Knowledge Diffusion with Complex Cognition*

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ABSTRACT: This paper aims to understand some of the mechanisms which dominate the phenomenon of knowledge diffusion in the process that is called 'interactive learning'. We examine how knowledge spreads in a network in which agents interact by word of mouth. We define a social network structured as a graph consisting of agents (vertices) and connections (edges) and situated on a wrapped grid forming a torus. The target of this simulation is to test whether knowledge diffuses homogeneously or whether it follows some biased path, and its relation with the network architecture. We also investigate the impact of a modelled ICT platform on the knowledge diffusion process.

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1. BACKGROUND

Modern economy has been described as knowledge-based, or a learning economy, due to the central role that knowledge and learning play for economic development (OECD1996). Nonetheless, the processes of learning and knowledge diffusion are still largely undiscovered and require substantial theoretical and empirical efforts to be properly understood.

From the premise that learning is a complex and interactive process which can take place at all times (we learn at school, we learn at work, we learn reading a book, we learn watching TV, we learn talking with people, we learn while using ICT) we operate a logical simplification to understand this phenomenon. Following the theoretical structure defined in previous work (Morone, 2001; Morone and Taylor, 2001), we divide learning into two categories: *formal learning* and *informal learning*. We define *formal learning* as the kind of learning that occurs in certain environments which are meant for learning such as schools, workplaces, and training groups. On the other hand we call *informal* those learning processes that occur 'spontaneously', simply by interacting with peers. Following the more traditional approach, we could define the knowledge acquired by formal learning as a standard economic good (for which I'm paying a price; i.e. tuition fees, foregone earnings); and the knowledge acquired by informal learning as an unconventional public good. Some authors have defined the latter kind of knowledge as a *club* good (Cornes and Sandler, 1996; Breschi and Lissoni 2003) which is non rival and non excludible only for restricted groups of people (i.e. the *members of the club*).

Formal learning has been extensively investigated both theoretically and empirically (Becker, 1964; Mincer, 1974; Psacharopoulos, 1994). Whereas, the second process has only recently captured the attention. Studies of innovation diffusion (Clark, 1984 and Rogers, 1995) are often viewed as good examples of informal learning processes because they tend to occur through interaction within geographical and other informal networks involving social externalities. Several researchers have investigated the patterns through which different agents adopt new technologies by means of theoretical as well as simulation models. (Ellison and Fundenberg, 1993, 1995; Bala and Goyal, 1995, 1998). Another common way of modelling the mechanisms of social learning and technology diffusion makes use of evolutionary game theory (Chwe, 2000; Ellison, 1993, 2000; Anderlini and Ianni 1996; Berninghaus and Schwalbe, 1996; Goyal, 1996; Akerlof,1997; Watts, 2001).

Along with the speed of new technologies' diffusion, several researchers have focused on the impact of peers' behaviour upon individual decisions in areas such as propensity to crime, use of drugs, school dropout and school attainments (Brock and Durlauf, 1995; Benabou, 1993; Durlauf, 1996; Gleaser, Sacerdote and Scheinkman, 1996).[1] What all the studies considered so far have in common is the fact that learning from neighbours occurs and that under certain conditions it leads to the desirable stable equilibrium. However, none of these studies go beyond a binary definition of learning.

Jovanovic and Rob (1989) proposed for the first time a model in which incremental improvements in knowledge were defined as a complex process of assembling deferent ideas by means of information exchange by heterogeneous agents. The new insight brought by the authors is that knowledge was defined as something more complex than a binary variable and, therefore, growth of knowledge could be defined as an interactive process tightly linked to its diffusion.

[1] For a more detailed review see Morone and Taylor, 2004.

Cowan and Jonard (1999) made a subsequent attempt to study the effects of incremental innovations and their diffusions over a network of heterogeneous agents. Knowledge in their model is considered as a vector of values and is exchanged via a simple process of barter exchange. Depending on the network structure, the authors found that there is a trade-off between the speed of knowledge diffusion and the variance of knowledge. In other words, there is a spectrum of states of the world varying from a situation of high knowledge inequality and fast knowledge diffusion (i.e. small-world), to the opposed situation, more equal in terms of knowledge variance but less efficient in terms of knowledge diffusion.

Along the lines of these works, Morone and Taylor (2001) defined a model in which agents exchanged knowledge exclusively by means of face-to-face interactions. The network structure was endogenous to the model and could vary over time. The authors showed how small-world networks emerged and coexisted with both a very unequal and a very equal diffusion of knowledge, different outcome depending upon the initial conditions.

The objective of this paper is to shed some light on informal learning by means of an agent-based simulation model in which we investigate the knowledge diffusion dynamics amongst agents interacting through a process of face-to-face knowledge exchange. Departing from previous works on knowledge diffusion we aim to develop a model which takes into consideration the complexity of the process of knowledge acquisition. In doing so we define a complex cognitive structure for each agent (cognitive map) which regulates the processes through which knowledge diffuses. The paper is organised as follows: section 2 presents our model of knowledge diffusion and section 3 discusses how learning is defined in a framework of complex cognition. Section 4 explains how network properties of the model are calculated.

Section 5 presents the results of a simulation exercise based on the model. Section 6 reviews the findings of an investigation applying this model to a case study based upon the data and geography of the Greater Santiago region in Chile, and finally, section 7 concludes the paper.

2. THE MODEL SPECIFICATIONS

We assume a population of N agents and a global environment consisting of a grid of cells. Each agent is initially assigned a random position in the grid, and interacts with her/his closest neighbours. Not all the cells of the grid are occupied by agents, and those occupied contain only one agent. We specify a wrapped grid (i.e. a torus) so that there are no edge effects - where we might have different behaviour due to the boundaries of the grid (peripheral agents have smaller neighbourhoods: hence fewer neighbours and fewer opportunities to interact).

The local environment of the agent is called the *local-network* of the agent and it is defined as the region on the grid that includes those cells adjacent in the four cardinal directions and within the agent's visible range (i.e. von Neumann neighbourhood structure). We also define a *cyber-network* as the ideal network connecting all those agents which have access to ICT. The *cyber-network* generates a second system which has no geographical dimension but connects all agents who have access to it. The two networks have different configurations: the *local-network* is defined as a regular system in which each agent represents a node and each connection represents an edge, while the *cyber-network* is structured with a central agent (star agent), external to the simulation, who works as a server and connects all other agents to one another. Each agent has an initial list of acquaintances including members of the *local-network* and (if the agent has access to ICT) the *cyber-network*.

Each connection has an associated strength, $t \in (0.05,1)$, which is a measure of the strength of the relationship from the agent to her/his acquaintance. Note that this model is not constrained to have symmetry of relationships between agents: in general, more prestigious agents (with higher levels of knowledge) will be the object of strong relationships from more peripheral agents (with lower levels of knowledge), which may be unreciprocated or reciprocated only weakly. At the beginning of the simulation, all strength values are set equal to 1.

