t} = E_t$$

It is obvious in view of the resulting functional form that a number of non-linearities would affect the output, in particular via the quadratic terms in the sequence of dominant extrema. In addition, each time shocks to the input are large enough to erase part of the previously memorised sequence the functional form for N_t will change dramatically. This also suggests that a generalisation of such dynamics to cases where the input follows a random process - to make the comparison with unit-root processes more directly relevant - would presumably lead to complex properties for the resulting output.

2.4 Implied probability distribution of the output in the case of random input

In practice, an additional problem, not addressed in the related literature, arises from the fact that an economic variable, such as the exchange rate, generally follows a random process which is therefore not fully consistent with the input-output model, where E is an exogenously controlled variable. In the case where the input is random, the functional form of the transition between input and output would then be associated with quite a complex expression for the

probability distribution of output, since the latter should reflect the likelihood of some non-trivial input sequence.
Take for instance the output value consistent with a very simple case, where the memory after t observations is reduced to (E_M^1, E_m^1, E), namely one maximum and one minimum only, i.e. a sequence similar to the simplest ones shown on Chart 6. For T observations, there are T-2 possible positions left where E_M^1 could be located - the other two being for E and E_m^1. Moreover E_m^1 has to occur after the maximum E_M^1 took place, since the maximum would otherwise wipe it out. Let j denote the location of the memorised minimum. The likelihood of the above mentioned sequence can then be computed as the sum of the probability of having a single minimum memorised at time j where $1<j<T$.
The input has to satisfy a number of additional restrictions for this given event to occur. First, the input has to stay in between the two dominant extrema before reaching E_m^1, otherwise the corresponding realisations would not have been wiped out. This restriction should hold until time j-1, with the exception of the maximum which can be located anywhere before the minimum, namely j-2 observations. Second, all observations between the minimum and E (namely T-j-1 observations) have to correspond to a value within that range since they would be memorised otherwise. Finally, the input must continuously increase between time j and the end of the sample, otherwise another maximum would be memorised. The latter restriction bears on observations as of j+2 until T-1.
Taking into account these three restrictions, respectively reflected in the terms P_a to P_c below, the resulting elementary probability, denoted by dP_j, for the minimum to be located at j where $1<j<T$, should satisfy the following:

$$dP_j = P(e = E).P(e = E_M^1).P(e = E_m^1)P_a P_b P_c$$
$$P_a = P(E_m^1 < e < E_M^1)^{j-2}$$
$$P_b = P(E_m^1 < e < E)^{T-j-1}$$
$$P_c = \prod_{i=j+2,\dots,T-1} P(e_i > e_{i-1})$$

The elementary probability for the (E_M^1, E_m^1, E) sequence is then equal to the sum of the dP_j 's, where j can take values in between 2 and T-1. For each value of j there is moreover a different order for the product term P_c, which reflects the monotonously increasing part of the sequence. The algebra for the likelihood and the empirical moments of output are therefore likely to be intractable, to the extent that a general approach should of course not only consider such a

simplistic two extrema sequence but sequences possibly comprising up to as many elements as the number of observations.
On the basis of those illustrative calculations, it seems more appropriate to analytically examine the properties of the output with the help of stochastic simulations. Results of such simulations are documented in the following section.

