

A comparison of different trading protocols in an agent-based market

Paolo Pellizzari^{1,2} and Arianna Dal Forno^{1*}

Dept. of Applied Mathematics
“Ca’ Foscari” University
Dorsoduro 3825/E
30123 Venice, Italy.

SSAV
School of Adv. Studies in Venice
Dorsoduro 3488/U Rio Novo
30123 Venice, Italy

Previous draft: March 2005. This draft: November 2-nd 2005

Abstract

We compare price dynamics of different market protocols (batch auction, continuous double auction and dealership) in an agent-based artificial exchange. In order to distinguish the effects of market architectures alone, we use a controlled environment where allocative and informational issues are neglected and agents do not optimize or learn. Hence, we rule out the possibility that the behaviour of traders drives the price dynamics. Aiming to compare price stability and execution quality in broad sense, we analyze standard deviation, excess kurtosis, tail exponent of returns, volume, perceived gain by traders and bid-ask spread. Overall, a dealership market appears to be the best candidate in this respect, generating low volume and volatility, virtually no excess kurtosis and high perceived gain.

1 Introduction

The observation that stock exchanges are run under a diverse set of market architectures and protocols raises some obvious and yet intriguing questions. If we do not content ourself with the explanation that this is due to historic accidents or mere temporal and casual path-dependence, then we could investigate reasons explaining why exchanges developed so differently and which are the main differences in the dynamic properties of the prevailing price or return time-series.

*Corresponding author: paolop@unive.it. We thank the participants to seminars at the City University of London, Venice, Palermo for useful discussion. Marco LiCalzi is gratefully acknowledged, all the remaining mistakes are sadly ours. This work is partially supported by MURST, grant “Agent-based models of trading mechanisms and efficiency of informational dissemination”

We propose a terse model where allocative and informational effects are ruled out in order to be able to ascertain what is causing what, focusing on the price dynamics statistics. The use of agent-based methodology is particularly suited to control for unwanted side-effects that can otherwise blur the results. We develop an artificial market where simple agents trade a risky asset using different market protocols. As they receive no information, have no learning capability and act in a strictly controlled framework¹, we hope to ascribe unambiguously the effects on price dynamics to the market protocol, which fundamentally is a blind mechanism to translate the demand of traders into actual orders and transactions. Even in the absence of price-discovery considerations or allocative efficiency remarks, this attribution might be of great interest to regulatory agencies or other policy makers in charge to devise and enforce trading rules.

Some important measures of execution quality and price stability can be investigated in this framework, as suggested in [Madhavan, 2000] and [Audet et al., 2001]: execution quality can be assessed by quantification of volume, bid-ask spread, liquidity, volatility, fatness of tails of returns distribution, kurtosis, market depth to mention some. Quite often, to borrow Madhavan’s words, we would expect from a market some key functions, namely good “provision of liquidity, continuity and price stabilization”. Although the overall execution quality of a market is a multidimensional concept, “price stability” in broad sense encompasses a wide set of previously mentioned measures and is quite often a targeted objective of regulators. Excess volatility, kurtosis and tail exponents can be related to relevant financial shocks and to the emergence of bubbles in equity price. The burst of a price bubbles can adversely affect the economy and is sometimes deemed responsible of a recession, see [Bernanke and Gertler, 1999]. Institutions should be concerned about price volatility to limit financial fragility of banks and insurance firms whose risk exposure is dependent on price level and volatility, see [Vila, 2000]. Broad stability of markets could also enhance price-discovery and news interpretation as signals can be perceived more easily when the background noise (i.e. the volatility intrinsically generated by the trading protocol) is low.

Different markets could serve to provide different performances with regard to the previously referenced criteria but a comprehensive comparison of different market protocols is, to the best of our knowledge, still lacking in the literature. There are obvious difficulties in comparing markets or institutions in that it’s virtually impossible to “repeat” the experiment under different circumstances, as it is often the case in social sciences. If a sequence of trades takes place in some market at some time because of some order

¹This setting is suited to describe a calm period in the market, with little informational and liquidity flows and no tensions to be resolved (mergings, IPOs...). Hence the results that we discuss in the sequel might be different from the ones generated in the presence of events like, say, September 11th.

flow, there is no hope to rewind the film changing the market protocol to see how different the outcomes would had been. Clever ideas have been used to overcome these design problems, see [Amihud and Mendelson, 1987] and [Amihud and Mendelson, 1991], using real exchanges data but some of their conclusions can be questioned in several ways.

The effects of one specific market protocol are rather indirectly inferred in [Maslov, 2000] and [LiCalzi and Pellizzari, 2003] where simple agents (no information flows, adaptation or herding) interact in a Continuous Double Auction (CDA). Though there is no comparison among different markets, their argument is the following: the behavioral characteristics of the agents are so simple and innocuous that the resulting returns properties can *chiefly* be attributed to the mechanism that forms the price. While, there is an appreciable effort to stress the notion that dynamics features of the price time-series could be related to institutional devices rather than to the behaviour of agents, as is most often done in the literature (see [Day and Huang, 1900] or [Lux and Marchesi, 1999]²), it is clear that their tentative conclusion needs much corroboration. In [LiCalzi and Pellizzari, 2005] different protocols are compared in their ability to converge to the unique allocative equilibrium.

A good deal of experimental economics papers (see [Smith, 1988]) have faced the comparison of market protocols directly running real experiments with human subjects. These works often study small scale markets, that can be reproduced *in vitro* more easily and with reasonable costs. Being the result of real experiments, this body of work is largely immune to the previous mentioned “repeatability” criticisms. Some of the results might be not easily generalized to large-scale financial markets but we feel that other drawbacks could be pointed out. As noted elsewhere, experiments with real subjects might be subtly affected by the skill of the agents, the way in which they have been instructed or their understanding of the problem. Additionally and more importantly for our purposes, many experiments show that the agents learn quite rapidly and naturally to more and more effectively cope with their task. Again, the reported results might not be imputed to the market mechanism alone if agents are adapting and learning in time. These considerations should increase the appeal of running computational experiments to sketch some hopefully clear-cut comparisons among markets, by means of an artificial and controllable agent-based environment.