The unit of time we define in our model is called the *cycle*. In each cycle, all individuals are sorted into a random order, and then each is permitted to interact with one acquaintance. Each interaction is aimed at diffusing knowledge. Each agent is endowed with a *cognitive map* (*CM*), which contains information on the level and the kind of knowledge possessed by her/him. The structure of the *CM* is that of a tree, where each node corresponds to a bit of potential knowledge and each edge corresponds to acquired knowledge. We will return to the *CM* in the next section.

In our simulation vertices correspond to agents and edges are agents' connections. Formally, we have $G(I, \Gamma)$, where $I = \{1,\ldots, N\}$ is the set of agents, and $\Gamma = \{\Gamma(i),\ i \in I\}$ gives the list of agents to which each agent is connected. This can also be written $\Gamma(x) = \{(y \in I \setminus \{x\} \mid d(x, y) \leq n) \cup (y \in w)\}$, where $d(x, y)$ is the length of the shortest path from agent x to agent y (i.e. the path which requires the shortest number of intermediate links to connect agent x to agent y), n (visibility) as already mentioned, is the number of cells in each direction which are considered to be within the agent's spectrum, and w defines the *cyber-network*, which by definition encompasses all those agents endowed with ICT facilities. Intuitively, Γ_x (we will use this notation rather than $\Gamma(x)$ from now on) defines the neighbourhood of the agent (vertex) x.

Initial acquaintances in the *local-network* are the immediate neighbours (i.e. those within the visible spectrum). Subsequently, an agent can learn of the existence of other agents through interactions with her/his acquaintances (i.e. she/he can be introduced to the acquaintances of her/his acquaintances). If the acquaintance selected for interaction is connected to other individuals of which the agent is not aware, then a new connection is made from the agent to the acquaintance of her/his acquaintance. If there is more than one unknown acquaintance, than the contacting agent will choose the one with highest strength (this would tend to avoid the situation where the agent is introduced to an acquaintance that is not considered to be a good choice). The new acquaintance will be added to the acquaintances list of the agent who initiated the interaction and the strength value will be equal to that the new acquaintance had with the original acquaintance. Moreover, agents can stop interacting with some of their acquaintances if the connection does not tend to result in gain interactions and is therefore no longer useful. Therefore the number of acquaintances changes over time, but does not necessarily increase over time. In this way we introduce a dynamic element into the network structure.

Having defined Γ_x as the set of initial acquaintances of agent x (or *first generation connections*), we define $j_{x,t}$ as the set of acquaintances of the acquaintances at time t (or *next generation connections*), and the individual $m_t \in j_{x,t}$ who is added at each t. We also define $J_{x,t}$ as the set of acquaintances dropped at time t (or *next generation connections*) and the individual $n_t \in J_{x,t}$ who is dropped at each t. Now we can define the total set of acquaintances for individual x at time $t=T$ as:

$$\Phi_{x,T} = \left(\Gamma_x \cup j_{x,T}\right) \setminus J_{x,T} \tag{1}$$

We also define a rule governing how an agent chooses an acquaintance to interact with. In doing so, we make the assumption that an agent prefers interacting with acquaintances with whom she/he has strong relations. Agent *y* will be selected for interaction with agent *x* with probability given by:[2]

$$p^x(y) = \frac{t_y^x}{\sum_{i \in \Phi} t_i^x}, \tag{2}$$

In other words, the probability that x selects y for interaction can be understood as the relative strength of all the potential interactions. The selection mechanism is not based on the assumption that each agent has, at any moment of time, full information about other agents' knowledge levels. Rather, we introduce a mechanism whereby an agent adapts strength of relations depending upon previous experience of interaction.

Each cycle, the strength of the relationship between each agent and her/his acquaintances t_i, (where $i = \{1, \ldots, F\}$), is adjusted (we drop for simplicity the index of the agent and use it only when strictly necessary) as follows:

$$t_{i,t} = e t_{i,t-1} - b \tag{3}$$

$$where \begin{cases} e = 1.5 \quad \text{and } b = 0 \quad \text{if learning takes place;} \\ e = 0.6 \quad \text{and } b = 0 \quad \text{if learning does not takes place;} \\ e = 1 \quad \text{and } b = 0.05 \quad \text{if an agent is not selected for interactio n.} \end{cases}$$

As already mentioned, t_i is bounded between 0.05 and 1. Whenever the t_i attached to any acquaintance reaches the lower threshold of 0.05, the acquaintance is dropped from the acquaintances list. However, acquaintances that are members of the *local-network* are never dropped due to the fact that they are geographical neighbours with whom we keep meeting unless we move to different neighbourhood (an option which is not considered in our simulation model).

[2] In this way we assume that agents are constrained by 'bounded rationality' in the sense that they respond to utility signals without this meaning that they maximize utility (Katz, 2001).

In this way the agent will develop preferences for selecting for an interaction with acquaintances with which it has previously experienced positive learning interactions. In other words, the agent builds *internal models of preference* represented by the strength values t_i. The strengthening of relationships serves to make interactions between the same agents more likely in subsequent periods.

3. COGNITIVE MAPS AND COMPLEX COGNITION

We will now discuss how learning takes place. One of the main limitations of simulation models that aim to formalise our understandings of knowledge diffusion processes (Cowan and Jonard, 1999; Morone and Taylor, 2001) is the oversimplifying assumption that knowledge is accumulated as a stockpile (i.e. a vector of cardinal numbers indicating the level of knowledge). The roots of this problem are to be found in the distinction between economics of information and economics of knowledge. As pointed out by Ancori *et al.* (2000) the economics of knowledge differs from the economics of information in the sense that knowledge is no longer assimilated to the accumulation of information in a stockpile. The distinction between these two concepts has been repeatedly ignored by a certain branch of the economic literature (economics of information), which does not consider the cognitive structure that agents use to elaborate knowledge.

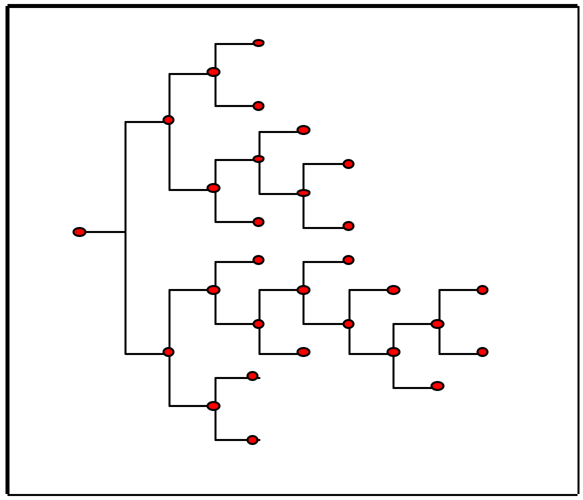
Following this distinction, Ancori e*t al.* (2000) develop an appreciative model in which the process of knowledge accumulation is disentangled into four major stages: identification of crude knowledge, learning how to use knowledge, learning how to transmit knowledge, and learning how to manage knowledge. The theoretical background of this model is the debate over the difference between tacit and codified knowledge. Three general observations are at the basis of the model: first, knowledge is closely dependent on the cognitive abilities of actors who hold it; second, knowledge cannot be considered separately from the communication process through which it is exchanged; and finally, knowledge demands knowledge in order to be acquired and exchanged.