3. SIMULATIONS OF STRONG HYSTERESIS: ALMOST NO UNIT-ROOTS?

Beyond illustrating what can be the output of such a hysteretic system resulting from a random input, an additional interesting outcome of such a simulation exercise is also to provide an opportunity to assess whether, in practice, such non-linear dynamics would deliver processes that would resemble unit-root processes. As already mentioned, this is especially of interest in the context of the labour market economics where econometric work dealing with hysteresis mostly focused on unit-root tests. In spite of quite obviously very different underlying formal dynamics, unit root and hysteresis might still be observational-equivalent.
In order to investigate this issue, we have conducted the following experiments. We have simulated 2000 realisations of 100 observations - thereby replicating a 25-year span of quarterly data, i.e. a standard sample for macroeconometricians. We have used alternatively white noise or random walk inputs, to illustrate the impact of hysteresis on both stationary and non-stationary inputs. Each observation of the input is equivalent to the exchange rate E_t in the equation defining the output (see section 2). It is crucial to realise that the magnitude of the input as such is irrelevant to the analysis, to the extent that the focus is here on the time-series properties of the resulting output, in terms of the autocorrelation of the process rather than of its mean.
White noise inputs have been assumed to have a mean of 10^4, with a variance of either 10^3 or 10^6 - those two variants being simulated in order to assess the potential impact of the variability of the input on the output. The latter case, where the mean is not much larger than the standard error, can be deemed more realistic and in line with actual data. It compares e.g. to the deviation to mean ratio for the exchange rate of the French Franc against the Dollar or the Deutsche Mark which, for the sample 1975-1995, is around 30%. Another comparison can be made with the properties of the annual growth rate for the euro area GDP\ over the sample 1970 to 1998, which was on average at 2.4%, with a standard deviation of around 0.2 percentage points.

For the random walk simulations, the input has two components: the initial observation, set at 10^4 to be in line with the white noise experiment, and also a random walk without drift. The underlying innovation has a standard deviation of 10^2, which implies that the unconditional standard deviation of the input - which increases with the number of observations - ranges from 10^2 to 10^4. The simulation horizon comprises 10^2 observations hence the 10^4 standard deviation at the end of the simulation. This calibration makes the random walk simulations comparable to those based on a white noise / high variance input, since the (constant) standard error (10^3) of the latter lies in between the two extreme values obtained with the random walk input.

In all simulations, the initial value of the output is equal to 5.10^7, i.e. to the area of the triangle defined by the B axis, the line B=10^4, and the 45 degree line. In the subsequent analysis, the output has been rescaled after simulation - divided by 5000 - to set it equal to the input at t=0, which facilitates the reading of charts comparing the input and output paths. Finally, the 5 initial observations are dropped, which seemed sufficient to avoid initial condition dependency of the simulation results which appeared otherwise in many cases.[8]

A specific routine had to be designed for the purpose of this exercise, namely the procedure to select the dominant extrema. For each observation, the output can be computed only once this observation-specific sequence of extrema has been appropriately updated, in a non-trivial manner. The initial memory only comprises the initial observation for the input. Afterwards, only extrema of the input can be memorised - i.e. points at which the first derivative of the input changes sign. Moreover, as already explained, any occurrence of the input which is lower than a previously memorised minimum (or, similarly, higher than a previously memorised maximum) leads to the deletion from the memory of such previous extrema and of all subsequently registered ones. Such deleted extrema are dropped forever.[9]

3.1 Some illustrations of the implied input-output relations

A number of descriptive elements can be derived from such simulations. In the case of a white noise input, the output crucially depends on the variance to mean ratio, as can be seen from the resulting charts. If one simulates the output generated by a low variance input, the output memorises roughly only the

[8] We thank one of the referees for having spotted this dependency.

[9] The RATS\ 4.1 routine which performs the selection of extrema and the resulting output computation is available upon request. The expression and computation would have been more complex under the assumption of a non-uniform distribution of the parameters A and B across firms, i.e. with no uniform "Preisach weights", see e.g. Piscitelli et al. (2000) for such an extended approach.

maxima sequence of input (see Chart 7a). The impact of decreases in the input is then very small, since only a small number of them would become OUT. When the maximum maximorum has been reached very soon, the output path turns to be close to that of a constant variable, although a lot richer because no further "wiping out" of the maxima occurs. Subsequent minima are therefore much more likely to enter the time-varying expression for the output, with very little impact on its level however (see Chart 7b). If, on the contrary, one allows for a higher variance, and therefore a more realistic input, large decreasing shocks can also be observed which have persistent impact on the output (see Chart 7c).