The work in [Audet et al., 2001], [Bottazzi et al., 2005] and [Cliff, 2003] on the contrary, provide direct and explicit comparison of different market institutions. In the first paper, quite sophisticated retail agents are compared on two different market platforms, namely a dealership and a limit

²However, an early paper suggesting to investigate the effects of different protocols is [LeBaron, 2001]: “This can be both a curse and a blessing to market designers. On the bad side it opens up another poorly understood set of design questions. However, it may have the beneficial effect of allowing one to study the impact of different trading mechanisms”.

order market³. The findings by Audet and coauthors are rich and depend both on the features of the learning agents and the dealers, their risk aversion, heterogeneity and number (that is a proxy of market depth). If the utility of the final welfare is used to assess the quality, then the dealership market would be preferred in thin environments or by investors affected by large liquidity shocks, for example.

[Cliff, 2003] offers a novel way to compare different markets. A continuous space of auctions, including as special cases the usual double auction, the English and the Dutch auctions, is explored in order to minimize Smith's α dispersion index by a genetic algorithm. The fresh idea is that agents, equipped with some sort of machine learning capabilities, *coevolve* optimizing their own profit together with the auction. Hence, the procedure really finds at the same time the best type of agents suited for the most efficient market structure. This is interesting as it can be argued that agents would most probably react to different institutional architectures changing their behaviour for self-interested purposes. As far as we know, no other papers investigate the mutual adaptation of traders and market mechanisms.

The paper by [Bottazzi et al., 2005] is a careful comparison of three alternative architectures and follows a line similar to our approach. In their model, myopic, quadratic utility maximizing agents trade in different setups, namely a Walrasian auction, a batch auction and a 'order-book' mechanism. Ecologies of agents are shaped by the fraction of chartist traders, extrapolating the future from past returns, that coexists with the remaining part of purely noisy investors. As well known, the chartist traders can destabilize the market and the deterministic skeleton of the dynamical system describing the price shows a sequence of slowly exploding bubbles and sudden crashes. The conclusion is that the properties of the time series "are largely shaped by the specific architectures of trading mechanisms" and, to a smaller extent, by the actual traders ecology. Quite differently, the allocative efficiency is more sensitive to behavioral parameters like the fraction of market orders issued.

In our contribution, we simplify the way agents issue limit orders using a reduced form that avoids the need to specify a full utility function and, more importantly, to devise a necessarily arbitrary procedure to generate some limit orders from it. Moreover, the standard dynamics of prices in our model is homeostatic and does not intrinsically show regular bubbles or crashes that are indeed to be ascribed to the behaviour of the agents more than to the trading protocol. Our work differs notably from all the previous papers in that we model much simpler roundedly rational agents and are interested in quantifying the impact of different market types on the macroscopic peculiarities of prices and returns generated by our agents,

³This is essentially identical to the batch auction presented in the following section and is not to be confused with a continuous double auction.

with the aim to compare price stability and execution quality.

The paper is organized as follows. In the Section 2 the model is described, splitting the treatment of the (minimalist) behavioral features of the agents from the description of the different market mechanisms: batch auction, CDA, competitive dealership. Section 3 gives some visual evidence that the return time series are diverse and presents the results of the simulations. In the first part, that can be intended as a sort of executive summary, we run a multidimensional scaling analysis to simply and graphically show the effects of distinct trading protocols. In the second part a more detailed analysis is presented and discussed. Finally, we comment our findings and draw some conclusions.

2 The model

The structural assumptions follow [LiCalzi and Pellizzari, 2003]. The economy we consider has two assets, one bond, paying a riskless yearly interest rate r , and one stock. The total amount of stock and cash in the economy is constant⁴ because we assume that the interest earned on the bond is spent elsewhere or consumed. However, since the number of active traders may vary over time, the quantities of cash and stock available on the market are not constant. Furthermore, we assume that no new information is ever released. Therefore, except for the number of active traders, the fundamentals of the economy are essentially unchanged over time.

In the following, a detailed description of the artificial agents and the markets where they participate is given.

2.1 The agents

We consider N heterogeneous traders who enter or exit the market independently from each other. They either buy or sell stock in exchange for cash. Short selling or borrowing money are not allowed. Thus, traders are budget constrained both on the buy and the sell side: they can place buy orders only if they have sufficient cash and sell orders only for the stock they possess. Traders' strategies rely on two basic assumptions. First, traders want to buy at low price and sell at high price; second, they estimate the fundamental value v of the stock and this is never revised during the activity period.

When active, trader i is endowed with s_i units of stock and a quantity c_i of cash. Each agent has an investing horizon h_i and expects that the risky asset value will reach the fundamental value v_i (s)he estimates by the time

⁴The dealership is an exception as a small cash quantity is gained by the dealer for each round-trip transaction. Hence the cash in the market is very slowly decreasing in this market.

$t + h_i$. Traders wish to maximize their gains over the remaining time span $h_i - t$, which is their activity period. At time t a risk neutral agent buys the stock at price p only if

$$\frac{v}{p} \geq (1 + r)^{(h_i - t)}. \quad (1)$$

More generally, agents are risk-averse. Since the bond has a riskless yearly rate of return r while investment in the stock is risky, we model the risk-return trade-off incorporating both a positive yearly risk premium π_i^B to buy and a positive yearly risk premium π_i^S to sell in (1). Hence, traders will buy stock at time t if they expect a return sufficiently higher than the return of the riskless bond, namely if

$$\frac{v_i}{p} \geq (1 + r + \pi_i^B)^{(h_i - t)}. \quad (2)$$

Consequently, we obtain the highest bid price β_i that an agent is willing to offer at time t :

$$\beta_i(t) = \frac{v_i}{(1 + r + \pi_i^B)^{(h_i - t)}}. \quad (3)$$

Similarly, the smallest ask price α_i (s)he is willing to offer at time t is

$$\alpha_i(t) = \frac{v_i}{(1 + r + \pi_i^S)^{(h_i - t)}}. \quad (4)$$

In order to ensure that $\beta_i(t) \leq \alpha_i(t)$ we set $\pi_i^B \geq \pi_i^S$ for any i . Hence, if active, agents submit orders as couples (Q_i, P_i) , where Q_i is the quantity they are willing to buy/sell and P_i is the relative limit price. We assume that Q_i is strictly positive (negative) for a buy (sell) order. Whenever agents can trade they randomly decide with equal probability if their orders are on the buy or sell side.