For our purposes it is of a great interest to understand how people can exchange knowledge and how it is acquired once we dismiss the stockpile hypothesis. According to Ancori *et al*. new knowledge is acquired "by a backward process through which the new knowledge is confronted and articulated with previous experience. […] the appropriation of crude knowledge – i.e. its integration in one's cognitive context – is not the result of a transmission, but rather the result of a re-engineering process" (Ancori *et al*., 2000: 267). What the recipient agent is basically doing is de-codifying the knowledge received in order to be able to position it in her/his own cognitive map.

Particularly useful is the following example: "when the receiver knowing 'blue' and 'green' received the message 'red', the result in his/her cognitive context is not to replace 'blue', 'green' by 'blue', 'green', 'red', but to replace 'blue', 'green', 'blue and green' by 'blue', 'green', 'red', 'blue and green', 'blue and red', 'green and red', and 'blue, green and red'" (Ancori *et al*., 2000: 267). This example leads to the idea that cognition follows combinatory rules and not additive rules.

The theoretical framework created by Ancori *et al*., in spite of its strictly appreciative nature, is of a great interest for the development of our model, establishing the theoretical guidelines to characterise and construct the cognitive map that we will use in our simulation. We can think of the cognitive map as a tree in which each vertex (node) represents a piece of crude knowledge and each edge (link) represents knowledge that we have already mastered and learned how to use.

Figure 1. COGNITIVE MAP



In the graphical representation above we present a possible cognitive map which shows only mastered knowledge in the active part of this map (the coloured nodes), while all the other possible nodes which would complete the tree represent knowledge that at present is not in our cognitive map but could be activated through individual as well as interactive learning.

As assumed by Ancori *et al.*, knowledge demands knowledge in order to be acquired; hence, in order to activate a new node it would have to be directly connected to active (coloured) nodes. Moving from left to right in the cognitive map we move from less to more specialised knowledge, where each subsequent column corresponds to a higher level of knowledge. This observation justifies the assumption that new nodes can only be activated (i.e. new knowledge can be acquired) if they are directly connected to active nodes.

Each agent is initially endowed with a cognitive map determined by a random process. The number drawn at random from the uniform distribution corresponds to the 'column depth' up to which nodes are activated in the initial CM of that agent. Up to and including the first four columns, all nodes are fully activated. However, if the initial endowment exceeds the first four columns, then subsequent columns will not be fully activated, but will be activated according to the rule for endowment of specialised knowledge. We define *specialisation* as knowledge accumulation only in certain areas of the cognitive map. Agents will be specialised in one of two areas: the *scientific* area and the *technical* area.

The agent's interaction/exchange of knowledge can now be formalised as follows: each time an agent receives a message she/he will activate a new node, but only if this new knowledge can be pegged to pre-existing knowledge. The reason is that every new piece of knowledge has to be integrated with existing knowledge in order to be used. From this analysis it follows that agents with a similar kind of knowledge (i.e. agents with similar patterns in the cognitive map) are more likely to have fruitful interactions. This fact is theoretically supported by the literature on 'epistemic communities' or 'communities of practice'.[3] Using this new approach will improve the simulation model, overcoming some of the limits of previous models.

To sum up, the main differences between a model which uses a 'knowledge vector' and a model which uses a 'knowledge structure' is that in the former cognition follows additive rules while in the latter cognition follows combinatory rules. Moreover, in the 'knowledge vector' model, knowledge accumulation does not depend upon the structure of previously accumulated knowledge, as it does with the 'knowledge structure' model. Formally, we have: $CM(X, \mathrm{N})$ where X is the set of the whole possible knowledge available (i.e. the set of vertices), and N identifies the piece of knowledge activated (i.e. edges of the graph).

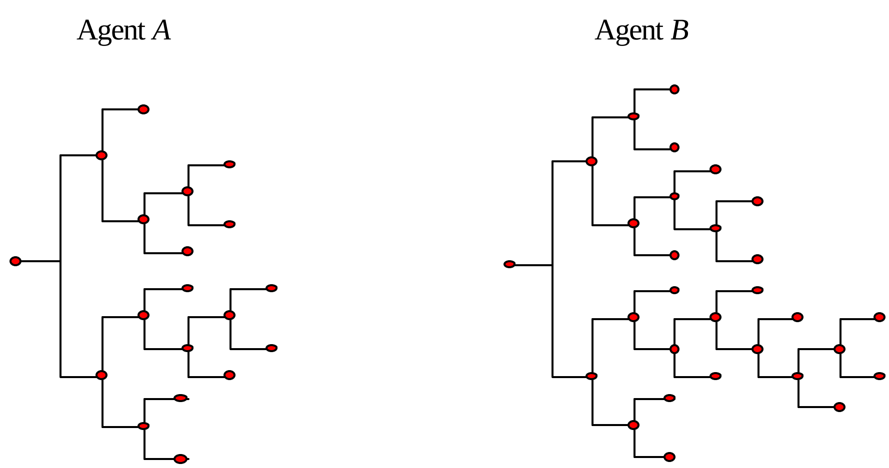
[3] See, for instance, Wenger (1998).

We will now explain how the process of knowledge diffusion takes place. An agent, whom we shall call A, contacts an acquaintance, B, in accordance with equation (2). Once the contact has been established the algorithm compares the two cognitive maps subtracting the cognitive map of A from that of B. This can produce one of two possible results:[4]

$$CM_A(X, \mathrm{N}) \setminus CM_B(X, \mathrm{N}) \begin{cases} = \varnothing \\ \\ \neq \varnothing \end{cases} \tag{4}$$

If the difference between the two sets is a non-empty set there is possibility for interaction; if not, agent A will have no interest in interacting with agent B as there is no possible gain.

Figure 2. Comparing two Cognitive Maps

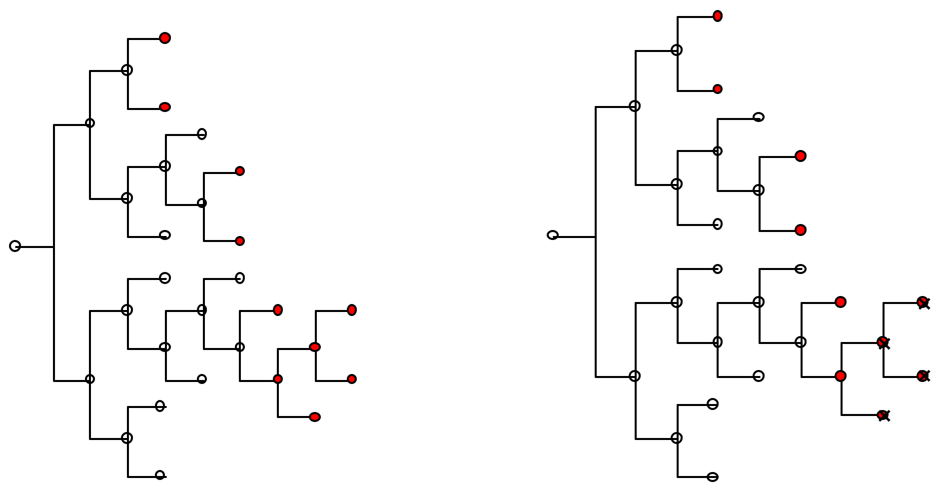


We present an example that will clarify the issue. The two graphs below represent the cognitive maps of agent A and an acquaintance, agent B. Now, let us assume that agent A contacts agent B. If we calculate the distance between the two maps we get $CM_A(X, \mathrm{N}) \setminus CM_B(X, \mathrm{N}) \neq \varnothing$ (this can be clearly observed in figure 3 below).