Two main remarks can be made to summarise the main features of the outputs derived from white noise inputs, both suggesting that those outputs may not resemble random walks. First, some discrete level shifts are observed, as could be expected from the theoretical framework, given that threshold effects play a major role. There is in addition some asymmetry, which was less easy to anticipate prior to carrying out such simulations. This asymmetry corresponds to the observed upward stickiness, whereby the output remains close to an upper limit, with moreover an increasing tendency over time to reach levels closer to that value. Intuitively, this is explained by the fact that a maximum maximorum is more likely to persist than a minimum minimorum, to the extent that the former can be wiped out only by a higher value whereas the latter disappears from the memory as soon as a higher value is observed for the input.[10]

When the input is a random walk - therefore with a continuously increasing unconditional variance - the output can vary quite a lot across simulations. It can roughly follow the input or, on the contrary, stay at a high level in spite of an input returning to its initial value or having a trend (see Chart 8b). The degree of persistence in output seems also in that case to differ to a large extent across the various inputs. The role of the variance to mean ratio becomes secondary in such a case, however, to the extent that for a unit-root process the "mean" matters only as a measure of initial conditions.

In all cases, the resulting output does not seem similar to a standard stationary process. For instance, the outputs derived from random walk inputs are very smooth and with protracted fluctuations, a stylised behaviour resembling that of standard I(1) non-stationary series. It is the case however that output sometimes seems to follow - albeit roughly - the input path (e.g. Chart 8a), suggesting that some stable long-run relation may link the two, in other words cointegration may exist between input and output. For comparison purposes, it should be recalled that a purely unit-root system would have generated I(2) outputs from random walk inputs, since the input would simply cumulate over time to deliver the corresponding output.

[10] In the case of a symmetric distribution, such as a Gaussian, for any given E_m and E_M centered around the mean, $P(E>E_{\_m}) = 1 - P(E > E_M)$, which for high values of E_M would be very close to one.

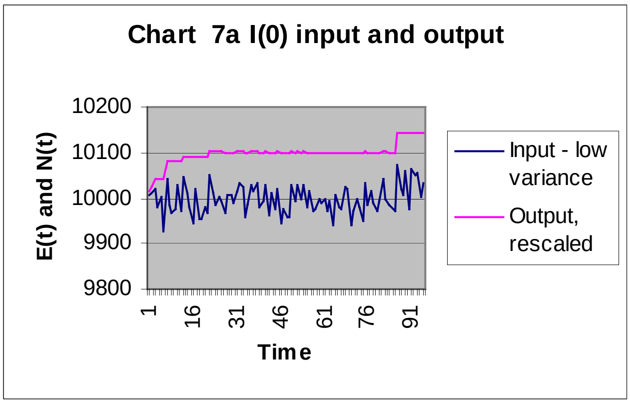


Chart 7a I(0) input and output
10200
10100
10000
9900
9800
E(t) and N(t)
1
16
31
46
61
76
91
Time
Input - low variance
Output, rescaled

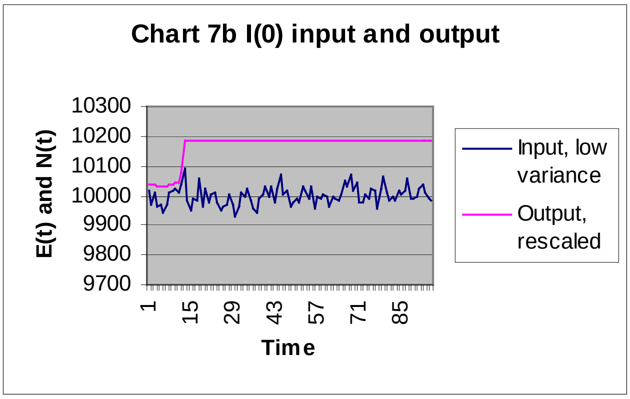


Chart 7b I(0) input and output
10300
10200
10100
10000
9900
9800
9700
E(t) and N(t)
1
15
29
43
57
71
85
Time
Input, low variance
Output, rescaled