When agents enter the market at time \bar{t} , $\alpha_i(t)$ and $\beta_i(t)$ are increasing functions defined in $[\bar{t}, h_i]$. During this activity period and subject to the budget constraints, agents buy stock if the current price p is smaller than $\beta_i(t)$ or sell if it is greater than $\alpha_i(t)$, and hold their position in between. Since the difference between bid and ask price decreases in the residual time $(h_i - t)$, agents are likely to trade more actively towards the end of their activity period, providing liquidity to the market. Notice that the bond provides only the riskless rate of interest r used in the definition of $\alpha_i(t)$ and $\beta_i(t)$, and plays no other role. When $t = h_i$ agents have reached their investment horizon and exit the market keeping their current endowment. These endowments are transferred to new traders entering the market some time later, with new investment horizons and new estimates for the fundamental value of the stock. Therefore, since investment horizons are randomly distributed across agents, a staggered entry of new traders occurs but the total amount of cash and stock is constant (this is only approximately true

for the dealership, see Section 2.2.3). The time elapsed between the exit of an agent and the subsequent entry of a new trader is a random variable τ . Operationally, when i -th agent exits at h_i , we sample a random τ_i and let a new agent enter at time $h_i + \tau_i$, with the same cash and stock but newly and independently sampled horizon and fundamental value.

2.2 Market architectures

In this section we describe three market architectures to compare how the price time series generated by the previously described agents are affected by institutional features. The three market protocols we take into account are thus a *Batch Auction* (BA), a *Continuous Double Auction* (CDA) and a *Dealer market* or *dealership* (DEA) whose descriptions are given below. Some features are common to all trading protocols. Trading is organized in sessions (*days*), in which all active agents submit bids or asks in random order. For concreteness, limit prices are discretized taking the ceiling (floor) of $\alpha(t)$ ($\beta(t)$), the minimum tick is $\Delta = 1$ and all unfulfilled orders are canceled at the end of the trading day. Hence, each day is beginning with no outstanding orders and if an agent was not able to trade, he must resubmit his/her limit orders.

2.2.1 Batch auction

In a Batch Auction agents can submit limit orders (Q_i, P_i) , where Q_i is the desired quantity and P_i is the maximum (minimum) price the agent is willing to pay (receive) to buy (sell) one unit of stock. Once all agents with adequate endowments have submitted limit buy and sell orders, a unique clearing price is selected in order to maximize the total quantity of exchanged units. In detail, define the demand and supply at price p as

$$D(p) = \sum_{Q_i > 0, P_i \geq p} Q_i \text{ and } S(p) = - \sum_{Q_i < 0, P_i \leq p} Q_i.$$

The clearing price p_t , if it exists, is to be chosen in the set

$$\mathcal{P} = \arg \max_p \min\{D(p), S(p)\}.$$

If $\min\{D(p), S(p)\} = 0, \forall p \in \mathcal{P}$, then no trading takes place, as the biggest bid is smaller than the smallest ask. We then set for convenience $p_t = p_{t-1}$ and there is no exchange of cash and assets.

If on the other hand, $\min\{D(p), S(p)\} > 0, \forall p \in \mathcal{P}$, we must select one element in \mathcal{P} . A common solution in this case is the κ -auction that sets $p_t = \kappa \inf \mathcal{P} + (1 - \kappa) \sup \mathcal{P}$. Observe that prices being quoted discretely, we use the a slightly modified version and compute

$$p_t = \lfloor \kappa \min \mathcal{P} + (1 - \kappa) \max \mathcal{P} \rfloor,$$

where $\lfloor x \rfloor$ is the biggest integer not larger than x . Orders of agents that have submitted adequate bids (with $P_i \geq p_t$) and asks (with $P_i \leq p_t$) are then satisfied, possibly with rationing, and cash and stock endowments are accordingly updated.

2.2.2 Continuous Double Auction

The second market model is a *Continuous Double Auction* (CDA). The agent can submit limit sell or buy orders that are stored in two different books if they cannot find an immediate execution. When an agent issues a buy order, the book containing yet unmatched asks is scanned. If the best ask in the book is smaller than the agent's bid the execution price is the ask in the book and the exchanged quantity is the smallest between the two orders. This procedure is then repeated until the incoming order is fully executed or no compatible ask is found. In this case, the unmatched part of the incoming order is stored for future use in the bid book, with the usual price-quantity-time priority. The same, *mutatis mutandis*, happens when a limit sell order is submitted to the market.

This procedure mimics the following behavior by an agent willing to issue an order at a certain price: he first checks the appropriate book for a compatible price, exchange units at this price if possible and then submit a limit order for the quantity that was not exchanged, if needed. While all units are traded for the same price in a BA, in a CDA different prices can be observed in a single trading day (and even in the execution of the same order).