[4] We define the cognitive map only as a function of X and N because at this stage we are not interested

Figure 3. Knowledge interaction and the occurrence of learning

$CM_A \setminus CM_B$ *Learning region*



The left-hand picture of figure 3 illustrates the difference between the two *CMs*. Once we have identified this difference, we need to identify the possible learning region where knowledge can be gained (i.e. additional nodes can be activated). To do so we recall the requirement that new knowledge has to be pegged to already existing knowledge, and thus we can cross out several of the coloured nodes in the first diagram. We conclude that the only knowledge that agent A can learn from agent B is that connected to activated nodes.

Defining the nodes of the learning region as W, then the actual learning can be expressed as pW, where p represents the percentage of nodes of the learning region that will be activated as a consequence of the interaction. In other words, the agent that has started the interaction will activate (learn) p percent of the nodes, selected randomly (rounding always to the highest integer in the case of decimal numbers) from the *learning region*.[5] Since the number of nodes increases exponentially, it

in the depth of knowledge.

[5] In the simulation model p is set equal to 0.1.

implies that the higher is the level of knowledge of the interacting agents, the higher will be the learning opportunity. This mechanism reflects the idea that the 'absorptive capacity'[6] of each agent is a positive function of her/his level of education.

A final note has to be made on the 'incentive' mechanisms that generate knowledge flows. The model is structured as a 'gift economy' in which agents give away information for free. This model then might better replicate behaviours which take place in particular environments such as research groups or university communities within which knowledge flows are generated not by direct payments but by a tacitly agreed reciprocity.

4. NETWORK CALCULATIONS

As discussed earlier one of the targets of this work is to investigate the nexus between network architecture and knowledge diffusion dynamics. In order to address this question we will study the network properties of the model. More precisely, we will calculate the average path length and cliquishness of our network in different stages of the simulation:

$$L(t) = \frac{1}{N}\sum_{x=1}^{N}\sum_{x\neq y}\frac{d(x,y)}{N-1}; \tag{5}$$

and the average:

$$C(t) = \frac{1}{N}\sum_{x=1}^{N}\sum_{y,z=1}^{\Phi}\frac{X(y,z)}{|\Phi_x|(|\Phi_x|-1)/2}, \tag{6}$$

where $X(y, z) = 1$ if y and z are connected at time t (no matter whether the connection is a first generation or next generation connection), and $X(y, z) = 0$ otherwise.

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[6] We refer explicitly to the work of Cohen and Levinthal (1989) on returns from R&D. The concept of individual absorptive capacity has already been developed in Morone (2001).

We shall compare our dynamic network with a random one at different stages throughout the simulation to show whether or not the small worlds architecture is emerging in our system. Since the number of connections in our network is changing over time (due to the mechanism by which agents make acquaintances of their acquaintances), in order to make an appropriate comparison we need to construct the random network with an equivalent number of connections. For calculating the average path length and cliquishness of a random network, we shall use the same approximation as Watts and Strogatz (1998) that $L_{random}(t) \cong \ln N / \ln n$ and $C_{random}(t) \cong n/N$, where n is the average number of connections of each agent and N is the total number of agents. The criteria for identifying the network as small worlds are that $L(t) \cong L_{random}(t)$ and $C(t) >> C_{random}(t)$.

If, when comparisons are made with the random network, we find that the Watts-Strogatz (Watts and Strogatz, 1998) criteria are observed, this will be evidence to suggest that a small worlds network structure is emergent from our model.

5. SIMULATION RESULTS AND INTERPRETATIONS

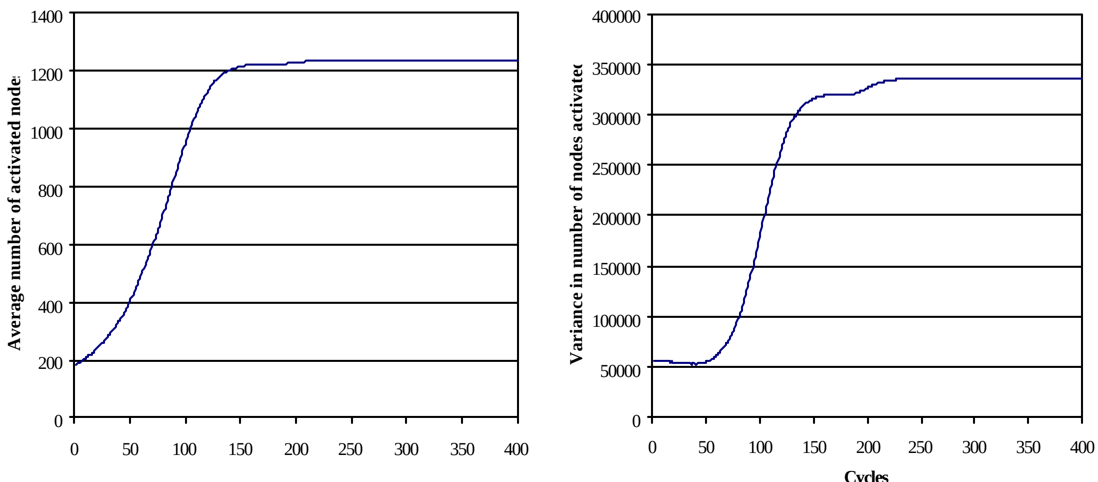
We run several batches of simulations and we examined both learning behaviours and network properties. We performed simulation experiments with a population of 100 agents allocated over a wrapped grid of dimension 20 by 20 cells. Hence, the grid had an approximate overall density of one agent per 4 cells. Each agent has a visibility parameter that we tuned to study changes in learning behaviours as well as network structure. We started with $\nu = 2$, meaning that each agent can see the two cells situated in the four cardinal directions. Moreover, we endow 10% of the overall population with ICT platforms, meaning that approximately 10 agents will be members of the *cyber network*.

The same random number seed was used for all the simulation runs ensuring fewer artefacts present in the results. The model was programmed in the Strictly Declarative Modelling Language (SDML) developed at the CPM (Wallis and Moss, 1994) to support the modelling of social processes with multi-agent systems. The results were analysed using the graphical output capabilities of SDML platform and the network analysis software toolkit UCINET 5.0 (Borgatti, Everett, and Freeman, 1999).