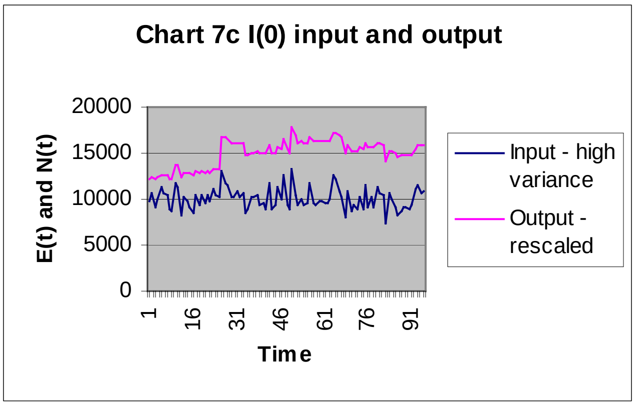
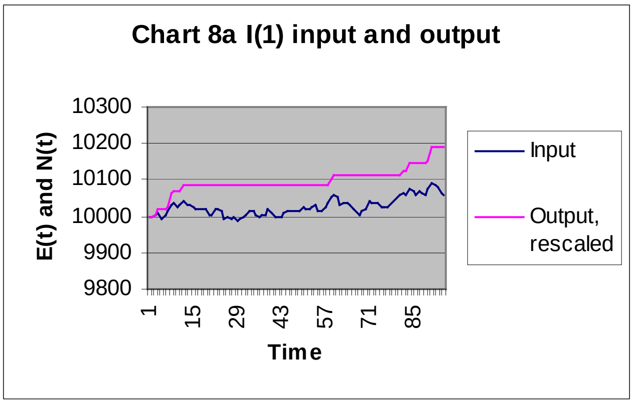
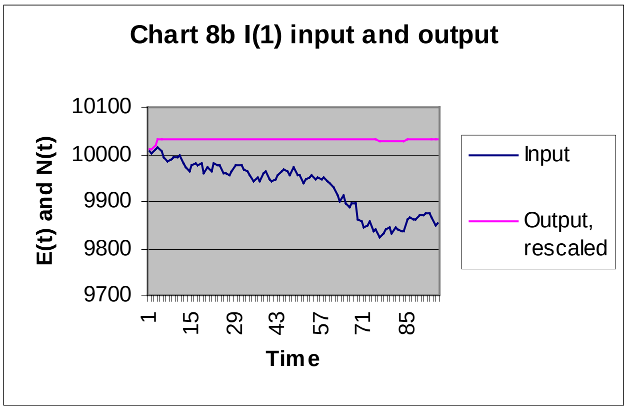


Chart 7c I(0) input and output
20000
15000
10000
5000
0
E(t) and N(t)
1
16
31
46
61
76
91
Time
Input - high variance
Output - rescaled





3.2 Stationarity tests on the resulting outputs

It is a well-established practice for applied econometricians to run tests such as the Dickey-Fuller test, irrespective of the well-known power problems affecting such tests. Accordingly, to assess the degree of non-stationarity present in our artificial data, we have plotted the resulting distributions of the Dickey-Fuller tests - with intercept - for each kind of input. According to the degree of integration of the latter the null hypothesis of interest varies. In the case of an I(0) input, it is more interesting to check whether output is non-stationary, whereas for an I(1) input, it seems appropriate to test for the null of non-cointegration between input and output. On Chart 9a, the histogram for the

Dickey Fuller t-statistic for the null of non-stationarity of output generated by I(0) input is reported, whereas the corresponding histogram for the Dickey Fuller t-statistic for the null of no cointegration between an I(1) input and its output is reported on Chart 9b. Both charts show therefore a proxy to the actual probability distribution of the corresponding Dickey Fuller test for the particular type of input considered.[11]

The main two results are the following. First, for white noise inputs, at the 5% risk of wrongly rejecting the null, 77% of the outputs have no unit root (the critical value - denoted by c.v. - is -2.89), and as much as 86% are found to be stationary if the risk is 10% (c.v. -2.58).[12] Second, in the case of random walk inputs, the number of cases in which cointegration between the input and the output cannot be rejected amounts to 34% for a 95% confidence interval (c.v. -3.17) and as much as 39% for a 90% confidence interval (c.v. -2.91).

It seems first that a hysteretic system does not in general generate unit rooted output from an I(0) input. In addition, such a system does not systematically filter out the I(1)-ness of the input - in spite of generating a bounded output. These two findings illustrate the complex nature of a process which cannot be captured in either of the two cases by a simple time-series model, to the extent that the order of integration amy differ between the input and the output.