2.2.3 Dealer market

The third market model we consider is a *Dealership Market* (DEA). Any agent willing to trade must do it through the intermediation of a dealer that quotes the price at which it will buy and sell. In this market the dealer supplies all the liquidity. The agent willing to buy (sell) at price P_i checks for the smaller ask (higher bid) provided by the dealer. If this is the case, all the units are exchanged at the dealer's quote; else the agent's order remains unfulfilled. After a successful trade, the dealer adjusts immediately his bid and ask prices mechanically increasing (decreasing) both quotes by the same unitary amount if he sold (bought) the risky asset. This is a rough device to keep the inventory under control, encouraging other traders to take deals on the other side. The bid-ask spread Γ offered by the dealer is then constant and fixed at the beginning of the simulation experiment. The bid-ask spread is the unique distinctive parameter of the dealer and is clearly related to the liquidity of the market. Some reflection shows that the dealer is continuously extracting some cash from the market as a round trip trade produce a gain of $\Gamma - 1$, at the expense of the agents. Hence, this is the only market where the

total amount of cash available to traders is (slowly but) steadily decreasing with time. Moreover, observe that our dealer is completely mechanical and has no strategic behavior. As in the other markets, the role of the dealer could be accomplished by a program as no intelligence or expertise is needed to implement his duties. The quotes available at the closure of one trading day are kept and used to open the following day.

3 Simulation results

Our simulations are not meant to replicate the price dynamics observed in real markets, but to compare the performance of the three different mechanisms we considered. For all the three architectures a trading session is thought, for convenience, as one day. It is then handy to speak of *years* of 250 trading days. We collect price dynamics statistics replicating 25 times each market for 2500 trading days. To avoid transient effects, we discard systematically the first 500 prices to compute returns. Therefore, we are left with summary statistics for 75 time series of 2000 observations each, corresponding to 8 years of trading data.

We take into account the multidimensional nature of market execution quality collecting data about a set of features possibly occurring with different market architectures. Aiming to discuss price stability and related indicators, see the Introduction, we focused on volatility (standard deviation), excess kurtosis and tail exponent of logarithmic returns. When multiple prices are observed in one trading sessions (CDA and DEA), we use as reference price the average the over the whole day. We also report volume of trades, a measure of perceived gain by the agents and the average bid-ask spread prevailing in the second half of the trading day. A more detailed description of these six measures is deferred to subsection 3.3.

In order to get a more meaningful comparison of the various markets, the simulations are run using the same random seed across different protocols. This means that the very *same* random sequence of buy/sell order is used by the agents submitting orders to *different* market mechanisms.

The material of this Section is presented in three subsections. The first one is showing the parameters used in the simulation. The second one is meant to be a brief and compact account of the main results, with emphasis on the overall diversity of price dynamics in different market frameworks. This subsection can be regarded as a sort of executive summary that enhances the most evident facts, postponing a more thorough discussion to the third part that contains a detailed analysis of each statistical indicator.

3.1 The parameters

We used in the simulation the parameter values listed in Table 1. Global parameters are: the number N of potentially active agents in a trading

session; the riskless interest rate r , assumed constant over time; the initial stock price p_0 . All traders are endowed with cash c_i and stock s_i . They are moreover characterized by their estimate of the fundamental value v_i of the risky asset, a premium to buy π_i^B and a premium to sell π_i^S , and their activity period h_i . Traders that exit the market are then replaced after a period τ_i by ‘new’ traders that inherit their cash and stocks. The dealer⁵ is described by his bid-ask spread Γ (and by the initial quotes that have very little importance given that we get rid of the first 500 days).

	Parameters	Initialization
Global	N	1500
	r	0.02
	p_0	1000
Dealer	Γ	4
Trader	c_i	2000 (first activation only)
	s_i	1 (first activation only)
	v_i	$\sim U[900, 1100]$
	π_i^B	$\sim U[0, 0.06]$
	π_i^S	$\pi_i^B/2$
	h_i	$\sim t + \lceil \exp(1/250) \rceil$ days
	τ_i	$\sim h_i + \lceil \exp(1/250) \rceil$ days

Table 1: Parameters used in the simulations.

U and \exp denote respectively the uniform and the exponential distribution, while ‘ \sim ’ denotes an independent draw from a probability distribution. Global parameters and dealer’s Γ are set once and for all at time $t = 0$. Traders’ initial endowments c_i and s_i are set at the first activation (and then inherited). The other traders’ individual parameters are reset at each time traders are activated. While active traders have initially the same endowment of cash and stock, after replacing exiting agents they may later have different endowments. The yearly rate of return $r = 0.02$ we chose is close to the current rate of inflation in the Euro zone, a reasonable lower bound for the nominal riskless rate of return. About the risk premium to sell, since bid prices must be smaller than ask prices we must have $\pi_i^B < \pi_i^S$ and therefore we restrict our choice to $\pi_i^S \in U[0, \pi_i^B]$. In particular, to simplify things and reduce the number of parameters, for our simulations we set $\pi_i^S = \pi_i^B/2$ for all $i = 1, \dots, N$.

⁵We have investigated markets with multiple dealers in a previous version of this paper. The main effect of the presence of many dealers is the reduction of the effective bid-ask spread. However, as this effect can be proxied by decreasing Γ with a unique dealer and the results are qualitatively the same, we describe the simplest situation here.

3.2 Executive summary

The summary statistics are reported in Table 2 where the averages over 25 passes (of 2000 trading days) are shown for each market, with estimated standard deviations in brackets. The columns contain the volatility, excess kurtosis and tail exponent of returns, average volume, perceived gain and bid-ask spread of quotes.

Vol	EKurt	TExp	Volume	PercG	Spread
Batch					
3.00%	1.00	-7.62	5.51	174.34	-
(0.063%)	(0.39)	(1.96)	(0.07)	(2.87)	-
CDA					
0.53%	39.36	-2.60	8.19	372.38	3.49
(0.054%)	(26.94)	(0.54)	(0.12)	(5.82)	(0.08)
Dealership					
0.24%	0.067	-	5.28	376.43	[4.00]
(0.005%)	(0.10)	-	(0.08)	(6.86)	-

Table 2: Averaged statistics (25 simulations) for each market, based on 2000 daily returns, standard deviation in brackets. The columns show volatility, kurtosis, tail exponent α , volume, perceived gain and bid-ask spread in the second half of the trading day. See a more detailed description in the following subsection.