5.1 Knowledge Diffusion Dynamics

We ran 400 cycles for each simulation, obtaining a long-term stationary state. When ν is set equal to two we observe substantial increases in both mean and variance, suggesting a polarisation of knowledge distribution and an increase in the knowledge gap. Given the structure of knowledge expressed by the cognitive map, we calculate mean and variance based on the total number of activated nodes for each agent. Figure 4 shows these dynamics: first we plot m against time and we observe that the average number of activated nodes grows substantially over the first 50 cycles, the pace of learning being approximately 4 nodes per cycle. Then, it speeds up remarkably, almost tripling the pace of learning (reaching approximately 11 nodes per cycle). This dynamic reflects the fact that agents first start interacting with their geographical neighbours, then they learn of the existence of acquaintances of their initial acquaintances and are therefore able to make better choices for interaction. Moreover, after several interactions they learn valuable information about their acquaintances' level of education through the individual model of preference. In other words, they understand with whom it is worth interacting. After the first 120 cycles the average level of knowledge flattens out and then barely grows in the following 100 cycles until finally at about 230 cycles reaches its maximum value. This is due to the fact that the CM of some agents has become saturated. Finally, after about 230 cycles the mean curve levels-off, meaning that the system has reached a stable equilibrium.

Figure 4. Changes in the mean and variance of knowledge (ν = 2).

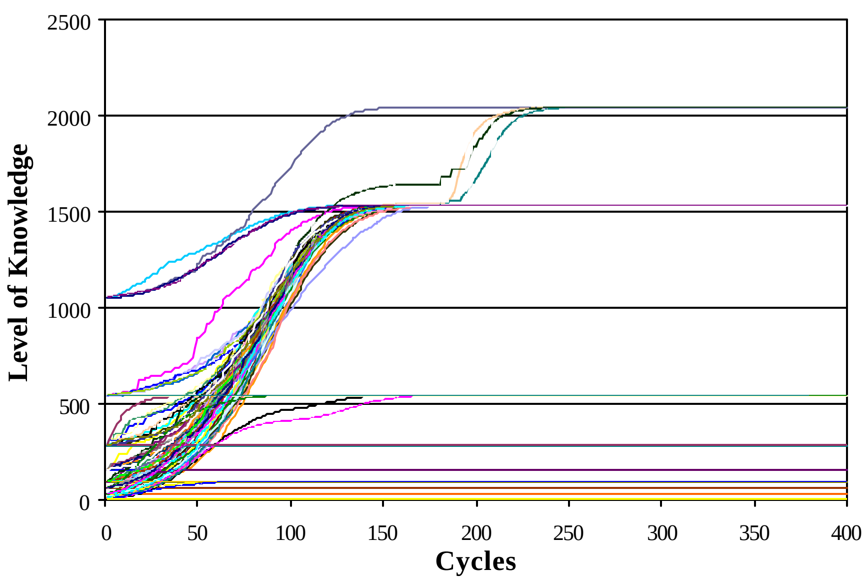


Source: Simulation results

Subsequently, we plot the variance in knowledge against time and we observe that s^2 first decreases slightly over the first 50 cycles or so, whereupon it reaches a turning point. After the first 150 cycles the variance slows down considerably and finally reaches a stationary state after 230 cycles. The variance pattern adds some useful information to our understanding: in the beginning, when everybody is interacting only with their closest neighbours there are similar learning opportunities for each agent, the learning path being rather homogeneous. On the other hand, when agents learn about of the existence of other acquaintances, and the network structure evolves, the society starts dividing into clusters[7] or sub-groups of fast- and slow-catching-up agents, and the learning path becomes heterogeneous and unequal.

Looking at individual dynamics corroborates this interpretation. We can clearly see how the model generates multiple equilibria, suggesting the existence of unconnected sub-clusters of agents. The groups converge to separate equilibria at very different intervals, one at 2044, one at 1532, one at 540, and several smaller groups at lower values. This is responsible for the high variance observed in the graph above.

Figure 5. Changes in the average level of knowledge by individuals ($\nu = 2$).



Source: Simulation results

To explain the agent learning behaviour illustrated by figures 4 and 5, we must consider the dynamics underlying the structure of knowledge in the model. The number of agents with fully saturated CMs increases over time, and as agents approach this state they have a reduced potential for learning, i.e. the learning region

[7] We will come back to this point in the following section while studying the network structure.

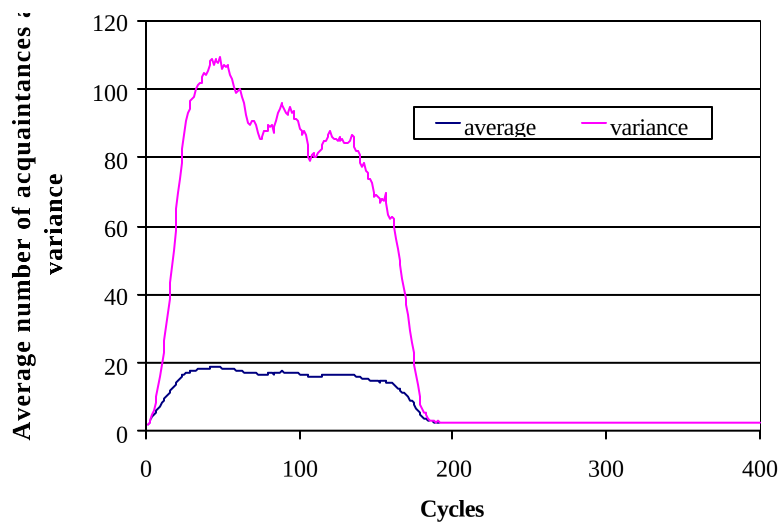
becomes smaller. However, on the other hand, in the early stages of the simulation this region tends to widen in the *CM* of the majority of agents, giving the potential for greater gains. In addition, agents will have increased opportunities to gain from interactions as *CMs* become more heterogeneous. For example, two agents with identical schooling will not be able to gain from an interaction in cycle 0, whereas later in the simulation they most likely will experience a small gain. This begs the question: to what extent is the observed increase in knowledge due to the widening of the learning region (i.e. the structure of the *CM*), and to what extent is it due to agents making better choices for interaction (i.e. the preferential model of acquaintance selection).

The efficiency of the learning mechanism has been demonstrated through exploration of a very similar model presented elsewhere (Morone and Taylor, 2004) (see section 7), where the authors discovered a more rapid diffusion process compared with simulations where there is no model of 'preferential acquaintance selection'. We leave this test of different learning mechanisms for further investigation.

More information on the structure of the network can be gathered by studying the dynamics through which agents make new connections. We will do so by looking at the average number of acquaintances and its variance. We expect to observe a monotonic increase in the number of acquaintances over the first few cycles: every interaction presents the opportunity to meet a new acquaintance, whilst agents will not start disconnecting non-gainful relationships until several cycles have passed (i.e. when the strength level has fallen below the threshold value).

Starting values describe the state of the system with the *local-network* in conjunction with the *cyber-network*: this situation is one of very low average number of acquaintances and variance as shown in figure 6. As anticipated, during the early part of the simulation the average number of acquaintances increases sharply, moving from an average of approximately two acquaintances, to an average of almost 20 after 30 cycles.[8] However, the variance behaves even more dynamically. It starts quite low, skyrockets over the first 50 cycles, then decreases (with several ups and downs) and eventually stabilizes at the initial low level after approximately 200 cycles. Thus when all agents have attained their maximum possible knowledge and learning has finished, the majority of acquaintances are dropped and we return to a system very similar to the *local-network* configuration with low mean and variance.