Moreover, hysteretic dynamics do not systematically generate a stable relation between an I(1) input and its output. The latter conclusion is also consistent with the expected theoretical result, namely a non-linear function with respect to the input dominant extrema would only be poorly captured by a linear relationship, such as the ones used in the cointegration framework. Perhaps "fractional" integration may be a better proxy to such processes, in which case it could be interesting in further work to try to estimate the distribution of the implied order of integration of output for both types of input.

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[11] We used the ADF test with two lags, to remove autocorrelation in the output values. The Johansen tests have been performed with the same autoregressive order. In practice, econometricians would most probably experiment with richer lag structures. Indeed standard lag truncation tests may end up overparametrising the model, to the extent that in the extreme case of an early extremum, even very remote in time values of the exchange rate may affect outcome. Critical values for the univariate and bivariate DF test are those for 100 observations reported in Fuller (1976) and Engle and Yoo (1987) whereas Table A3 of Johansen and Juselius (1990) is used for the VAR results.

[12] The results and the given critical values should be interpreted as follows: at the x% risk, there is a 1-x% probability that, under the null, the test statistic would be higher than the corresponding c.v. As a result, when the test statistic is lower than the c.v. the null can be rejected, with an x% risk of making a mistake and there is a 1-x% confidence interval.

Chart 9a Histogram of DF tests for I(1) output

from I(0) input

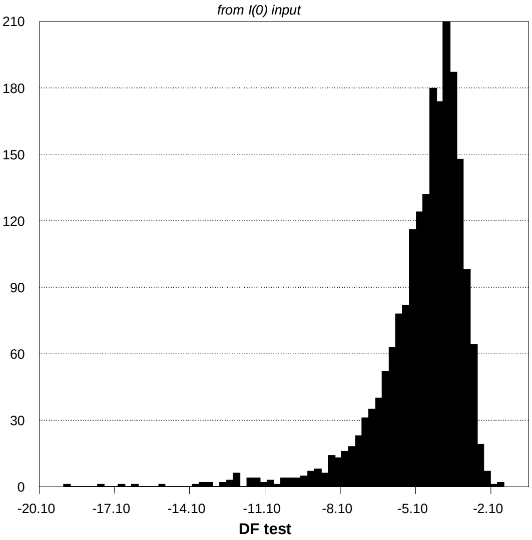
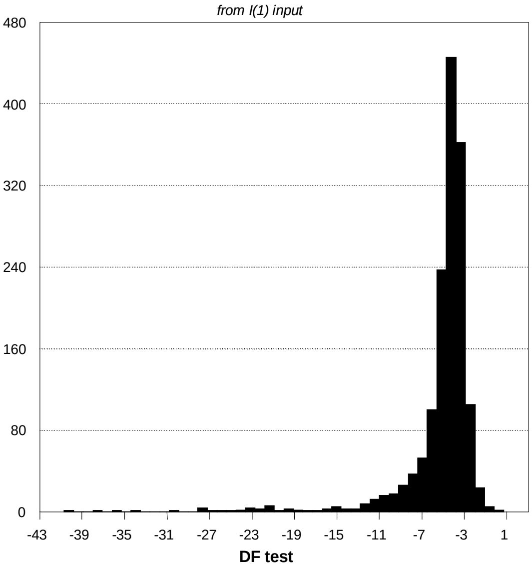


Chart 9b Histogram of DF tests for no cointegration

from I(1) input



For each of the input processes, as a supplementary check, and also in line with currently employed applied econometric practices, we have also computed the multivariate Johansen Trace VAR-based test for two null hypotheses on the (input,output) vector, first for no stationary combination in the vector and, second, for one or less stationary combination.[13]

In the case of a stationary input, the hypothesis of zero cointegration relation in the bivariate system (input,output) is rejected in all of the cases at both the 5% and the 10% risk, which implies that there is at least one stationary component in the vector. The latter is found to be fully I(0) in 68% of the cases at the 5% risk, which implies that a single cointegration relation is found in the remaining cases, i.e. 32% of the simulations. The corresponding figures are 75% for two relations and 25% for one relation at the 10% level. Given that the input is by construction I(0), the result of a pure I(0) vector (i.e. two cointegration relations) is needed to draw the conclusion that the output is stationary too. This conclusion seems therefore to hold slightly less often (roughly two thirds vs. three quarters of the cases) with VAR-based tests than in view of the DF tests.