The first three columns are directly related to variants of price stability. While volatility is a common measure of dispersion, the kurtosis is more sensitive to local shocks and the tail exponent is a possible assessment of returns riskiness. The fourth and sixth columns contain other widely used indicators about depth and liquidity of the market. The fifth column is measuring the absolute accumulated difference of execution and limit prices and can be regarded as the perceived gain by the agent. The comparison of the three markets (BA, CDA and dealership) shows the following main features:

1. The batch auction produces very volatile returns coupled with low kurtosis and fast tail decay (i.e. extreme returns are rare). The volume ranks in between the CDA and the dealership and the agents perceive a low gain from BA markets;
2. The CDA market has low volatility (one order of magnitude less than the BA) but frequent extreme events. Moreover, very slow tail decay makes it a risky environment. The market generates the biggest amount of trades and is liquid due to the low bid-ask spread. Agents perceive high gain when trading (statistically equal to the one perceived in a dealership);

3. The dealership return dynamics is the least volatile, with insignificant kurtosis (no extreme events) and exponentially fast tail decay. The volume of trades is low but the overall perceived gain is high (it is the biggest on a per-share basis). The bid-ask spread was fixed at 4 to be somehow close to a CDA.

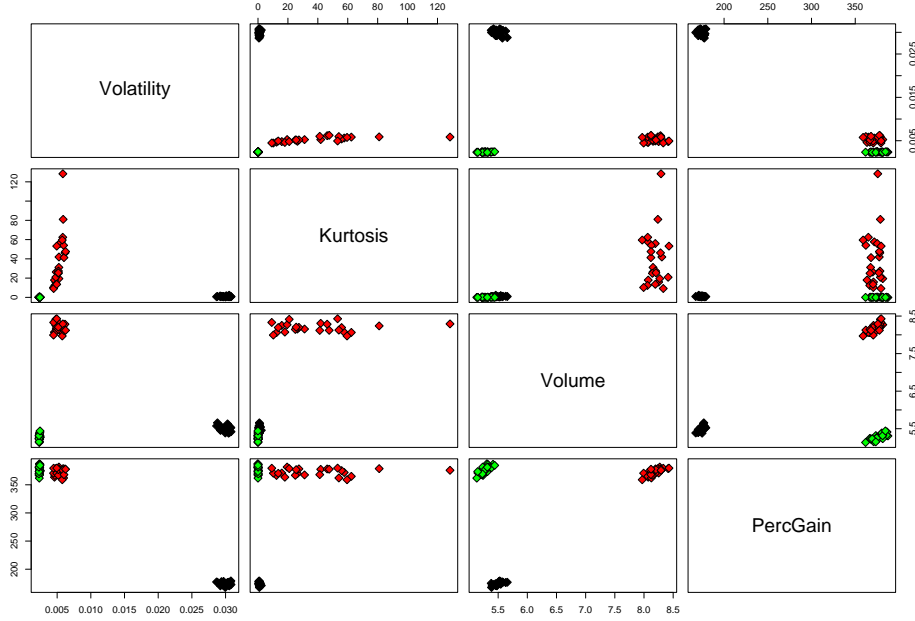


Figure 1: Pairwise statistics for BA (black), CDA (red), DEA (green) markets.

The four statistics common to all the markets are depicted pairwise in Figure 1. It can be noted that markets, in different colors, can be clearly separated by joint observation of different indicators.

To stress the fact that different market architectures produce markedly dissimilar price dynamics, we process our data (75 vectors in \mathbf{R}^4 , omitting the tail exponent and spread that are not always defined) in order to reduce their dimensionality to ease graphical comparison. Multidimensional scaling is a well-known technique used to preserve as much as possible the distances between the original datapoints in a lower dimensional subspace, see [Cox and Cox, 2000] for a full account. Figure 2 shows a planar representation of our simulations where distinct clusters are corresponding to the three markets under scrutiny. We used Principal Coordinate Analysis (PCoA), as described in [Edwards and Oman, 2003]. This graph, where the different markets can be clearly distinguished, reinforces the issue that institutional architectures really matter in shaping the multidimensional nature

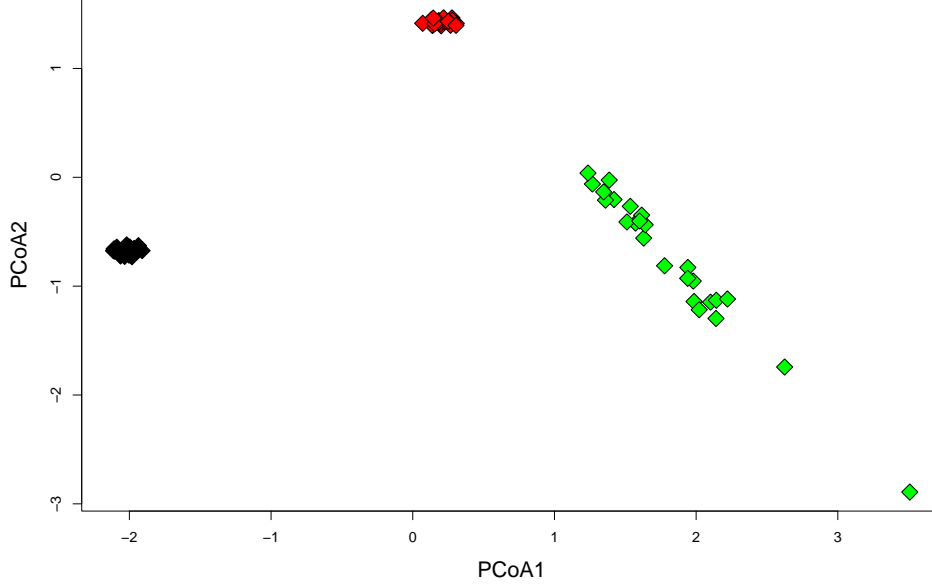


Figure 2: Multidimensional scaling of simulation data. The planar distance of points in the graph is approximately the same as in the full 4-dim space of statistics: BA (black), CDA (red), dealership (green).

of execution quality and price stability. It also suggests that most (but not all) realizations of a CDA might produce price dynamics closer to a dealership rather than to a batch market. A very similar pictorial result can be obtained by a standard Principal Component Analysis (PCA) as well.