Figure 6. Average number of acquaintances and variance ($\nu = 2$).



Source: Simulation results

The variance behaviour during the learning period of the first 150-200 cycles is easily explained by considering the many disconnected agents and sub-groups in the network. As the density of connections in the main population increases, these agents remain with relatively very few (or zero) acquaintances, and this largely accounts for

[8] It is worth noting that the average number of acquaintances reported here do not include the cyber acquaintances as it is constant over time.

the high variance seen in figure 6. In this simulation experiment, few agents have got a very small number of acquaintances, whilst the vast majority of agents are extremely well connected. Clearly, this is not facilitating the equality of knowledge flows, keeping the wrapped grid as a whole a rather un-cohesive environment.[9]

5.2 Enhancing Knowledge Flows

One possible way to facilitate knowledge flows would be to make the global simulation environment more cohesive by increasing the density of the network. We could achieve this target either reducing the grid size or alternatively increasing the visibility range of each agent. These two options are technically very similar, as they increase the initial connectivity (and make possible more subsequent connections), practically reducing the geographical distance between agents. A useful example of the importance of the cohesiveness of environments to enhance knowledge flows is provided by the literature on industrial districts. Several authors[10] pointed out the importance of cohesiveness and geographical proximity in determining the overall efficiency of a district.

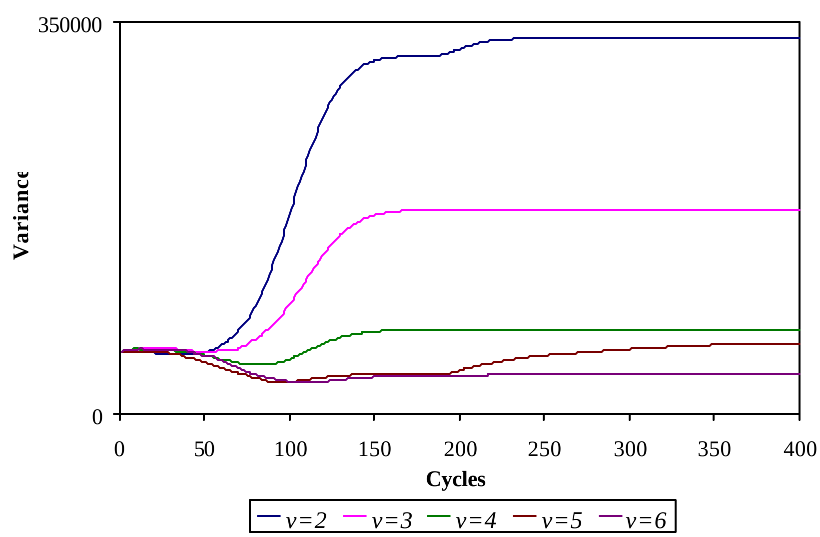
In our first simulation we had an overall density of the graph of one agent per 4 cells with a visibility equal to two. This produced a rather un-cohesive environment where groups of agents were isolated from each other. By raising the value of ν from two to six we increased the cohesiveness of the global environment. In figure 5 we report changes in the variance dynamics after changing the visibility value. We can clearly see how the variance behaves very differently according to the tuning of the visibility parameter: rising ν from two to three the model diverges at a slower pace

[9] It is worth clarifying this point: prestigious agents are facilitating knowledge flows but the network structure is not facilitating it.

[10] See among others: A. Marshall, 1952; G. Becattini, 1990; G. Dei Ottati, 1994.

and towards a much less unequal equilibrium. If we raise the ν value to four and five, we can observe a short-term behaviour during which the variance decrease describing a converging pattern. Subsequently, after the first 100 cycles, the variance starts growing again and the model stabilises around a value of the variance not to dissimilar from the original one. Finally when ν is set higher than five, the model shows a converging behaviour both in the short-term as well as in the long run steady-state.

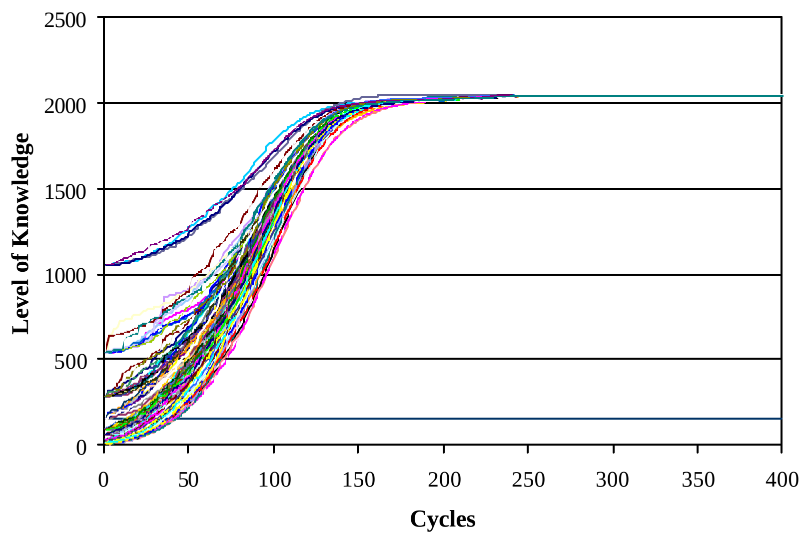
Figure 7. Knowledge variance transition.



Source: Simulation results

Increasing ν, we decrease the number of isolated agents and the number of isolated sub-groups. Nevertheless, when the visibility is set equal to six, our simulation shows that convergence is not always complete (i.e. the model does not converge to zero variance and maximum mean) solely because there is one agent who is totally isolated and hence unable to be engaged in any interaction. Nonetheless, 99% of the population reach the highest possible level of knowledge in less than 250 cycles. Likewise, when visibility is increased the distribution of acquaintances is more even.

Figure 8. Changes in the average level of knowledge by individuals (ν = 6).