With the random walk input, the null of no cointegration relation is accepted for 55% of the cases at the 5% risk - only 45% at 10%. The hypotheses of a fully stationary vector (i.e. two cointegration relations) is accepted for 4% only of the cases at the 5% risk (9% at 10%), but this result is not significant since this frequency of rejection of the null is roughly equal to the size of the test. This outcome is in fact not surprising, since at least the input should have been identified as a non-stationary variable, which should render by construction the hypothesis of a fully I(0) vector almost not likely. The remaining case, i.e. a single cointegration relation between input and output, still represents about half of the simulations (41% at 5% and 46% at 10%), a ratio slightly higher than what was found on the basis of the DF tests solely, for which the corresponding figures were 34% and 39% respectively.

All in all, the VAR results are broadly in line with those reported for DF tests, indicating however less stationarity in the analysed output for the white noise inputs but more stationarity for the random walk inputs. In any event, the VAR based tests confirm and strengthen the main previous findings, namely that hysteretic dynamics can generate a sizeable number of I(0) output from a

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[13] The 10% critical values - c.v. - are 7.6 and 18.0 for the null of at most one and zero cointegration relation, respectively, while the corresponding values at 5% are 9.2 and 20.1. The various (input,output) vectors are attributed a cointegrating rank between 0 and 2, according to the following procedure. We need first to test for the absence of cointegration. If the Trace statistic for no cointegration relation is lower than its c.v., there is no cointegrating vector and the system is I(1). In the opposite case, the other statistic has to be checked. If it is also higher than its c.v, the whole vector is I(0) and both the input and the output are then stationary. There is a single cointegration relation otherwise.

stationary input, contrary to what would happen if the same input were used to feed in unit root processes. It is also confirmed that there are many cases where no stable relation between a random walk input and its output can be detected, although cointegration can be found rather frequently in spite of the very non-linear features of the hysteretic system employed.

4. SUMMARY AND CONCLUSIONS

We have recalled how the aggregation procedure developed by Mayergoyz (1991) can be used to oppose, on the one hand, weak hysteresis at the micro level to strong hysteresis at the macro level, and, on the other hand, hysteresis to unit-root dynamics. Hysteretic dynamics involve a selection of the extrema of the exogenous variables, so that only an increasing (decreasing) sequence of minima (maxima) is memorised. Any occurrence of a lower minimum (greater maximum) lead to an update of the memory. We found that analytical computation of moments or likelihood of such a process appeared intractable. We have instead performed stochastic simulations. Results confirm that the theoretical distinction between hysteresis dynamics and unit-root processes should not be overlooked, to the extent that the two processes are found to be significantly not observationally equivalent. In particular, the simulations carried out have shown that:

First, hysteresis dynamics generate in many cases - more than two-thirds of the experiments - stationary processes from stationary exogenous inputs, whereas unit roots would by construction almost surely yield an non-stationary output;

Second, hysteresis dynamics can generate quite often - around 40\% of the experiments - an output which is cointegrated with a non-stationary input, also something not to be expected from a unit-root process.

In view of these results, some renewed attention should be given to the analysis of remanence as opposed to persistence in both labour market and international trade econometrics. However, for the latter to be feasible, a number of further steps have to be on the research agenda. First, some form of testing and detecting methodology should be defined to allow econometricians to better identify processes possibly generated by hysteresis dynamics, with presumably some emphasis on threshold effects and asymmetries, as those observed in the above mentioned simulations. In addition, it may be appropriate to develop theories introducing time dimension and possibly interacting agents in such non-linear modelling, to the extent that e.g. the discouraged worker hypothesis or habit formations effects on demand curve imply a time-dimension not fully captured by the presented model of hysteresis. In such a context, there is finally, from an empirical perspective, a need for using individual or sectoral data - to analyse actual entry/exit decisions and their link to the macroeconomic outcome. Some of the significant papers in this stream of literature focusing on a non-linear approach to hysteresis have indeed already explored these promising areas (e.g. Piscitelli et al. (2000), Vendrik (1998), Giovannetti and Samiei (1996)).

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