3.3 A detailed analysis

We give in this section a more complete description of our simulation results. Where meaningful, we computed for each experiment the following 6 relevant statistics of returns or indicators.

Volatility. Volatility is measured as the standard deviation of the returns.

Excess kurtosis. It describes the peakedness and tails of a distribution with respect to a normal distribution (that has null excess kurtosis). It is computed as $E[(R - \mu_R)^4]/\sigma_R^4 - 3$, where R is the random return and μ_R, σ_R denote mean and standard deviation.

Tail exponent. Assuming the cumulative distribution functions of returns is $F(x) \sim 1 - x^\alpha$, the exponent provides a widely used measure of the speed of tail decay.

Volume. The cumulated number of transactions occurred in every trading day is collected.

Perceived gain. For each trading day, it is the cumulated excess gain, computed as the quantity traded times the absolute difference of the limit price order P_j submitted by an agent and the price p_j actually paid/received in the transaction. We have

$$PG_t = \sum_{j \in \mathcal{E}_t} q_j |P_j - p_j|,$$

where the sum is over the set of transactions \mathcal{E}_t of day t and q_j denotes the actual number of traded stocks. Observe that p_j might vary in a single day and even in the execution of a single order in the case of the CDA. The greater PG , the higher is the perception by trader to have made good bargains.

Bid-ask spread. The difference between the best ask and the best bid is a very common measure of liquidity in the book-based or quote-driven markets. Clearly, the best ask and bid are simply the dealer's quotes in a dealership. In a CDA, some caution is needed as the spread might be very large at the beginning of a trading day, due to the temporary lack of orders recorded in the book. Hence, we compute, for each day, the average bid-ask spread in the *second* half of the trader session, when the books contain many orders.

Whenever needed, the estimates of the fundamental value v are uniformly distributed around 1000 with an offset of $\pm 10\%$. The risk premium to buy π_i^B is in the range from 0 to 3 times the rate of interest, in order to span a large set of attitudes to risk. As mentioned above, we set $\pi_i^S = \pi_i^B/2$.

Entries and exits from the markets follow independent Poisson processes. Both activity and inactivity periods are exponentially distributed with parameter 250, that is, the expected agents' lifetime is 1 year. The entry-exit process is stationary and the number of active traders is constant on average. Notice that, since time is discrete, the values of the investment horizon h_i and the inactivity period τ_i are truncated to be integers. The same holds for the limit price $\alpha_i(t)$ and $\beta_i(t)$, due to the discrete nature of ticked prices.

We show, in Figure 3, representative time series of the trading price in one simulation for each of the three mechanisms. Some differences are immediately visible.

A glance at the price time series shows that the CDA returns are leptokurtic. The small number of isolated shocks in the BA well explains the extremely thin tails of its returns distribution, while their complete absence in the DEA makes this protocol rather peculiar. In fact, the support of the returns distribution in the dealership is a moving compact interval of

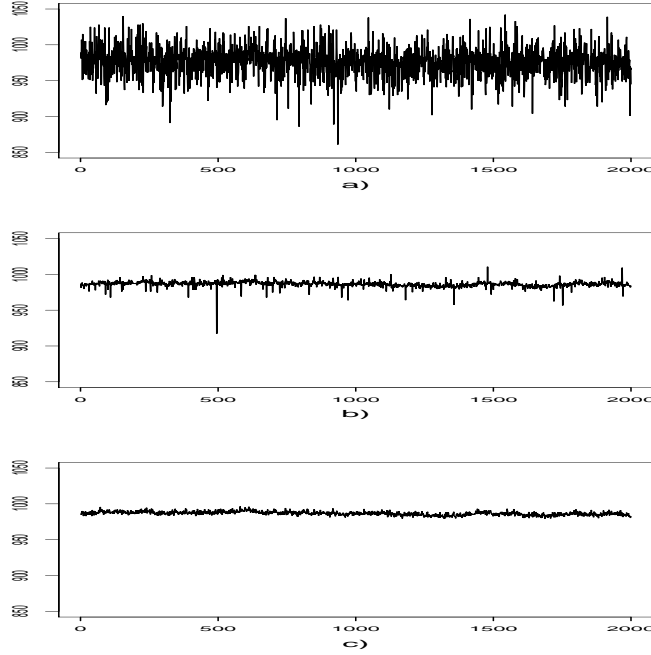


Figure 3: Time prices series of *a*) Batch Auction (BA), *b*) Continuous Double Auction (CDA), *c*) Dealer market (DEA).

length Γ that moves around the mean value. This nicely agrees with the exponentially fast decay of the tails.

Shocks on prices also affect volatility of the returns of the three protocols, as shown in a comparison in Figure 4. The BA shows an average standard deviation of 3%, while for both the CDA and DEA market the values are much smaller, about 0.5% and 0.2%, respectively. This is evident as well from the price series presented in Figure 3.

When comparing the excess kurtosis in the distributions of returns one observes that it is not significantly different from zero under the DEA and moderate in a BA, with average values respectively equal to 0.067 and 1.00. In the CDA the empirical excess kurtosis has a huge average value of 39.36 (see Figure 5). This is not very surprising since the CDA typically exhibits many big isolated shocks in the price time series.