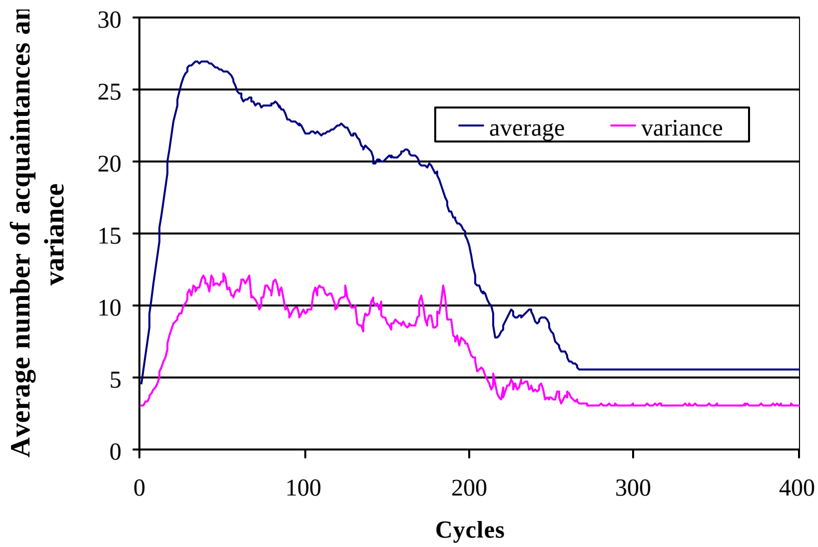
Source: Simulation results

In figure 9 we can see that the average number of contacts per agent is higher than in the case *v=2*. Throughout the simulation agents maintain more connections: this number peaks at about 27 acquaintances and remains at a high level for nearly 200 cycles, producing a very dense network. Interestingly however, the variance is much lower than in the case *v=2*, implying that agents are almost uniformly maintaining a high number of personal contacts. As in the previous case, the average number of acquaintances starts decreasing as soon as the model converges towards the long run steady state around cycle 270.

In this second simulation, the mean number of acquaintances is much higher than the variance, reversing the result of the first simulation. The difference is largely attributable to the reduced number of disconnected agents, and the result illustrated in figure 9 therefore gives us a more accurate picture of the typical connectivity of agents following the preferential acquaintance selection model.

In conclusion, increasing the visibility range generates a more interconnected environment, which in turn produces improvements both in terms of overall efficiency (i.e. speed of knowledge diffusion) as well as in terms of equality of distribution.

Figure 9. Average number of acquaintances and its variance (ν = 6).



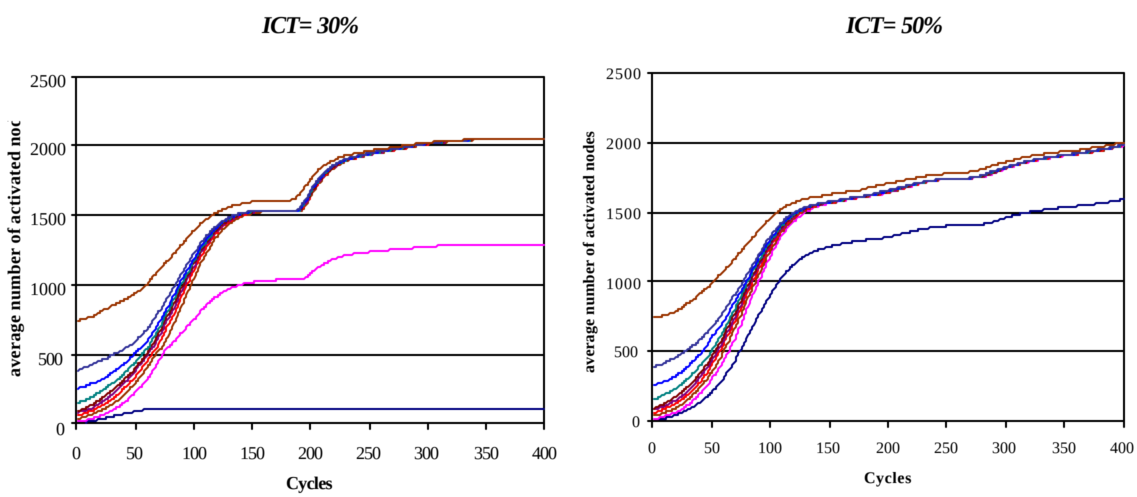
Source: Simulation results

Given the structure of our model, an alternative way of enhancing knowledge diffusion would be increasing the percentage of agents endowed with ICT. So far, we have assigned an ICT platform to just 10% of the population. Raising this value would represent an alternative way of bridging over the physical distance among agents and making the environment more cohesive.

When ICT penetration is boosted up to 30%, we can observe a rapid increase in the number of agents able to converge to the absolute maximum in the long run steady state. Nonetheless, almost 15% of the overall population is unable to converge and appears to be fully disconnected and hence unable to interact at all. This implies that the overall variance would increase quite rapidly, converging to a high value.

Further increasing the ICT penetration to 50%, however, we have the situation where almost all agents are able to converge to the highest possible level of knowledge exactly as we saw in the case where visibility was set equal to six.

Figure 10. Changes in the average level of knowledge by deciles.
Different values of ICT penetration.



Source: Simulation results

As we can see in figure 10, when we push ICT penetration up to 50% (meaning that every second agent has access to internet platform), the model will converge to a long run steady state in which more than 90% of the agents will reach the highest possible level of knowledge. Nonetheless, 400 cycles did not prove sufficient to reach the steady state, meaning that the pace of convergence is much slower if compared to a model with high degree of visibility. In other words, both actions to enhance knowledge flows (i.e. increasing the cohesiveness of the network by means of higher visibility, and increasing the ICT diffusion) will generate a long-term stationary state in which almost every agent converges.

Nonetheless, increasing the visibility seems to be a more efficient tool to reach this target. Knowledge flows distribute more equally if we increase the size of each agent's neighbourhood (and hence *local-network*) rather than superimposing one large *cyber-network* over half of the population. In fact, this second option will account for greater chances of social exclusion even though it results in initially very much lower average path length and high cliquishness as we will see in the following section.

5.3 Calculating network properties

In this section we present our results for the small world calculations. We have calculated average path length and cliquishness at different stages of simulations and for three different scenarios.[11] In this way we can compare network properties with the knowledge diffusion results presented above. More precisely we have calculated small world properties at cycles 0, 10, 50, 100, 150, 200, 250, 300, 350 and 400; for the network with visibility equal to two and ICT penetration equal to 10%, for the network with visibility equal to six and ICT penetration equal to 10%, and finally for the network with visibility equal to two and ICT penetration equal to 50%. These results are then compared with those characterising comparable random networks. In this way we can examine the robustness of small world structures following the test first introduced by Watts and Strogatz (1998) and described in section 5 above.

[11] C was calculated by taking the average over all agents of the proportion of an agent's acquaintances that are themselves acquainted. This was a straightforward calculation made by means of querying the database of SDML at the appropriate stage of the simulation. L was calculated by importing the relational data into UCINET 5.0 and using the Networks-Properties function to produce a matrix of path lengths between each node. The average path length was determined by finding the density of the matrix. More detailed information on these calculations are available upon request.

In the first two cases, where ICT penetration is set to 10% and *v* varies (we did the calculations for *v* equals two and *v* equals six) the same pattern is observed: cliquishness increases and average path length decreases over the first 150 cycles as the system becomes more densely connected and knowledge flows more intense. After this period the system starts converging towards the long-run steady state equilibrium and, as the system stabilises, there are fewer gainful interactions and agents start disconnecting from their acquaintances. At this point, the average number of connections falls and cliquishness decreases, whilst the average path length starts increasing. Eventually the network evolves back towards the initial configuration.

This is not the case in the third simulation, however, where ICT is set equal to 50% and *v* equals two. The initial system is much more densely connected due to the high level of ICT penetration. Nonetheless, agents can disconnect from their initial cyber acquaintances (unlike their geographical neighbour acquaintances) and therefore we observe that cliquishness goes considerably down after the first 150 cycles whilst the average path length rises.