The tail exponent α is one way to quantify the relevance of extreme events. It is not estimated for the DEA whose returns have nearly compact support. The exponent of the BA distribution is about -7.62 , pointing to very fast tail decay, while the exponent of the CDA is -2.63 , signalling quite fat tails. Similar values for the tail exponent imply that the theoretical excess kurtosis is infinite, thus explaining huge values for the empirical

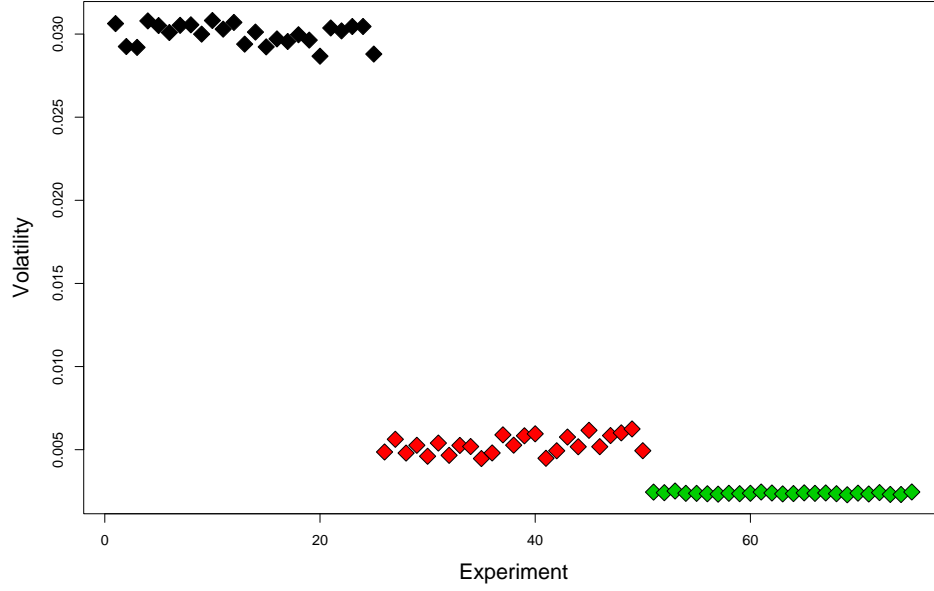


Figure 4: Comparison of volatilities of returns for BA (black), CDA (red), DEA (green) markets.

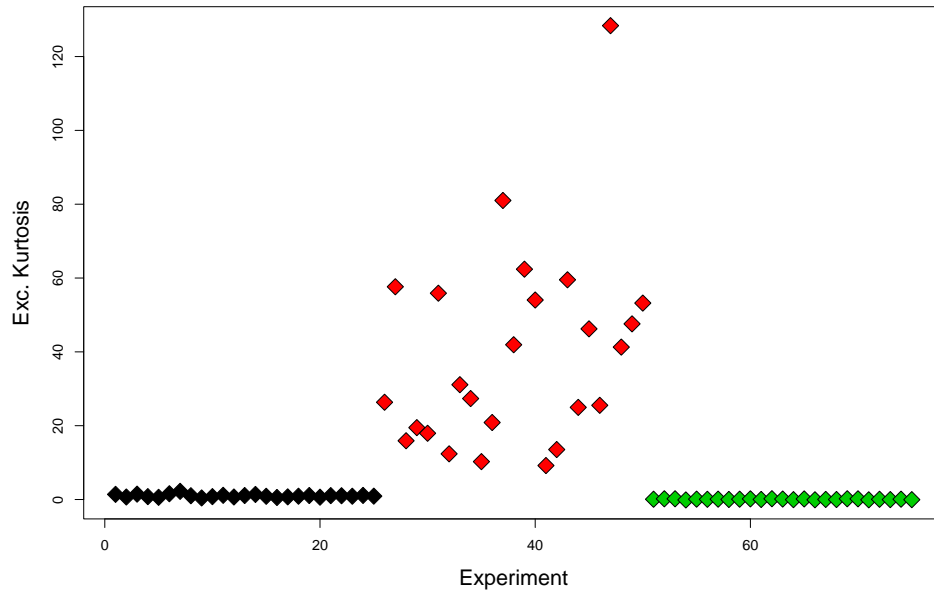


Figure 5: Excess kurtosis comparison for BA (black), CDA (red), DEA (green) markets.

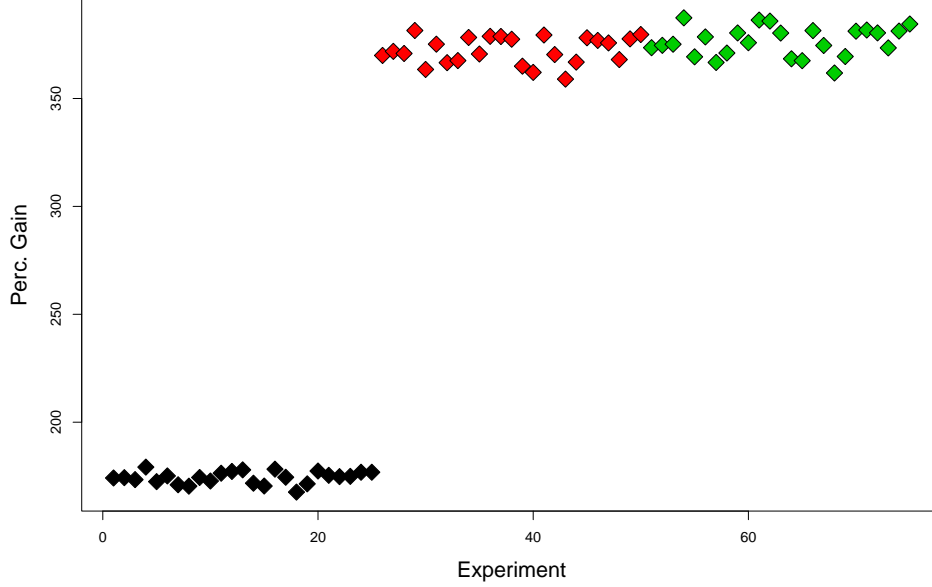


Figure 6: Comparison of the perceived gain in BA (black), CDA (red), DEA (green) markets.

estimate in the CDA.

The volume in BA and DEA are quite close, 5.51 and 5.28, respectively. The CDA average number of transactions (8.19) is notably bigger, consistently with the observation of excessive volume often found in the literature. We note that a lighter volume in the DEA might be justified by the reduction of wealth cashed by the dealer for every round trip transaction. Moreover, to avoid counting twice the same traded unit in a DEA (from one agent to the dealer and then from the dealer to another agent), we compute the volume halving the sum of units bought and sold by the specialist.

In real world, book based and dealership are the most commonly used market and several authors addressed the question why continuous trading is needed. A measure of perceived gain might help to understand this aspect. In fact, from the traders' point of view different trading protocols and, consequently, different available prices, might lead to a subjectively different perception of gain. As we can see in Figure 6, CDA and DEA have a comparable perceived gain, which is more than doubling the value in a BA. Hence, our results might give one reason why CDA and DEA markets are preferred. At the end of this discussion we will come back to this issue to add more.

In the order-book market, the bid-ask spread (in the second part of the day) is smaller than that observed in case of dealer market. In fact, the CDA

shows an average spread of about 3.50, to be contrasted with the constant spread (4) that was fixed in the dealership market.

Table 3 reports the ratio between the perceived gain and the volume of transactions. The dealership strikingly emerges in this evaluation with a ratio value much bigger than that of the CDA and almost twice as that of the BA. This implies that, between the order-book and the dealership market, the latter might be perceived as a better environment for trading.

	PercG/Vol
BA	31.64
CDA	45.47
DEA	71.29

Table 3: Ratio between the perceived gain and the volume for the three protocols.

Finally, in stable markets as ours, high volume means high liquidity and hence, low volatility. The matter is different in real markets where explosive volumes are coupled with new information arrival, high uncertainty, strong price movements and high volatility. In summary, the relation we find is reasonable in a calm and stable situation and this does not contrast with the usual claim that high volume is coming with volatility spikes that are appropriate for nervous markets.

4 Conclusion

We have studied three market protocols with respect to “price stability”, studying some relevant indicators like volatility, excess kurtosis and tail exponent of returns, volume, perceived gain and bid-ask spread. Computational experiments are run in a clean agent-based environment that controls for unwanted allocative or informational effects. The markets rank differently depending on the specific performance measure, thus revealing the multidimensional nature of market execution quality. The BA generates the smallest volume but the biggest dispersion in returns, the CDA is a liquid market with rather low standard deviation but has huge excess kurtosis. If one agrees that a regulatory agency might pursue stable, non-volatile, liquid markets with little excess volume and infrequent large shocks then standard deviation, kurtosis, bid-ask spread and tail exponent of price returns should be low. Volume is a more controversial measure, though there might be the general sentiment of too abundant transactions in financial markets. In the framework just sketched, the dealership protocol compares rather favorably to other architectures for various reasons. The volatility is low, virtually no excess kurtosis is present, ultra-fast tail decay is observed and the volume is lower than in a BA (but is clearly dependent on the choice of the spread

Γ). As an added bonus, agents might perceive high gains from trading in this market, both in absolute terms and on a per-transaction basis. We may argue that a dealership accommodates the total centralization of a BA with the freedom allowed in a CDA, providing a sort of mediation of two extremes that result in good overall performance. However, this is coming with some cost unique to this market. The dealership functioning is indeed slightly reducing the wealth of the agents as the dealer is cashing a spread on each round-trip transaction, even in the absence of any other transaction fee (that might be present in other mechanisms as well). Summing up, the mild rationality offered by a dealership allows to avoid severe pathologies in price dynamics and, in our view, has few drawbacks and reasonable cost.

References

- [Amihud and Mendelson, 1987] Amihud, Y. and Mendelson, H. (1987). Trading mechanisms and stock returns: an empirical investigation. *Journal of Finance*, 42(3):533–553.
- [Amihud and Mendelson, 1991] Amihud, Y. and Mendelson, H. (1991). Volatility, efficiency, and trading: evidence from the japanese stock market. *Journal of Finance*, 46(5):1765–1789.
- [Audet et al., 2001] Audet, N., Gravelle, T., and Yang, J. (2001). Optimal market structure: Does one shoe fit all? Technical report, Bank of Canada.
- [Bernanke and Gertler, 1999] Bernanke, B. and Gertler, M. (1999). Monetary policy and asset price volatility. *Economic Review, Federal Reserve Bank of Kansas City*, 0:17–51.
- [Bottazzi et al., 2005] Bottazzi, G., Dosi, G., and Rebesco, I. (2005). Institutional architectures and behavioral ecologies in the dynamics of financial markets. *Journal of Mathematical Economics*, 41(1–2):197–228.
- [Cliff, 2003] Cliff, D. (2003). Explorations in evolutionary design of online auction market mechanisms. *Electronic Commerce Research and Applications*, 2:162–175.
- [Cox and Cox, 2000] Cox, T. and Cox, M. (2000). *Multidimensional Scaling*. Chapman and Hall/CRC.
- [Day and Huang, 1900] Day, R. and Huang, W. (1900). Bulls, bears and market sheep. *Journal of Economic Behavior and Organization*, 14(3):299–329.
- [Edwards and Oman, 2003] Edwards, J. and Oman, P. (2003). Dimensional reduction for data mapping. *R News*, 3(3).

- [LeBaron, 2001] LeBaron, B. (2001). A builder’s guide to agent based financial markets. *Quantitative Finance*, 1(2):254–261.
- [LiCalzi and Pellizzari, 2003] LiCalzi, M. and Pellizzari, P. (2003). Fundamentalists clashing over the book: a study of order driven stock markets. *Quantitative Finance*, 3:470–480.
- [LiCalzi and Pellizzari, 2005] LiCalzi, M. and Pellizzari, P. (2005). Simple market protocols for efficient risk sharing. *Mimeo, submitted*.
- [Lux and Marchesi, 1999] Lux, T. and Marchesi, M. (1999). Scaling and criticality in a stochastic multi-agent model of a financial market. *Nature*, 397:498–500.
- [Madhavan, 2000] Madhavan, A. (2000). Market microstructure: a survey. *Journal of Financial Markets*, 3:205–258.
- [Maslov, 2000] Maslov, S. (2000). A simple model of an order-driven market. *Physica A*, 278:571–578.
- [Smith, 1988] Smith, V. (1988). Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica*, 56(5):1119–1151.
- [Vila, 2000] Vila, A. (2000). Asset price crises and banking crises: some empirical evidence. Technical report, Bank of International Settlements.