In each case, the initial network is small world due to the presence of the *cyber-network* which connects far-distant agents and reduces the path length of the network. What we can observe looking at the network calculations, is that in every case the system preserves the most efficient network structure (i.e. the small world) for the duration of the simulation, and in particular, the learning period (i.e. the first 150-200 cycles) is characterised by very low average path length and high cliquishness.

Table 1. Small world calculation results

First simulation results: v=2 and ICT=10%				
	simulation		random network	
Cycle	**average path length (among reachable pairs)**	**cliquishness**	**average path length**	**cliquishness**
0	5.9910	0.3497	4.2305	0.0297
10	2.5330	0.6203	2.0504	0.0945
50	2.2850	0.6910	1.6495	0.1631
100	2.0480	0.6999	1.6890	0.1528
150	2.1050	0.6903	1.7334	0.1425
200	3.1320	0.5995	3.0940	0.0443
250	4.3330	0.4609	4.5854	0.0273
300	4.0950	0.5015	4.6714	0.0268
350	3.5630	0.4677	4.8788	0.0257
400	3.9250	0.4135	4.8991	0.0256

Second simulation results: v=6 and ICT=10%				
	simulation		random network	
Cycle	**average path length (among reachable pairs)**	**cliquishness**	**average path length**	**cliquishness**
0	3.1280	0.4514	2.5375	0.0614
10	2.3390	0.4541	1.8583	0.1192
50	1.9090	0.6677	1.4224	0.2547
100	1.9830	0.6695	1.4663	0.2312
150	2.0180	0.7038	1.4806	0.2243
200	2.3560	0.4811	1.7853	0.1319
250	3.1630	0.3533	2.5678	0.0601
300	3.1780	0.3530	2.5678	0.0601
350	3.2200	0.3758	2.5702	0.0600
400	3.1910	0.3742	2.5726	0.0599

Third simulation results: v=2 and ICT=50%				
	simulation		random network	
Cycle	**average path length (among reachable pairs)**	**cliquishness**	**average path length**	**cliquishness**
0	1.7370	0.7734	1.2754	0.3699
10	1.5580	0.9712	1.2299	0.4228
50	2.0970	0.7397	1.5596	0.1916
100	2.2250	0.7233	1.6132	0.1737
150	2.2930	0.7901	1.6288	0.1690
200	3.5110	0.3798	2.8793	0.0495
250	5.9450	0.4088	4.1419	0.0304
300	6.0930	0.3732	4.2046	0.0299
350	5.7700	0.3863	4.1918	0.0300
400	6.6450	0.3929	4.2046	0.0299

Source: Simulation results

In conclusion, small world properties are observed both when knowledge flows lead the system to convergence, and also when they lead to non-convergence. In other words, the network structure doesn't affect directly the distributional aspects of knowledge flows. Convergency patterns will be determined solely by the existence of isolated agents and subgroups of agents.

6. AN EMPIRICAL APPLICATION OF THIS MODEL

An interesting exercise to test the usefulness of the model presented in this paper would be applying it to an empirical case study. This would allow investigating directly the risk of exclusion in a specific society and developing a model which might bring insight to the knowledge diffusion process in a well-identified context. This kind of exercise was carried out by the authors, who applied the model to the Chilean case (Morone and Taylor, 2004). As we will see, several interesting results were obtained.

The data, which were used to calibrate the model, were a sub-sample of the 1998 edition of the *Encuesta de Ocupación y Desocupación* (one of the most comprehensive household surveys collected in Santiago de Chile), providing us with the following useful variables: district of residence, years of schooling, kind of schooling, and use of computers at work. These variables were used to distribute agents over the geographical grid, to build the *CM* of each agent and to construct the *cyber network*.

The model environment was defined as a grid that resembled the geographical configuration of the metropolitan area of Greater Santiago de Chile. The grid was divided into 34 portions, each corresponding to a defined district of Santiago, having thus different dimensions and population densities. Defining the grid as a twodimensional geographical region added into the model a core-periphery aspect, with some districts being located in a central position and others in a peripheral one. Each agent was initially assigned a district and then allocated, randomly, to a cell within that district. Depending on the geographical location, agents were endowed with acquaintance lists, and - depending on the empirical data - few agents were selected as members of the *cyber network*. Moreover, each agent was initially endowed with a different cognitive map, which depended upon her/his level and kind of education (measured as years of schooling and kind of school attended). Each column corresponded to a higher level of education.

The results concerning the knowledge diffusion process were very interesting: in presence of high levels of (knowledge) inequality there was a high risk of exclusion for those agents initially endowed with low level of education – an *ignorance trap* where agents were never able to catch up. Moreover, looking into the spatial dimension of the exclusion process, we found that the *ignorance trap* mechanism is more likely to take place if an initial situation of low level of knowledge is coupled with geographical exclusion. In other words, those people who start with a high level of individual learning (i.e. schooling) will always be able to escape from the *ignorance trap* mechanism, while more backward people might be trapped f their low level of knowledge is cumulated with geographical exclusion.

These findings appear to be extremely important from a policy prescription perspective. Based upon the theoretical results obtained in this paper a twofold policy action could be suggested to avoid the occurrence of an *ignorance trap*: the policy maker should aim at reducing the geographical gap between centre and periphery. This policy could be implemented through the development of infrastructure, bridging the centre-periphery distance, which would correspond to an increase of the visibility range of our model population, as well as through the development and improvement of ICT connections. In other words, the *exclusion risk* could be minimised through the development of a more comprehensive cyber-network, so that also peripheral agents will have the same opportunity to interact with central and semi-peripheral agents.

7. CONCLUSIONS

In this paper we addressed the issue of knowledge diffusion, developing a simulation model to investigate the complex learning process which occurs among agents interacting in informal networks. In our model, agents exchange knowledge by means of face-to-face interactions, and every time a knowledge transfer occurs, the new knowledge acquired is confronted and linked with previous knowledge. In other words, knowledge is acquired not through a simple additive process, but by a more articulated combinatory process.

We studied how, within this framework, knowledge flows. Particularly, we investigated the occurrence of different long-run steady states for different levels of network cohesiveness and ICT penetration. We found a critical level, by tuning the visibility parameter, above which convergence in knowledge levels occurs. A converging long-run equilibrium was also achieved by increasing the ICT penetration. Nonetheless, we showed how this latter option was less efficient than the first one, as convergence was slower. We conclude from this that a more effective measure aimed towards generating more evenly-distributed knowledge flows should focus upon enhancing *local-network* connectivity rather than extending the *cyber-network* coverage.

Subsequently, we studied the network properties of different systems, showing how the model consistently preserved a small world structure, presenting desirable properties in terms of overall knowledge flows.

As a suggestion for further research, we would like to point out the importance of better investigating the real nexus between network cohesiveness and ICT penetration. In other words, we suggest studying the relation between geographical proximity and cyber proximity in order to understand if these two system properties are substitutable or, as we would foresee, complementary in promoting knowledge flows.

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